Acoustic Wave Propagation and Scattering in the Pipe with Uncertainties

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Pipebots

In collaboration with

The University Of Sheffield.
UNIVERSITY OF BIRMINGHAM
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Outline

- Partially filled water pipe
- Pipe wall roughness
- Scattering due to blockage:
  - detection and localization

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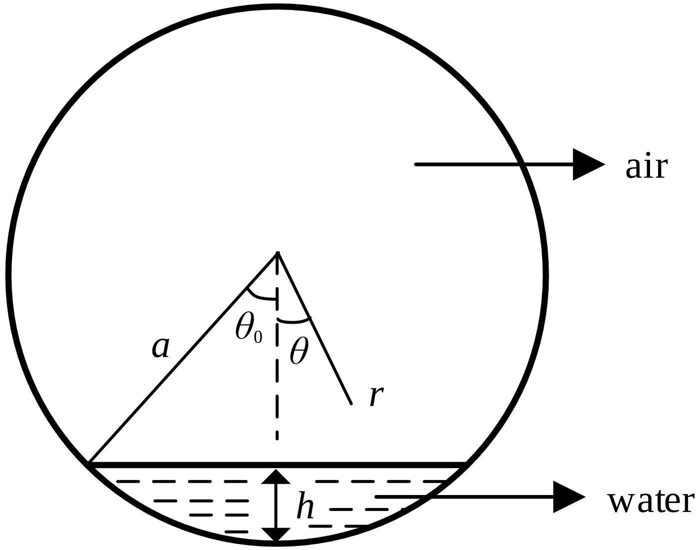
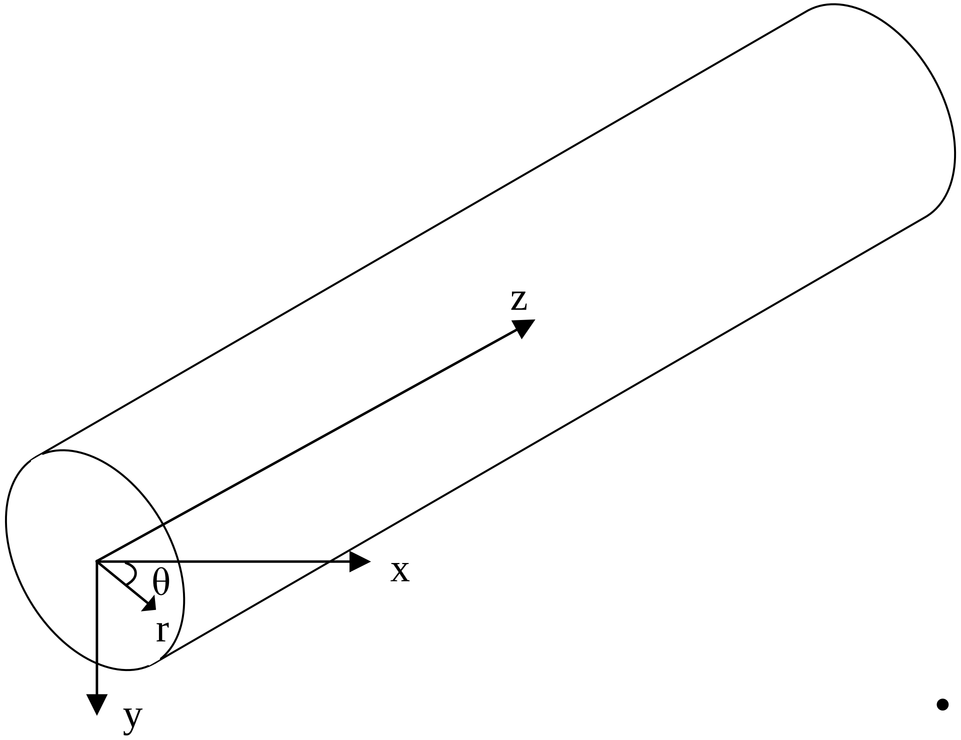
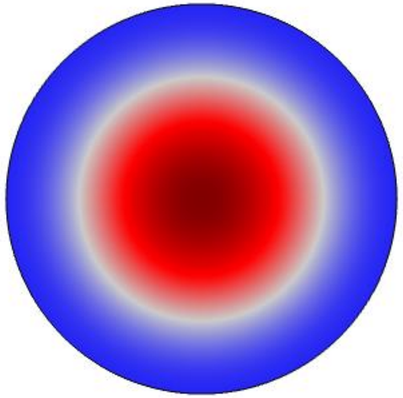
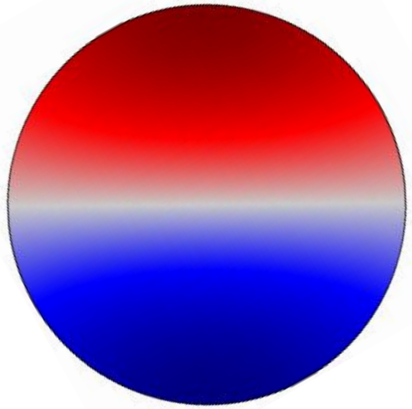


