Rect Function in Nonlinear Optics: Topological Control of Extreme Waves and Quantum Peregrine Solitons

12TH DECEMBER 2022

INI – ISAAC NEWTON INSTITUTE FOR MATHEMATICAL SCIENCES

UNIVERSITY OF CAMBRIDGE

PHYSICAL APPLICATION OF DISPERSIVE HYDRODYNAMICS

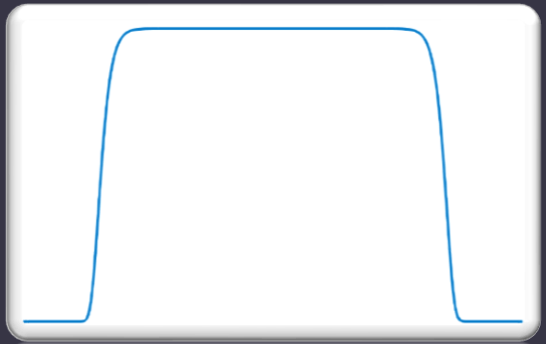
DR. G. MARCUCCI

CONFLICT OF INTEREST

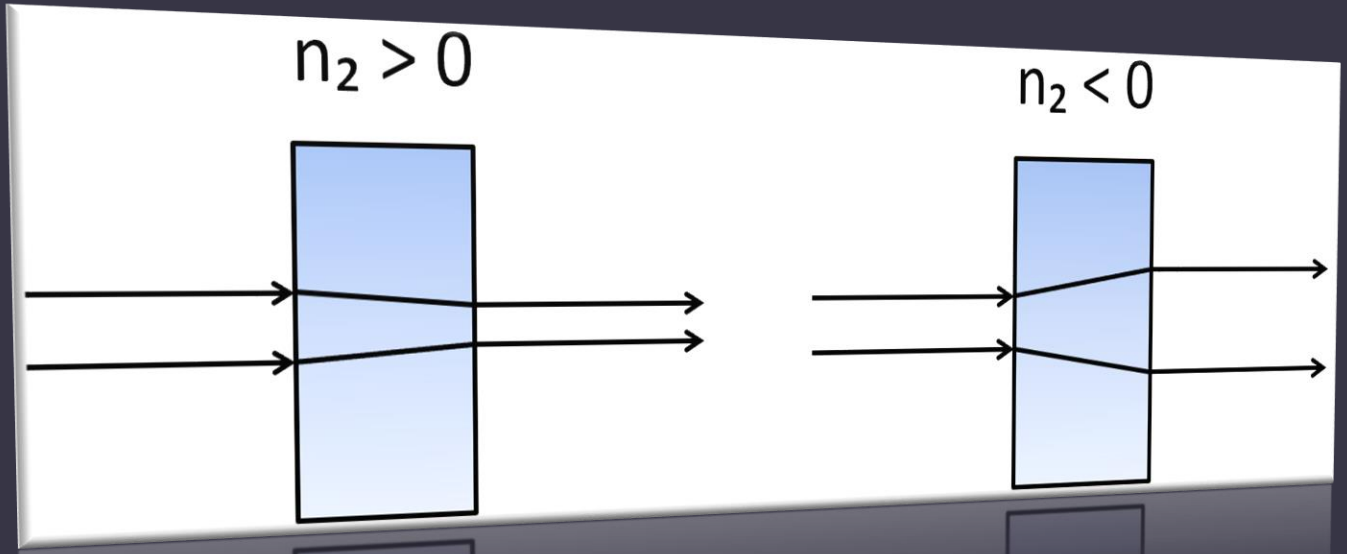
THE PRESENTER IS EMPLOYED BY APOHA LTD, A COMPANY DEDICATED
TO THE DEVELOPMENT OF BIOPHOTONIC COMPUTATIONAL DEVICES.

Rect Function in Nonlinear Optics

CW beam intensity distribution



Propagation in a Kerr medium



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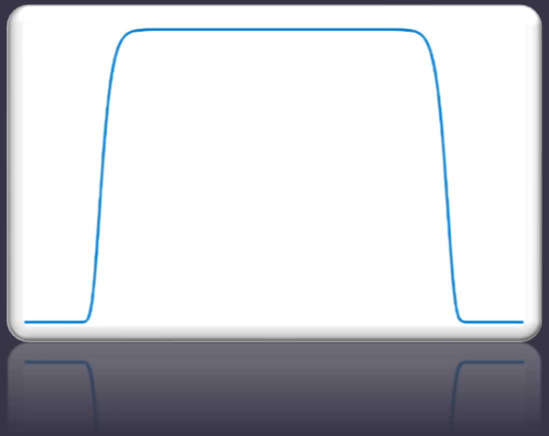

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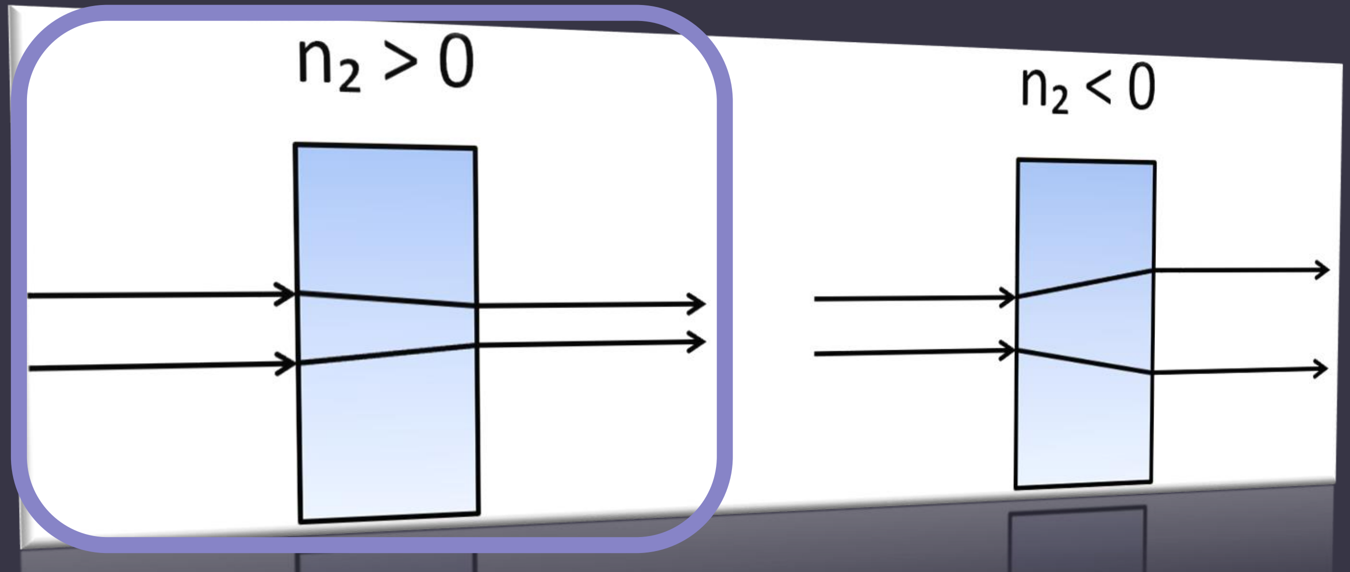

Self-focusing nonlinearity

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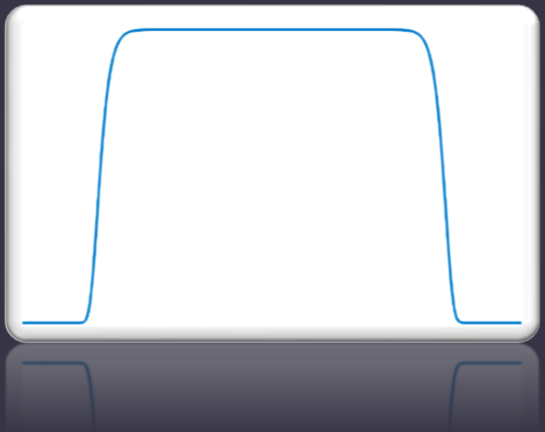

Nonlinear Schrödinger Equation

$$\imath\epsilon\partial_\zeta\psi + \frac{\epsilon^2}{2}\partial_\xi^2\psi + |\psi|^2\psi = 0,$$

$$\psi(\xi,0) = \begin{cases} q & \text{for } |\xi| < l \\ 0 & \text{elsewhere} \end{cases}$$

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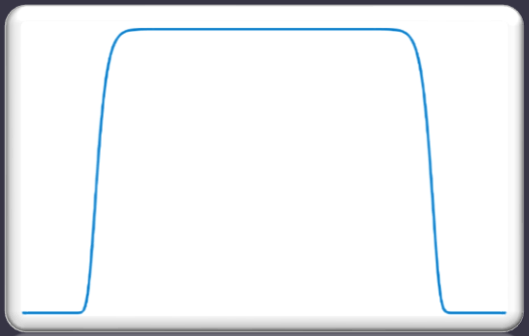

IOP Publishing | London Mathematical Society Nonlinearity

Nonlinearity **29** (2016) 2798–2836 doi:10.1088/0951-7715/29/9/2798

Dam break problem for the focusing nonlinear Schrödinger equation and the generation of rogue waves

G A El[1], E G Khamis[2,3] and A Tovbis[4]

[1] Department of Mathematical Sciences, Loughborough University, Loughborough LE11 3TU, UK
[2] Instituto de Física, Universidade de São Paulo, 05508-090 São Paulo, Brazil
[3] Center for Weather Forecasting and Climate Studies-CPTEC, National Institute for Space Research (INPE), Cachoeira Paulista, São Paulo, Brazil
[4] Department of Mathematics, University of Central Florida, Orlando, FL, USA

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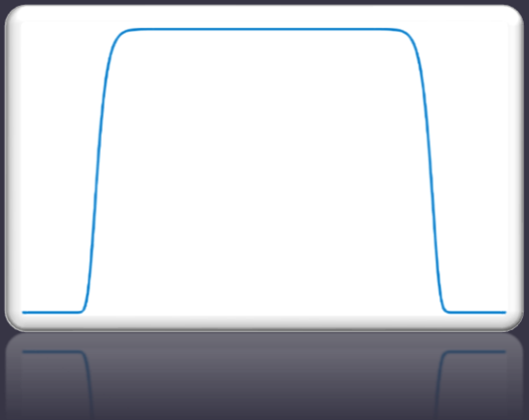
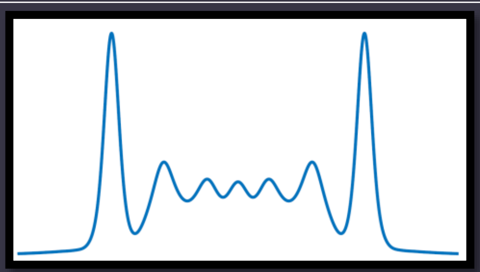


