Wave amplification via asymmetries in nonlinear photonic systems

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Physical Applications of Dispersive Hydrodynamics, Cambridge

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Outline

Introduction: modulation instability (MI) in the NLSE

Frequency unbalanced losses for signal and idler:
"gain-through-loss" (GTL) and MI in the defocussing NLSE

GTL examples in nonlinear optics and applications for optical frequency combs generation

The relation of asymmetry mediated wave amplification with converse symmetry breaking (CSB) for coupled oscillators

Conclusions and future directions

The NLSE for optical fibres

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + i\gamma|A|^2 A$$

$A(z,t):$ *electric field slowly varying envelope*

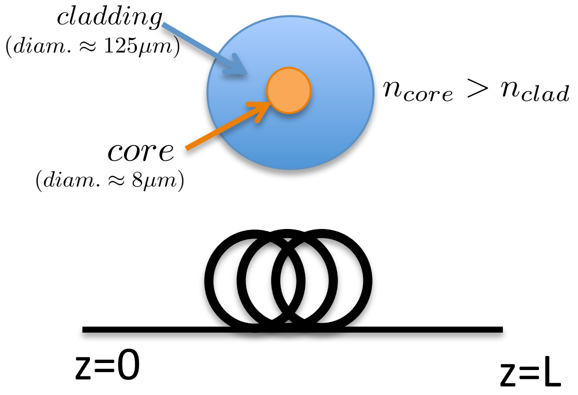
$\gamma:$ *Kerr nonlinearity*

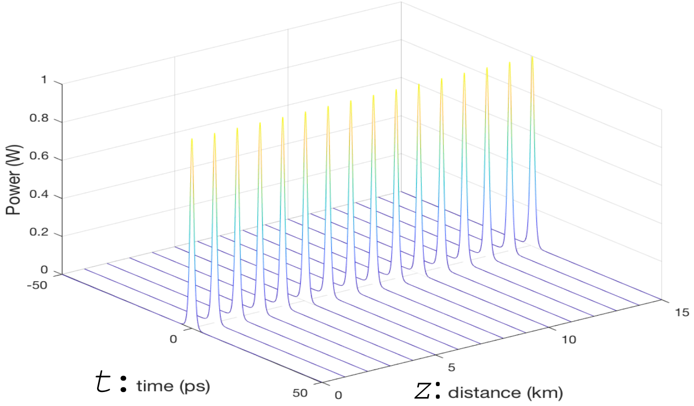
$\beta_2:$ *group velocity dispersion*

$z:$ *propagation coordinate along the fibre*

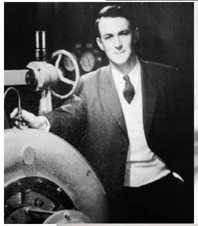
$t:$ *time in a reference frame co-moving with the pulse*

fibre section





Modulation Instability (MI) in the NLSE

In the focussing case (anomalous dispersion regime for fibres) a powerful monochromatic wave (pump/condensate) is unstable with respect to perturbations oscillating at frequencies symmetrically detuned by $\pm\Omega$ resulting in wave destruction and sidebands amplification

MI is ubiquitous due to the universality of the NLSE and its generalised forms: it occurs in hydrodynamics, photonics, plasmas, Bose-Einstein condensates…

MI is related to:
FPUT Recurrences, solitons, turbulence, rogue waves…

N. Bogoliubov, Journal of Physics 11, 23 (1947)
T. Benjamin and J. E. B. Feir, J. Fluid Mechanics 27, 417-430 (1967)
V.E. Zakharov and L.A. Ostrovsky Physica D 238, 540-548 (2009)

Linear Stability Analysis

Homogeneous solution

$$A_s = \sqrt{P} e^{i\gamma P z}$$

Perturbed homogeneous solution

$$A(z,t) = A_s \left[1 + a_+(z) e^{-i\Omega t} + a_-(z) e^{i\Omega t}\right]$$

Subsitution into NLSE and linearisation with respect to a

$$\frac{\partial A}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A \qquad \beta_2 < 0$$

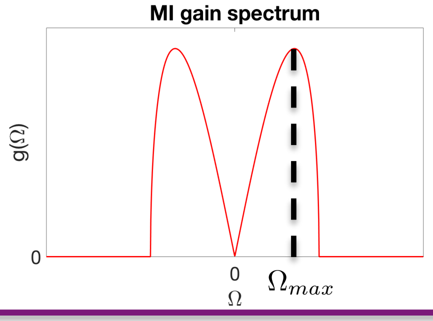
Critical frequency

$$\Omega_c = \sqrt{\frac{4\gamma P}{|\beta_2|}} \qquad |\Omega| < \Omega_c$$

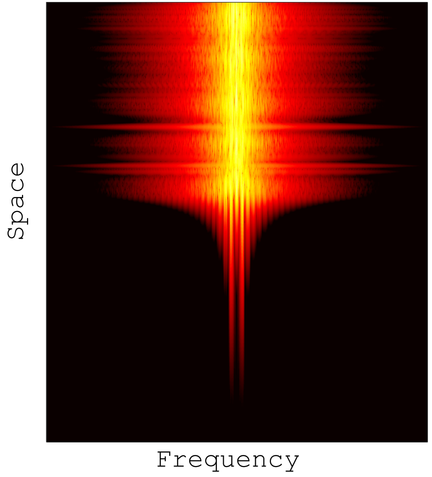
MI gain

$$g(\Omega) = |\beta_2 \Omega| \sqrt{(\Omega_c^2 - \Omega^2)}$$

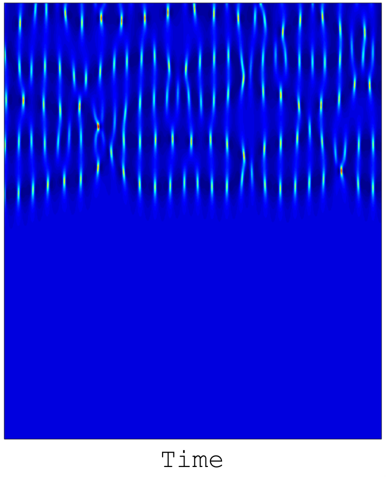
Homogeneous solution becomes modulated by a wave at frequency Ω_{max}