Acoustic wave modes in pipes





- First axisymmetric mode

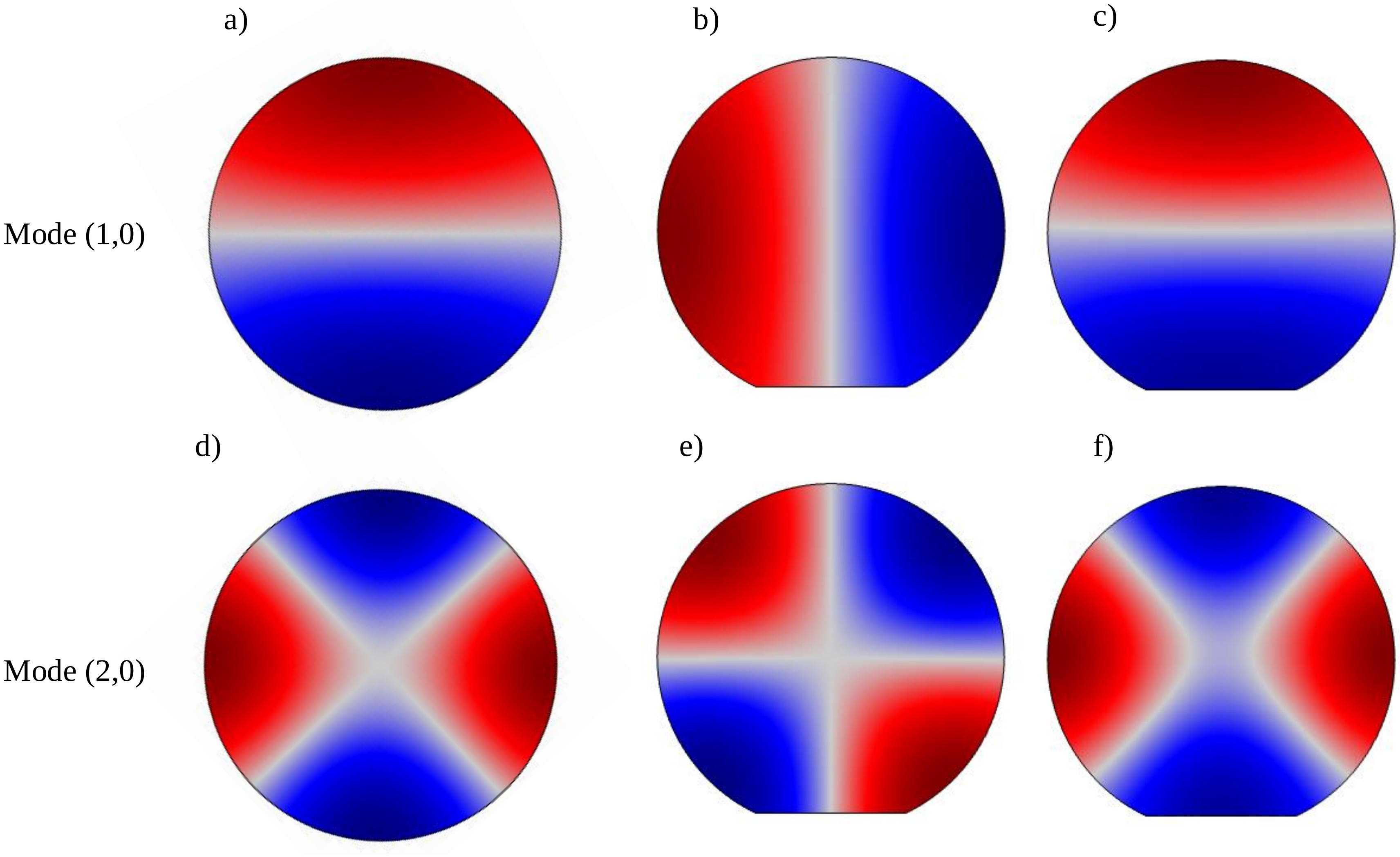


- First non-axisymmetric mode

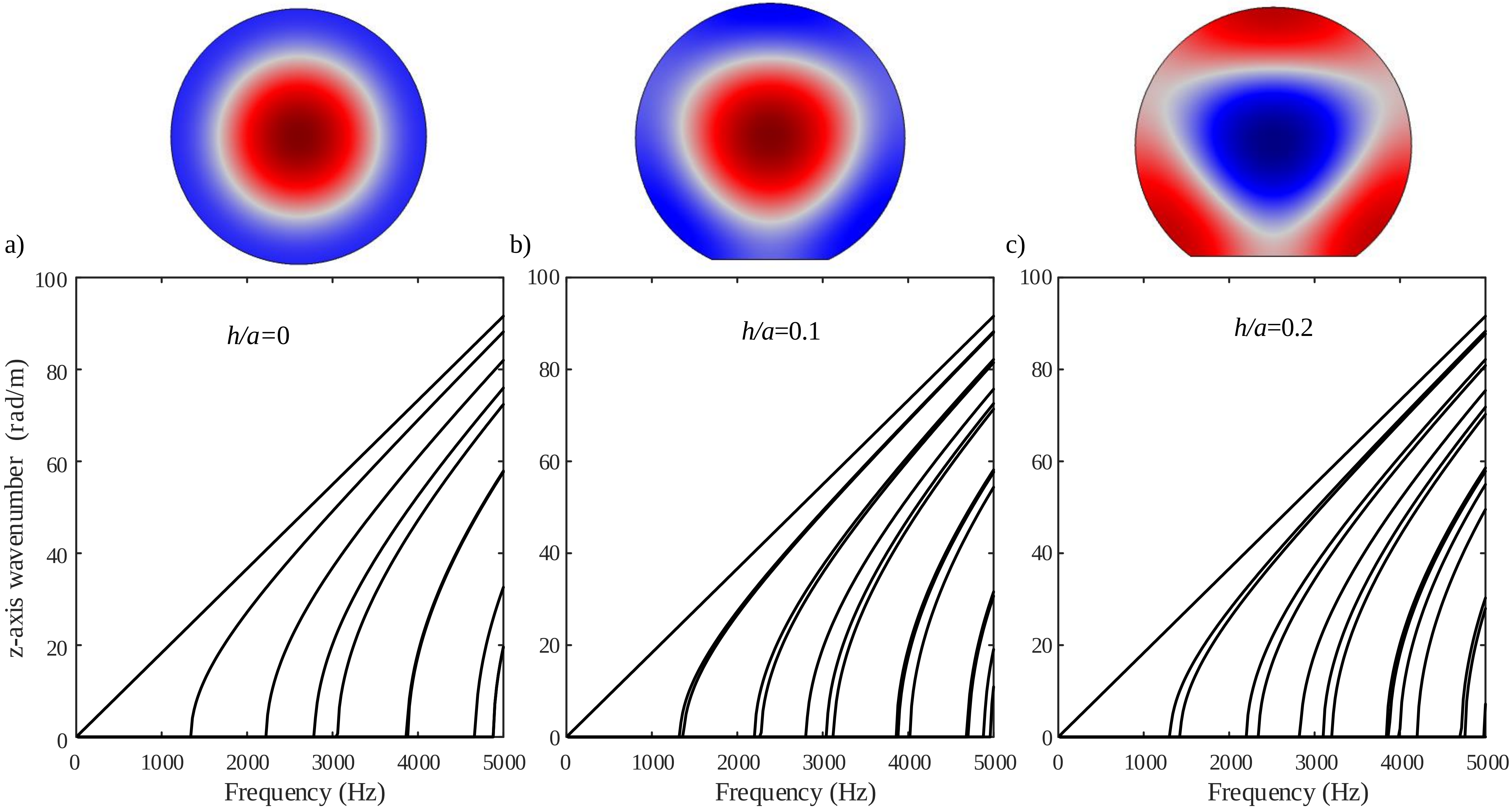
Dispersive curve of acoustic waves in partially filled sewer pipe: split of non-axisymmtric modes

Empty pipe

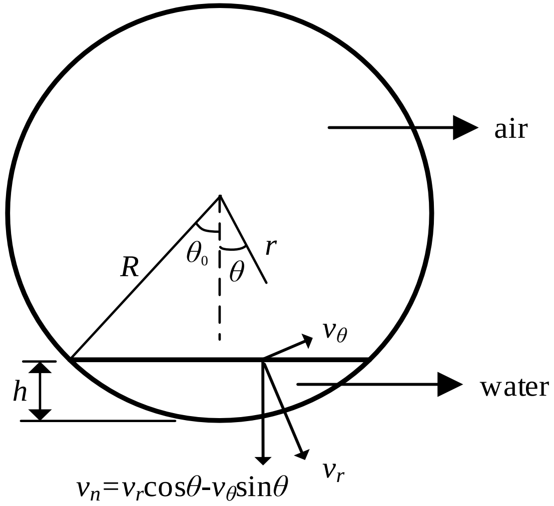
Partially filled pipe: mode split



Dispersive curve of acoustic waves in partially filled sewer pipe: axisymmetric mode shapes (first axisymmetric mode) and dispersive curves

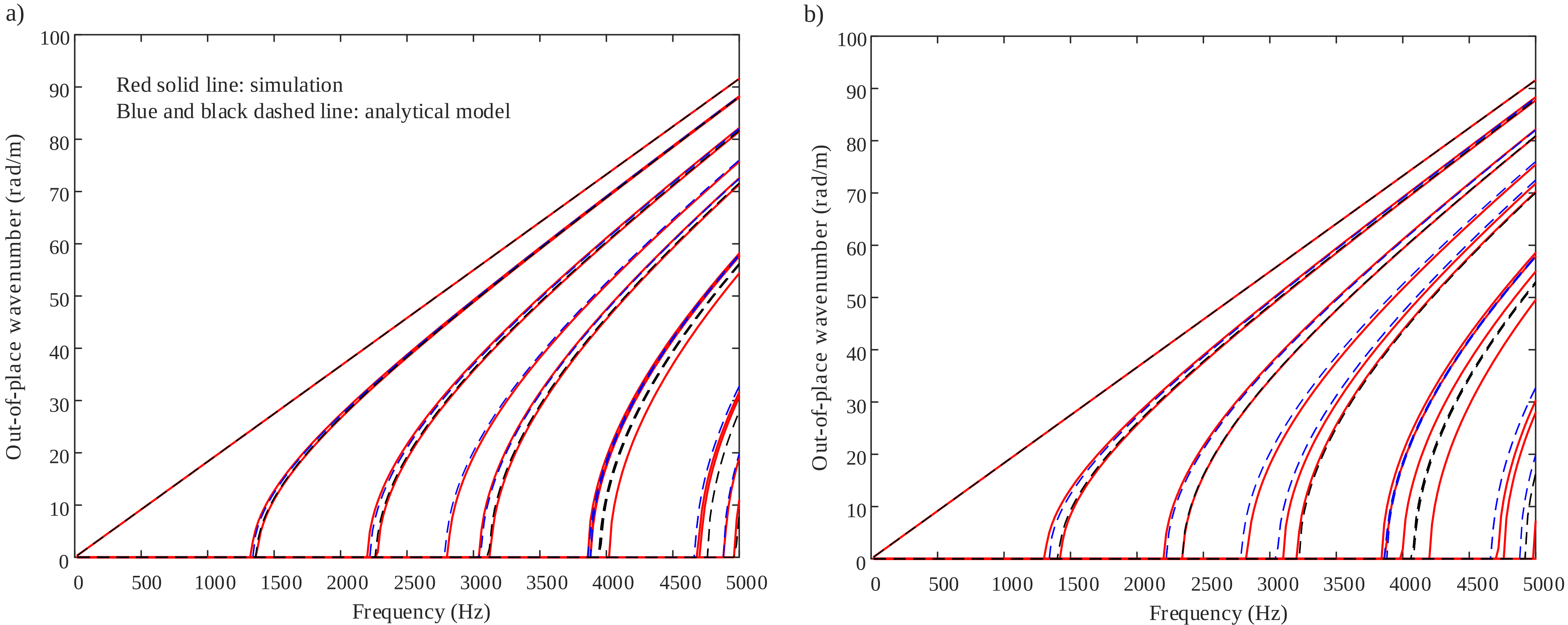


Dispersive curve of acoustic waves in partially filled sewer pipe: analytical model



