REINFORCED RANDOM WALK
&
A HYPERBOLIC SPIN MODEL ON THE REGULAR TREE

(Infinite-Order Transition and an Intermediate Phase)

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The Models

1) The VRJP (= vertex-reinforced jump process)

Definition: The VRJP is a continuous-time jump process $(X_t)_{t\geq 0}$ on a locally finite graph $G=(V,E)$. Started at some vertex $X_0 = i_0$, at time t it jumps from $X_t = x$ to a neighbour $y \sim x$ at rate

$$\beta[1+L_t^y] \quad \text{with} \quad L_t^y := \int_0^t 1_{X_s=x} \mathrm{d}s,$$

for a fixed *inverse temperature* $\beta > 0$.

2) The $\mathbb{H}^{2|2}$-model (= spin system w/ values in the hyperbolic superplane $\mathbb{H}^{2|2}$)

up to subtleties on the super-manifold $\mathbb{H}^{2|2}$.

"Definition": Consider a finite graph $G=(V,E)$. Fix an *inverse temperature* $\beta > 0$ and a *magnetic field* $h > 0$. For a functional $F \in C^\infty((\mathbb{H}^{2|2})^V)$ over *spin configurations* $\underline{\mathbf{u}} = (\mathbf{u}_i)_{i\in V} \in (\mathbb{H}^{2|2})^V$ the $\mathbb{H}^{2|2}$*-model* defines its expectation as

$$\langle F(\underline{\mathbf{u}})\rangle_{\beta,h} := \int_{(\mathbb{H}^{2|2})^V} \mathrm{d}\underline{\mathbf{u}}\, F(\underline{\mathbf{u}})\, e^{\sum_{ij\in E}\beta(\mathbf{u}_i\cdot\mathbf{u}_j+1)+\sum_{i\in V} h(\mathbf{u}_i\cdot\mathbf{u}^{(0)}+1)},$$

with Haar measure $\mathrm{d}\underline{\mathbf{u}} = \prod_{i\in V}\mathrm{d}\mathbf{u}_i$ and $\mathbf{u}^{(0)} \in \mathbb{H}^{2|2}$ the direction of the magnetic field.

WHY IS THIS INTERESTING?

- VRJP: recurrence/transience transition in β.

connection via
Dynkin isomorphism

- $\mathbb{H}^{2|2}$-model: spontaneous symmetry-breaking transition in β.

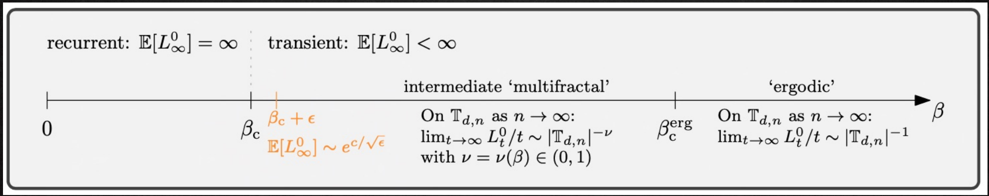
- Zirnbauer introduced $\mathbb{H}^{2|2}$-model as toy model for the Anderson transition (1991).

RESULTS FOR VRJP ON REGULAR TREES

- $\mathbb{T}_{d,n}$: rooted $(d+1)$-regular tree up to generation n.
- $\mathbb{T}_d := \mathbb{T}_{d,\infty}$

$\mathbb{T}_{2,3} =$ 

- Consider VRJP on $\mathbb{T}_d$ ($/\mathbb{T}_{d,n}$) started at the root o.
- L_t^o: time spent at o up to t; L_∞^o: total time at o.
- "THEOREMS" (Poudevigne-W., 2023, to be published)



critical point was known [Basdevant-Singh, 2012]