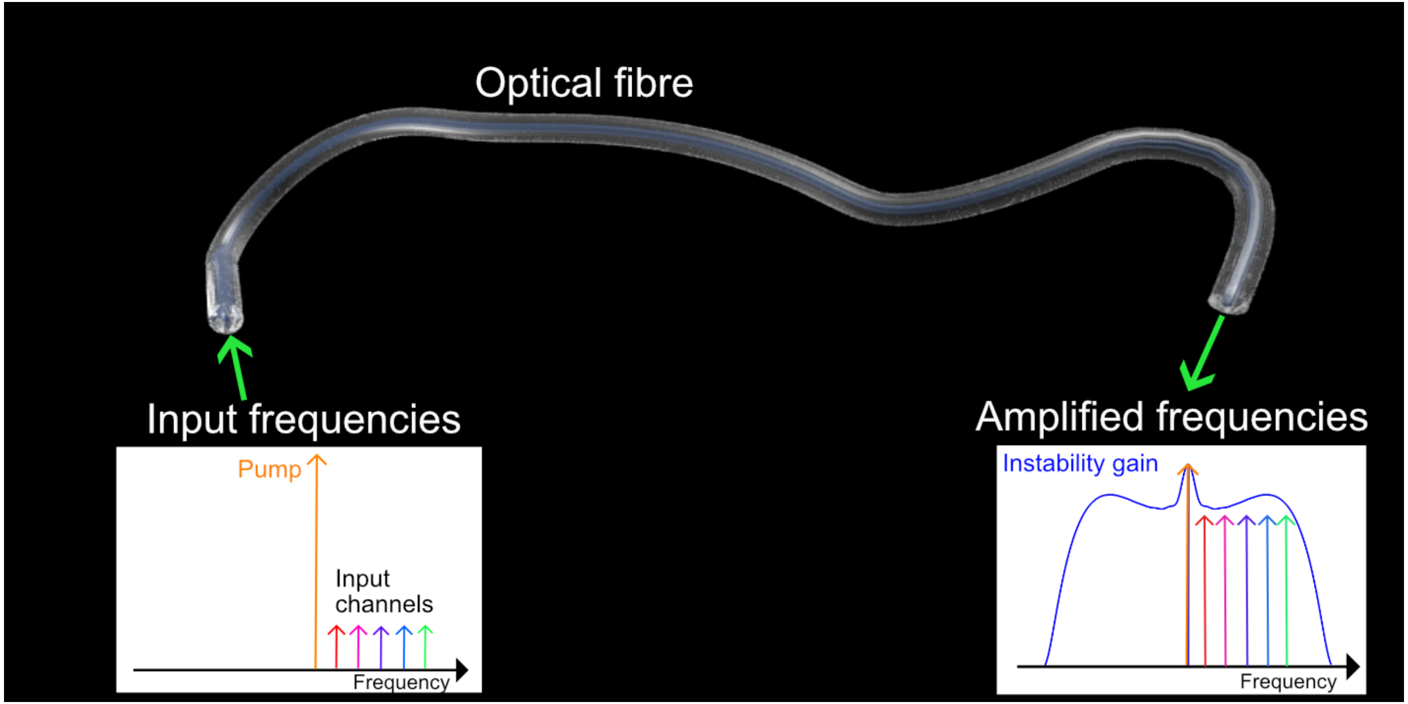
Spectral evolution



Time evolution



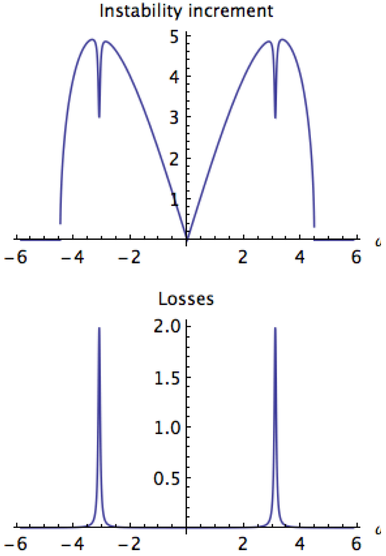
MI application: optical parametric amplifiers



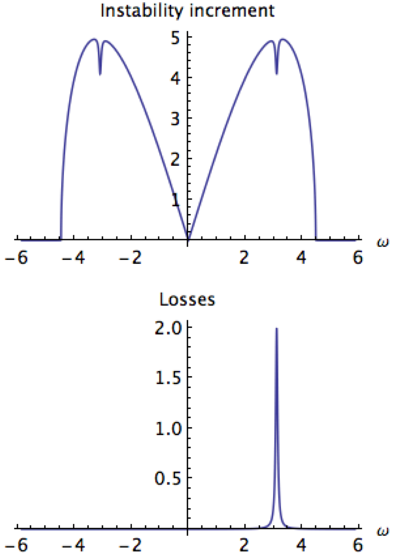
Losses: a stabilising factor?

Focussing NLSE

Symmetric losses

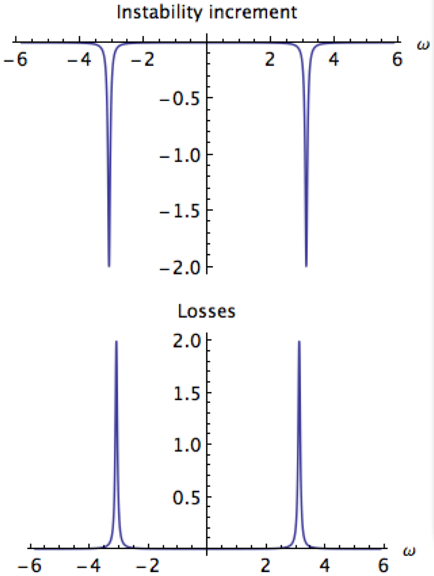


Asymmetric losses

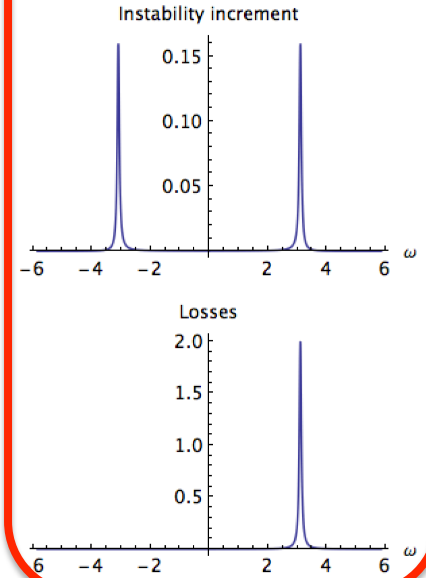


Defocussing NLSE

Symmetric losses

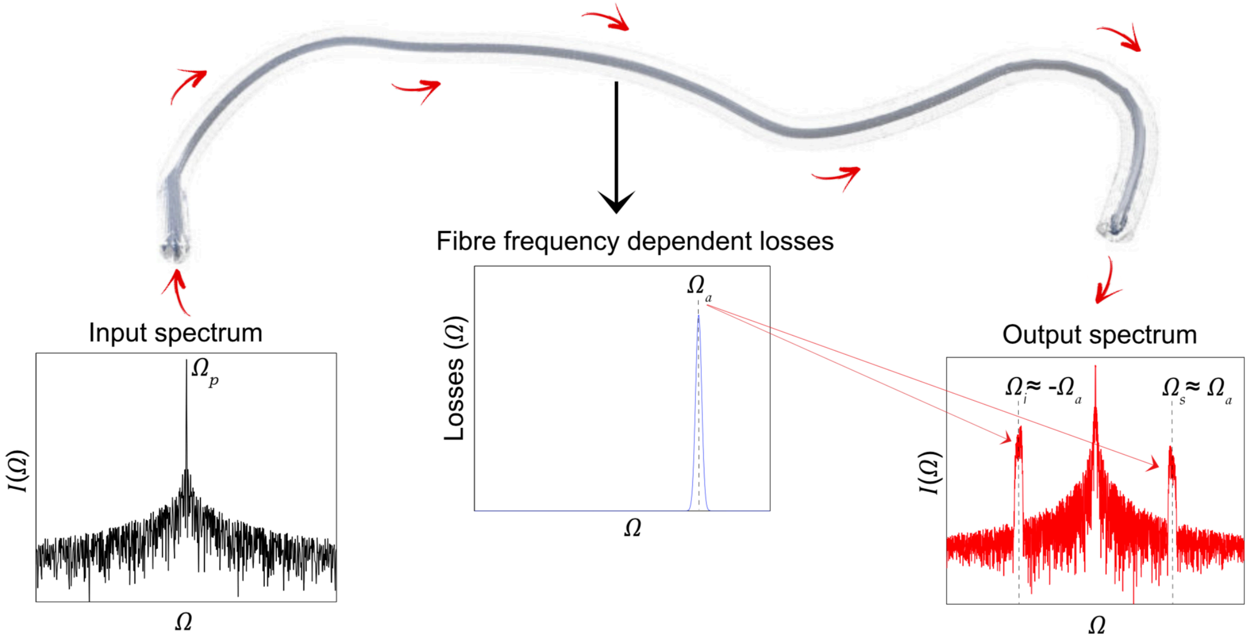


Asymmetric losses



Asymmetric losses for signal and idler waves destabilise the homogeneous solution