$$v_n\big|_{r=R} = 0 \xrightarrow{\text{mode orthogonality}} \nabla_{\mathrm{r}}\Psi_{mn}^{(q)}\big|_{r=R} = 0 \qquad \theta_0 < \theta < 2\pi - \theta_0$$

$$v_n\Big|_{r=\frac{R-h}{\cos\theta}} = 0 \xrightarrow{\text{mode orthogonality}} \left(\nabla_r\Psi_{mn}^{(q)}\cos\theta + \frac{\nabla_\theta\Psi_{mn}^{(q)}}{r}\sin\theta\right)\Bigg|_{r=\frac{R-h}{\cos\theta}} = 0 \qquad -\theta_0 < \theta < \theta_0$$



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- Scattering due to blockages in the pipe
- Blockage detection and localization

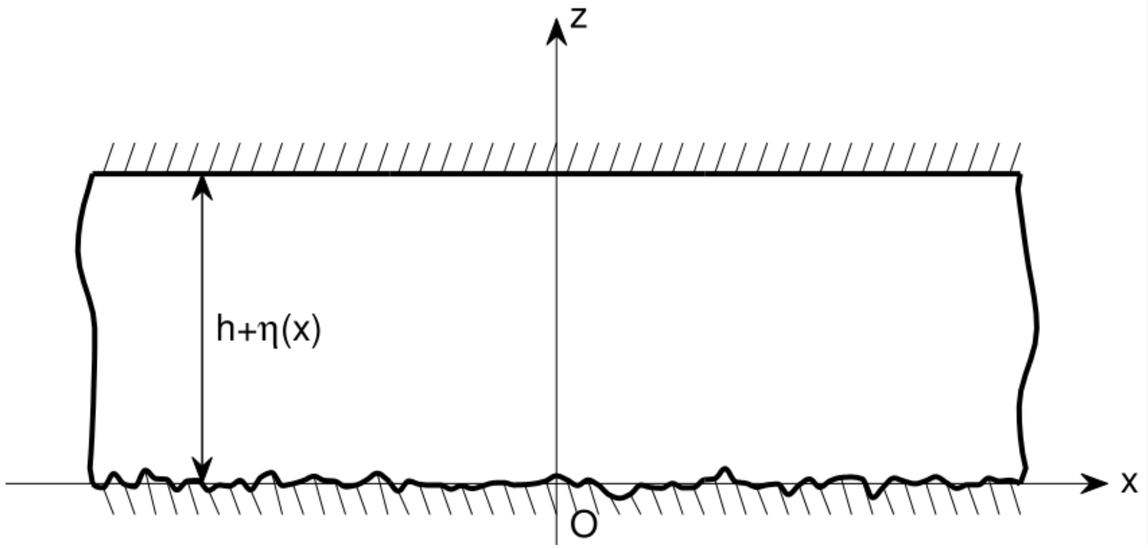
Rough surface

- Statistically 1D rough surface

$$\overline{\eta(x)} = \int_{-\infty}^{+\infty} \eta(x) w(\eta, x) \mathrm{d}\eta = 0$$

  - ○ $\eta(x)$ - random function
  - ○ Gaussian pdf

$$w(\eta, x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\eta^2}{2\sigma^2}\right)$$



Small perturbation theory

- Deterministic p_a and random p_r components

$$p = p_a + p_r$$

with

$$\overline{p} = p_a \quad \text{and} \quad \overline{p}_r = 0$$

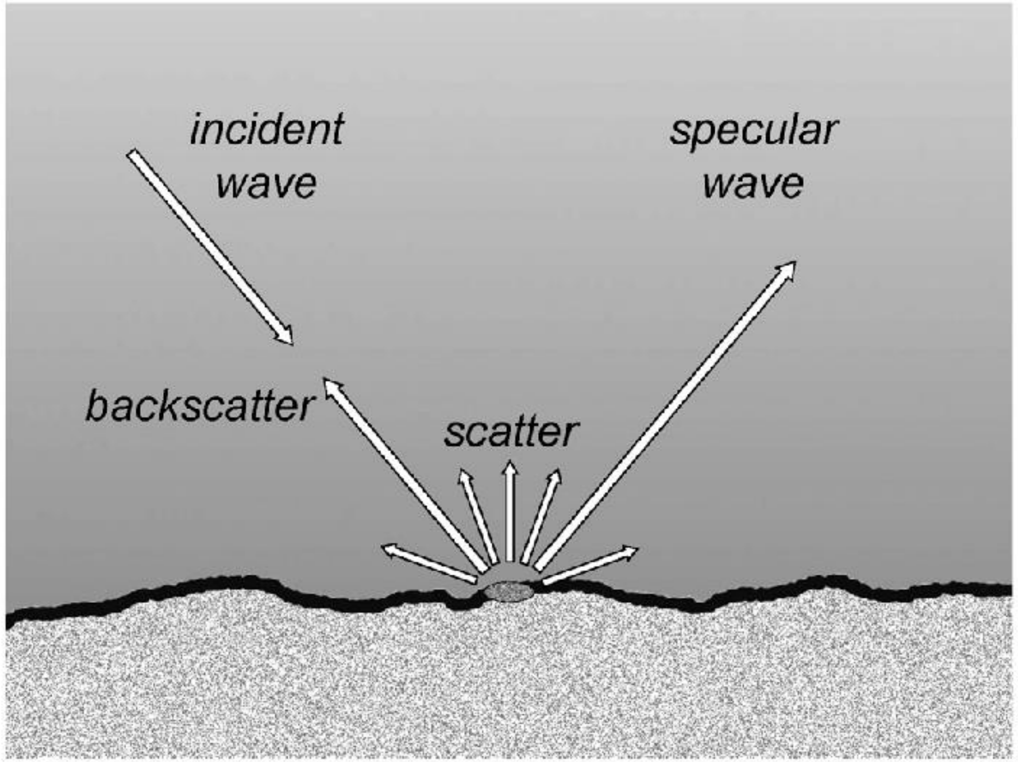
- Small perturbation

$$\epsilon = \frac{\sigma}{h} << 1 \quad \text{and} \quad h \sim L$$

with

$$\sigma = \left[\overline{\eta(x)^2}\right]^{1/2}$$

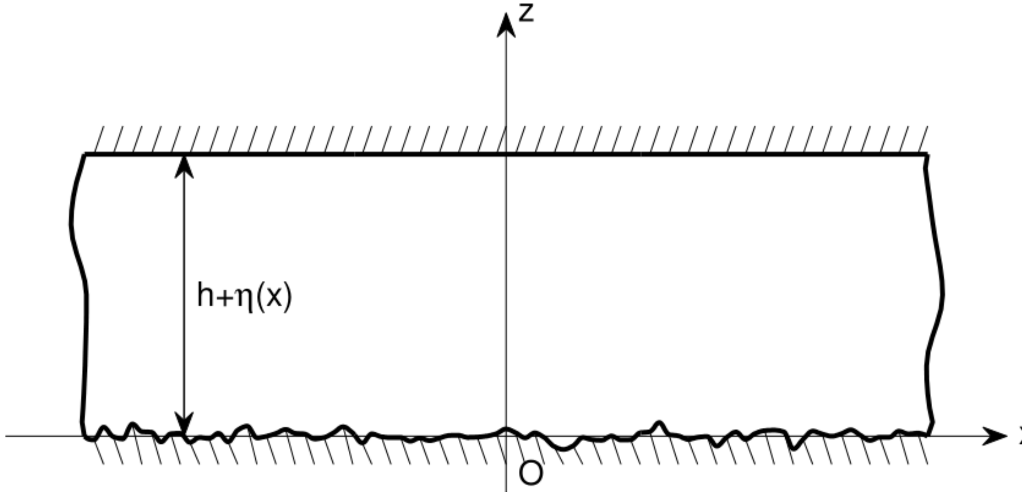
L - characteristic wavelength along waveguide axis