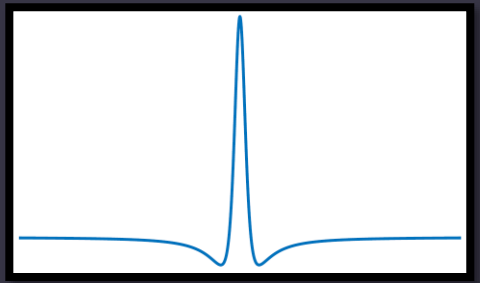
Shock-like Waves

$$\zeta \sim 0$$



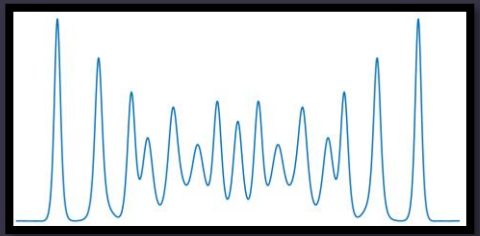
Rogue Wave

$$\zeta = \zeta_0 := \frac{l}{2\sqrt{2q}}$$



Soliton Gas

$$\zeta >> \zeta_0$$



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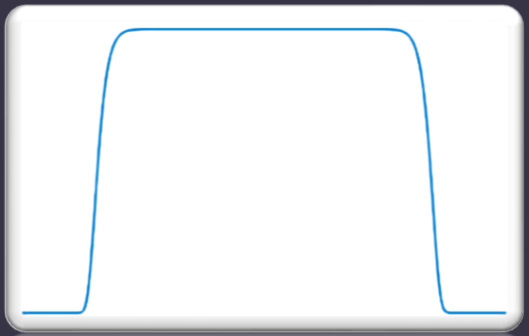

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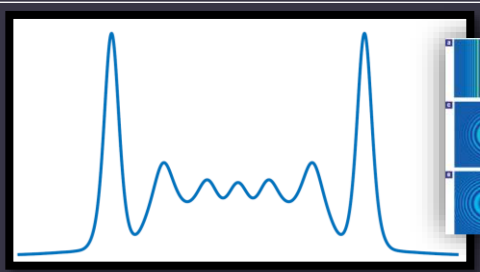

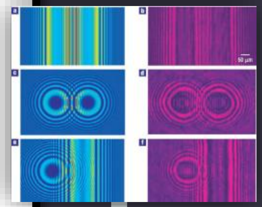

W. Wan *et al.*, *Nat. Phys.* **3** (2007)

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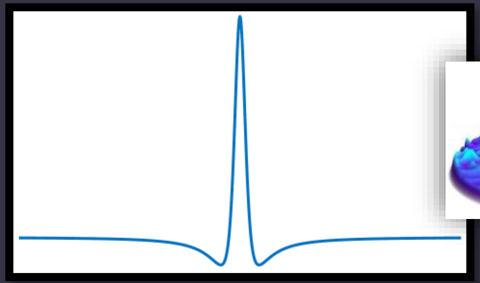

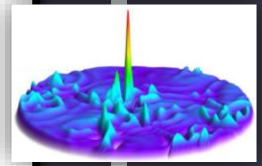

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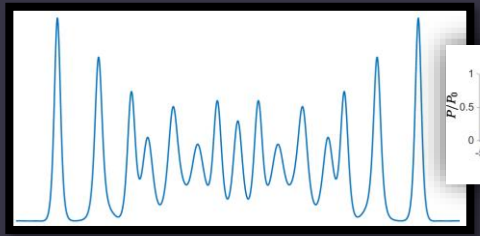
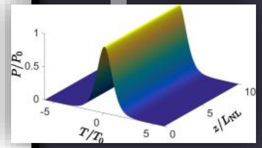




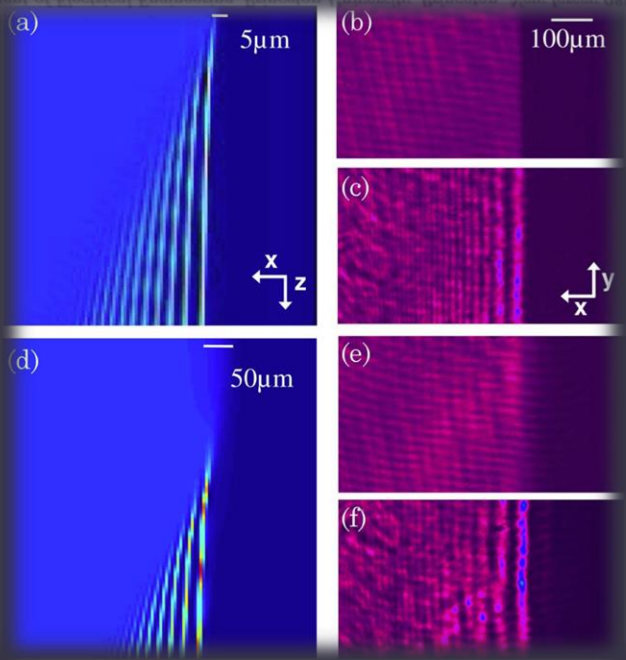
F. Mitschke *et al.*, *Appl. Phys.* **7** (2017)

Rect Function in Nonlinear Optics

August 15, 2010 / Vol. 35, No. 16 / OPTICS LETTERS

Diffraction from an edge in a self-focusing medium

Wenjie Wan, Dmitry V. Dylov, Christopher Barsi, and Jason W. Fleischer*
Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA
**Corresponding author: jasonf@princeton.edu*



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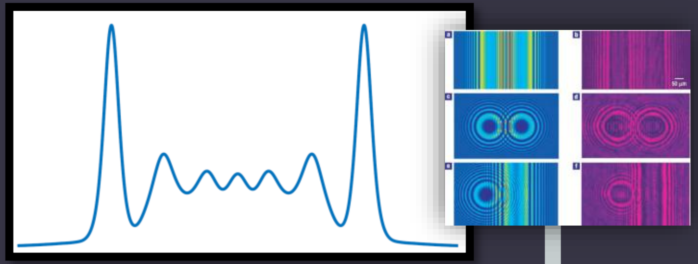

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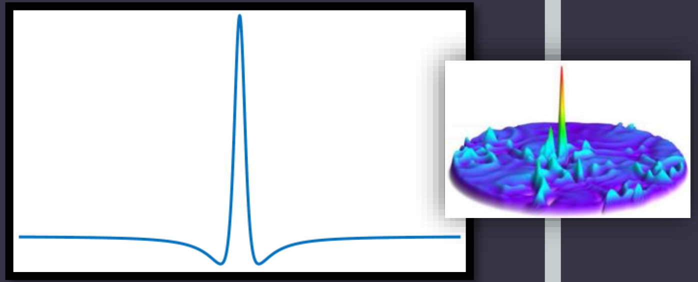

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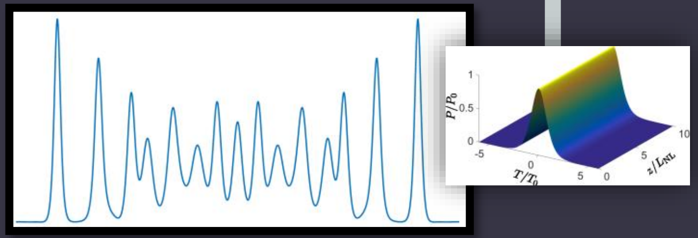

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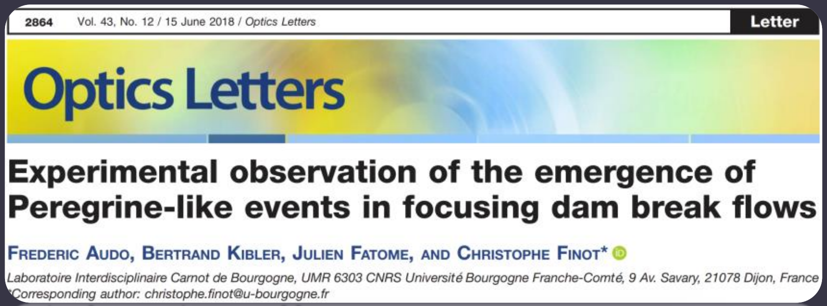

2864 Vol. 43, No. 12 / 15 June 2018 / Optics Letters Letter

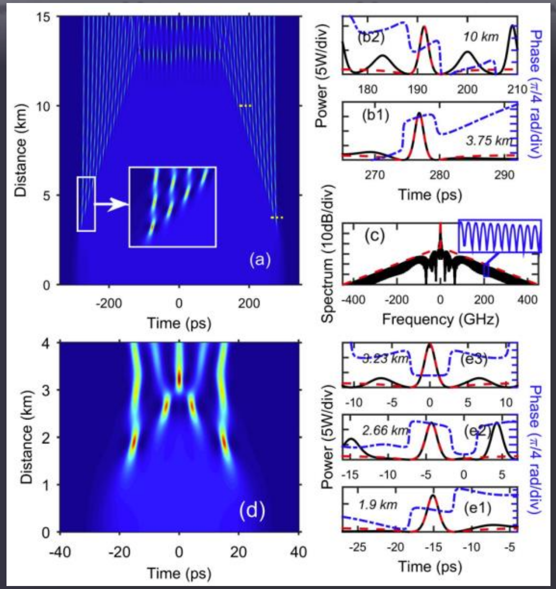
Optics Letters

Experimental observation of the emergence of Peregrine-like events in focusing dam break flows

Frederic Audo, Bertrand Kibler, Julien Fatome, and Christophe Finot*

Laboratoire Interdisciplinaire Carnot de Bourgogne, UMR 6303 CNRS Université Bourgogne Franche-Comté, 9 Av. Savary, 21078 Dijon, France

*Corresponding author: christophe.finot@u-bourgogne.fr



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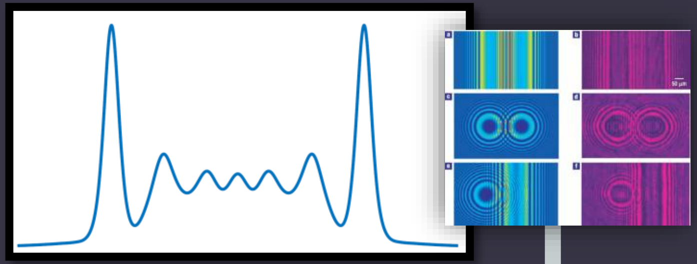

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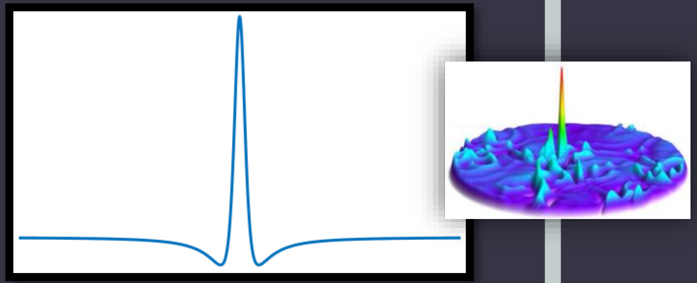

