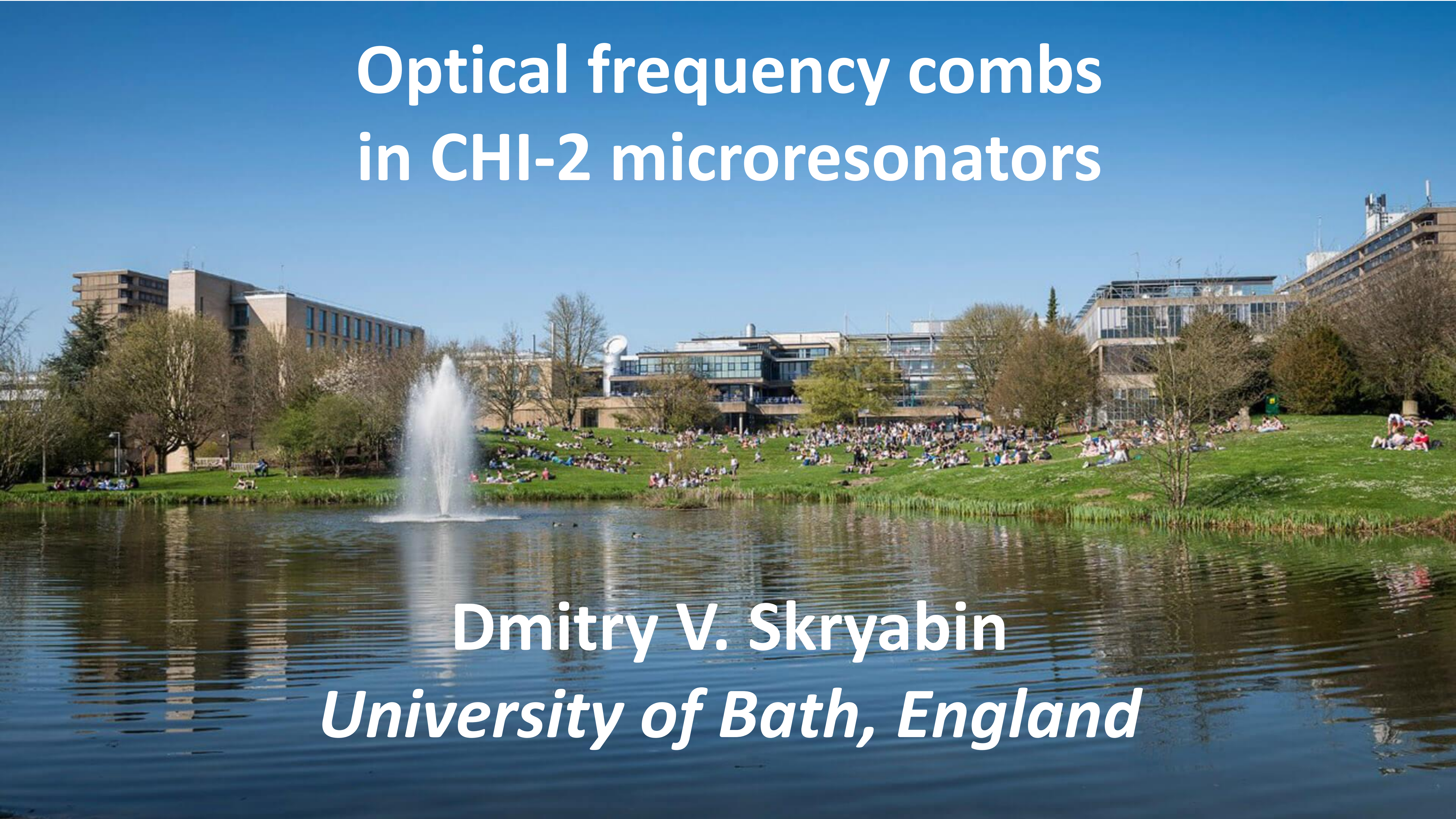


Optical frequency combs in CHI-2 microresonators

Dmitry V. Skryabin

University of Bath, England

Bath group and sponsors

Danila

Zhiwei

Vlad

Alberto

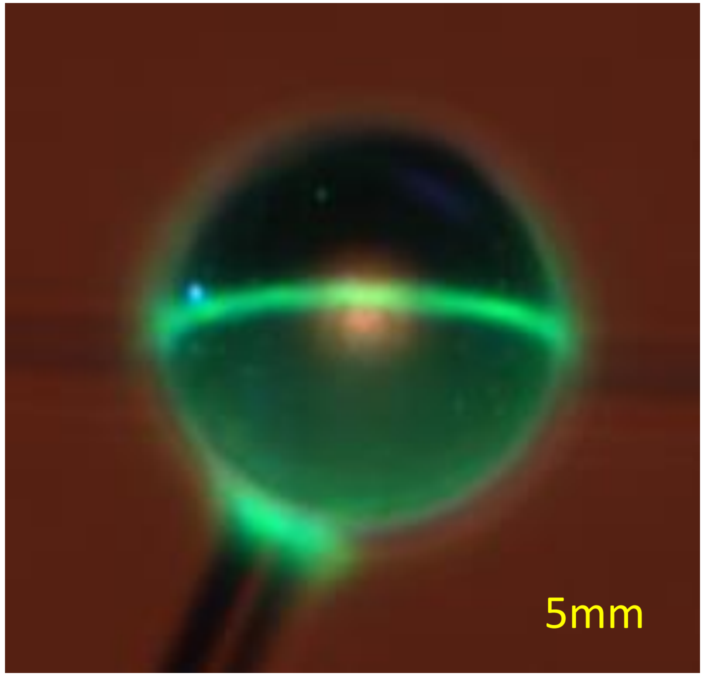
Dmitry

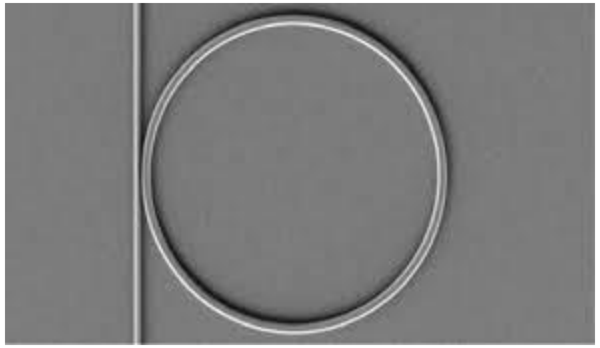
THE FRAMEWORK PROGRAMME FOR RESEARCH AND INNOVATION

HORIZON 2020

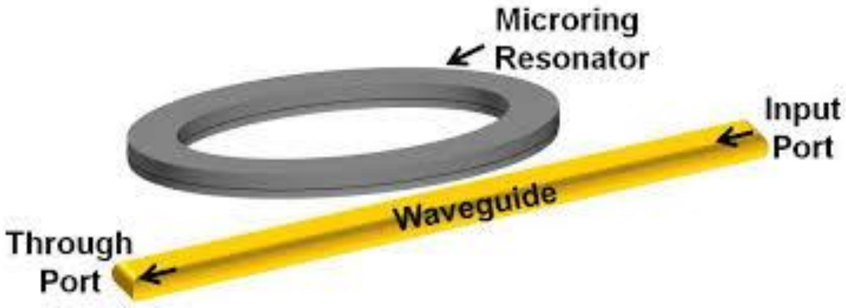
INNOVATIVE TRAINING NETWORKS

Microresonators





$50\mu m$



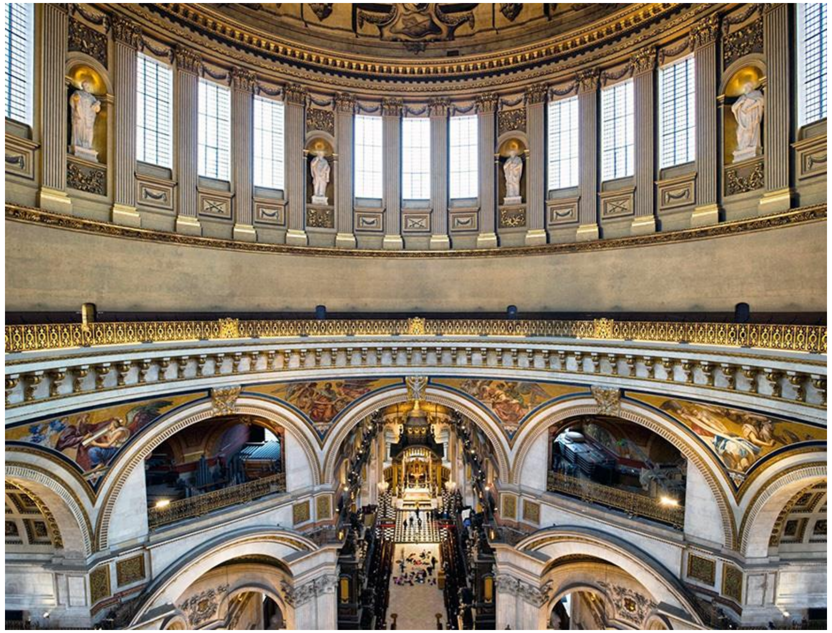
CaF_2, SiO_2

1000000

$Si_3 N_4$

1000