Reflection (coherent part) and scattering of an acoustic wave incident a rough surface. By Masetti et al

Fourier transform

- $\hat{F}(z,\xi) = \int_{-\infty}^{+\infty} F(x,z)\mathrm{e}^{-\mathrm{i}\xi x}\mathrm{d}x$



- Correlation function of Gaussian distribution

$$W(x_1,x_2) = \overline{\eta(x_1)\eta(x_2)} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \eta(x_1)\eta(x_2)w(\eta_1,x_1;\eta_2,x_2)\mathrm{d}\eta_1\mathrm{d}\eta_2,$$

$$\lim_{|x_1-x_2|\to\infty} W(x_1,x_2) = 0$$

with joint pdf

$$w(\eta_1,x_1;\eta_2,x_2) = \frac{1}{2\pi\sqrt{1-W(x_1,x_2)}}\exp\left[-\frac{\eta_1^2 - 2W(x_1,x_2)\eta_1\eta_2 + \eta_2^2}{2(1-W(x_1,x_2)^2)}\right]$$

Wavenumber correction

- Back to dimensional variables
- $M = 0$, $q_0 = 0$ and $\xi_0 = k$
- Scattering backwards

$$\xi_{0,2} = \frac{\mathrm{i}}{2}\frac{\sigma^2}{h^2}k^2\hat{W}(2k)$$

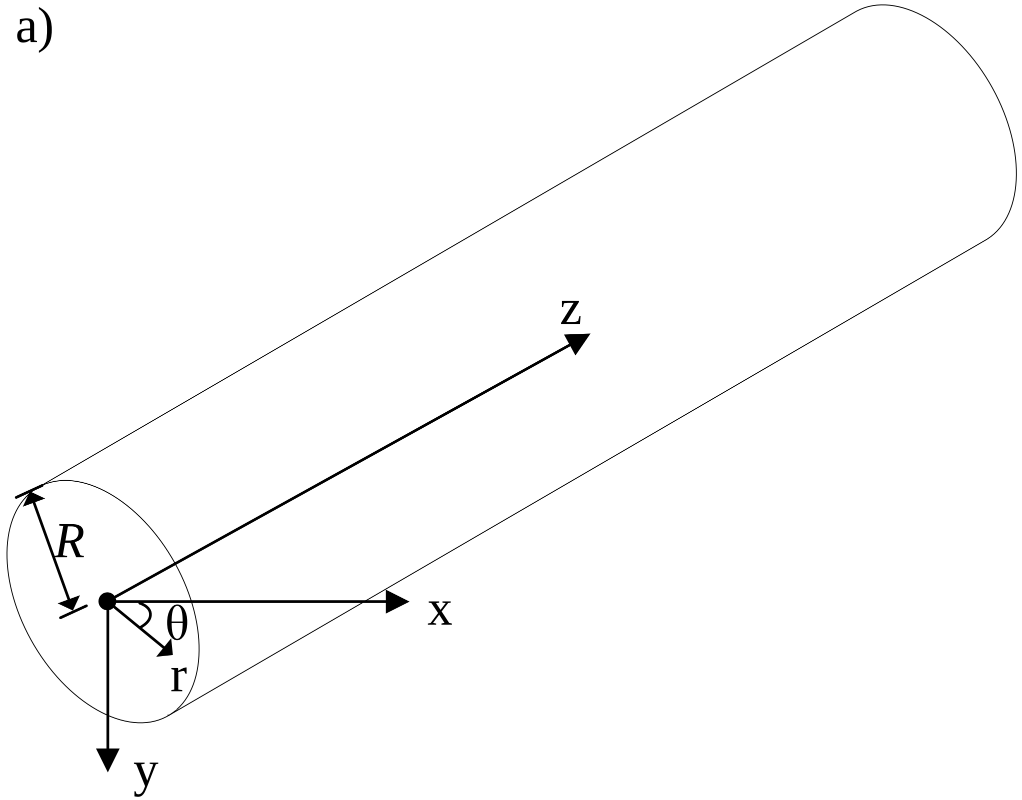
- Amplitude of the propagating plane wave $(x > 0)$

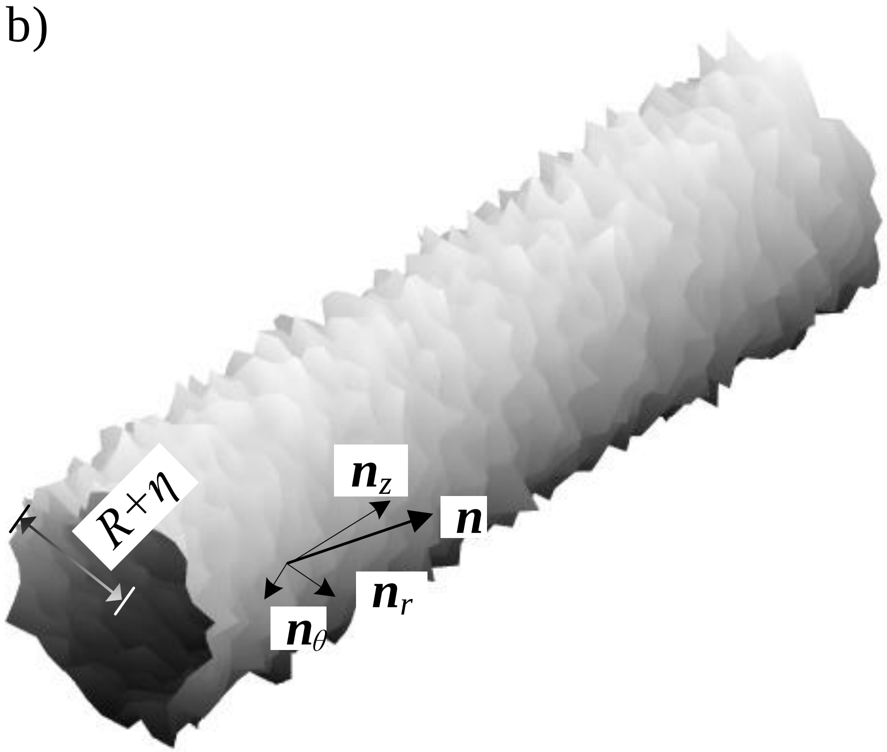
$$|p_a| \approx A_0 \exp(-|\xi_{0,2}|x)$$

- Correlation function and its Fourier transform

$$W(x) = \mathrm{e}^{-x^2/l^2} \quad \text{and} \quad \hat{W}(\xi) = \sqrt{\pi}l\mathrm{e}^{-\xi^2 l^2/4}$$

Surface roughness in cylindrical pipe





$$\bar{\eta}(\theta,z) = \int_{-\infty}^{\infty} \eta(\theta,z)\,w(\eta,\theta,z)\,d\eta = 0$$

Wavenumber correction

General equation for wavenumber

- $\xi = \xi_{mn} + \frac{i\sigma^2}{2R^2} \sum_{q=0}^{Q} \sum_{s=0}^{S} \frac{k_{mn} J_m(k_{mn})}{\xi_{mn}\xi_{qs}^{+} f_m'(k_{mn})(1+\delta_{m0,n0,q0,s0})} \frac{J_q(k_{qs})}{J_q''(k_{qs})} \left(-qm + q^2 - \xi_{qs}^{\pm}\xi_{mn} + {\xi_{qs}^{\pm}}^2 - \frac{J_m''(k_{qs}^{\pm})}{J_m(k_{qs}^{\pm})} k_{qs}^2\right)\left(-qm + m^2 - \xi_{qs}^{\pm}\xi_{mn} + \xi_{mn}^2 - \frac{J_m''(k_{mn})}{J_m(k_{mn})} k_{mn}^2\right)\widehat{W}(m-q, \xi_{mn} - \xi_{qs}^{\pm})$

Plane wave

$$\boxed{\xi_{00,2} = \frac{i\sigma^2}{2R^2}\Bigg[\Bigg[k^2\widehat{W}(0,2k)} + \frac{J_1(k_{10})}{4k\xi_{10}J_1''(k_{10})}\left({\xi_{10}}^2 - k\xi_{10} - \frac{J_1''(k_{10})}{J_1(k_{10})} k_{10}^2\right)(k^2 - k\xi_{10})\widehat{W}(1, k - \xi_{10})$$