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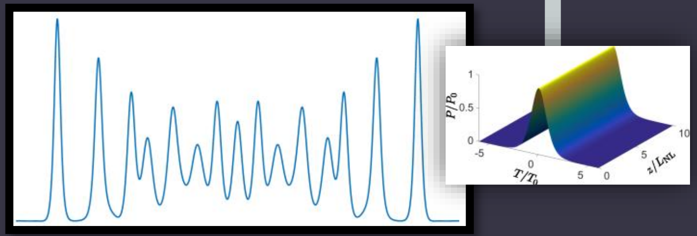

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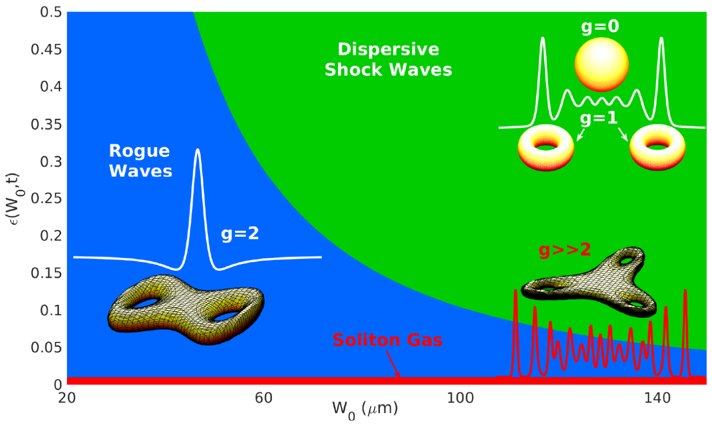
Rect Function in Nonlinear Optics: Topological Control of Extreme Waves and Quantum Peregrine Solitons

- Topological control of extreme waves:
  - Geometry of NLSE solutions
  - Framework in a photorefractive crystal
  - Propagation of a beam with large waist: Akhmediev breathers
  - Propagation of a beam with small waist: Peregrine solitons

- Quantum Peregrine soliton generation:
  - Quantum nonlinear waves
  - Our approach to quantum NLSE
  - Effect of quantum noise on rogue waves

Topological Control of Extreme Waves



nature COMMUNICATIONS

ARTICLE

https://doi.org/10.1038/s41467-019-12815-0

OPEN

Topological control of extreme waves

Giulia Marcucci[1,2*], Davide Pierangeli[1,2], Aharon J. Agranat[3], Ray-Kuang Lee[4], Eugenio DelRe[1,2] & Claudio Conti[1,2]

research highlights

EXTREME WAVES

Tamed by topology

Nat. Commun. **10**, 5090 (2019)

Topological Control of Extreme Waves: Geometry of NLSE Solutions

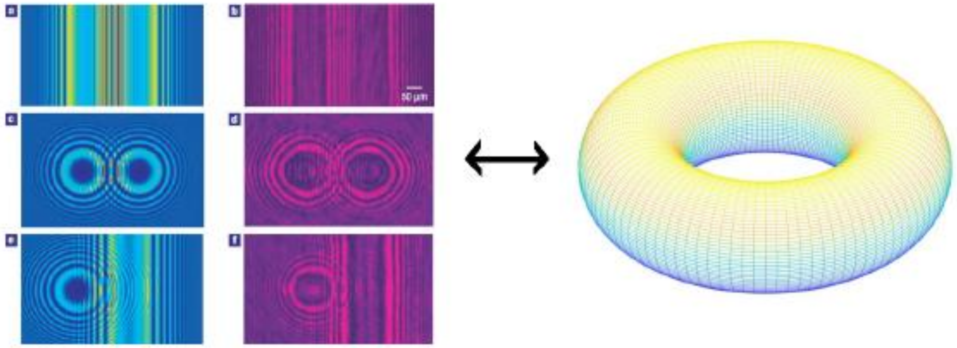
Theta functions and non-linear equations

To cite this article: B A Dubrovin 1981 *Russ. Math. Surv.* **36** 11

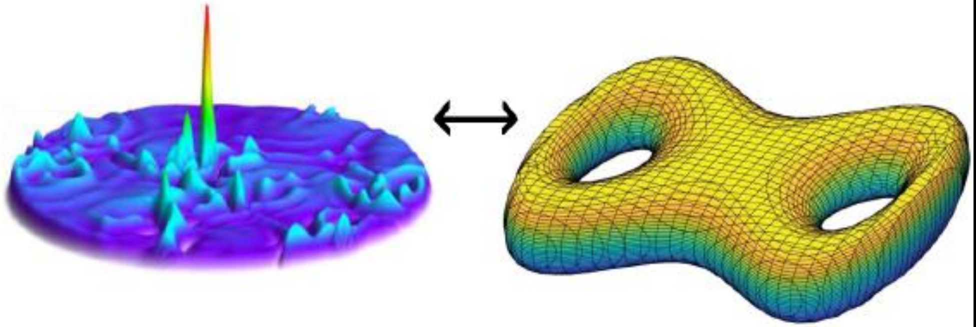
NLSE: $\imath \partial_z \psi + \partial_t^2 \psi + 2|\psi|^2 \psi = 0$

$$\psi(t,z) = q \frac{\Theta_g(t,z,\nu_-^0)}{\Theta_g(t,z,\nu_+^0)} e^{\imath q^2 z}$$

$q \in \mathbb{R}$, $\nu_\pm^0 \in \mathbb{R}^g$ Riemann theta-function phases.



By finite-gap theory (counterpart of IST for periodic problems), solutions of NLSE are ratios between Riemann theta-functions associated to hyperelliptic Riemann surfaces of same genus g.



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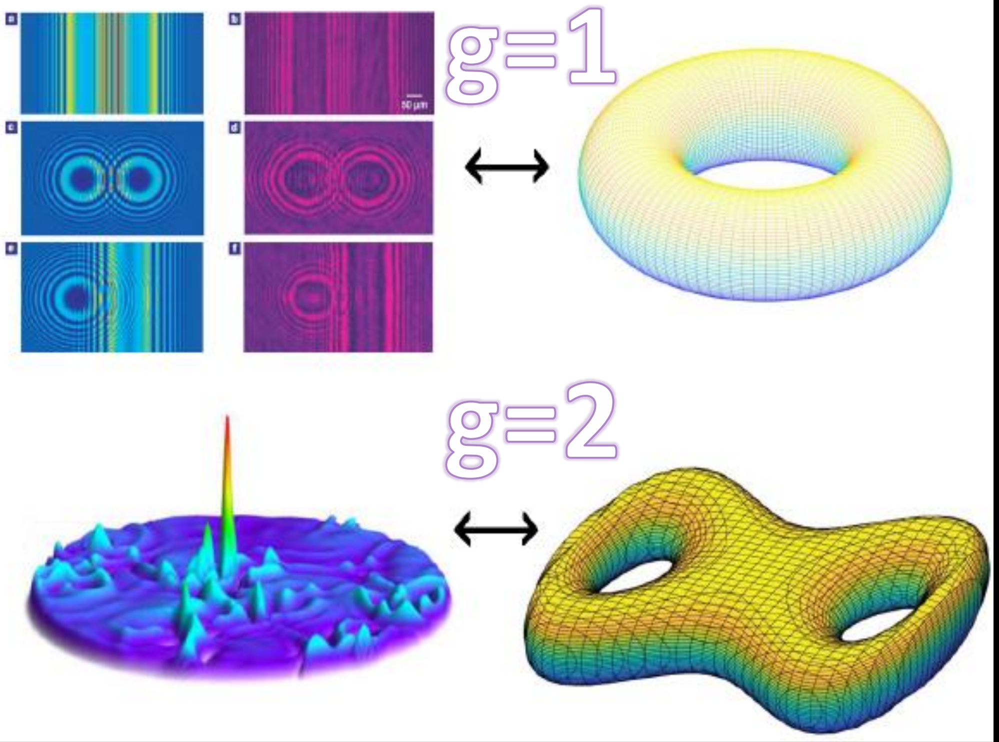

Isomorphism between wave classes and genera: by a simple integer number, we characterize complex wave regimes.

Topological Control of Extreme Waves: Geometry of NLSE Solutions

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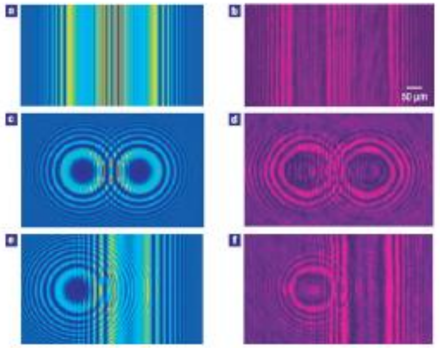
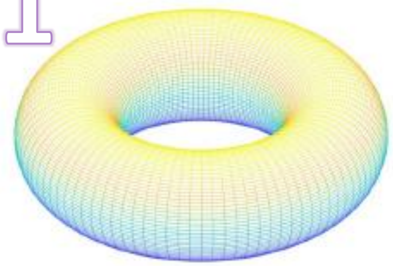
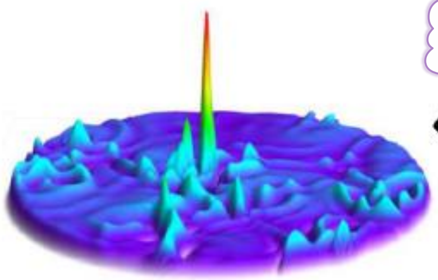
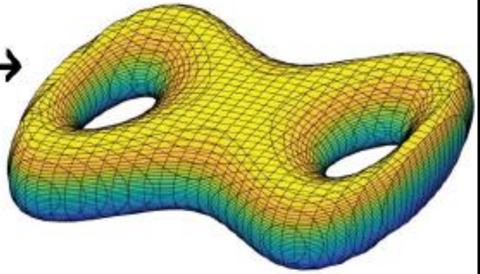


g=1 ⟷





g=2 ⟷



Can we control the genus?
Does this mean to control the output of an experiment?

