Cutoff for random walk on random graphs with a community structure

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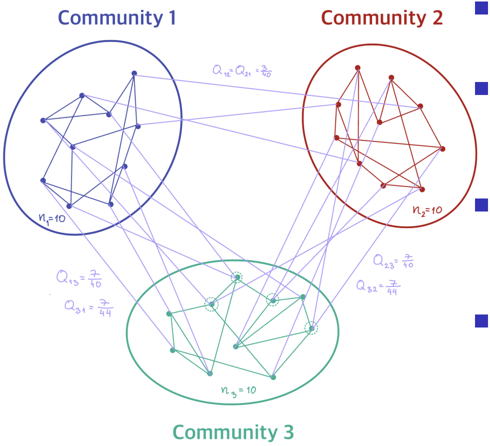
Graphs with community structure

- Community structure: there is a partition of vertices such that vertices in the same group are more likely to be connected than vertices in different groups
- Social networks: acquaintance networks, collaboration networks
- Technological networks: Internet and power grids
- Biological networks: neural networks, food webs, metabolic networks

m communities graph



- n vertices split into m communities
- for all vertices v, $\deg^{\text{int}}(v), \deg^{\text{out}}(v)$ are given
- the proportion of edges of community i that lead to j, $Q_n(i,j)$ is given
- connect vertices uniformly at random satisfying these constraints

Main result

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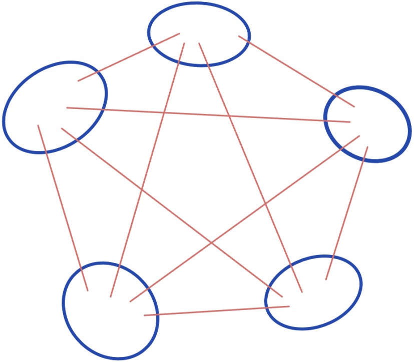
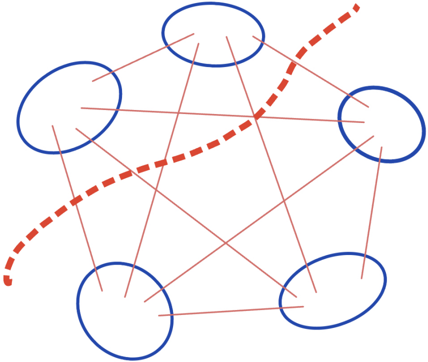
- Mixing time is of order $\log(n) + 1/\alpha_n$
- If $\alpha_n \gg \frac{1}{\log n}$ the random walk exhibits a cutoff with high probability
- If $\alpha_n \lesssim \frac{1}{\log n}$ with high probability there is no cutoff

Intuition



- mixing in one community takes $\asymp \log n$ and has cutoff (configuration model)
- if $\alpha_n \gg \frac{1}{\log n}$ we jump quicker between any sets of communities

Intuition



- mixing in one community takes $\asymp \log n$ and has cutoff (configuration model)
- if $\alpha_n \lesssim \frac{1}{\log n}$ there is a partition of communities into two sets such that it takes longer to jump between them than to mix in a community

Thank you!