Whispering Gallery in StPaul's Cathedral



Lord Rayleigh
(~1870)

Soliton-crystals

Soliton crystals in Kerr resonators

Daniel C. Cole[1,2*], Erin S. Lamb[1], Pascal Del'Haye[1,3], Scott A. Diddams[1] and Scott B. Papp[1]

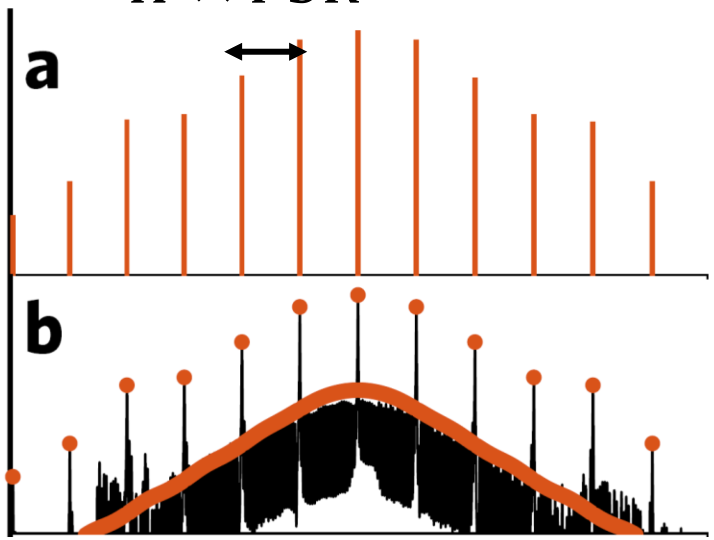
NATURE PHOTONICS | VOL 11 | OCTOBER 2017 | 671–676 |

Dynamics of soliton crystals in optical microresonators

Maxim Karpov[1], Martin H. P. Pfeiffer[1], Hairun Guo[1,2], Wenle Weng[1], Junqiu Liu[1]
Tobias J. Kippenberg[1*]

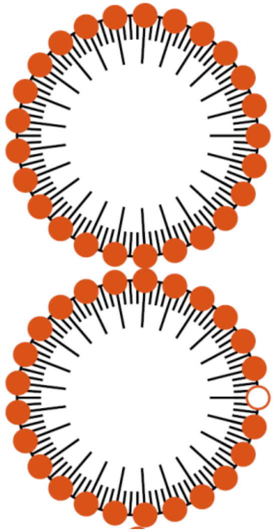
NATURE PHYSICS | VOL 15 | OCTOBER 2019 | 1071–1077