Gain through losses concept



A. M. Perego et al. *Light: Science & Applications* 7, 43 (2018)

Gain through losses: basic amplifier

Theory: linear stability analysis

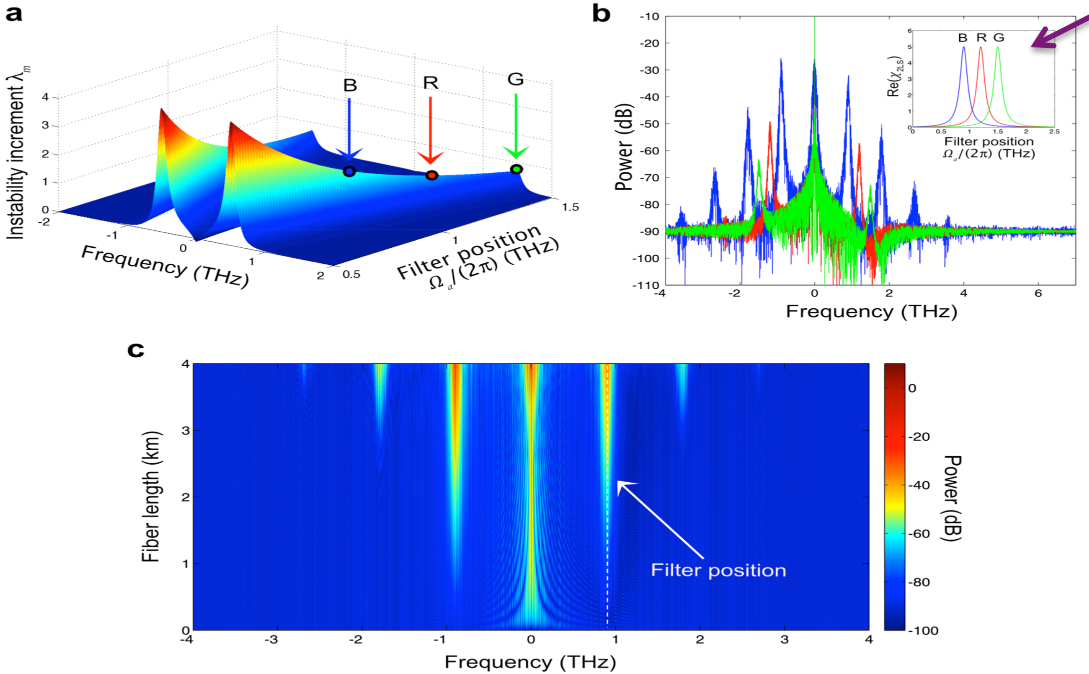
Simulations

Distributed filters

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + i\gamma|A|^2 A - \theta \star A$$

$$\theta = \mathcal{F}^{-1}\left\{\chi_{2LS}(\Omega)\right\}$$

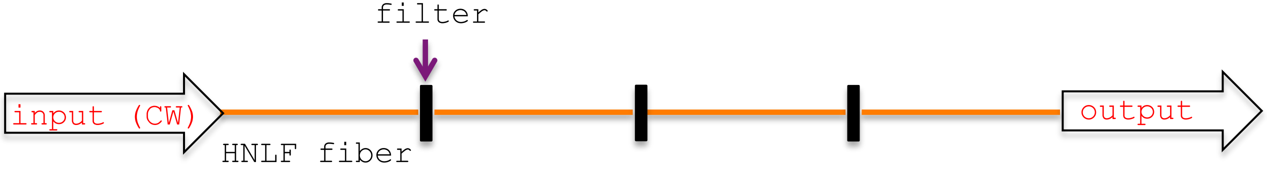
$$\chi_{2LS}(\Omega) = g\frac{\gamma_\perp}{\gamma_\perp + i(\Omega_a - \Omega)}$$



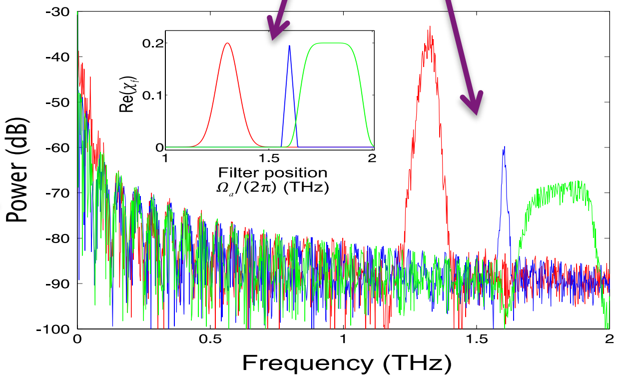
energy transfer from the pump field to the spectral region of losses (signal) and to the symmetric one (idler)

Imaging of spectral losses into gain

Concatenated fibres with lumped filters



The filter's shape is mapped into the gain profile



A. M. Perego et al. *Light: Science & Applications* 7, 43 (2018)

Quadratic nonlinearity medium (OPO)

$$\frac{\partial A_s}{\partial z} = -i\Delta k A_s + M_s A_i^* - \alpha_s A_s$$

$$\frac{\partial A_i^*}{\partial z} = i\Delta k A_i^* + M_i A_s - \alpha_i A_i^*$$

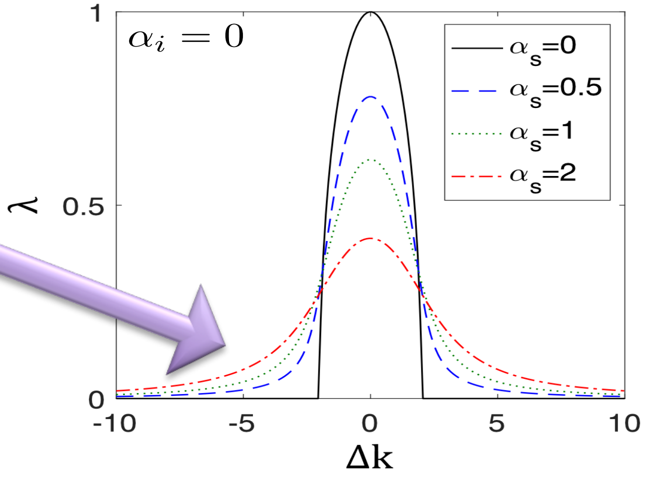
$$M_s \cdot M_i = \rho |A_p|^2$$

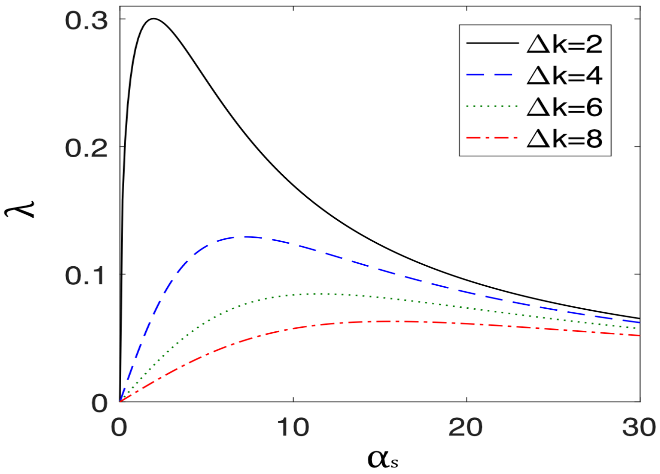
$$\Delta k = (k_p - k_s - k_i)/2$$

$$\lambda_{\pm} = \frac{1}{2}\left[-(\alpha_s + \alpha_i) \pm \sqrt{(\alpha_s - \alpha_i - i\Delta k)^2 + 4\rho |A_p|^2}\right]$$

$$\lambda = max[Re(\lambda_+), Re(\lambda_-)]$$

Gain induced by loss asymmetry outside of phase-matching





Some key works (photonics+BEC)

