Phase transition for random walks on graphs with added weighted random matching

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Motivation

- A sequence of graphs might have a property *cutoff*.

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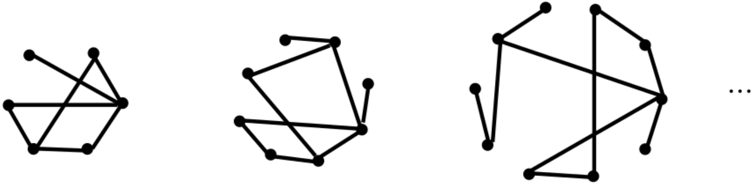
Question: Given a sequence of graphs (G_n), can we make a small modification to get cutoff?

Motivation

[Hermon, Sly, Sousi 2020]

- General sequence of graphs (G_n).



G_1 G_2 G_3 $\cdots$

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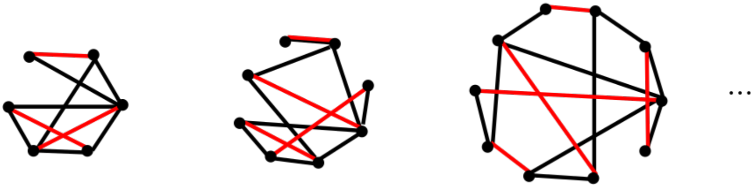
[Hermon, Sly, Sousi 2020]

- General sequence of graphs (G_n).
- Add edges of a uniform random matching $\rightsquigarrow$ random (G_n^*).



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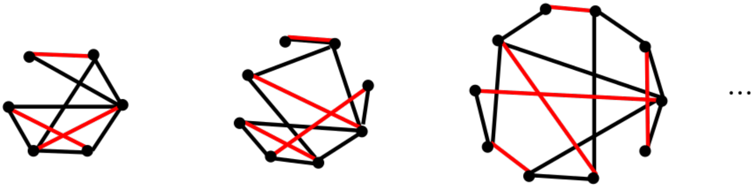

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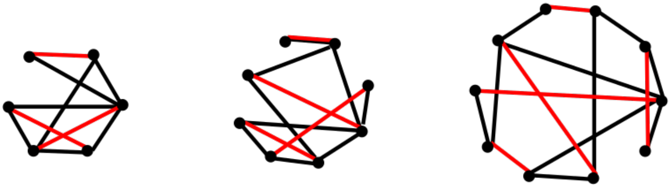

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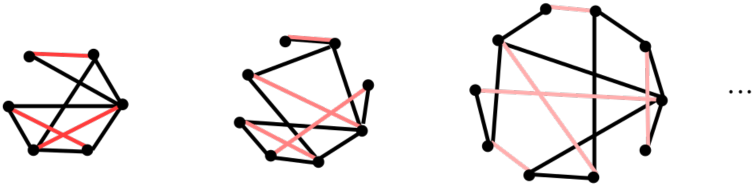

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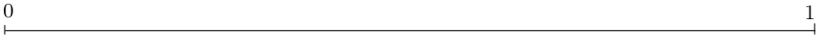
Question: What if we add the red edges with smaller weight?

- weight ε_n, allow $\varepsilon_n \to 0$

Intuition



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(G_n^*) has cutoff
iff (G_n) does

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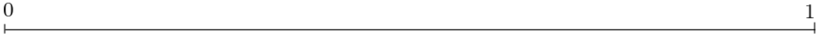
Question: What happens in-between?

Results - expanders

(G_n) expander family, bounded degrees



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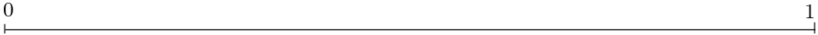

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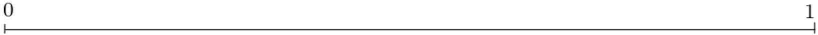

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whp (G_n^*) has cutoff

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$\varepsilon_n \asymp \frac{1}{\log |V_n|}$, (G_n) has no cutoff $\implies$ whp (G_n^*) has no cutoff

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$$\varepsilon_n \asymp \frac{1}{\log|V_n|},\ (G_n) \text{ has no cutoff} \implies \text{whp } (G_n^*) \text{ has no cutoff}$$

$$\exists\,(G_n):\ (G_n) \text{ has cutoff},\ \forall \varepsilon_n \asymp \frac{1}{\log|V_n|} \text{ whp } (G_n^*) \text{ has cutoff}$$

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whp (G_n^*) has cutoff

$\varepsilon_n \asymp \dfrac{1}{\log|V_n|}$, (G_n) has no cutoff $\Longrightarrow$ whp (G_n^*) has no cutoff

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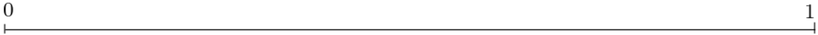

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Thank you for your attention!

Also check out my poster!

References

Jonathan Hermon, Allan Sly, and Perla Sousi. *Universality of cutoff for graphs with an added random matching.* The Annals of Probability, 50(1):203 – 240, 2022.