Topological Control of Extreme Waves: Framework in a Photorefractive Crystal

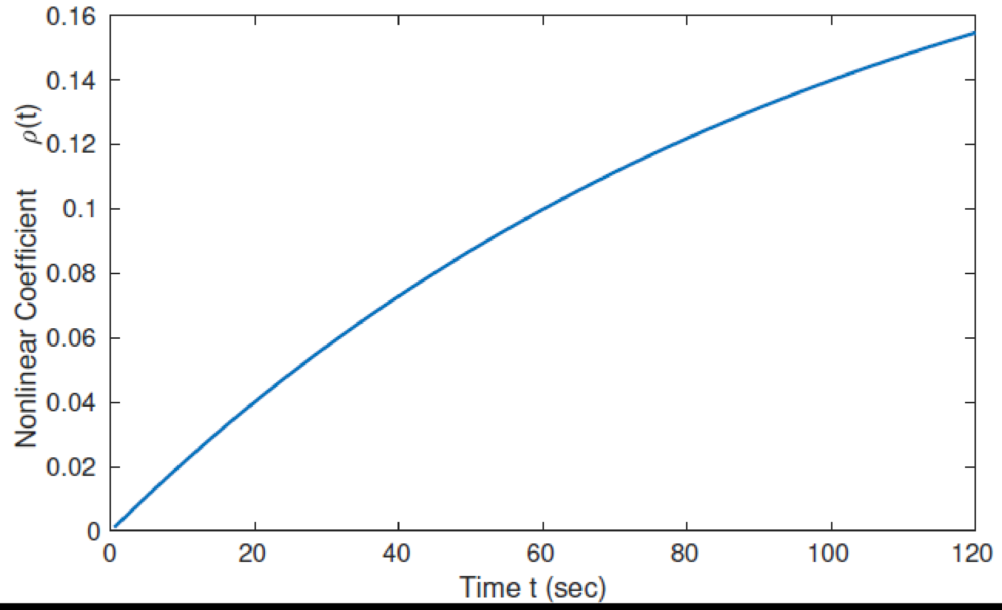
$$\zeta = \frac{z}{\epsilon z_D}, \quad \xi = \frac{2x}{W_0}, \quad \psi = \frac{A}{\sqrt{I_0}}, \quad \epsilon(t) := \frac{W_0}{4z_D}\sqrt{\frac{2n_0}{\delta n_0 f(t)}}$$

$$\Downarrow$$

Photorefractive media NLSE in the Kerr-like regime:

$$\imath\partial_z A + \frac{1}{2k}\partial_x^2 A + 2\rho(t)|A|^2 A = 0$$

$$\rho(t) = \frac{k\delta n_0}{n_0 I_0} f(t),\ f(t) = 1 - e^{-t/\tau},\ n = n_0 + \frac{2\delta n_0 I}{I_0} f(t),\ I = |A|^2,\ A(x,0) = \begin{cases} \sqrt{I_0} & \text{for } |x| \leq \frac{1}{2}W_0 \\ 0 & \text{elsewhere} \end{cases}$$



As the nonlinearity changes, also the genus changes.

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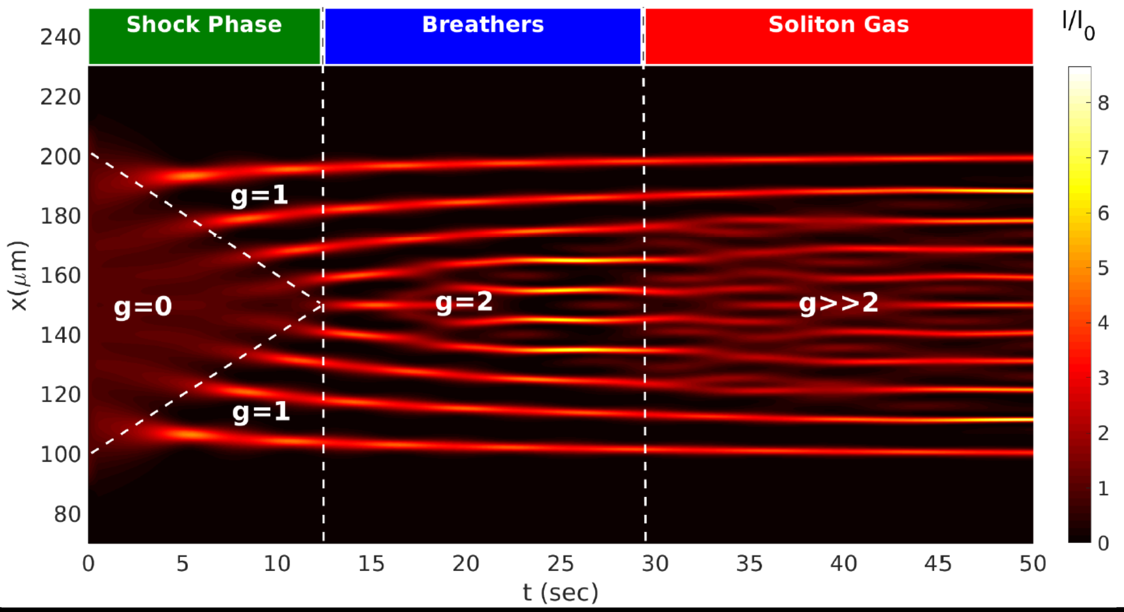

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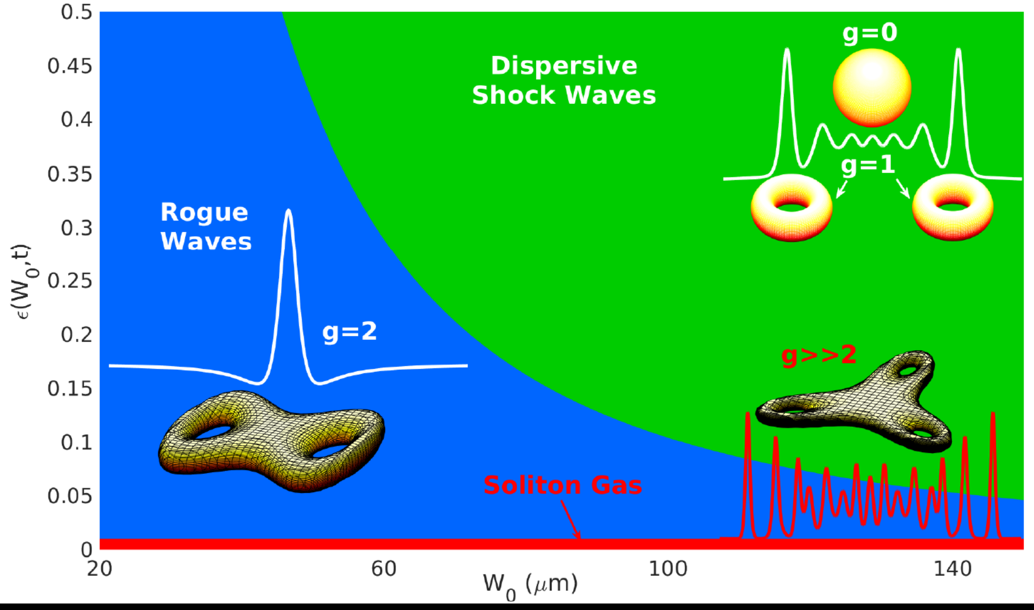

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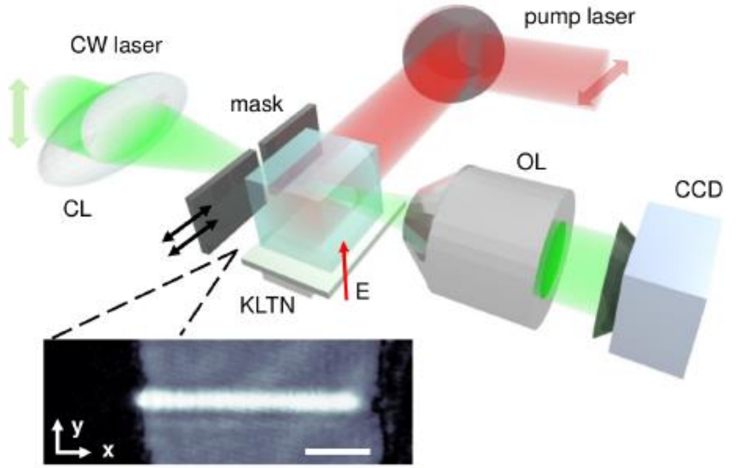
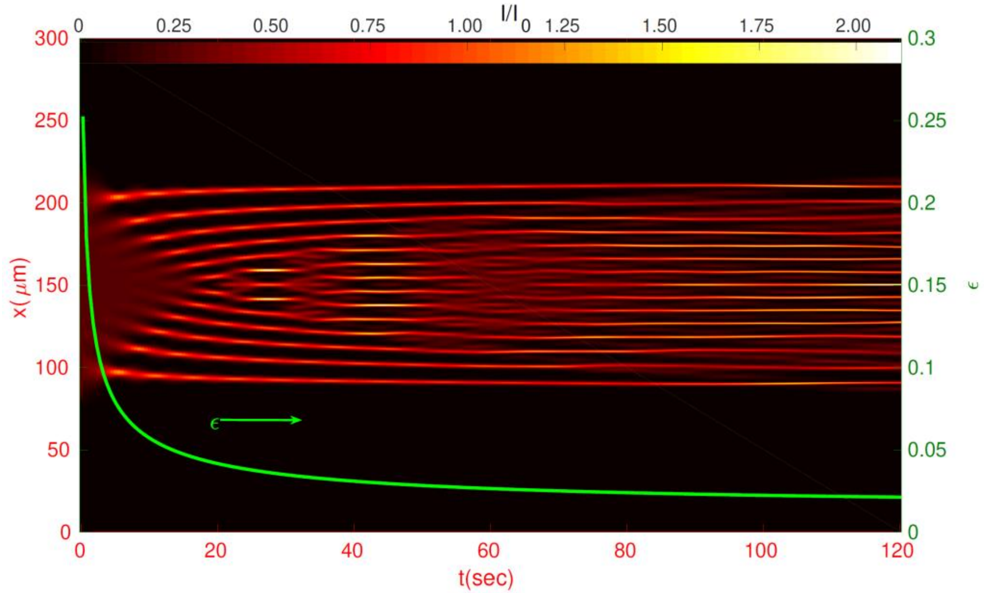
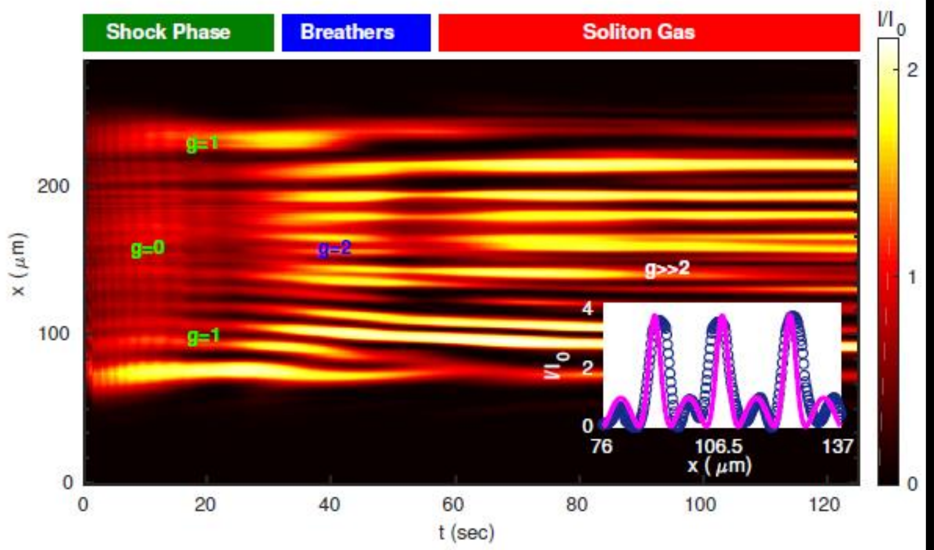
Topological Control of Extreme Waves: Propagation of a beam with large waist

A y-polarized optical beam, $\lambda = 532$nm, from a CW 80mW Nd:YAG laser, is focused by a cylindrical lens down to a quasi-one-dimensional beam. The initial box shape is obtained by a tunable mask, placed in proximity of the input face of the KLTN photorefractive crystal.







