A Second-Order Theory for Modelling Nonlinear Interactions in Wave Energy Systems

E. Renzi

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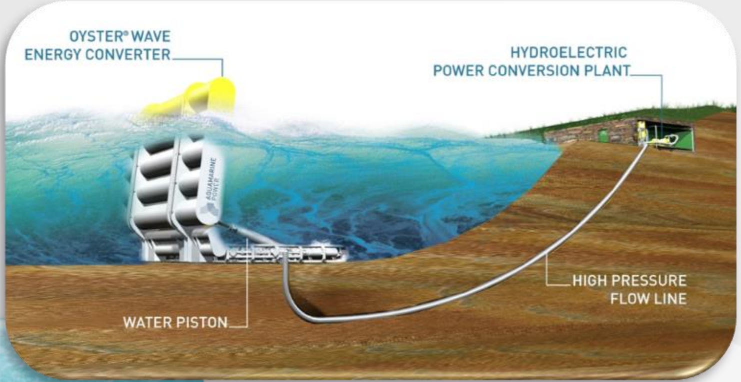
Wave Power

Oscillating Wave Surge Converter

AW Energy –
Wave Roller

Zyba – CCell



APL – Oyster

Today you will see…

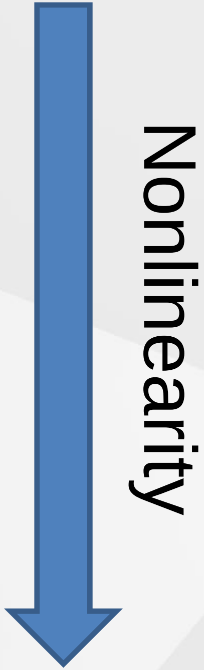
- Linear behaviour in small-amplitude waves

- Weakly nonlinear behaviour (2nd order)
  - Single flap
  - Finger-flaps
  - Influence of flap shape

Today you will not see…

- Subharmonic resonance of arrays (3rd order)

- Slamming effects and pressure impulse

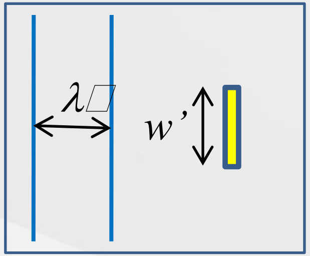


Linear Regime

- Small-amplitude waves: $A'/\lambda' \ll 1$
- Two characteristic dimensions:

  $k' = 2\pi/\lambda'$ wavenumber of incident wave

  w' = flap width



- Characteristic parameter: $k'w'$

Small	*Intermediate*	*Large*
$k'w' \ll 1$	$k'w' = O(1)$	$k'w' \gg 1$
Point absorber theory (Budal, 1977)	Full 3D diffraction – radiation problem	2D theory (Mei, 2005)

Model (Renzi E. & Dias F. 2012. *Journal of Fluid Mechanics* 701, 482 – 510)

Governing Equations - Linear potential flow theory

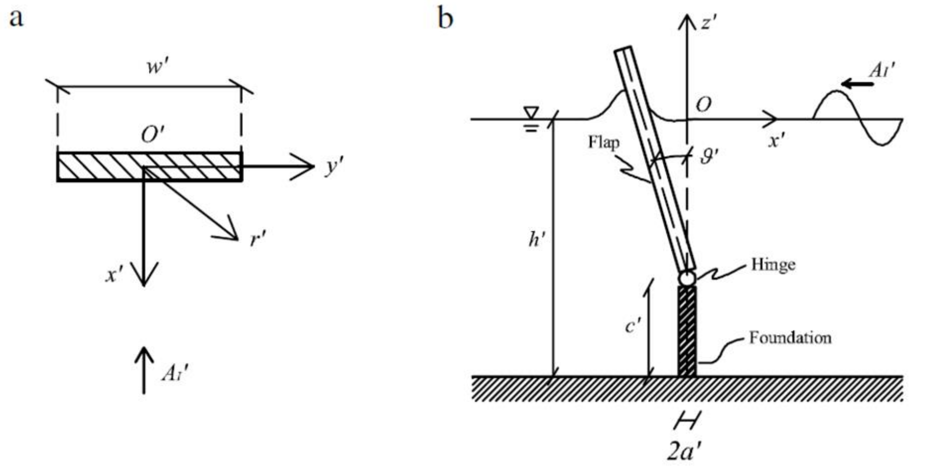


Fig. 1. Geometry of the system in physical variables; (a) plan view, (b) section.