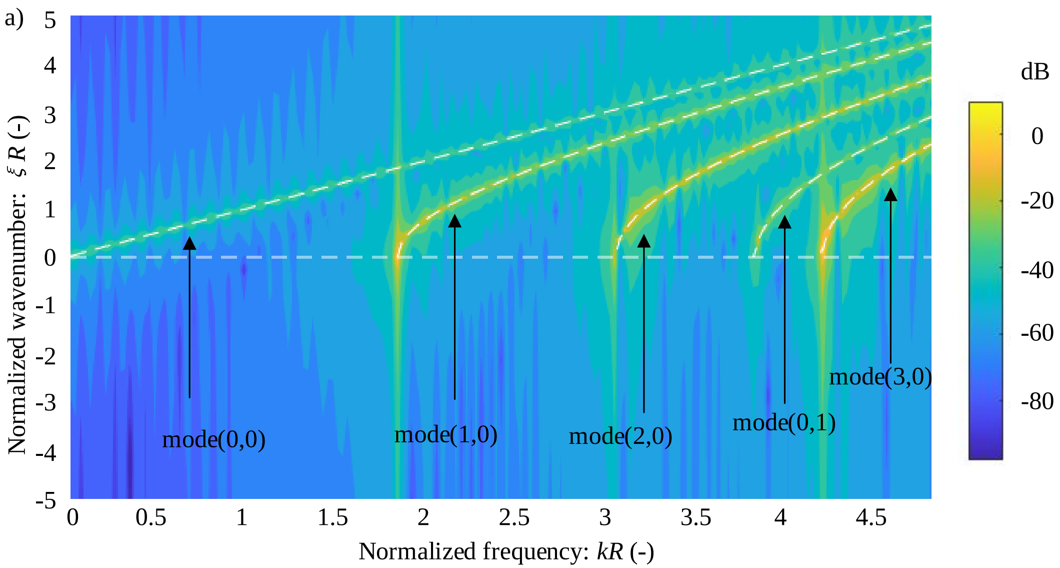
$$+ \frac{J_1(k_{10})}{4k\xi_{10}J_1''(k_{10})}\left({\xi_{10}}^2 + k\xi_{10} - \frac{J_1''(k_{10})}{J_1(k_{10})} k_{10}^2\right)(k^2 + k\xi_{10})\widehat{W}(1, k + \xi_{10})\Bigg]$$

$$+ \frac{J_2(k_{20})}{4k\xi_{20}J_2''(k_{20})}\left({\xi_{20}}^2 - k\xi_{20} - \frac{J_2''(k_{20})}{J_2(k_{20})} k_{20}^2\right)(k^2 - k\xi_{20})\widehat{W}(2, k - \xi_{20})$$

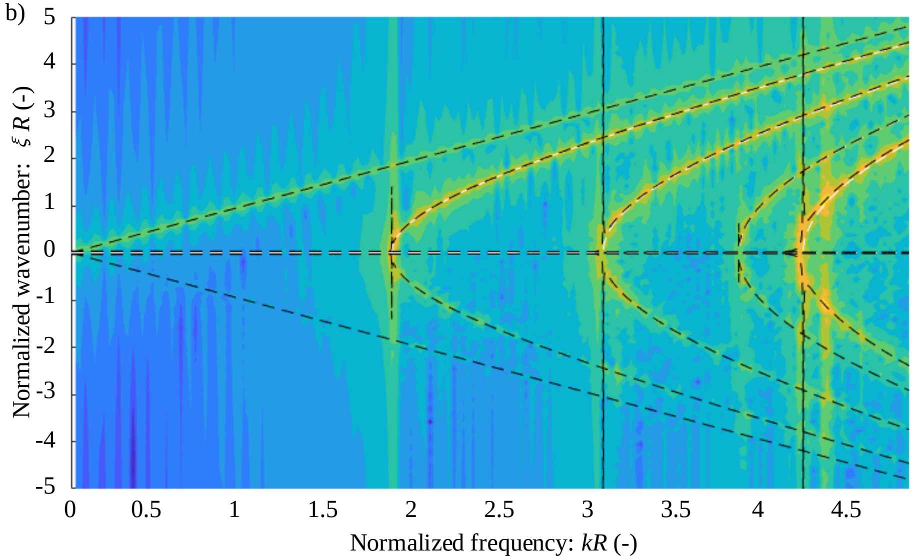
$$+ \frac{J_2''(k_{20})}{4k\xi_{20}J_2(k_{20})}\left({\xi_{20}}^2 + k\xi_{20} - \frac{J_2''(k_{20})}{J_2(k_{20})} k_{20}^2\right)(k^2 + k\xi_{20})\widehat{W}(2, k + \xi_{20})\Bigg]$$

Wave scattering

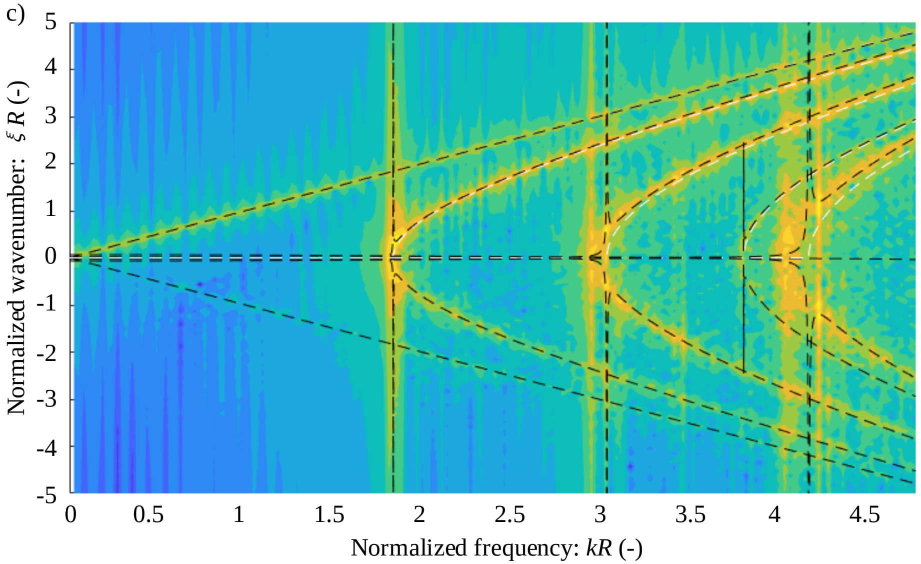
Empty pipe



σ=0.05



σ=0.1



- Wave scattering increases with σ
- Scattered wave propagates with modes
- The cut-off frequency gets smaller