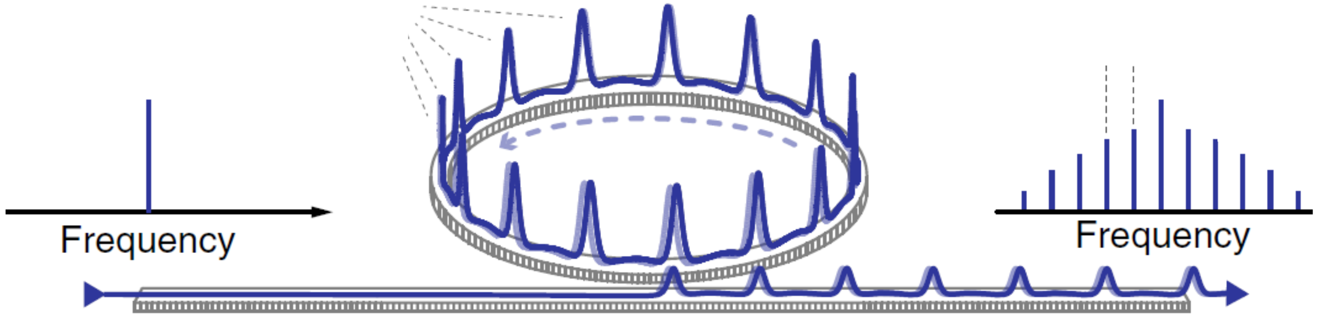
$K \times FSR$



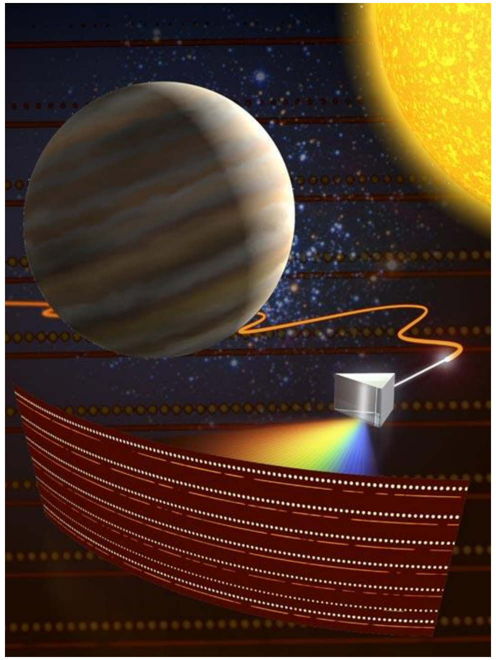
frequency

K solitons





$$P = \chi_1 E + \chi_2 E^3$$



Microresonators for Astro-Combs

An **Astro-Comb** (HARPS) detects spectral wobbles caused by small *exoplanets*

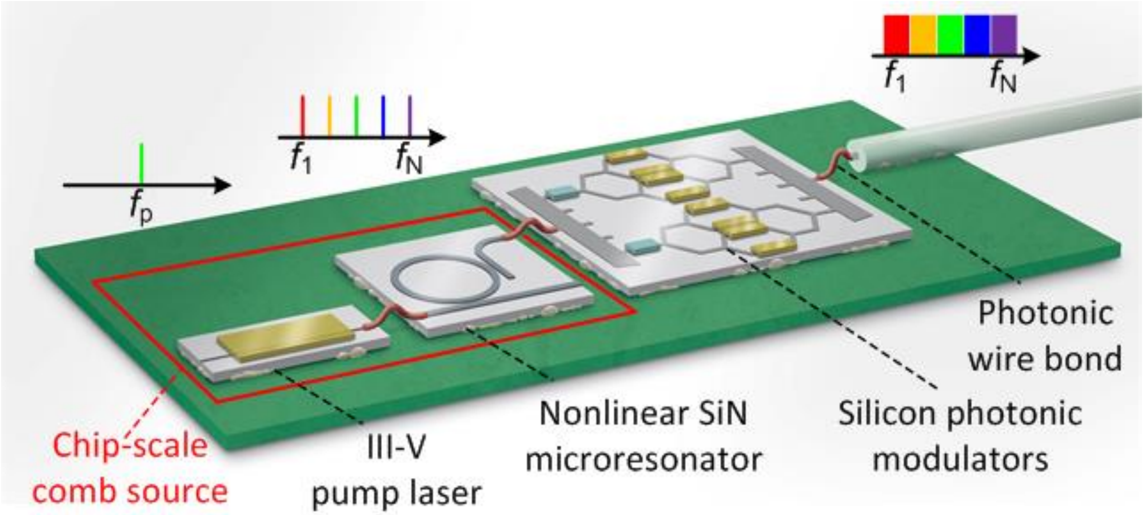
nature astronomy
LETTERS
https://doi.org/10.1038/s41550-020-1010-x

A crucial test for astronomical spectrograph calibration with frequency combs

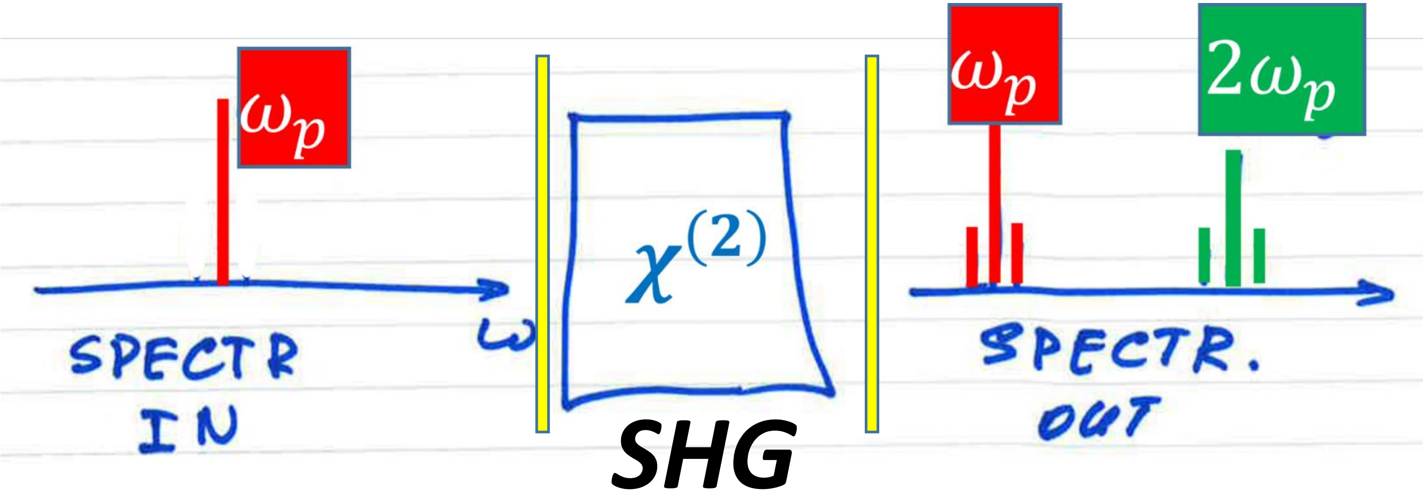
Rafael A. Probst[1,2*], Dinko Milaković[3], Borja Toledo-Padrón[4,5], Gaspare Lo Curto[3,6], Gerardo Avila[3],