$$\nabla'^2 \Phi'(x', y', z', t') = 0$$

$$\Phi'_{,t't'} + g\Phi'_{,z'} = 0, \quad z' = 0,$$

$$\Phi'_{,z'} = 0, \quad z' = -h',$$

$$\Phi'_{,x'} = -\theta'_{,t'}(t')(z' + h' - c')\,H(z' + h' - c'),$$
$$x' = \pm 0, \; |y'| < w'/2,$$

where the thin-plate approximation is applied (Linton & McIver, 2001)

Model

Governing Equations

- Introduce the following non-dimensional variables

$$(x, y, z, r) = (x', y', z', r')/w', \qquad t = \sqrt{g/w'}\,t', \qquad \Phi = (\sqrt{gw'}A')^{-1}\Phi', \qquad \theta = (w'/A')\theta'$$

- Separate time (harmonic oscillations of frequency ω)

$$\theta(t) = \Re\left\{\Theta\, e^{-i\omega t}\right\},$$

- Decompose potential

$$\Phi(x, y, z, t) = \Re\left\{(\phi^I + \phi^D + \phi^R)e^{-i\omega t}\right\}$$

Incident (ϕ^I) — Diffracted (ϕ^D) — Radiated (ϕ^R)

Model

Solution – Semi-analytical Galerkin Scheme

- Apply Green’s integral theorem to obtain a system of **hypersingular integral equations** for the jump in potential across the flap

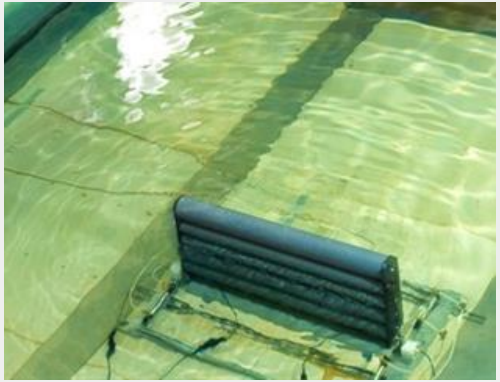
- Expand the jump in potential in series of **Chebyshev polynomials** of the second kind

- Obtain a linear system of equations for the expansion coefficients which are solved **numerically** via a collocation method (Linton & McIver, 2001)

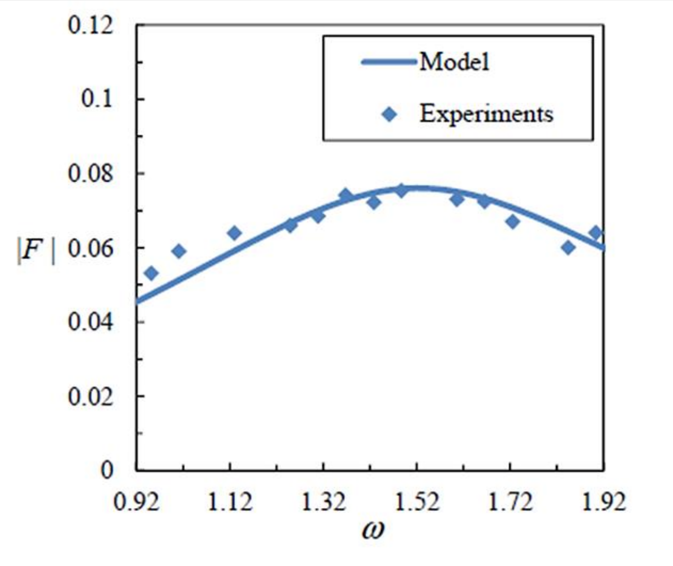
Results

Validation

Very good agreement with experimental data of **Oyster 1** WEC.