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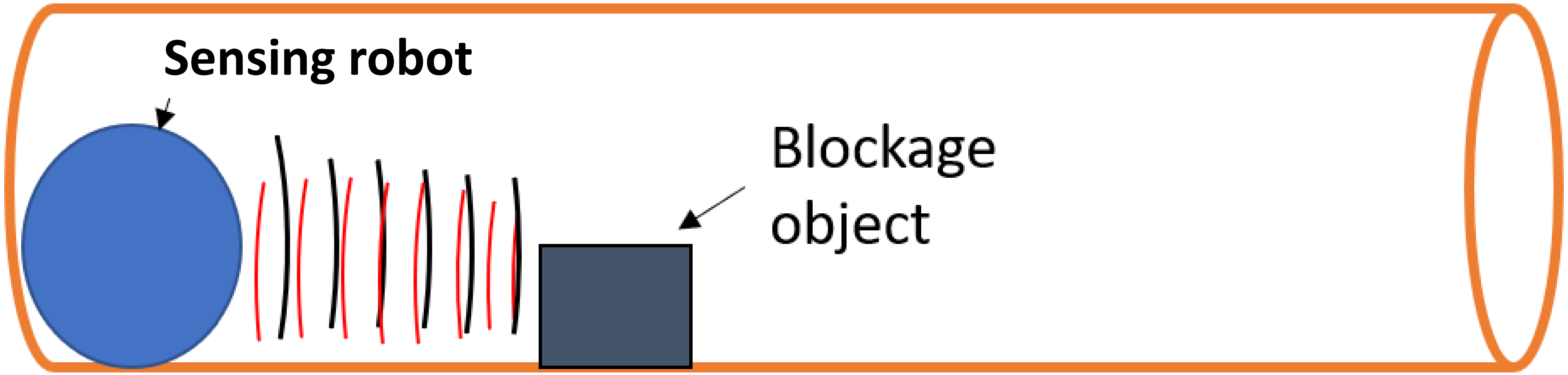
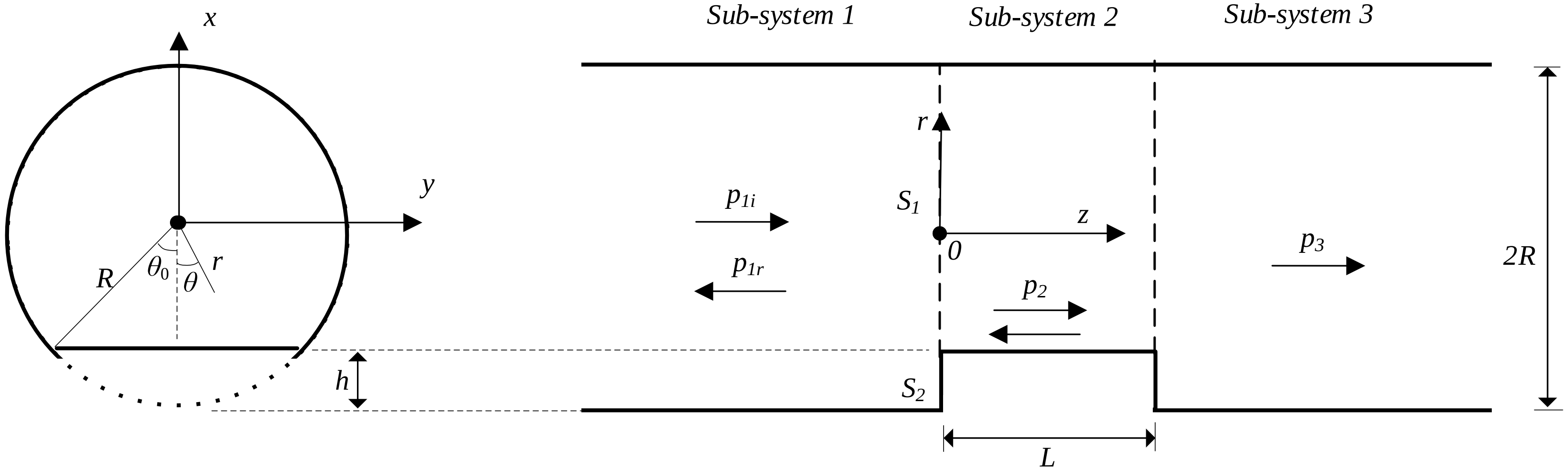


Mode Coupling theory



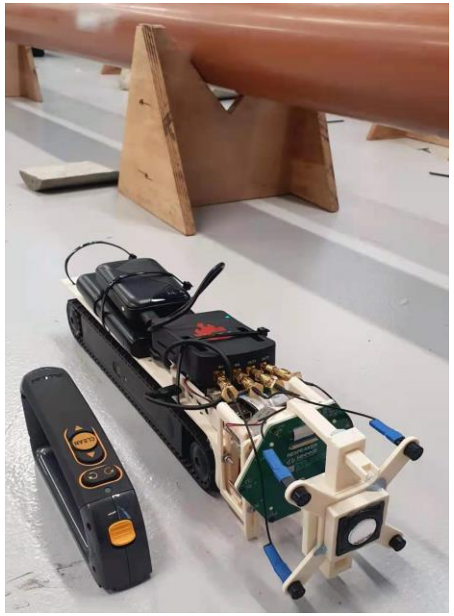
$$p_1 = p_{1i} + p_{1r} = \sum_{m,n} \left(a_{i,mn} e^{i\gamma_{mn} z} + a_{r,mn} e^{-i\gamma_{mn} z} \right) \Psi_{mn}(r, \theta)$$

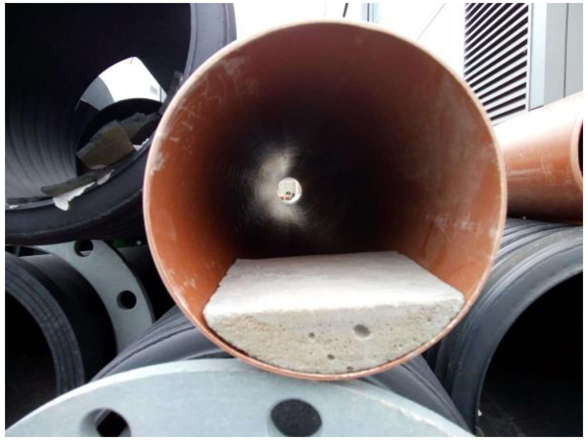
$$p_3 = \frac{1}{\sqrt{2\pi}} \sum_{m,n} c_{t,mn} e^{i\gamma_{mn}(z-L)} \Psi_{mn}(r, \theta)$$

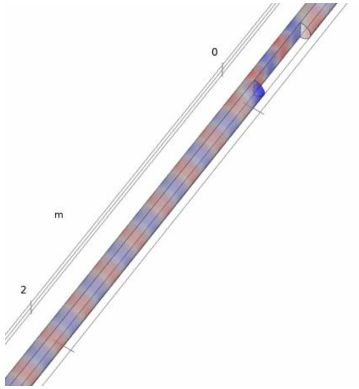
$$p_2 = \sum_{\mu,l} b_{\mu l} \Phi_{\mu l}(x, y, z)$$

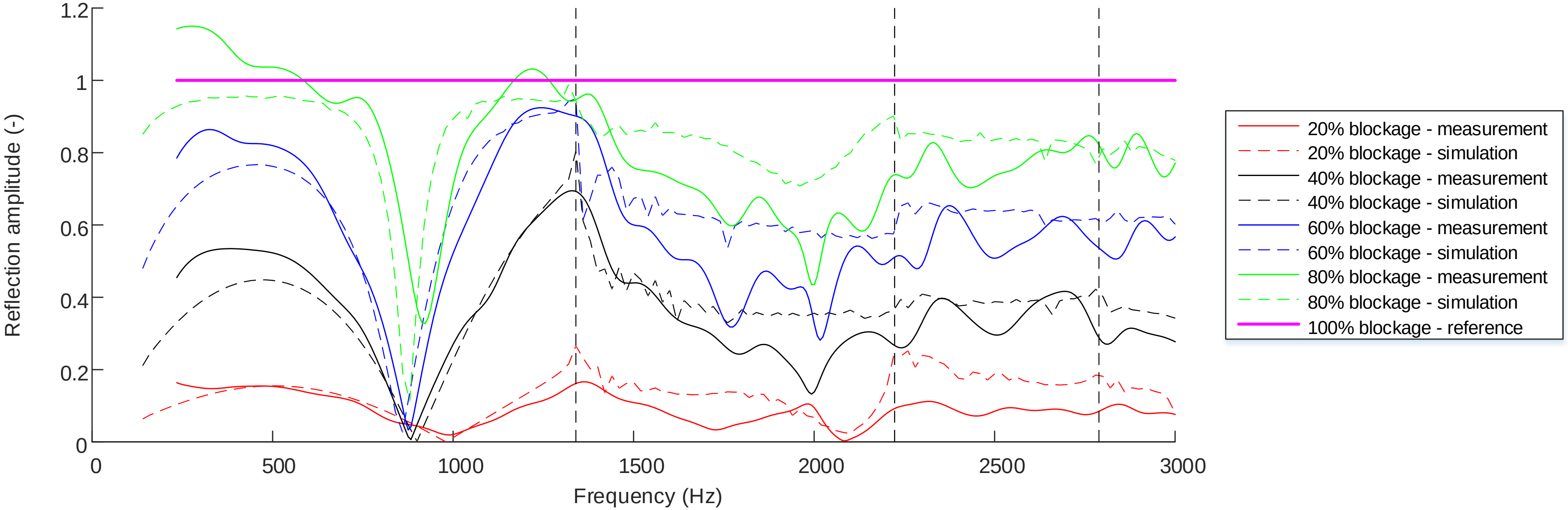
$$\iiint_V \left[p \nabla^2 \Phi_{\mu l} - \Phi_{\mu l} \nabla^2 p + \left(k_{\mu l}^2 - k_0^2 \right) \Phi_{\mu l} p \right] dV = i \rho_0 \omega Q_S(\omega) \Phi_{\mu l}(\boldsymbol{r}_s)$$