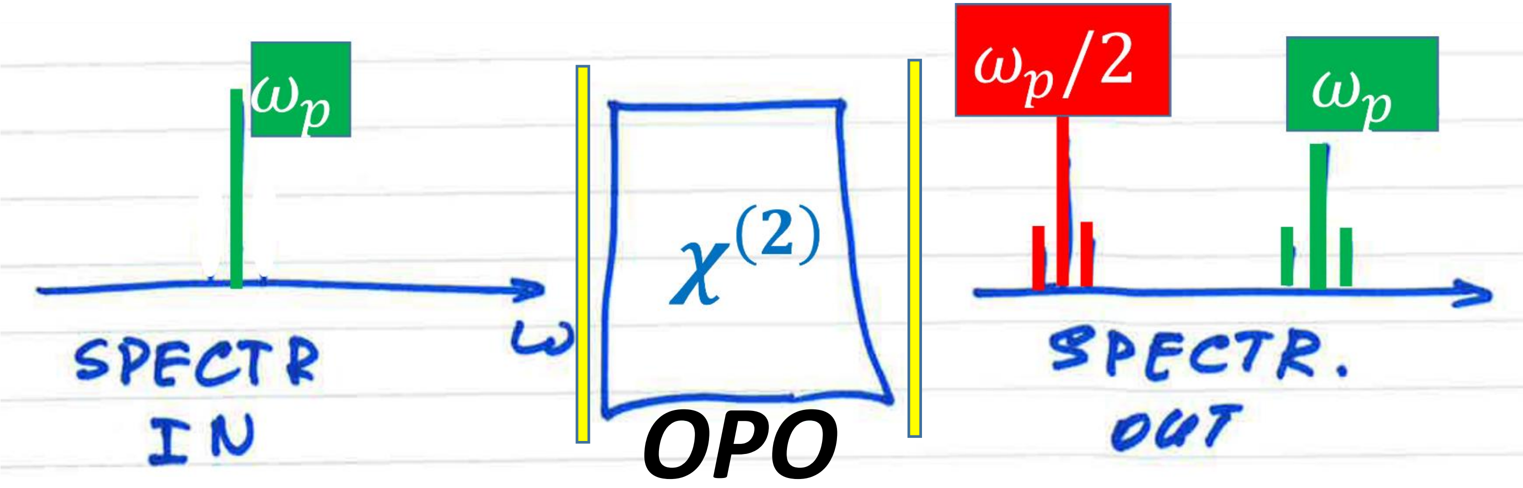
Coherent terabit communications with microresonator Kerr frequency combs

Pfeifle et al, Nature Phot 8, 375 (2014)