VOLUME 93, NUMBER 16 PHYSICAL REVIEW LETTERS week ending 15 OCTOBER 2004

Modulational Instability and Parametric Amplification Induced by Loss Dispersion in Optical Fibers

Takuo Tanemura,* Yasuyuki Ozeki, and Kazuro Kikuchi

Optics Communications 285 (2012) 790–794

Contents lists available at SciVerse ScienceDirect

ELSEVIER

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Synthesis of broadband and flat parametric gain by idler loss in optical fiber

Kun Xu *, Hongyao Liu, Yitang Dai, Jian Wu, Jintong Lin

5086 Vol. 40, No. 21 / November 1 2015 / *Optics Letters* Letter

Optics Letters

Optical parametric amplification via non-Hermitian phase matching

R. El-Ganainy,[1,*] J. I. Dadap,[2] and R. M. Osgood, Jr.[2]

PHYSICAL REVIEW A **96**, 013605 (2017)

Non-Hermitian matter-wave mixing in Bose-Einstein condensates: Dissipation-induced amplification

S. Wüster[1,2,*] and R. El-Ganainy[3,†]

IEEE Photonics Journal

An IEEE Photonics Society Publication

Open Access

Lumped Dissipation Induced Quasi-Phase Matching for Broad and Flat Optical Parametric Processes

Volume 11, Number 6, December 2019

Hanwen Hu
Lei Zhang
Chi Zhang
Yuntian Chen
Jing Xu
Xinliang Zhang

Perego et al. *Light: Science & Applications* (2018)7:43
DOI 10.1038/s41377-018-0042-9

Official journal of the CIOMP 2047-7538
www.nature.com/lsa

REVIEW ARTICLE Open Access

Gain through losses in nonlinear optics

Auro M. Perego[1], Sergei K. Turitsyn[1,2] and Kestutis Staliunas[3,4]

Optical frequency combs (OFCs)

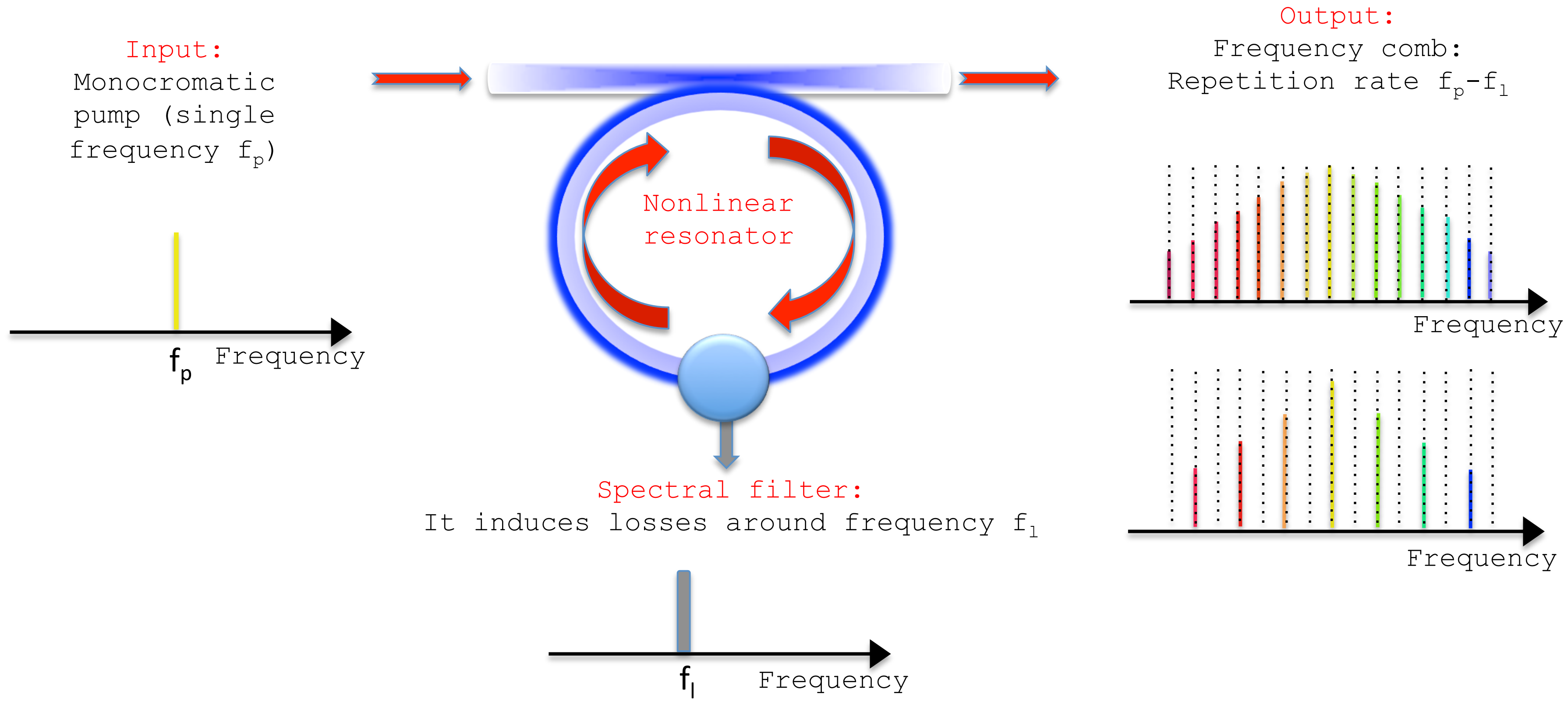
2005 Nobel Price for Physics to Hall and Hänsch

OFC:
Set of coherent equally spaced in frequency laser lines

Applications:
Powerful tool for metrology, spectroscopy, optical clocks, molecular fingerprinting, exoplanets search, optical communications and more

Sources:
Mode-locked lasers, driven passive resonators (Kerr and quadratic nonlinearity), nonlinear optical fibres…

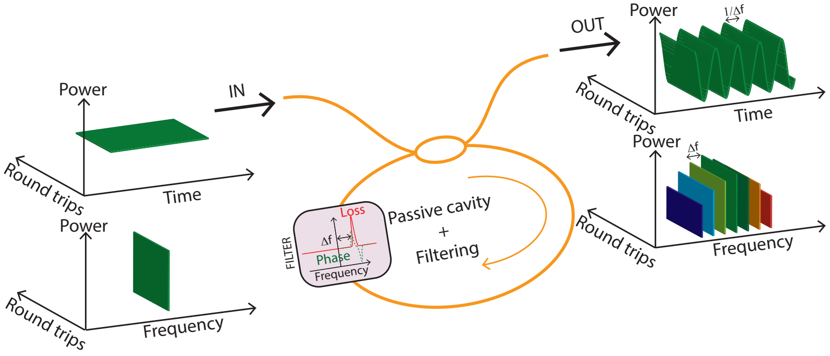
Application for OFC generation



-Changing pump and filter relative frequency position we control OFC repetition rate!