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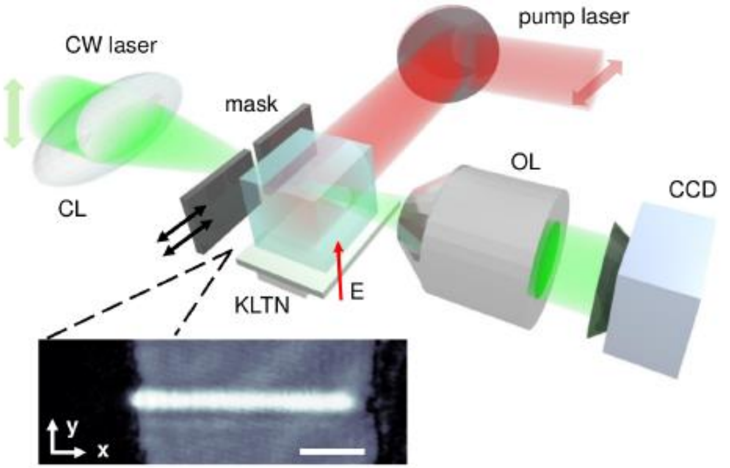

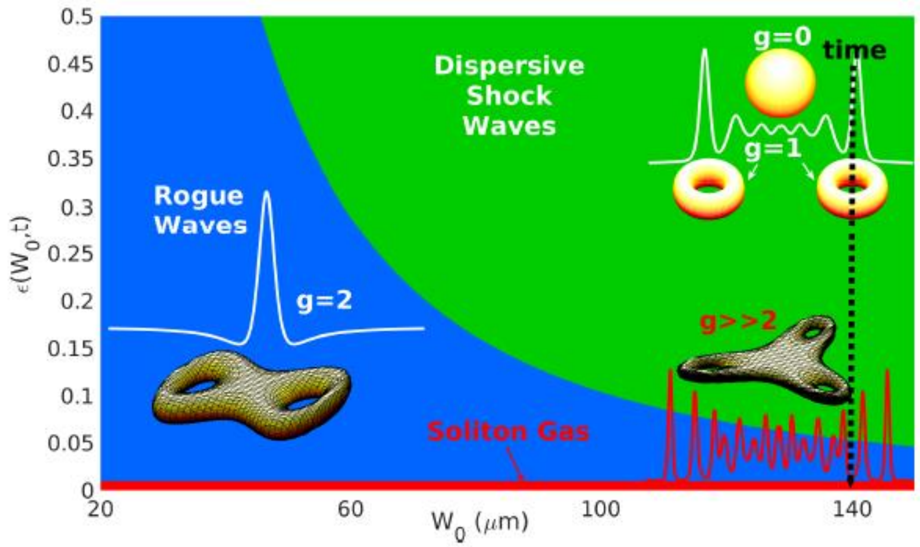

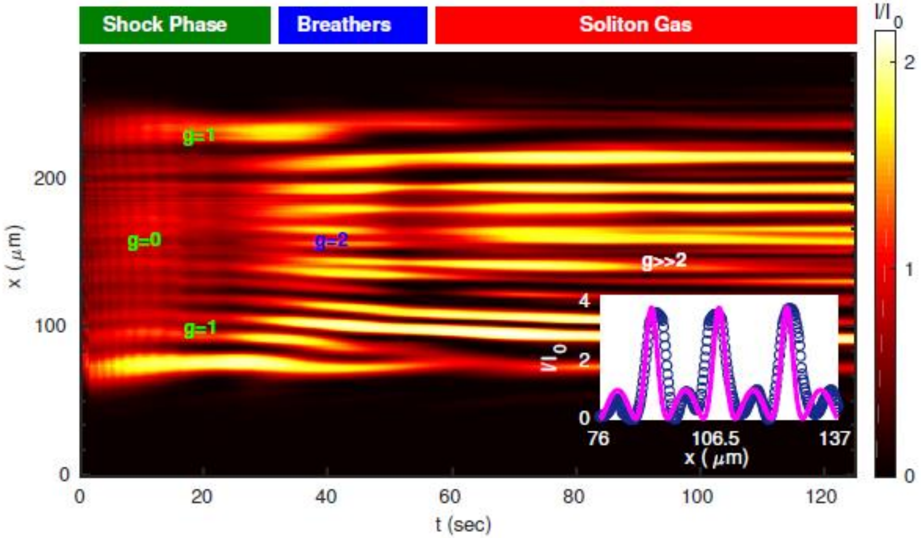

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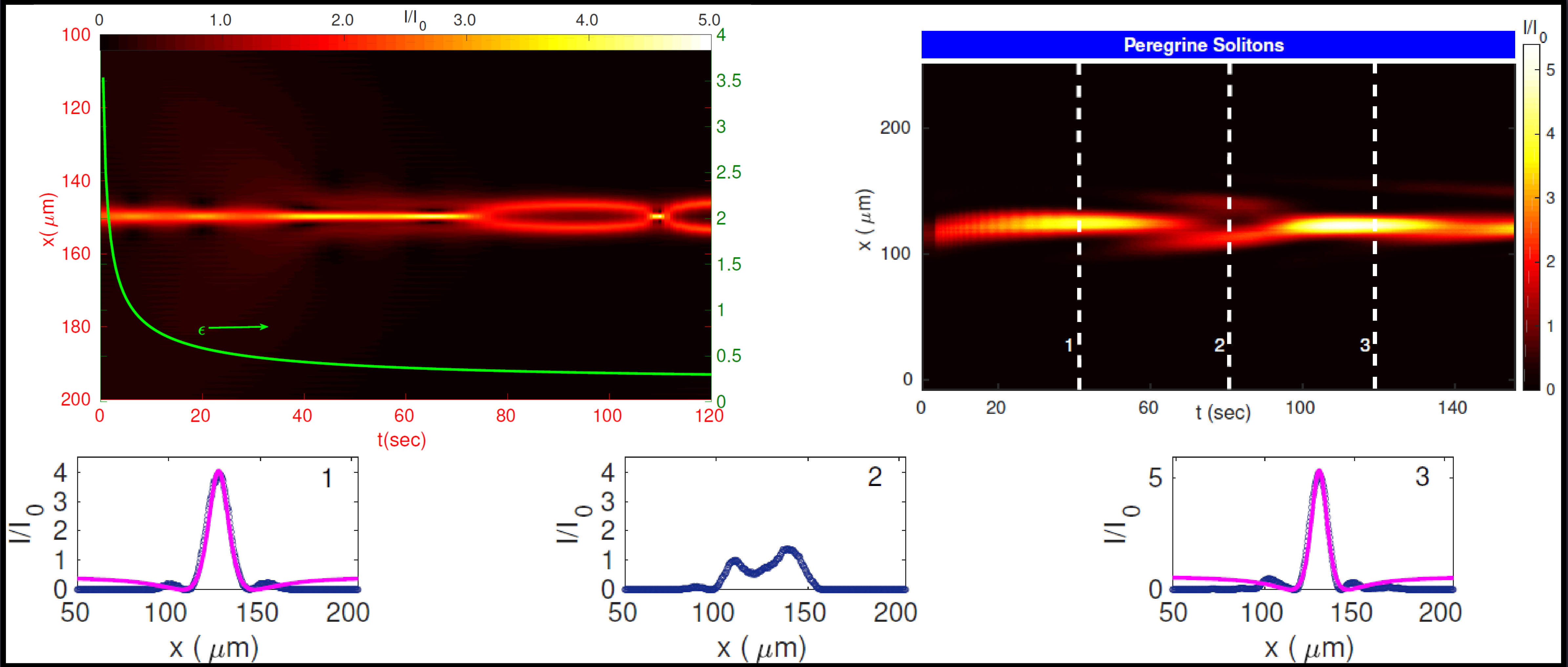

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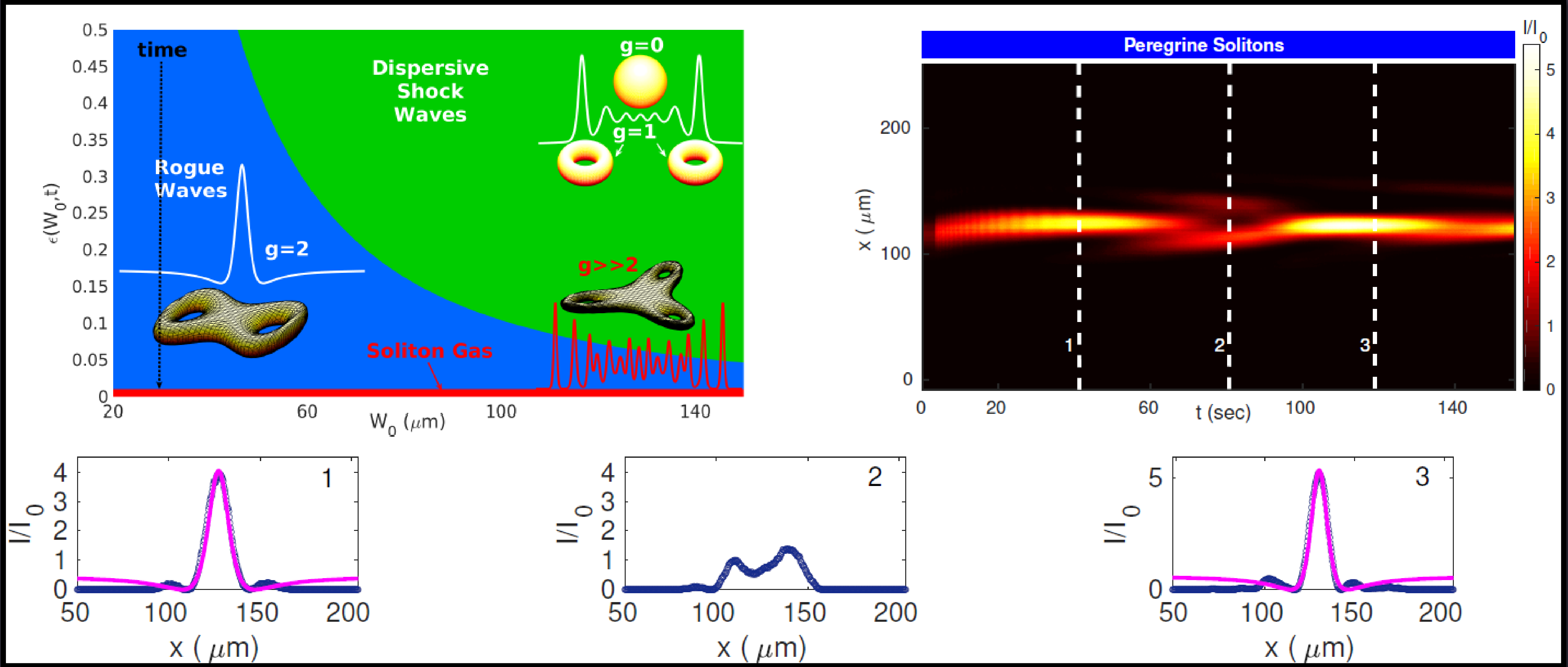