Experimental data provided by Aquamarine Power Limited & QUB



Wave torque vs. frequency of incident wave

Results

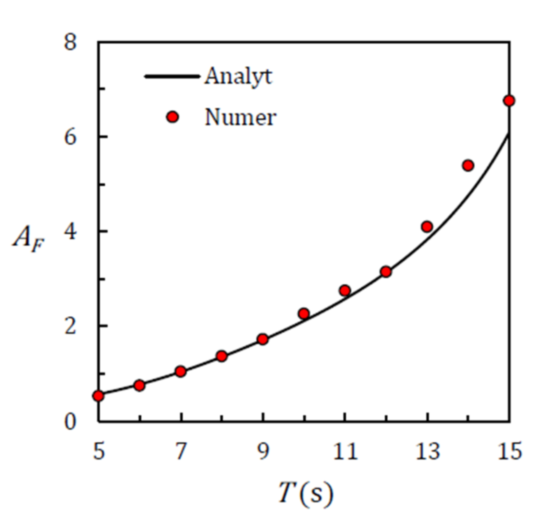
Validation

Comparison with numerical model (UR3, Italy)

Flap width	$w' = 26$ m
Depth	$h' = 13$ m
Foundation	$c' = 4$ m
Wave amp.	$A' = 1$ m

Model	Time	Periods
Numerical	12 hr	11
Semi-analytical	20 min	100

Amplitude factor = Max stroke/A'

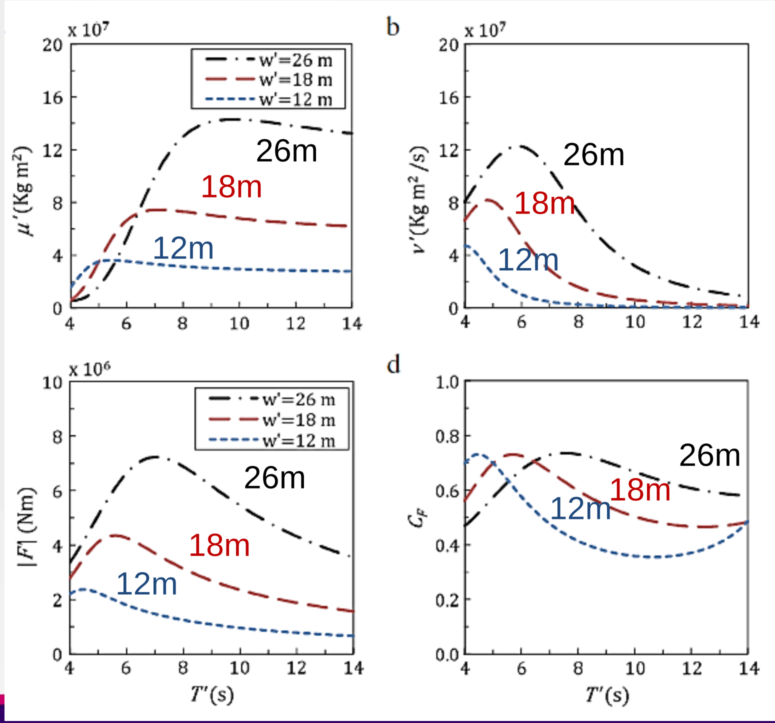


Results

Influence of Flap Width

Added inertia

Radiation damping



Exciting torque

Capture Factor

$$C_F = \frac{P}{\frac{1}{2}A_I^2 C_g}$$

Weakly Nonlinear Regime

- Three characteristic dimensions:

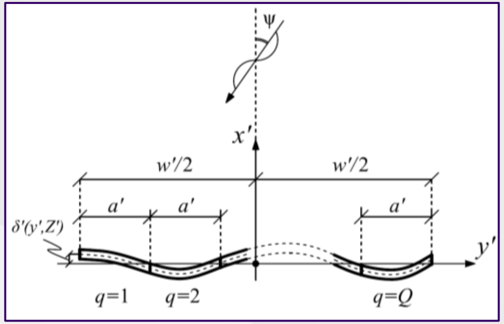
  λ' = wavelength of incident wave

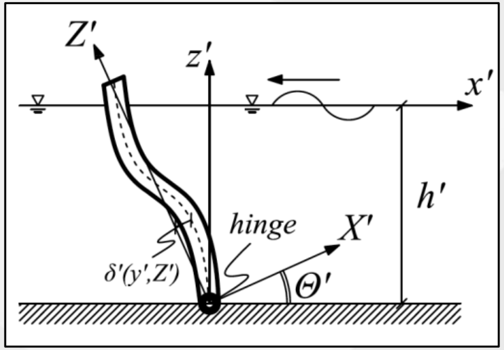
  A' = wave amplitude

  $\delta'(y',z')$ = flap shape function

- Characteristic parameters:
  - $\varepsilon = A'/\lambda' < 1$, with $\varepsilon^2 << 1$
  - $\mu = \max(\delta')/\lambda' = O(\varepsilon)$