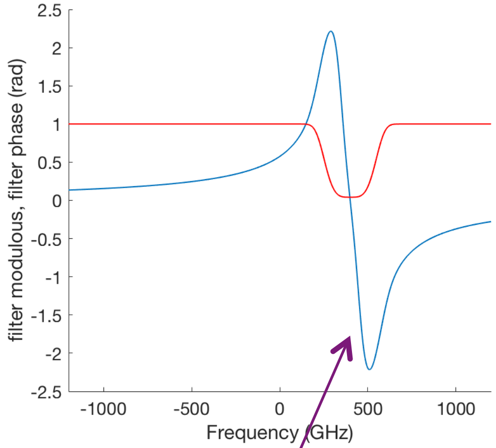
-We can have selective sidebands amplification (unlike standard parametric amplification)

Experimental setup



Normal dispersion fibre resonator with intracavity Fibre-Bragg-Grating filter

$$\beta_2 = 0.5\ \mathrm{ps}^2\mathrm{km}^{-1} \quad \gamma = 2.5\ \mathrm{W}^{-1}\mathrm{km}^{-1} \quad L = 104\ \mathrm{m}$$



Filter phase profile induced by frequency dependent losses through Kramers-Kronig relations!

Theory: Ikeda map

Propagation:

GVD

Kerr

$$i\frac{\partial A_n}{\partial z} - \frac{\beta_2}{2}\frac{\partial^2 A_n}{\partial t^2} + \gamma |A_n|^2 A_n = 0, \quad 0 < z < L$$

Boundary conditions:

Detuning

$$A_{n+1}(z=0,t) = \theta E_{\mathrm{IN}} + \rho e^{i\phi_0} h(t) \star A_n(z=L,t)$$

Pump

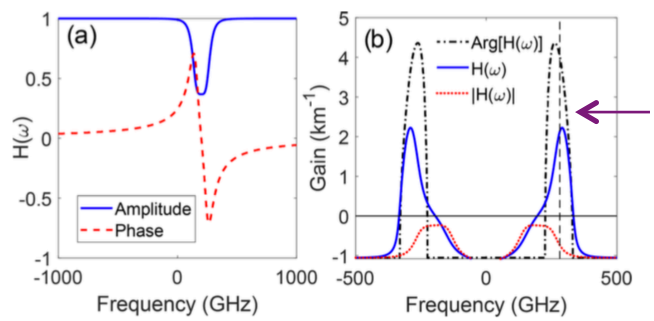
Coupler loss

Filter

Filter:

$$H(\omega) = \mathcal{F}[h(t)] = \exp[\alpha(\omega) + i\psi(\omega)] \qquad |H(\omega)| = 1 - R\exp\left[-(\omega-\omega_f)^4/(\sigma_f)^4\right] \qquad \psi(\omega) = \arg[H(\omega)] = -\mathcal{H}[\alpha(\omega)]$$

Hilbert transform



Filter phase plays key role in the gain

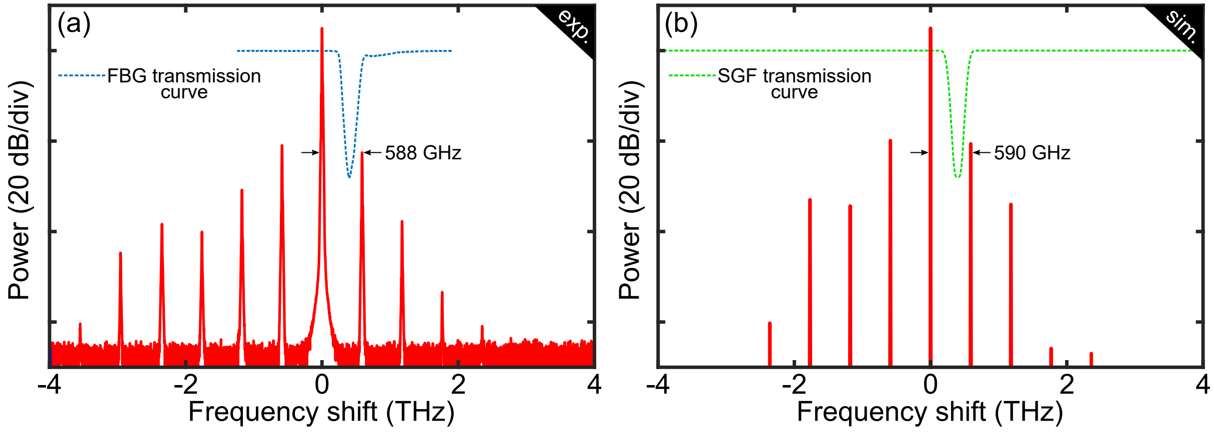
Frequencies that are amplified should satisfy the following phase-matching condition:

Even part of the filter-phase

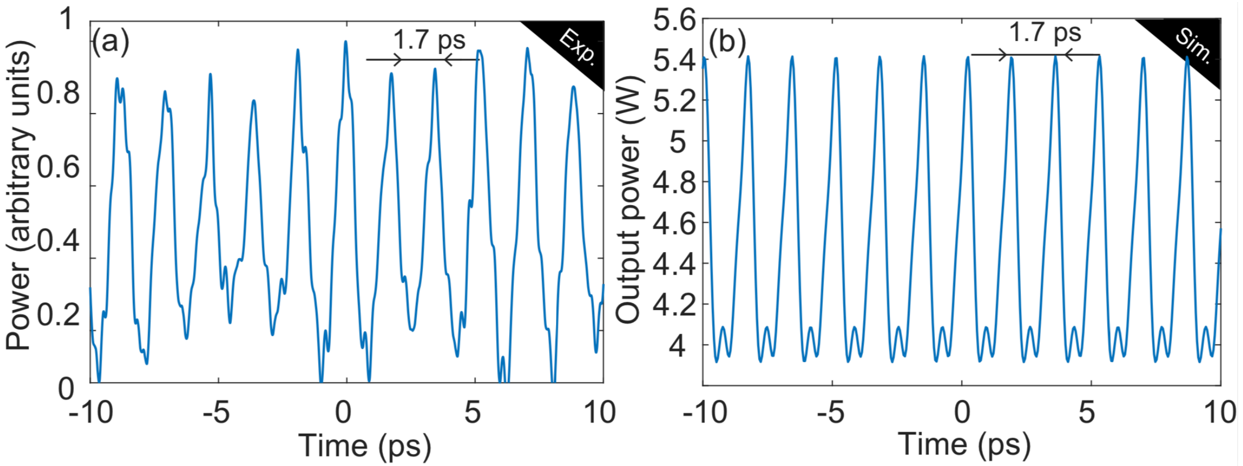
$$\frac{\beta_2}{2}\omega^2 L + 2\gamma P L + \phi_0 + \psi_e(\omega) = 0$$

Asymmetric losses for signal and idler waves induce a frequency dependent phase enabling wave amplification!