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Quantum Peregrine Soliton Generation

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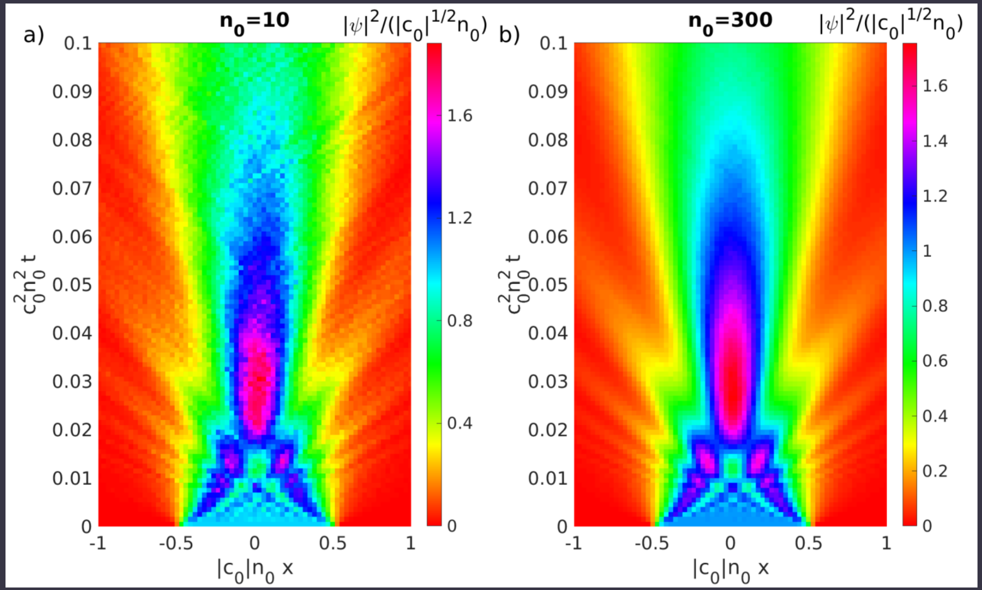


Quantum Peregrine Soliton Generation

Quantum Peregrine Soliton Generation: Quantum nonlinear waves

RESEARCH

QUANTUM OPTICS

Observation of three-photon bound states in a quantum nonlinear medium

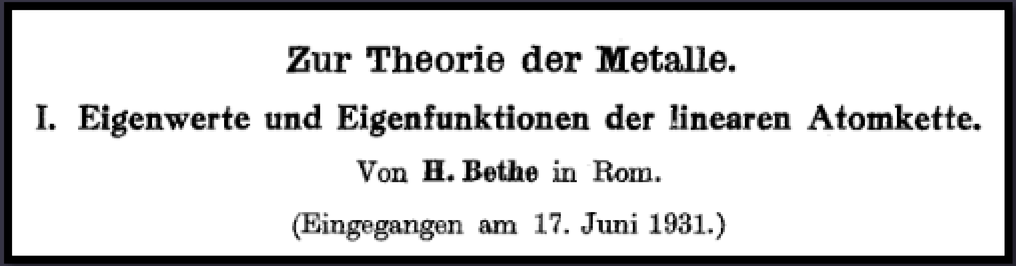
Qi-Yu Liang,[1] Aditya V. Venkatramani,[2] Sergio H. Cantu,[1] Travis L. Nicholson,[1] Michael J. Gullans,[3,4] Alexey V. Gorshkov,[4] Jeff D. Thompson,[5] Cheng Chin,[6] Mikhail D. Lukin,[2]* Vladan Vuletić[1]*

Liang, Science 359, 783 (2018)

- First experimental observation of a quantum soliton, following the definition of P. D. Drummond and H. He, *PRA* **56** (1997);

- Quantum nonlinear optical processes in fibers, microresonators and cavities.
- Generation of frequency combs, supercontinuum, solitons, squeezed light, etc.
- Highly entangled multimode states.

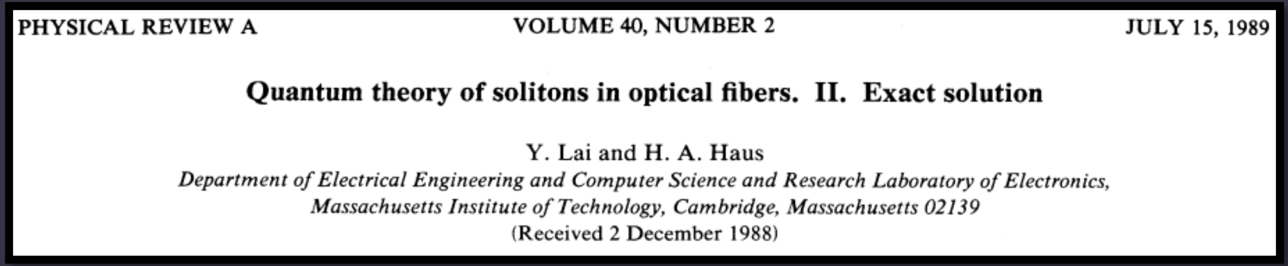
Quantum Peregrine Soliton Generation: Quantum nonlinear waves


Zur Theorie der Metalle.

I. Eigenwerte und Eigenfunktionen der linearen Atomkette.

Von **H. Bethe** in Rom.

(Eingegangen am 17. Juni 1931.)

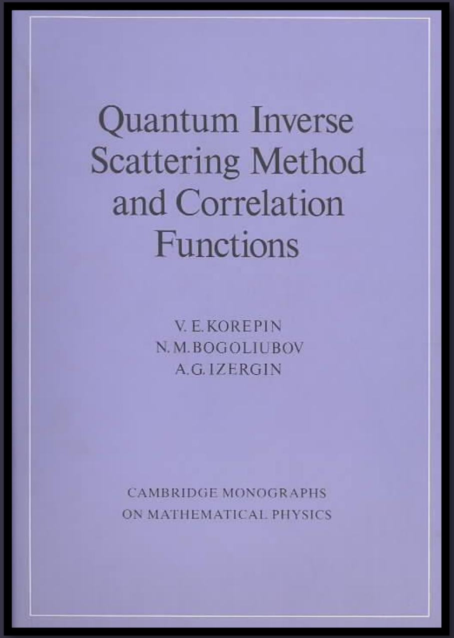

PHYSICAL REVIEW A — VOLUME 40, NUMBER 2 — JULY 15, 1989

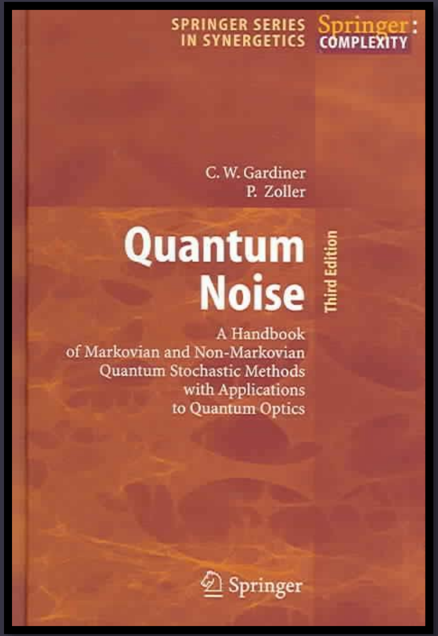
Quantum theory of solitons in optical fibers. II. Exact solution

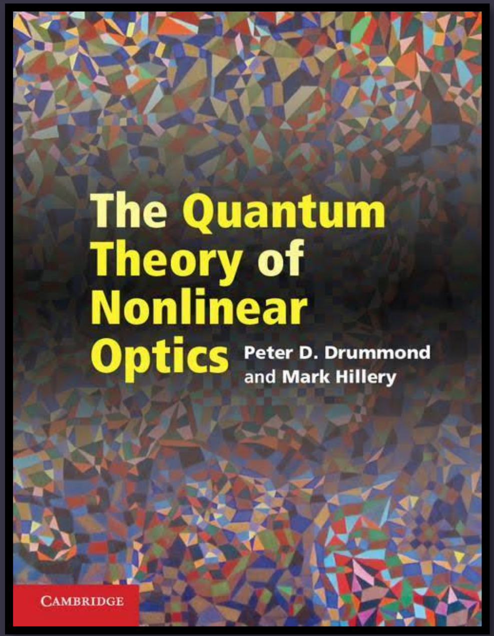
Y. Lai and H. A. Haus

Department of Electrical Engineering and Computer Science and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 2 December 1988)







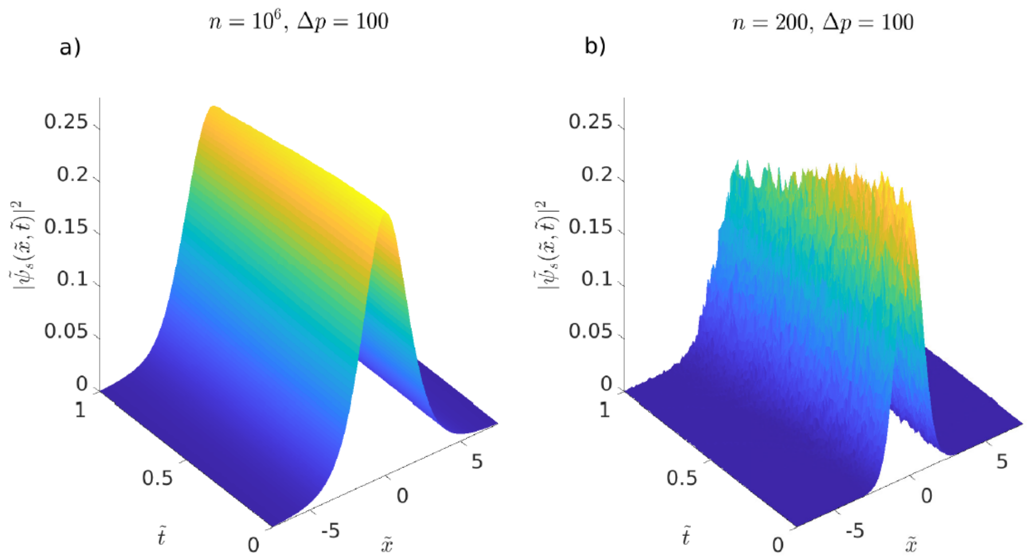
Quantum Peregrine Soliton Generation: Quantum nonlinear waves

Lai-Haus Quantum Soliton

$$|\psi_s\rangle|_{t=0} = \int \mathrm{d}p g(p)|\alpha(p)\rangle,$$

$$\psi_s(x) := \langle\psi_s|\hat{\Phi}(x,0)|\psi_s\rangle| \simeq \frac{n_0}{2}\sqrt{|c|} sech\left(\frac{|c|n_0}{2}x\right) e^{-(\Delta p x)^2},$$

Gaussian distribution: $g(p) = \exp(-p^2/2\Delta p^2)/(\sqrt{\pi}\Delta p)^{1/2}$.