In weakly nonlinear regimes the (small) curvature of the flap starts to have an effect on the hydrodynamics of the device





Model (Michele S. & Renzi E. 2019. *Journal of Fluids & Structures* 88, 315 – 330)

Governing Equations (non-dimensional)

$$\nabla^2 \Phi = 0, \quad (x, y, z) \in \Omega,$$

$$-G\zeta = \Phi_t + \epsilon \frac{1}{2} |\nabla \Phi|^2, \quad z = \epsilon\zeta,$$

where

$$G = \frac{g'}{\omega'^2 \lambda'}$$

$$\Phi_z = 0, \quad z = -h.$$

boundary condition on the flap:

$$\Theta_t \left\{ \sin \epsilon\Theta_q \left[-x + \mu\delta_Z (z + h)\right] + \cos \epsilon\Theta_q \left[(z + h) + \mu x \delta_Z\right]\right\} + \Phi_x \left(\cos \epsilon\Theta_q + \mu\delta_Z \sin \epsilon\Theta_q\right) - \Phi_y \mu\delta_y + \Phi_z \left(\sin \epsilon\Theta_q - \mu\delta_Z \cos \epsilon\Theta_q\right) = 0,$$

Model (Michele S. & Renzi E. 2019. *Journal of Fluids & Structures* 88, 315 – 330)

Governing Equations (non-dimensional)

Equation of motion of the qth flap:

$$
\begin{aligned}
& I\epsilon\Theta_{q,tt} + GC\sin\epsilon\Theta_q + \nu_{pto}\epsilon\Theta_{q,t} = \\
& \int_{y_q}^{y_{q+1}} \mathrm{d}y \Bigg\{ -\int_{-h}^{\epsilon\zeta^+} \mathrm{d}z \left(Gz + \epsilon\Phi_t^+ + \frac{\epsilon^2}{2}\left|\nabla\Phi^+\right|^2 \right) \\
& \times \left\{ (z+h) - \tan\epsilon\Theta \left[-\tan\epsilon\Theta\,(z+h)\,\frac{\mu\delta}{\cos\epsilon\Theta} \right] + \frac{\mu^2\delta\delta_Z}{\cos\epsilon\Theta} \right\} \\
& + \int_{-h}^{\epsilon\zeta^-} \mathrm{d}z \left(Gz + \epsilon\Phi_t^- + \frac{\epsilon^2}{2}\left|\nabla\Phi^-\right|^2 \right) \\
& \times \left\{ (z+h) - \tan\epsilon\Theta \left[-\tan\epsilon\Theta\,(z+h)\,\frac{\mu\delta}{\cos\epsilon\Theta} \right] + \frac{\mu^2\delta\delta_Z}{\cos\epsilon\Theta} \right\} \Bigg\}
\end{aligned}
$$

Model

Solution – Analytical method

- Assume a perturbation expansion of the physical quantities, e.g.

$$\Phi\,(x, y, z, t) = \Phi_1\,(x, y, z, t) + \epsilon \Phi_2\,(x, y, z, t) + O\left(\epsilon^2\right)$$

- Harmonic expansion at the single orders:

$$\left\{\Phi_n, \zeta_n, \Theta_n, \Theta_{q,n}\right\} = \sum_{m=0}^{n} \left\{\phi_{nm}, \eta_{nm}, \theta_{nm}, \theta_{q,nm}\right\} e^{-im\omega t} + *,$$

- The nonlinear set of governing equations is decomposed in a sequence of linear boundary-value problems of order ***n*** and harmonic ***m***