Spectrum

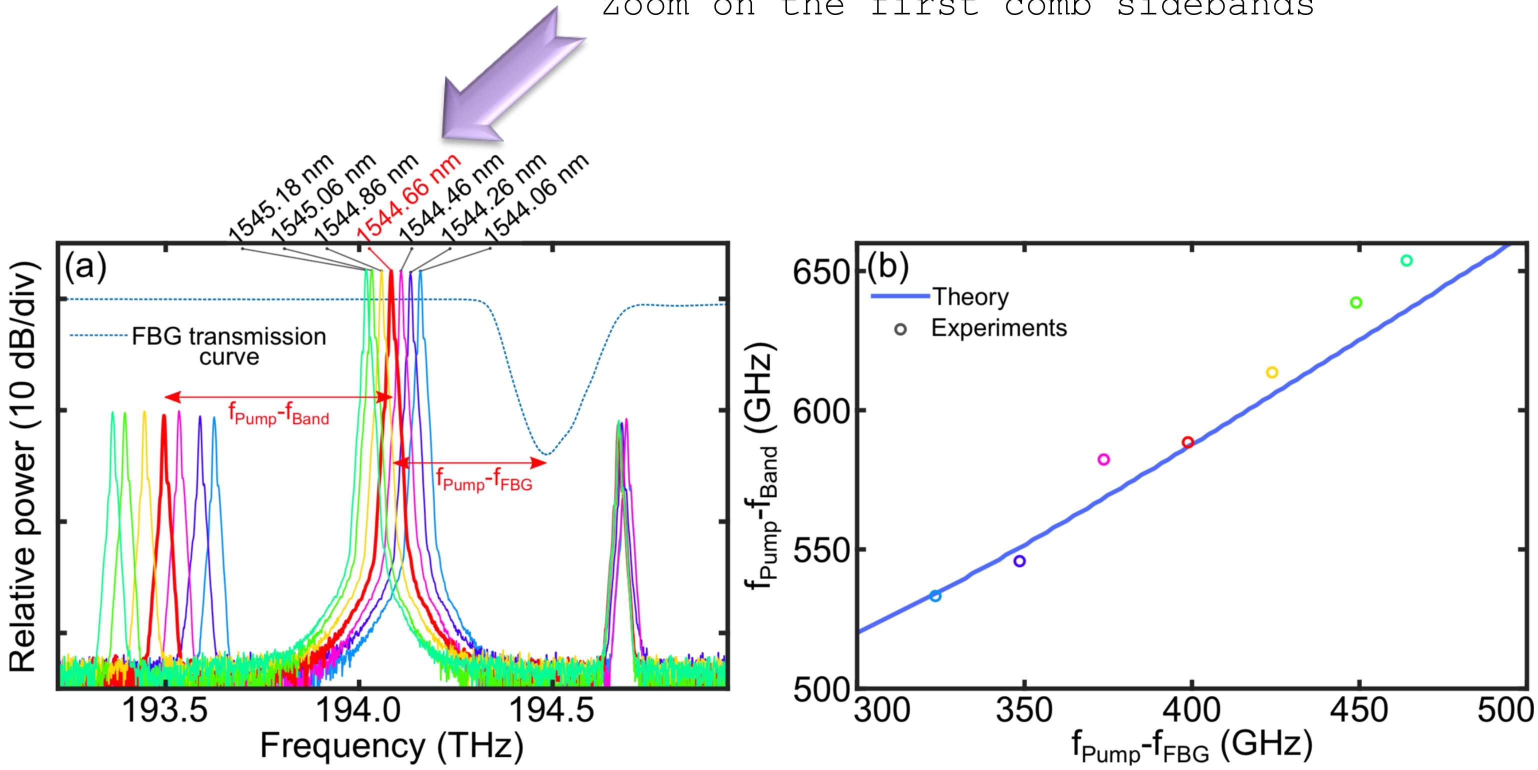


Time trace



Tuneability

Zoom on the first comb sidebands



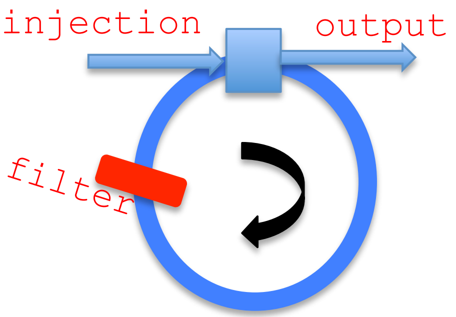
By changing the detuning between pump and filter frequency we can control the comb line spacing: e.g. using a tuneable laser for pumping

Generalised Lugiato-Lefever equation

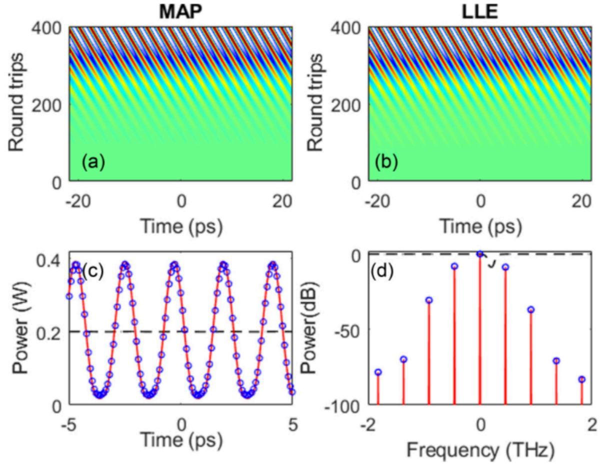
loss, detuning, filter loss, filter phase, injection

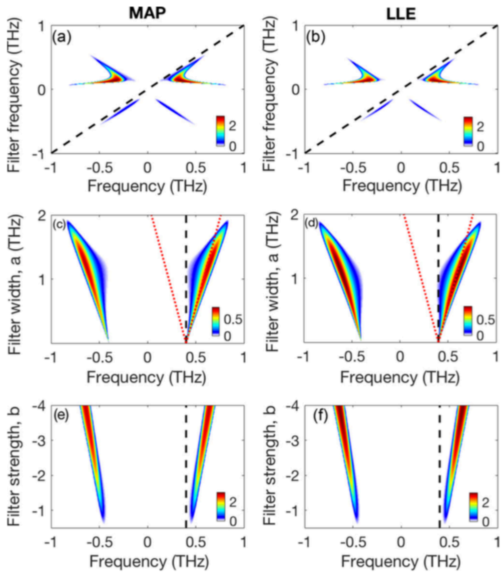
$$L\frac{\partial A}{\partial z} = [-\alpha + i\phi_0 + \Phi\star + i\Psi\star]A + \left[\frac{-iL\beta_2}{2}\frac{\partial^2}{\partial t^2} + iL\gamma|A|^2\right]A + \theta\sqrt{P_{IN}}$$

dispersion, nonlinearity



Motivation: A meanfield model simplifies the description of propagation equation & boundary conditions and enables extracting analytical information too.