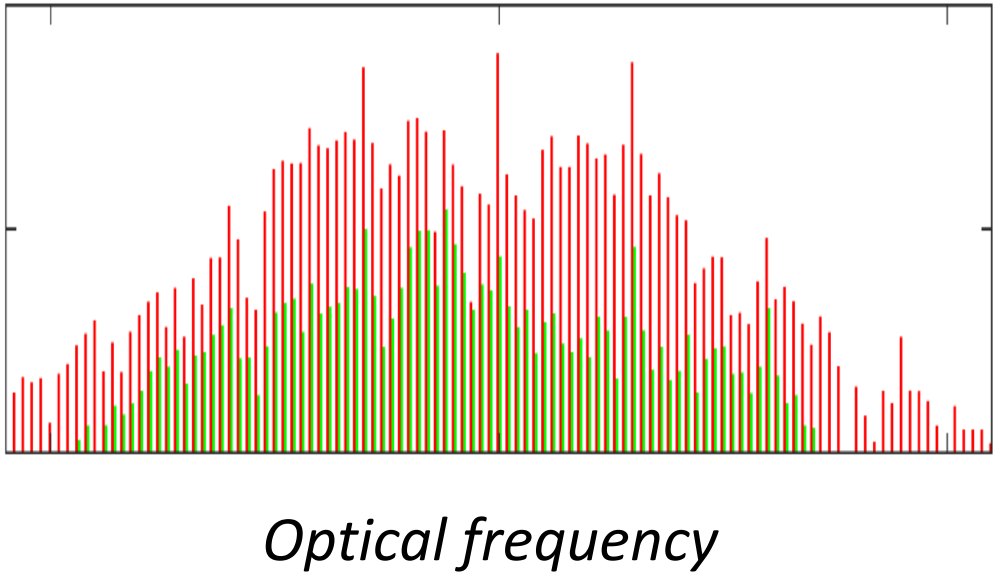


Concept of chi-2 combs

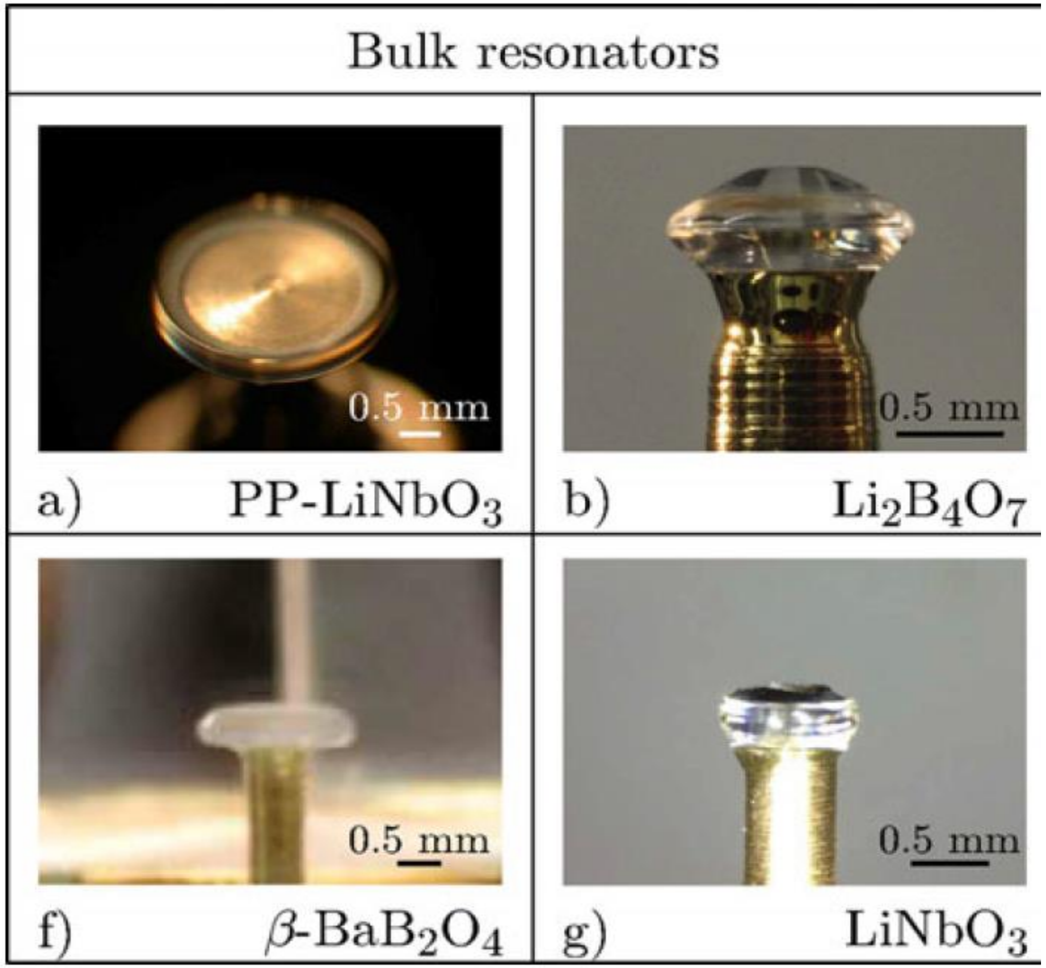




$$P = \chi_1 E + \chi_2 E^2$$



Bulk LiNbO3 ring resonators



Breunig, LasPhotonRev 10, 569 (2016)

Ilchenko …, PRL 92, 043903 (2004)
Furst …, PRL 104, 153901 (2010)

$LiNbO_3$

$Q \sim 10^8$ finesse ~ 10000 FSR ~ 20 GHz

Linewidth: $\kappa_{a,b} \sim 1 - 10$ MHz

Dispersion is normal till around 1.5um

$$e^{iM\theta - i\frac{1}{2}\omega_p t} \sum_{\mu} a_\mu(t)\Phi_\mu(\vec{r}\,)e^{i\mu\theta} + c.c.$$

Low-frequency modes

$$e^{i2M\theta - i\omega_p t} \sum_{\mu} b_\mu(\mathrm{t})\Psi_\mu(\vec{r})\, e^{i\mu\theta} + c.c.$$

High-frequency modes

$\mu = .., -2, -1, 0\,, +1, +2..$ (relative mode number)

$$c^2 \partial_\alpha \partial_{\alpha_1} \mathcal{E}_{\alpha_1} - c^2 \partial_{\alpha_1} \partial_{\alpha_1} \mathcal{E}_\alpha$$

$$+ \partial_t^2 \int_{-\infty}^{\infty} \hat{\varepsilon}_{\alpha\alpha_1}(t - t', r, \theta, z)\mathcal{E}_{\alpha_1}(t', \vec{r})\mathrm{d}t' = -\partial_t^2 \mathcal{N}_\alpha$$

$$\mathcal{N}_\alpha = \chi^{(2)}_{\alpha\alpha_1\alpha_2} \mathcal{E}_{\alpha_1} \mathcal{E}_{\alpha_2}$$

$$i\partial_t a_\mu = \delta_{\mu a} a_\mu - \frac{i\kappa_a}{2} a_\mu - \gamma_a \sum_{\mu_1 \mu_2} \widehat{\delta}_{\mu,\mu_1-\mu_2} b_{\mu_1} a^*_{\mu_2}$$

$$i\partial_t b_\mu = \delta_{\mu b} b_\mu - \frac{i\kappa_b}{2} \left(b_\mu - \widehat{\delta}_{\mu,\mu'} \mathcal{H}\right) - \gamma_b \sum_{\mu_1 \mu_2} \widehat{\delta}_{\mu,\mu_1+\mu_2} a_{\mu_1} a_{\mu_2}$$

$$\omega_{b\mu_1} - \omega_{a\mu_2} - \omega_{a\mu}$$

$$\omega_{a\mu_1} + \omega_{a\mu_2} - \omega_{b\mu}$$

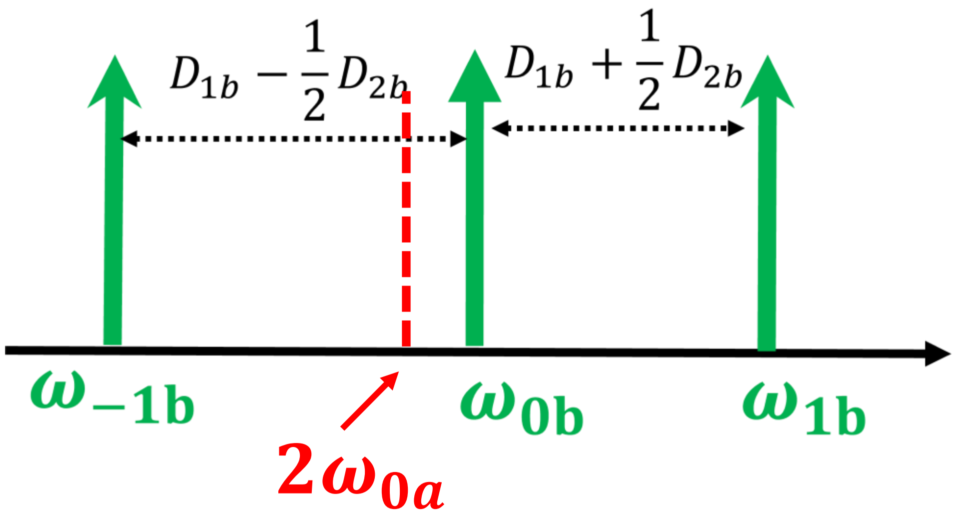
DVS …, JOSA B 37, 2604 (2020)