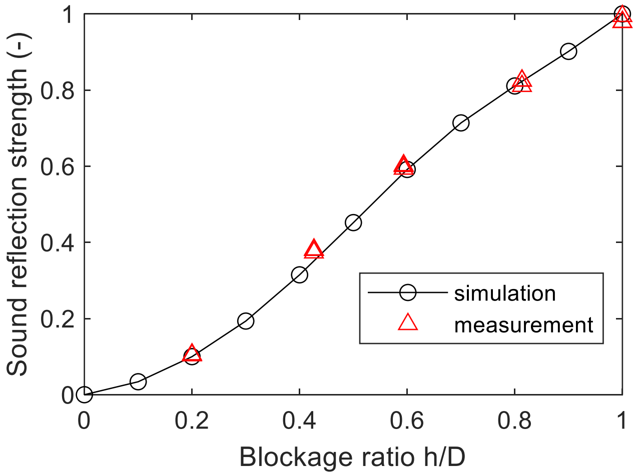
Frequency domain: blockages



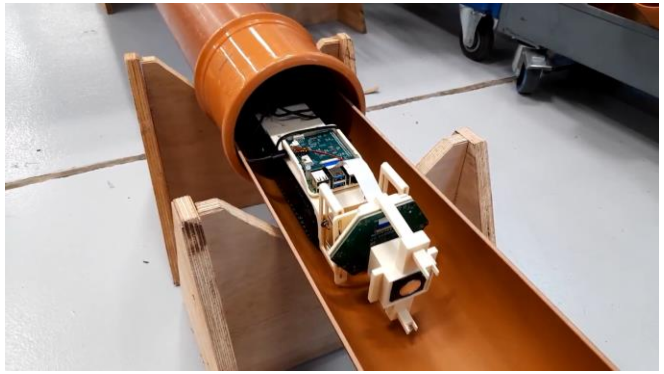


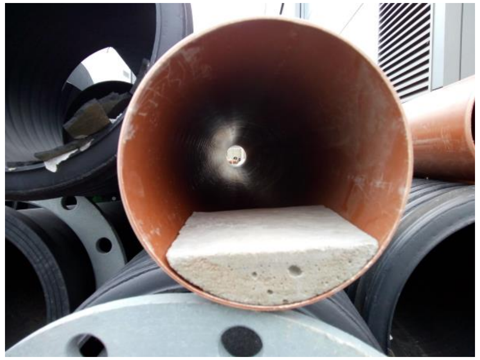


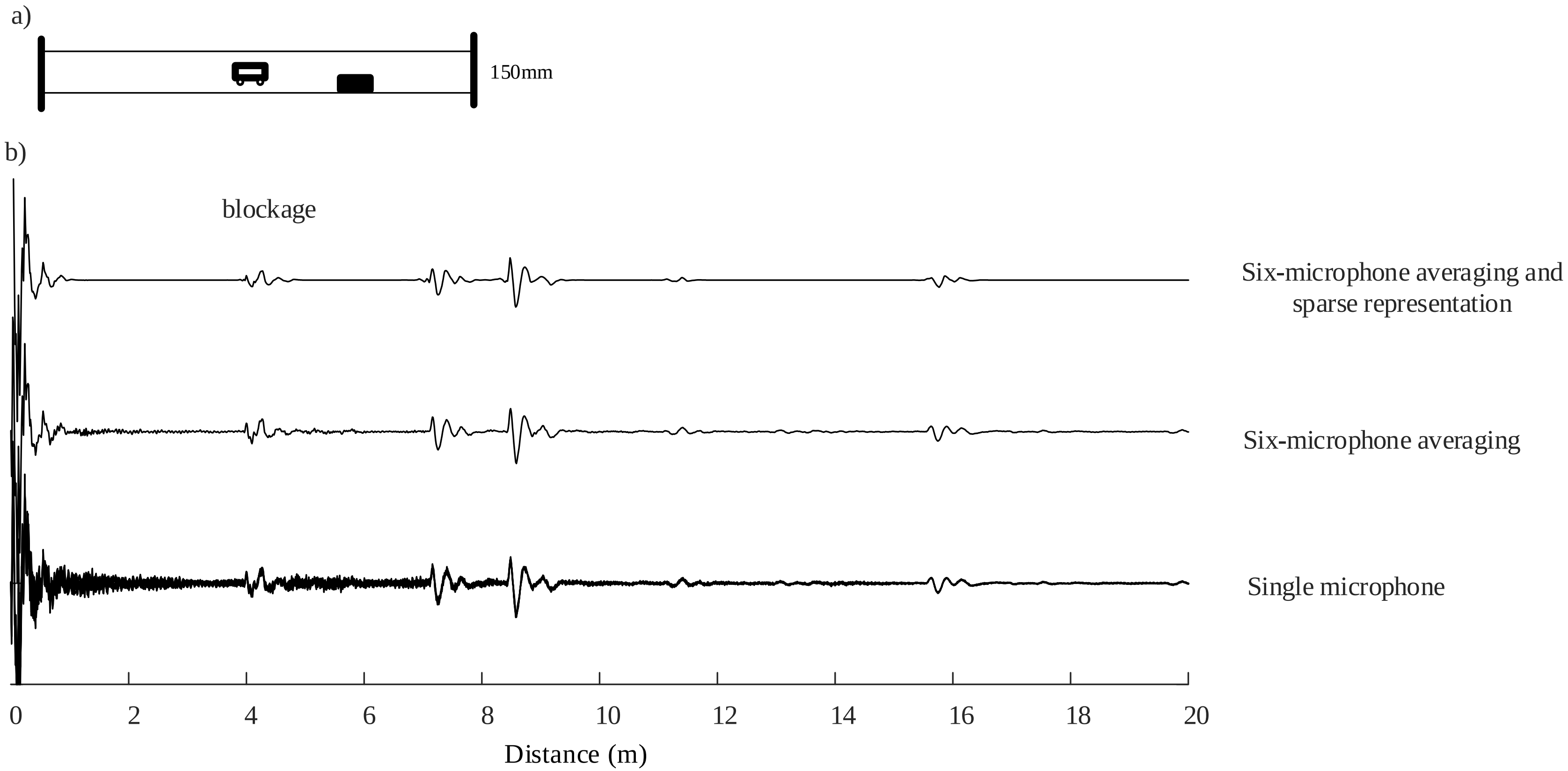




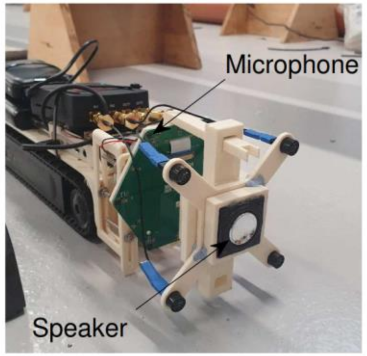
Microphone array for blockage/lateral detection on robot