Quantum noise $\left(\propto \frac{1}{\sqrt{n_0}}\right)$ causes phase fluctuations:
photons phase relation is broken and QSs spread out!
QSs are no more propagation invariant as CSs!

Quantum Peregrine Soliton Generation: Quantum nonlinear waves

$$\imath\psi_t = -\psi_{xx} + 2c|\psi|^2\psi \qquad H = \int \left(|\psi_x|^2 + c|\psi|^4\right) \mathrm{d}x$$

Second Quantized Nonlinear Schrödinger Equation:

$$\imath\hat{\Phi}_t = -\hat{\Phi}_{xx} + 2c\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi},$$

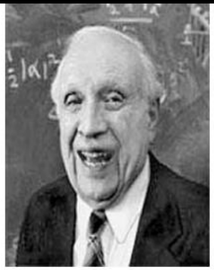
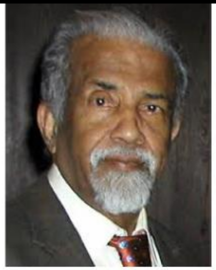
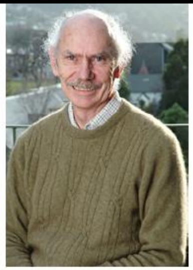
$\hat{\Phi}(x,t)$ is the field operator.

$$\psi(x,t) := \langle \mathrm{extreme}|\hat{\Phi}(x,t)|\mathrm{extreme}\rangle \xrightarrow{n_0>>1} \psi_{\mathrm{extr}}(x,t).$$

Quantum Peregrine Soliton Generation: Our approach to quantum NLSE

Positive Glauber-Sudarshan P-representation

It maps a nonlinear field theory into a system of stochastic differential equations.


Glauber & Sudarshan 1963,
Gardiner & Drummond 1980.

Density matrix $\hat{\rho}$ expanded in two sets of coherent states, spanned by complex parameters α and β:

$$\hat{\rho} = \int P(\alpha, \beta) \frac{|\alpha\rangle\langle\beta^*|}{\langle\beta^*|\alpha\rangle} \mathrm{d}^2\alpha \mathrm{d}^2\beta$$

$$\Downarrow$$

$$\partial_t \phi = -\imath \partial_x^2 \phi + \imath c \phi^2 \psi + \sqrt{\imath c}\,\xi(t,x)\phi,$$
$$\partial_t \psi = \imath \partial_x^2 \psi - \imath c \phi \psi^2 + \sqrt{-\imath c}\,\eta(t,x)\psi,$$

ξ and η are independent white noises.

Quantum Peregrine Soliton Generation: Our approach to quantum NLSE

$$\imath\hat{\Phi}_t = -\hat{\Phi}_{xx} + 2c\hat{\Phi}^\dagger\hat{\Phi}\hat{\Phi} \longrightarrow \imath\hat{\rho}_t = \left[\hat{H}, \hat{\rho}\right]$$

$$\hat{H} = \int \left(\Phi_x^\dagger \Phi_x + c\Phi^\dagger\Phi^\dagger\Phi\Phi\right) \mathrm{d}x$$

$$\langle\hat{\Phi}\rangle = \mathrm{Tr}\left(\hat{\rho}\hat{\Phi}\right)$$

$$\hat{\rho} = \int P(\alpha,\beta)\frac{|\alpha\rangle\langle\beta^*|}{\langle\beta^*|\alpha\rangle}\mathrm{d}^2\alpha\mathrm{d}^2\beta$$

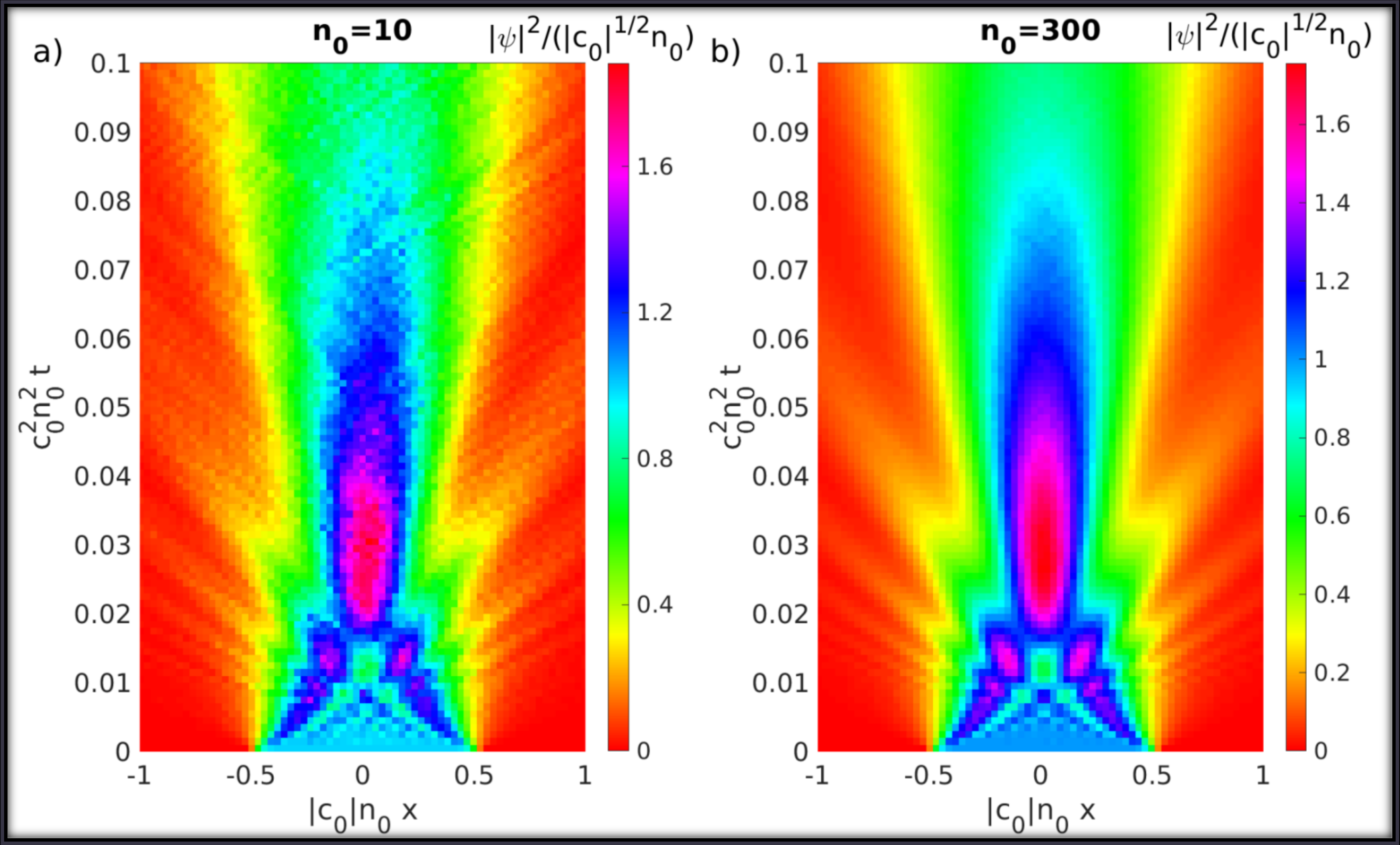
$$\begin{cases} \imath\alpha_t = \alpha_{xx} - c\alpha^2\beta + \imath\sqrt{\imath c}\xi\alpha \\ \imath\beta_t = -\beta_{xx} + c\alpha\beta^2 + \imath\sqrt{-\imath c}\eta\beta \end{cases} \longleftrightarrow \frac{\partial}{\partial t}P(\underline{\alpha},t) = \left[-\sum_i \frac{\partial}{\partial\alpha_i}V(\underline{\alpha}) + \frac{1}{2}\sum_{i,j}\frac{\partial^2}{\partial\alpha_i\partial\alpha_j}D_{ij}(\underline{\alpha})\right]P(\underline{\alpha},t)$$

$$\mathrm{d}\underline{\alpha} = V(\underline{\alpha})\mathrm{d}t + B(\underline{\alpha})\cdot\mathrm{d}W,$$

$$D(\underline{\alpha}) = B(\underline{\alpha})B^T(\underline{\alpha})$$

Quantum Peregrine Soliton Generation

$$\psi(x,t) := \langle\psi|\hat{\Phi}(x,t)|\psi\rangle \simeq \begin{cases} \sqrt{|c_0|n_0} & \text{if} \quad |x| \leq (2|c_0|n_0)^{-1} \\ 0 & \text{if} \quad |x| > (2|c_0|n_0)^{-1} \end{cases}$$



Area = number of photons

Quantum noise $\left(\propto \frac{1}{\sqrt{n_0}}\right)$ causes phase fluctuations:
photons phase relation is broken.
Does this affect the intensity peak per number of photons?