Model

Solution – Analytical method

$\nabla^2\phi_{nm} = 0,$	Fluid domain	in Ω,
$\phi_{nm_z} = \phi_{nm}\dfrac{m^2\omega^2}{g} + \mathcal{F}_{nm},$	Free surface	$z = 0,$
$\eta_{nm} = \phi_{nm}\dfrac{im\omega}{g} + \mathcal{B}_{nm},$	Free surface	$z = 0,$
$\phi_{nm_z} = 0,$	Bottom	$z = -h,$
$\phi^{\pm}_{nm_x} = im\omega\theta_{nm}(z+h) + \mathcal{G}^{\pm}_{nm},$	Array surface	$x = \pm 0, z \in [-h, 0]\,.$

$$-m^2\omega^2 I\theta_{q,nm} + C\theta_{q,nm} - im\omega v_{pto}\theta_{q,nm} = im\omega\rho\int_{y_q}^{y_{q+1}} dy \int_{-h}^{0} \Delta\phi_{nm}(z+h)\ dz + \mathcal{D}_{nm}.$$

The forcing terms $\boldsymbol{F_{nm}}$, $\boldsymbol{B_{nm}}$, $\boldsymbol{G_{nm}}$ **and** $\boldsymbol{D_{nm}}$ are defined for each order.

Model

Solution – Analytical method

- Each boundary-value problem is solved by transforming the relevant governing equations in **elliptical coordinates**

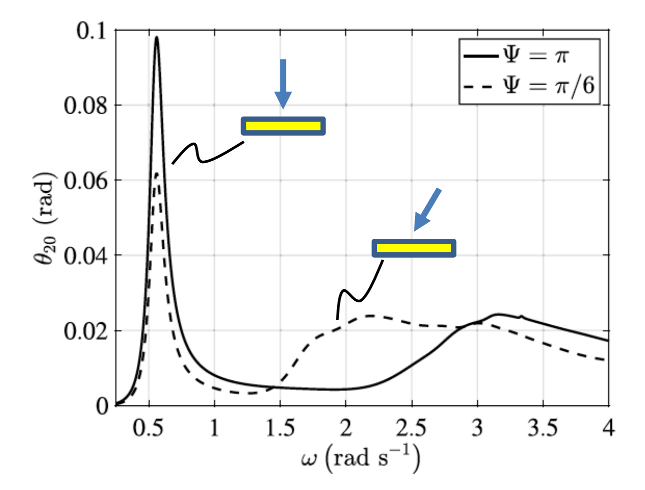
$$x = \frac{w}{2} \sinh \chi \sin \vartheta , \quad y = \frac{w}{2} \cosh \chi \cos \vartheta ,$$

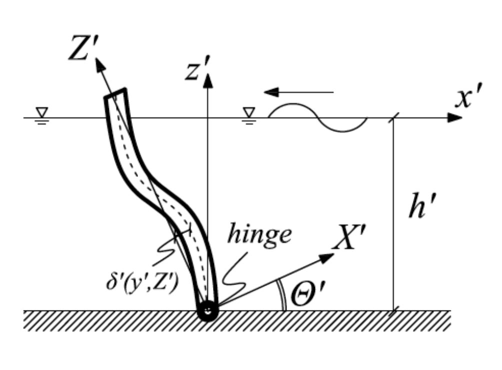
- The solutions can then be expressed in terms of **Mathieu functions**. E.g. at the first order we have:

$$\left. \begin{array}{c} \varphi_{qn}^{R}(\chi,\vartheta) \\ \varphi^{S}(\chi,\vartheta) \end{array} \right\} = \sum_{m=0}^{\infty} \left\{ \begin{bmatrix} \mathcal{A}_{qnm}^{R} \\ \mathcal{A}_{m}^{S} \end{bmatrix} \mathrm{ce}_{2m}(\vartheta)\, \mathrm{He}_{2m}^{(1)}(\chi) + \begin{bmatrix} \mathcal{B}_{qnm}^{R} \\ \mathcal{B}_{m}^{S} \end{bmatrix} \mathrm{ce}_{2m+1}(\vartheta)\, \mathrm{He}_{2m+1}^{(1)}(\chi) \right.$$
$$\left. + \begin{bmatrix} \mathcal{C}_{qnm}^{R} \\ \mathcal{C}_{m}^{S} \end{bmatrix} \mathrm{se}_{2m+2}(\vartheta)\, \mathrm{Ho}_{2m+2}^{(1)}(\chi) + \begin{bmatrix} \mathcal{D}_{qnm}^{R} \\ \mathcal{D}_{m}^{S} \end{bmatrix} \mathrm{se}_{2m+1}(\vartheta)\, \mathrm{Ho}_{2m+1}^{(1)}(\chi) \right\},$$

