Schwarzian Field Theory

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joint with Roland Bauerschmidt and Peter Wildemann

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2D Quantum Gravity

Liouville CFT: Schramm-Löwner Evolution, Random LQG Surfaces, Random Planar Maps, Ising Model, Random Cluster Models, Gaussian Multiplicative Chaos, etc.

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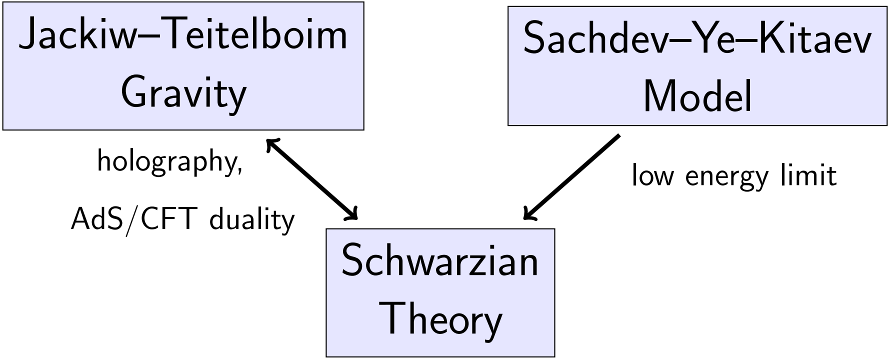
Other models:
(J. Maldacena, P. Saad, S. Shenker, D. Stanford, E. Witten, et. al.)



Path Integral

Path integrals are integrals over the space of functions which usually have the form

$$Z(\beta) = \int \exp\left[-\beta\, I(f)\right] \mathcal{D}f,$$

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Example: For $I(f) = \int (\nabla f)^2$ it corresponds to the Gaussian Free Field (in 1D — Brownian Motion).

Schwarzian Theory

Formally, Schwarzian Theory on $\mathrm{Diff}(\mathbb{S}^1)$ is given by path integral

$$\exp\left[\beta\int_0^1\left(\mathcal{S}\{\varphi,\tau\}+2\pi^2\varphi'(\tau)^2\right)d\tau\right]\mathcal{D}\varphi \tag{1}$$

where

$$\mathcal{D}\varphi \text{ is the Haar measure on } \mathrm{Diff}(\mathbb{S}^1),$$

$$\mathcal{S}\{\varphi,\tau\}=\frac{\varphi'''(\tau)}{\varphi'(\tau)}-\frac{3}{2}\left(\frac{\varphi''(\tau)}{\varphi'(\tau)}\right)^2.$$

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Path integral (1) is conformally invariant.

So, Schwarzian Theory is a finite measure on $\mathrm{Diff}(\mathbb{S}^1)/\mathrm{SL}(2,\mathbb{R})$, given by (1).

Questions

1. Definition of the Schwarzian Theory
2. Partition function
3. Observables
4. Characterization
5. Universality
6. Connections with JT gravity and SYK model

Thank you!