Phase mismatch and repetition-rate mismatch

Linear spectrum

$$\omega_{\mu\mathrm{a}} = \omega_{0\mathrm{a}} + \mu D_{1\mathrm{a}} + \frac{1}{2}\mu^2 D_{2\mathrm{a}}$$

$$\omega_{\mu\mathrm{b}} = \omega_{0\mathrm{b}} + \mu D_{1\mathrm{b}} + \frac{1}{2}\mu^2 D_{2\mathrm{b}}$$



$\mu = 0$ *frequency mismatch parameter*

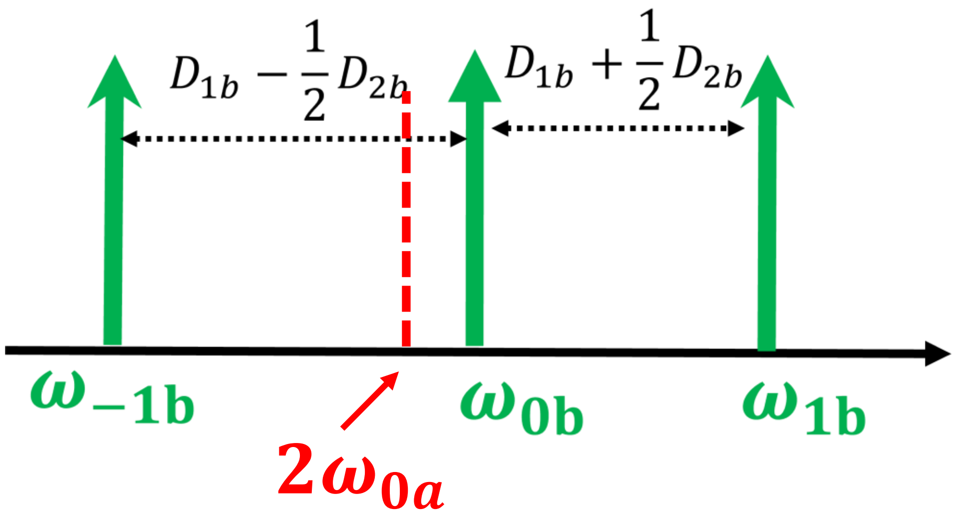
$$\epsilon = 2\omega_{0\mathrm{a}} - \omega_{0\mathrm{b}} = \frac{2Mc}{R}\left(\frac{1}{n_\mathrm{a}} - \frac{1}{n_\mathrm{b}}\right)$$
$$= 0 \text{ to } \pm 20\mathrm{GHz}$$

Phase velocity and group velocity mismatch

Linear spectrum

$$\omega_{\mu a} = \omega_{0a} + \mu D_{1a} + \frac{1}{2}\mu^2 D_{2a}$$

$$\omega_{\mu b} = \omega_{0b} + \mu D_{1b} + \frac{1}{2}\mu^2 D_{2b}$$



$\mu = 0$ frequency mismatch parameter

$$\epsilon = 2\omega_{0a} - \omega_{0b} = \frac{2Mc}{R}\left(\frac{1}{n_a} - \frac{1}{n_b}\right)$$
$$= 0 \text{ to } \pm 20\text{GHz}$$

$D_{1a} - D_{1b}$ = 1 GHz

Difference of linear repetition rates

Comb initiation equations

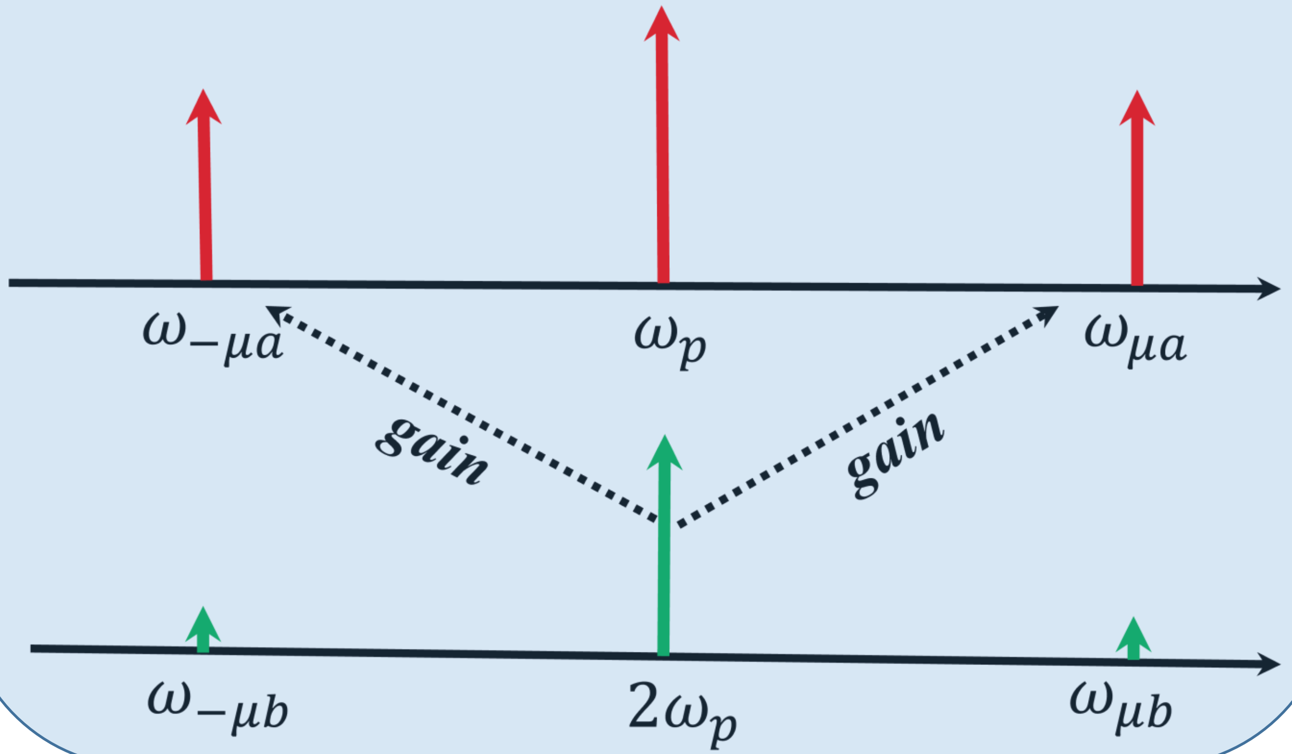
$$i\frac{\partial}{\partial t}\begin{pmatrix} a_\mu \\ b_\mu \\ a_{-\mu} \\ b_{-\mu} \end{pmatrix} =$$