Results

Second-order drift (0th harmonic)



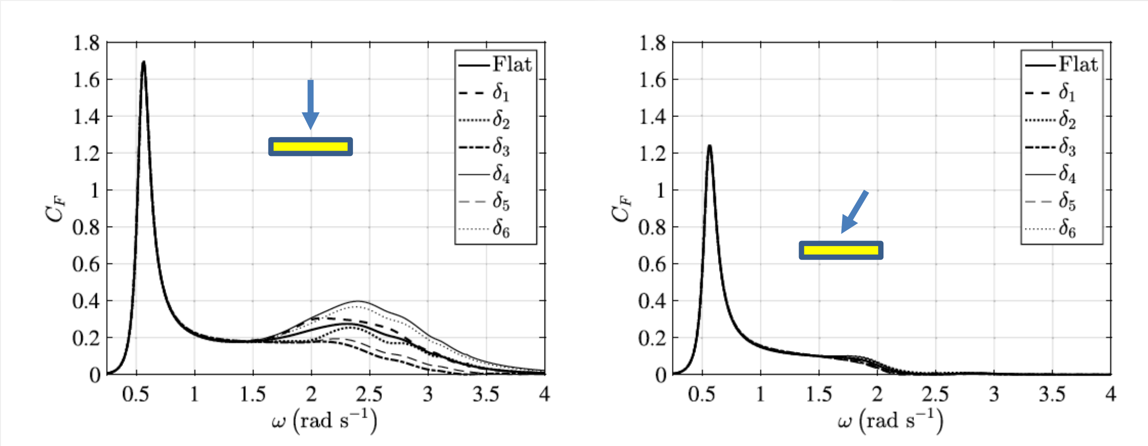


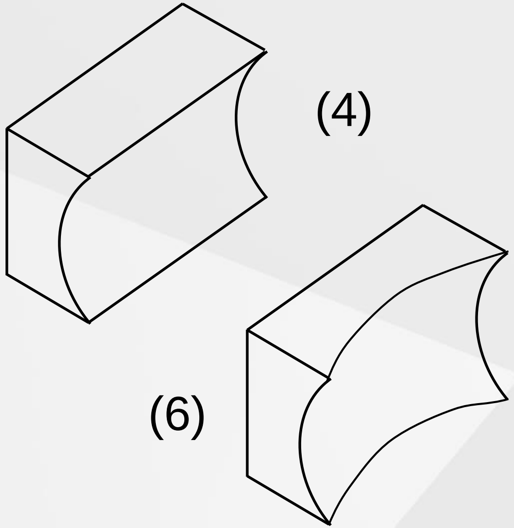
Flap width	$w' = 20$ m
Depth	$h' = 10$ m
Wave amp.	$A' = 1$ m

The drift at the second order is independent of the shape of the flap and induces a **static displacement**

Results

Shape Effects on Capture Factor (single flap)