Converse symmetry breaking

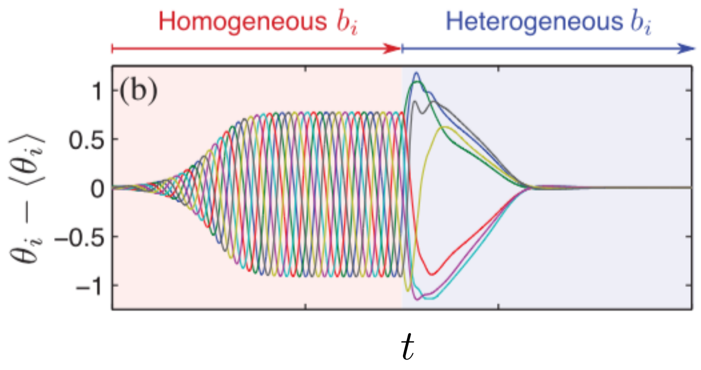
"the scenario in which complete synchronization is not stable for identically coupled identical oscillators but becomes stable when, and only when, the oscillator parameters are judiciously tuned to nonidentical values, thereby breaking the system symmetry to preserve the state symmetry."

Predicted for a set of coupled oscillators

PRL 117, 114101 (2016) PHYSICAL REVIEW LETTERS week ending 9 SEPTEMBER 2016

Symmetric States Requiring System Asymmetry

Takashi Nishikawa* and Adilson E. Motter



Demonstrated experimentally with electromechanical oscillators, it enhances stability and synchronization in various systems including in power grids!

Network experiment demonstrates converse symmetry breaking

Ferenc Molnar, Takashi Nishikawa & Adilson E. Motter

Nature Physics 16, 351–356 (2020) | Cite this article

Random heterogeneity outperforms design in network synchronization

Yuanzhao Zhang[a,b], Jorge L. Ocampo-Espindola[c], István Z. Kiss[c], and Adilson E. Motter[a,d,1]

PNAS 2021 Vol. 118 No. 21 e2024299118

nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-021-24459-0 OPEN

Check for updates

Heterogeneity-stabilized homogeneous states in driven media

Zachary G. Nicolaou[1], Daniel J. Case[1], Ernest B. van der Wee[1], Michelle M. Driscoll[1] & Adilson E. Motter[1,2]

nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-021-21290-5 OPEN

Asymmetry underlies stability in power grids

Ferenc Molnar[1,3], Takashi Nishikawa[1,2] & Adilson E. Motter[1,2]

Kerr nonlinearity:
Purely dispersive asymmetry example

Like in the previous OFC example we assume that asymmetric losses induce a frequency dependent phase-shift which rules the amplification dynamics (we neglect losses - Re(D)=0 - but keep the phase terms – Im(D) –)

$$\frac{\partial a_s}{\partial z} = i\frac{\beta_2}{2}\omega^2 a_s + i\gamma P(a_s + a_i^*) + D_s(\omega) a_s$$
$$\frac{\partial a_i}{\partial z} = i\frac{\beta_2}{2}\omega^2 a_i + i\gamma P(a_i + a_s^*) + D_i(\omega) a_i$$

$$\Omega_s = \frac{\beta_2}{2}\omega^2 + \gamma P - iD_s$$
$$\Omega_i^* = \frac{\beta_2}{2}\omega^2 + \gamma P + iD_i^*$$
$$\dot{a} = i\Omega a \quad c = \gamma P$$

$$\dot{a}_s = i\Omega_s a_s + ica_i^*$$
$$\dot{a}_i^* = -i\Omega_i^* a_i^* - ica_s$$

$$\dot{a}_s = i\Omega_s a_s + ica_i^*$$
$$\dot{a}_i^* = -i\Omega_i^* a_i^* - ica_s$$

Are symmetric/invariant under the transformation $(a_{s,i} \to a_{i,s}^*,\ i \to -i)$ if $D_s = D_i$

Diversity parameters

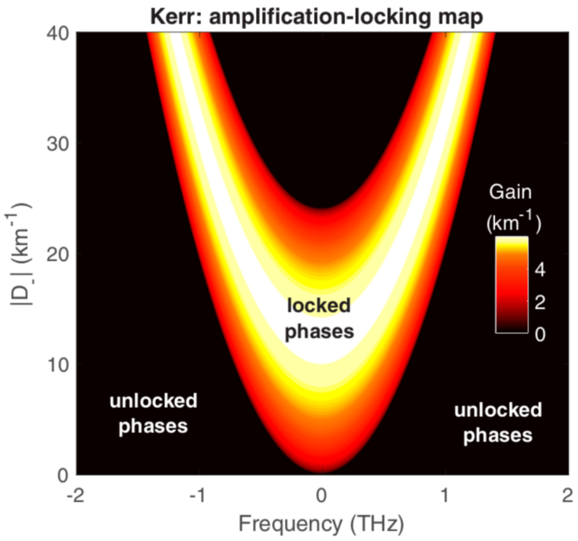
$$D_+ = \mathrm{Im}(D_s) + \mathrm{Im}(D_i)$$
$$D_- = \mathrm{Im}(D_s) - \mathrm{Im}(D_i).$$

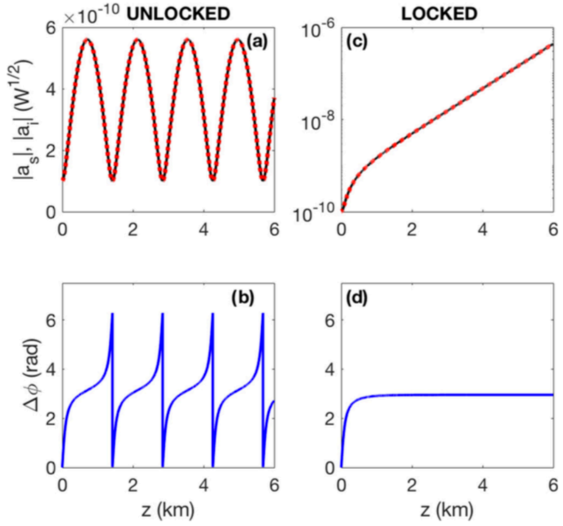
Eigenvalues (LSA of CW solution)

$$\lambda_\pm = \tfrac{1}{2}[iD_- \pm \sqrt{-(D_+ + \beta_2\omega^2)(D_+ + 4c + \beta_2\omega^2)}]$$

Locking associated to wave amplification

Diversity($D_- \neq 0$) enables locking and amplification





Quadratic nonlinearity (OPO): Purely dissipative asymmetry example

$$\frac{dA_i^*}{dz} = -i\omega_i^* A_i^* - ic_i^* A_s$$

$$\frac{dA_s}{dz} = i\omega_s A_s + ic_s A_i^*$$

Are symmetric/invariant under the transformation $(A_{s,i} \rightarrow A^*_{i,s}, i \rightarrow -i)$ if $D_s = D_i$

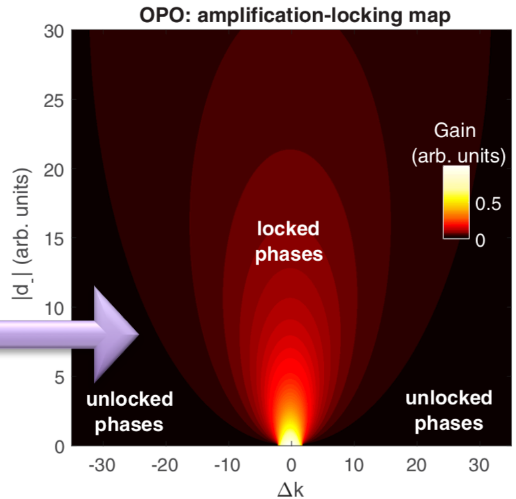
$$d_+ = \mathrm{Re}(D_s) + \mathrm{Re}(D_i) \quad \omega_i^* = -\frac{\Delta k}{2} + iD_i^* \quad c_i = \sqrt{\mu}A_p^*$$