Quantum Peregrine Soliton Generation

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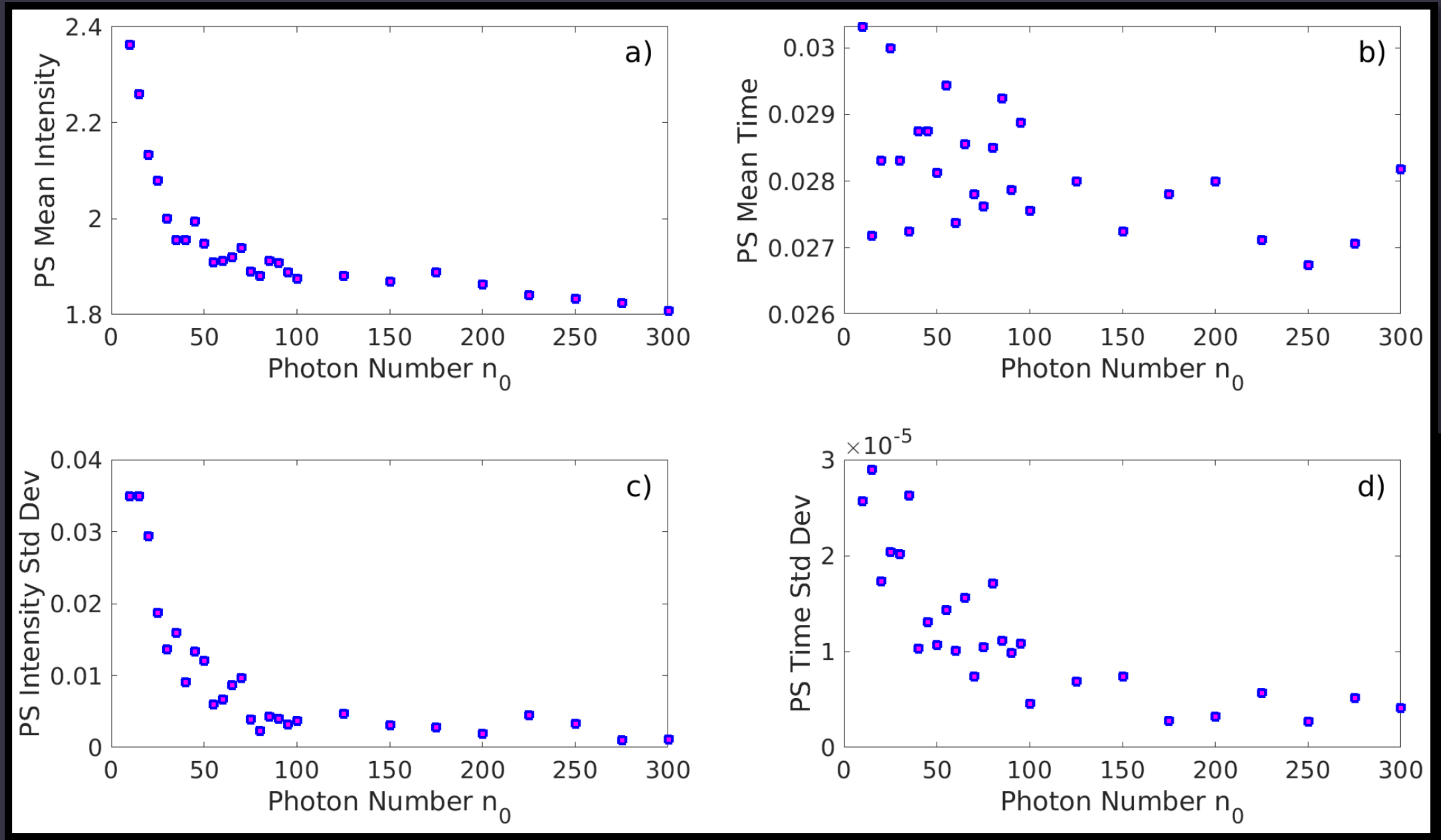

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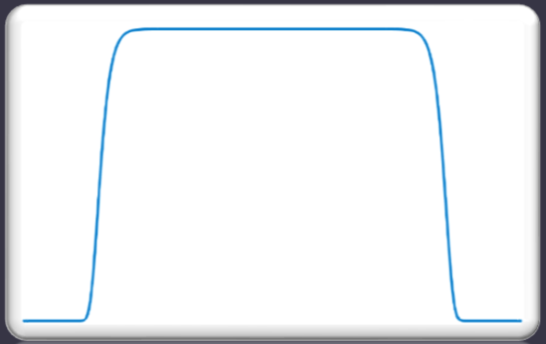

Quantum noise $\left(\propto \frac{1}{\sqrt{n_0}}\right)$ causes phase fluctuations:
photons phase relation is broken.
Does this affect the intensity peak per number of photons?
Yes, it does!

Conclusions

- We demonstrated that **topological invariants** describe **complex light regimes**.
- We showed theoretically and experimentally that **transitions between shocks, rogue waves, and soliton gases** can be supervised by **topological control**.
- We showed how to solve the **quantum nonlinear Schrödinger equation** by **phase-space methods** and stochastic simulations.
- We reported our results on **quantum noise effects** on **rogue waves generation efficiency**.

Conclusions

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Thank you for your attention!

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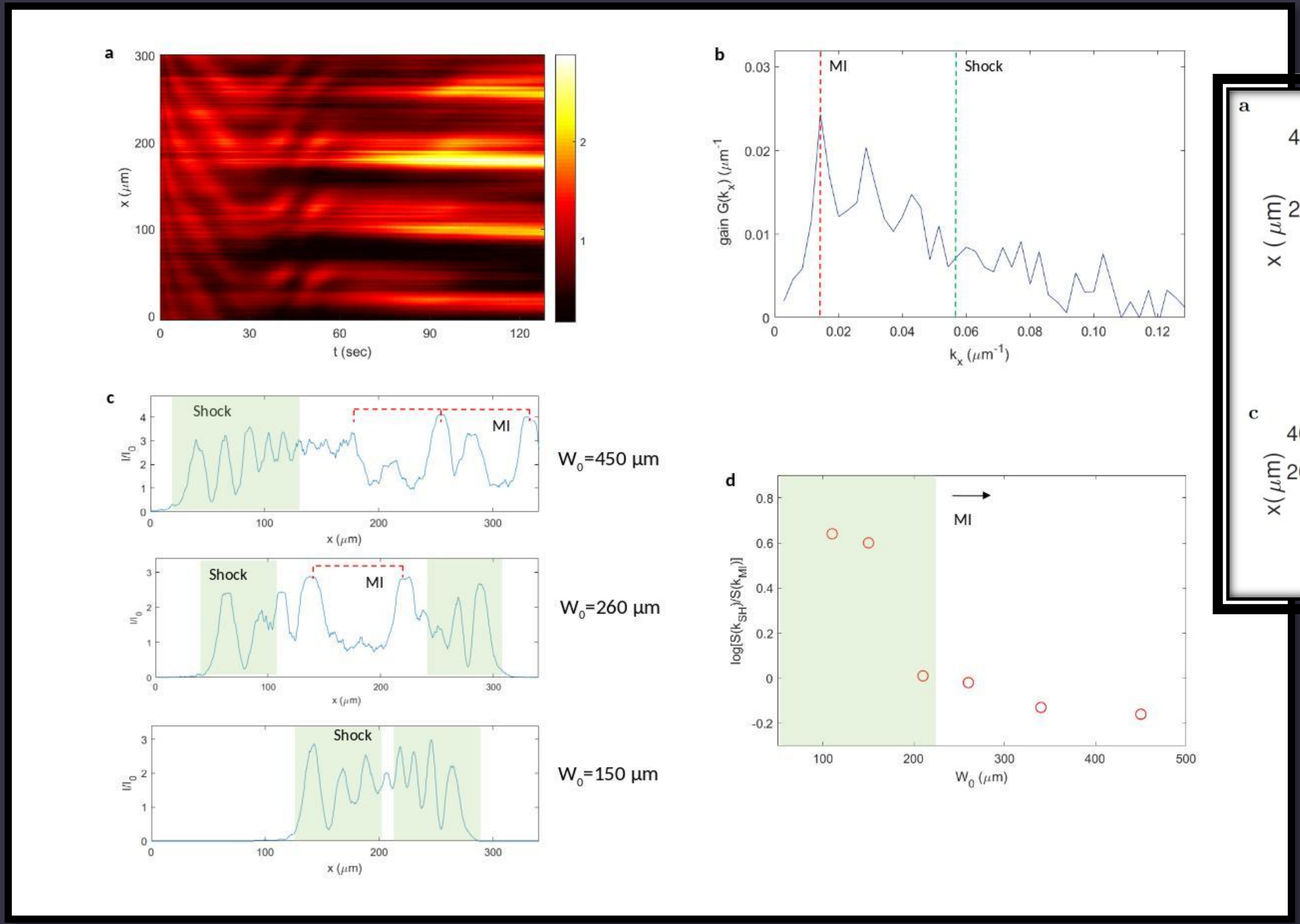
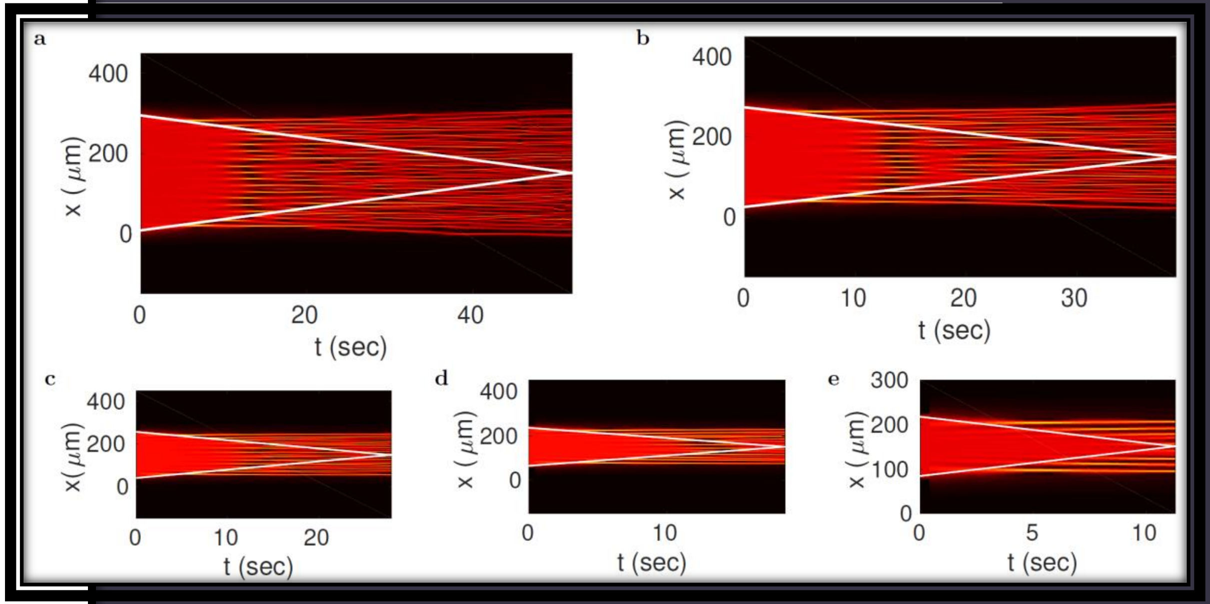
Topological Control of Extreme Waves: Propagation of a beam with larger waist





$$(a)W_0 = 300\mu m, (b)W_0 = 260\mu m, (c)W_0 = 220\mu m,$$
$$(d)W_0 = 180\mu m, (e)W_0 = 140\mu m$$

Topological Control of Extreme Waves: Propagation of a beam with small waist