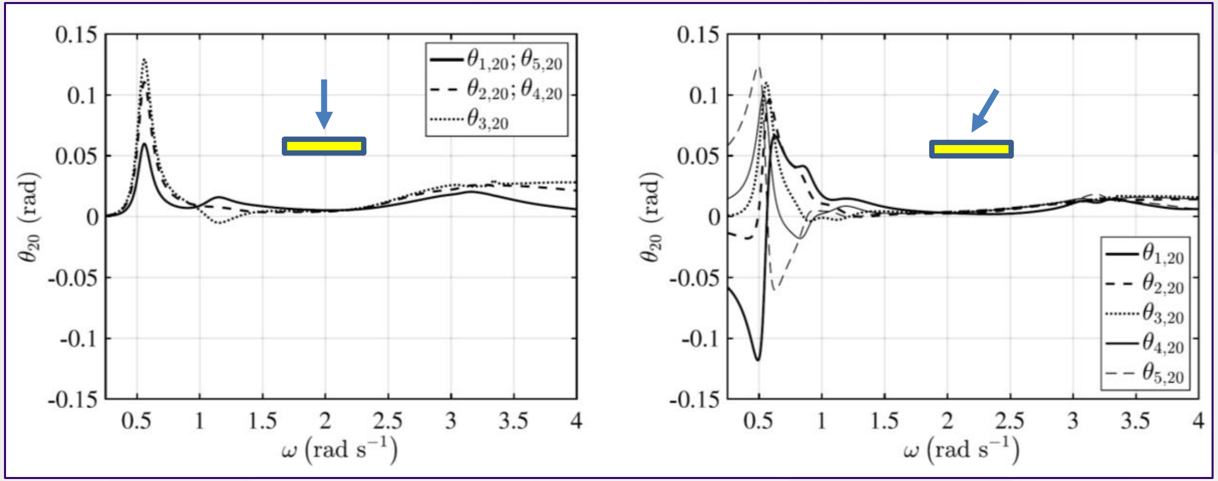




Concave shapes (4,6) enhance capture factor at high frequencies in head sea (focussing effect)

Results

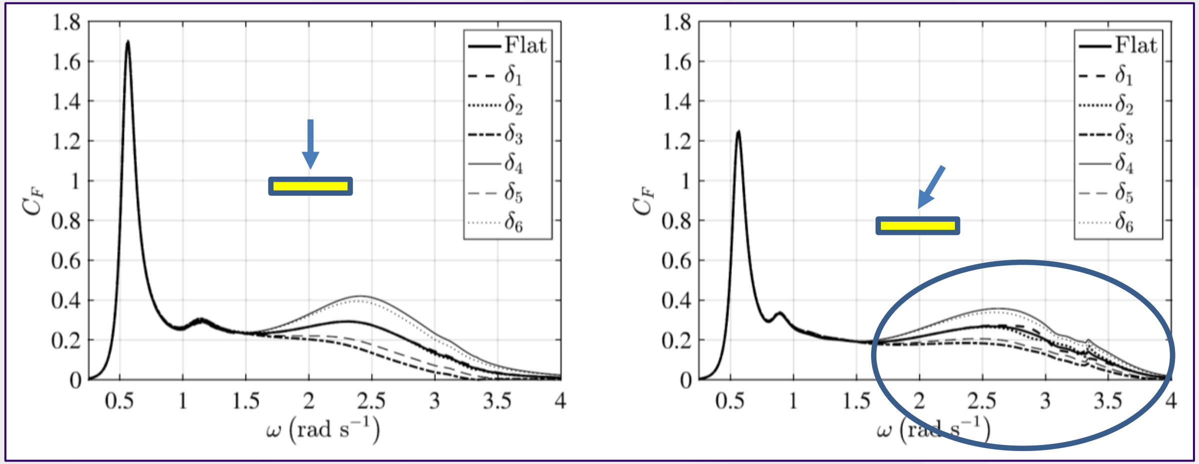
Shape + Array Effects on Capture Factor (5-flap array)



The static rotation in an array can be negative for some of the elements

Results

Shape + Array Effects on Capture Factor (5-flap array)



Shape and array effects enhance the capture factor in oblique incidence

Conclusions

- In the **linear regime**, the behaviour of the flap is not affected by shape effects (of the order of the wave amplitude).
- In the **weakly nonlinear regime** the second-order drift can change the neutral flap position and the shape of the device affects the capture factor
- Concave configurations determine a **20% increase** in capture factor at large frequencies
- **Finger-flaps** are more efficient than equivalent single flaps at large frequencies in oblique seas

Thank you for your attention

References

1. MICHELE, S. & RENZI, E. **2019** A second-order theory for an array of curved wave energy converters in open sea. ***Journal of Fluids and Structures***. 88, 315–330.
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3. MICHELE, S., SAMMARCO, P. & d'ERRICO, M. **2018** Weakly nonlinear theory for oscillating wave surge converters in a channel. ***Journal of Fluid Mechanics***. 834, 55–91.