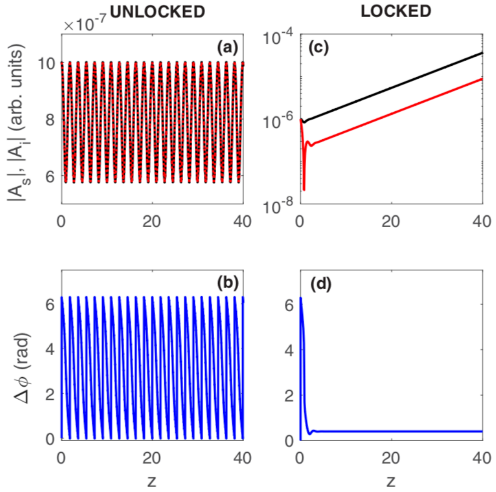
$$d_- = \mathrm{Re}(D_s) - \mathrm{Re}(D_i) \quad \omega_s = -\frac{\Delta k}{2} - iD_s \quad c_s = \sqrt{\mu}A_p$$

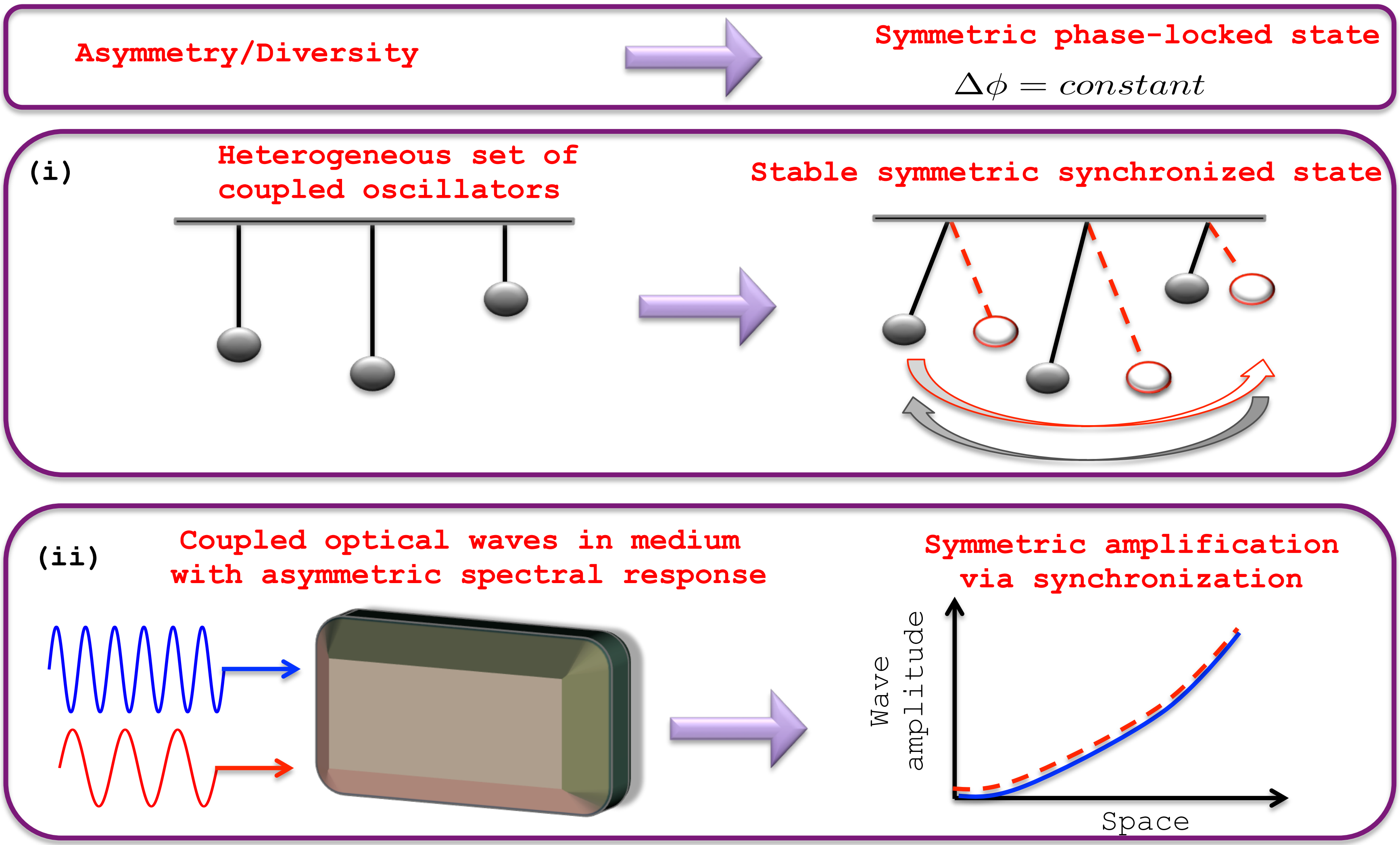
Eigenvalues (LSA)

$$\lambda_{1,2} = \frac{1}{2}[d_+ \pm \sqrt{(i\Delta k - d_-)^2 + 4\mu|A_p|^2}]$$

Diversity($d_- \neq 0$) enachances locking and amplification area



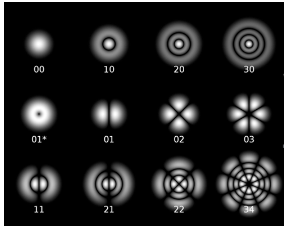



Asymmetry/Diversity
Symmetric phase-locked state
$\Delta\phi = constant$
(i)
Heterogeneous set of coupled oscillators
Stable symmetric synchronized state
(ii)
Coupled optical waves in medium with asymmetric spectral response
Symmetric amplification via synchronization
Wave amplitude
Space

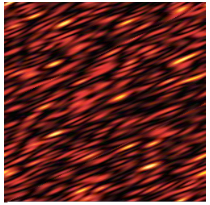
Future directions

Systems beyond photonics

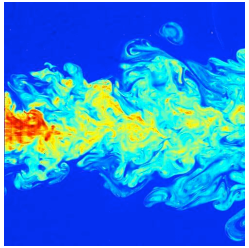
Multimode amplification



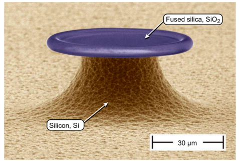
Multimode CSB in disordered photonic systems



Path to turbulence



Miniaturisation for photonic applications



Conclusions

Asymmetries enable wave amplification in nonlinear dispersive systems described by universal formalism

We can engineer dissipation, and “gain from losses”, developing novel photonic technologies

Asymmetries may enable synchronisation in scenarios where symmetry prevents it

References

Perego et al. *Light: Science & Applications* (2018)7:43
DOI 10.1038/s41377-018-0042-9
Official journal of the CIOMP 2047-7538
www.nature.com/lsa

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Gain through losses in nonlinear optics

Auro M. Perego[1], Sergei K. Turitsyn[1,2] and Kestutis Staliunas[3,4]

nature COMMUNICATIONS

ARTICLE
https://doi.org/10.1038/s41467-019-12375-3 OPEN

Gain-through-filtering enables tuneable frequency comb generation in passive optical resonators

Florent Bessin[1], Auro M. Perego[2*], Kestutis Staliunas[3,4], Sergei K. Turitsyn[2,5], Alexandre Kudlinski[1], Matteo Conforti[1] & Arnaud Mussot[1]

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Theory of filter-induced modulation instability in driven passive optical resonators

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Letter

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