Parametric
and
Sum-frequency
terms

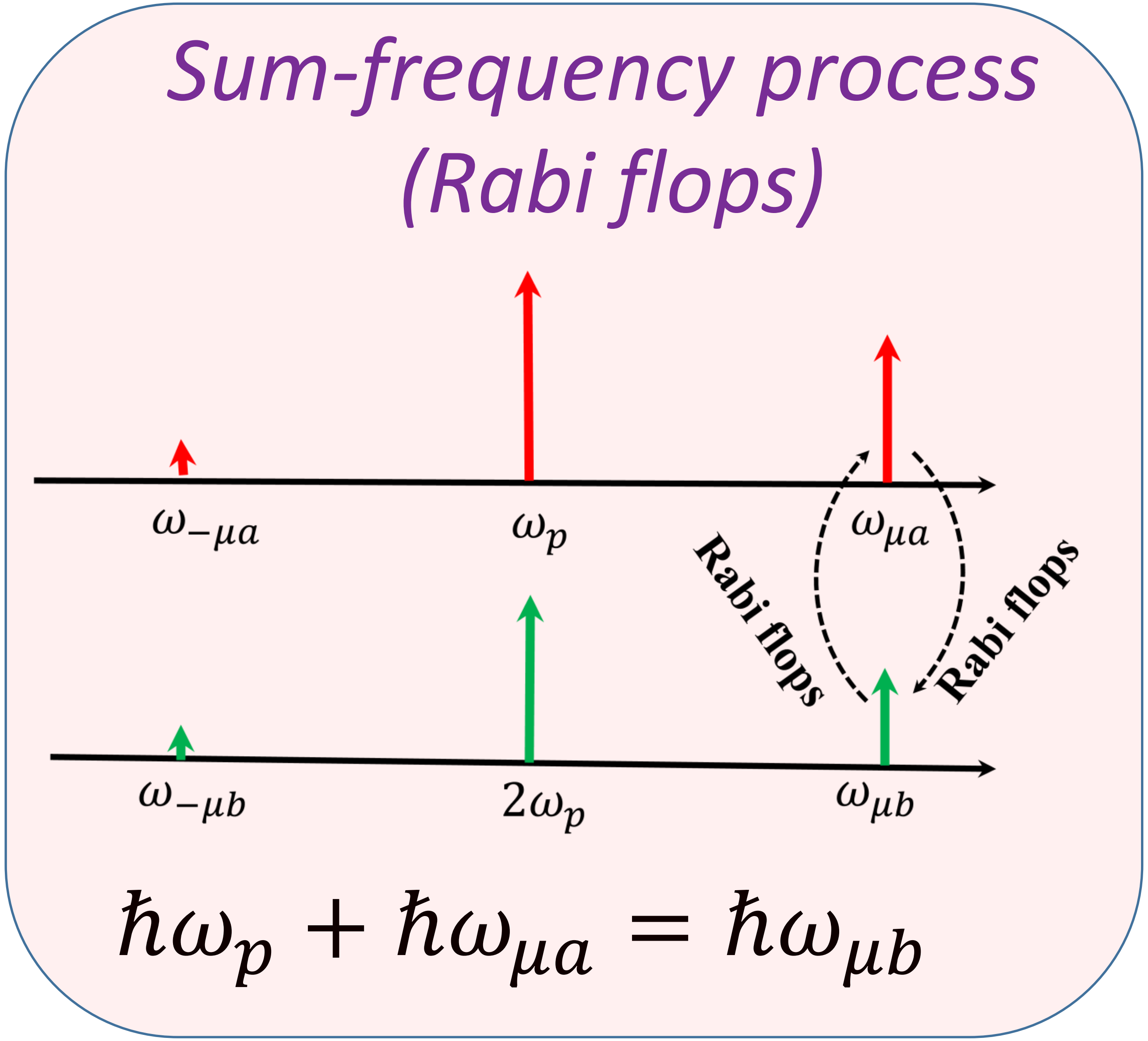
units of Hz

$$\begin{bmatrix} \delta_{\mu a} - \frac{i\kappa_a}{2} & -\gamma_a \widetilde{a}_0^* & -\gamma_a \widetilde{b}_0 & 0 \\ -2\gamma_b \widetilde{a}_0 & \delta_{\mu b} - \frac{i\kappa_b}{2} & 0 & 0 \\ \gamma_a \widetilde{b}_0^* & 0 & -\delta_{-\mu a} - \frac{i\kappa_a}{2} & \gamma_a \widetilde{a}_0 \\ 0 & 0 & 2\gamma_b \widetilde{a}_0^* & -\delta_{-\mu b} - \frac{i\kappa_b}{2} \end{bmatrix} \begin{pmatrix} a_\mu \\ b_\mu \\ a_{-\mu} \\ b_{-\mu} \end{pmatrix}$$

Parametric conversion (photon pair creation)