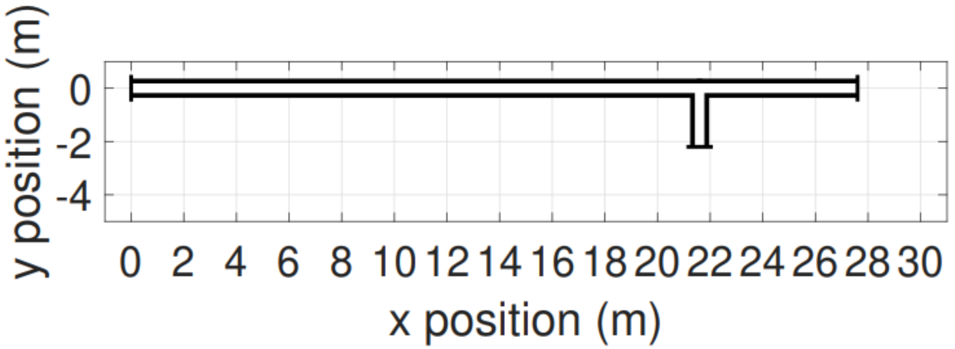


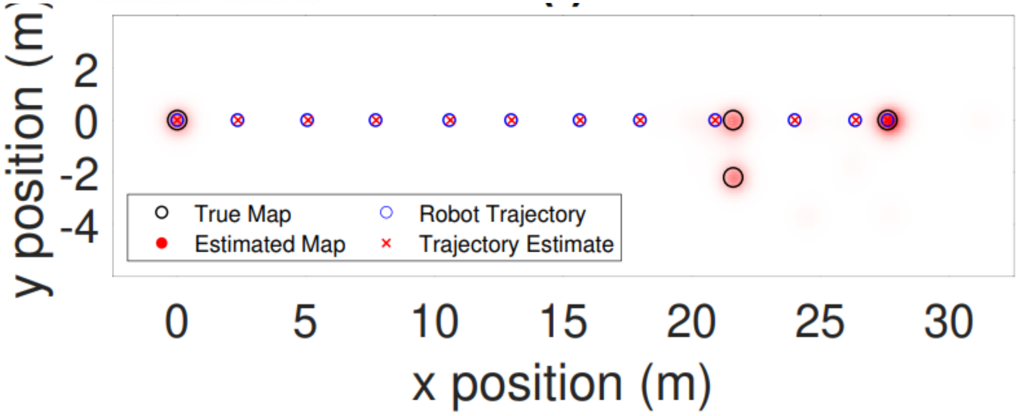


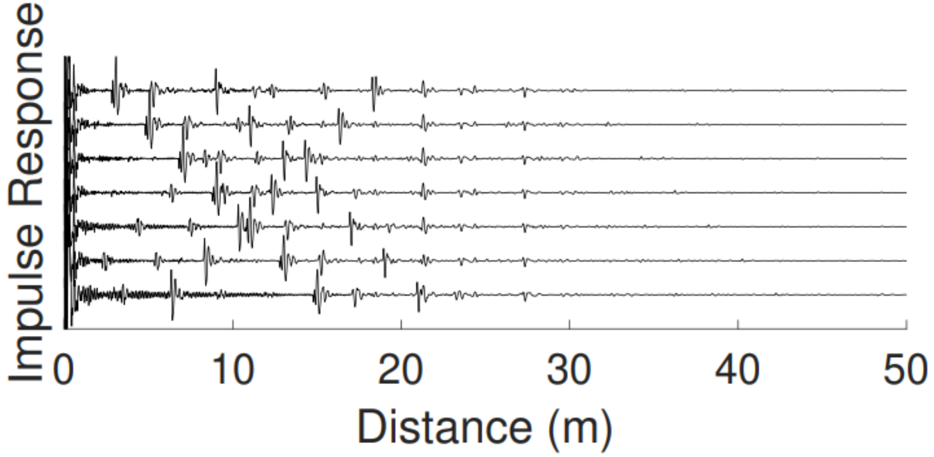


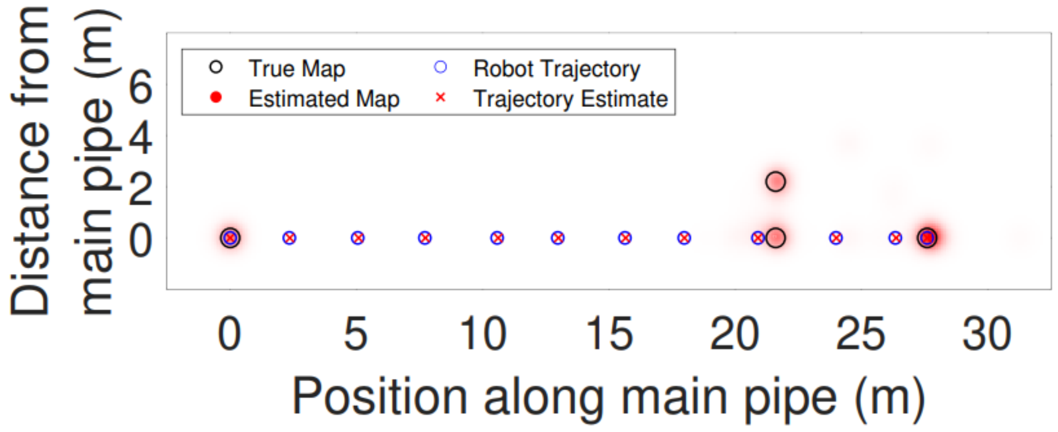
Acoustic localization and mapping of the pipe





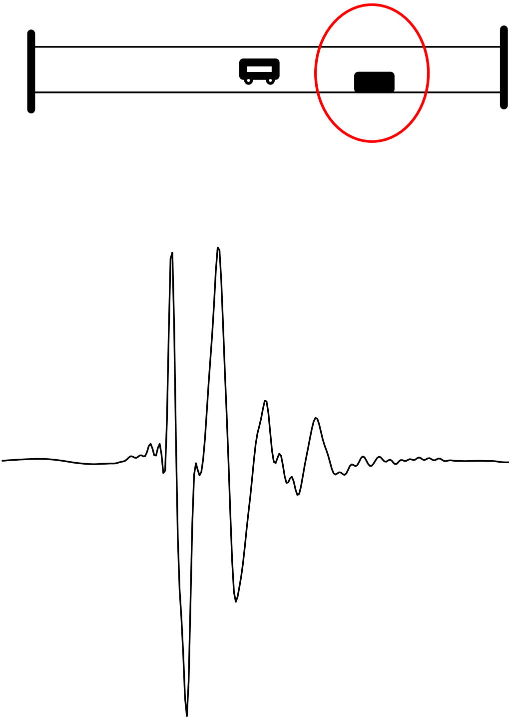




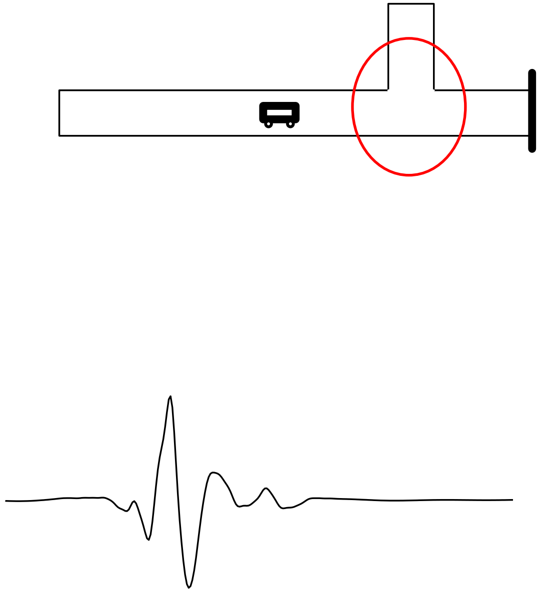


Acoustic classification of pipe features

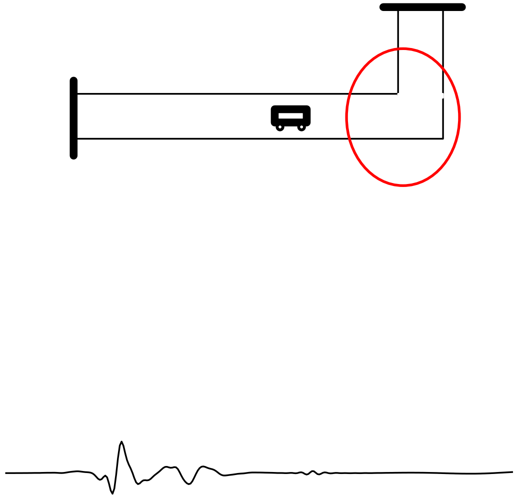
blockage



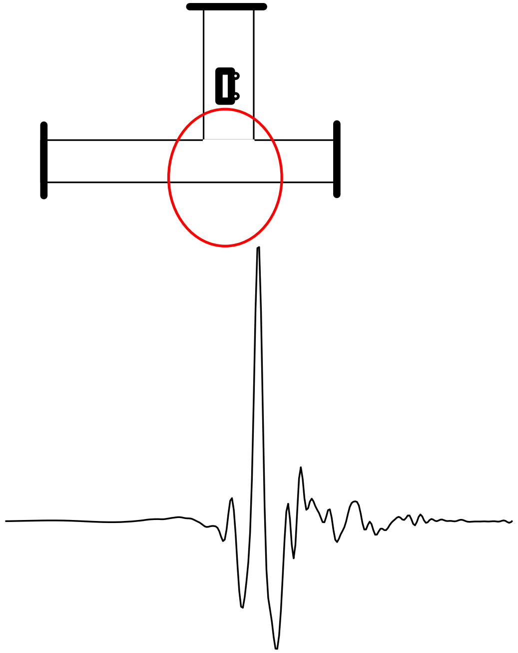
lateral



L-junction



T-junction



Acoustic classification

Metric	Time domain Linear SVM	Wavelet Linear SVM	Time domain RBF SVM	Wavelet RBF SVM
Accuracy	53%	65%	78%	88%
Precision	0.571	0.686	0.829	0.886
Recall	0.625	0.615	0.806	0.912
F1 score	0.597	0.649	0.817	0.899

- Thank you!