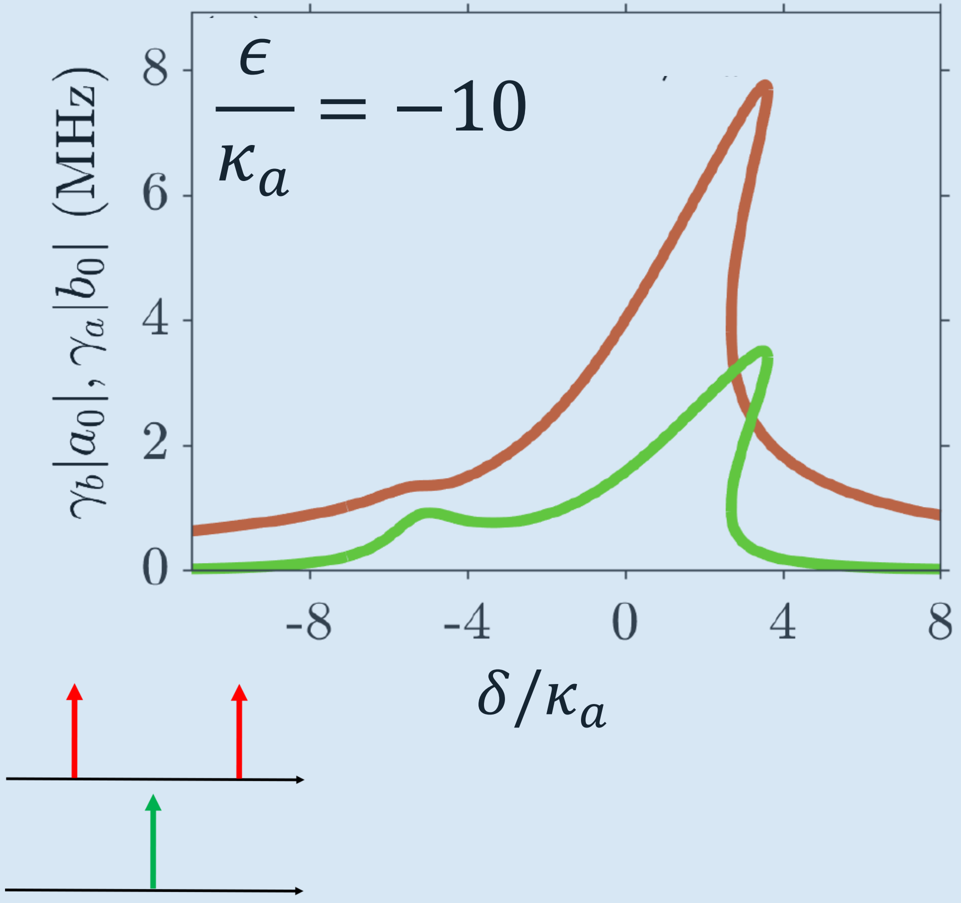
$$\hbar\omega_{\mu a} + \hbar\omega_{-\mu a} = \hbar 2\omega_p$$



$$\epsilon_\mu = \epsilon + \mu(D_{1a} - D_{1b}) \approx 0$$

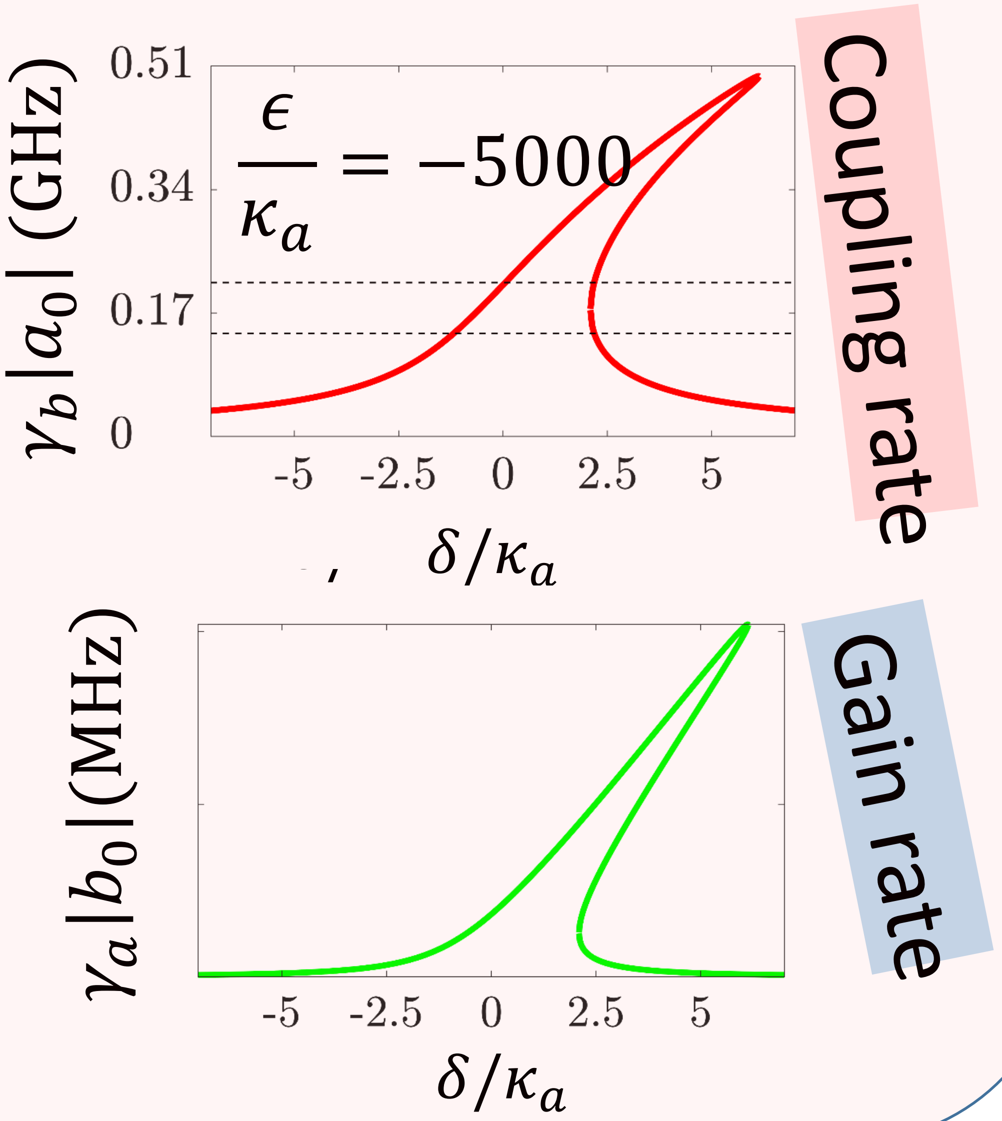
“Parametric” resonator

$$\mu|D_{1a} - D_{1b}| \gg |\varepsilon|$$

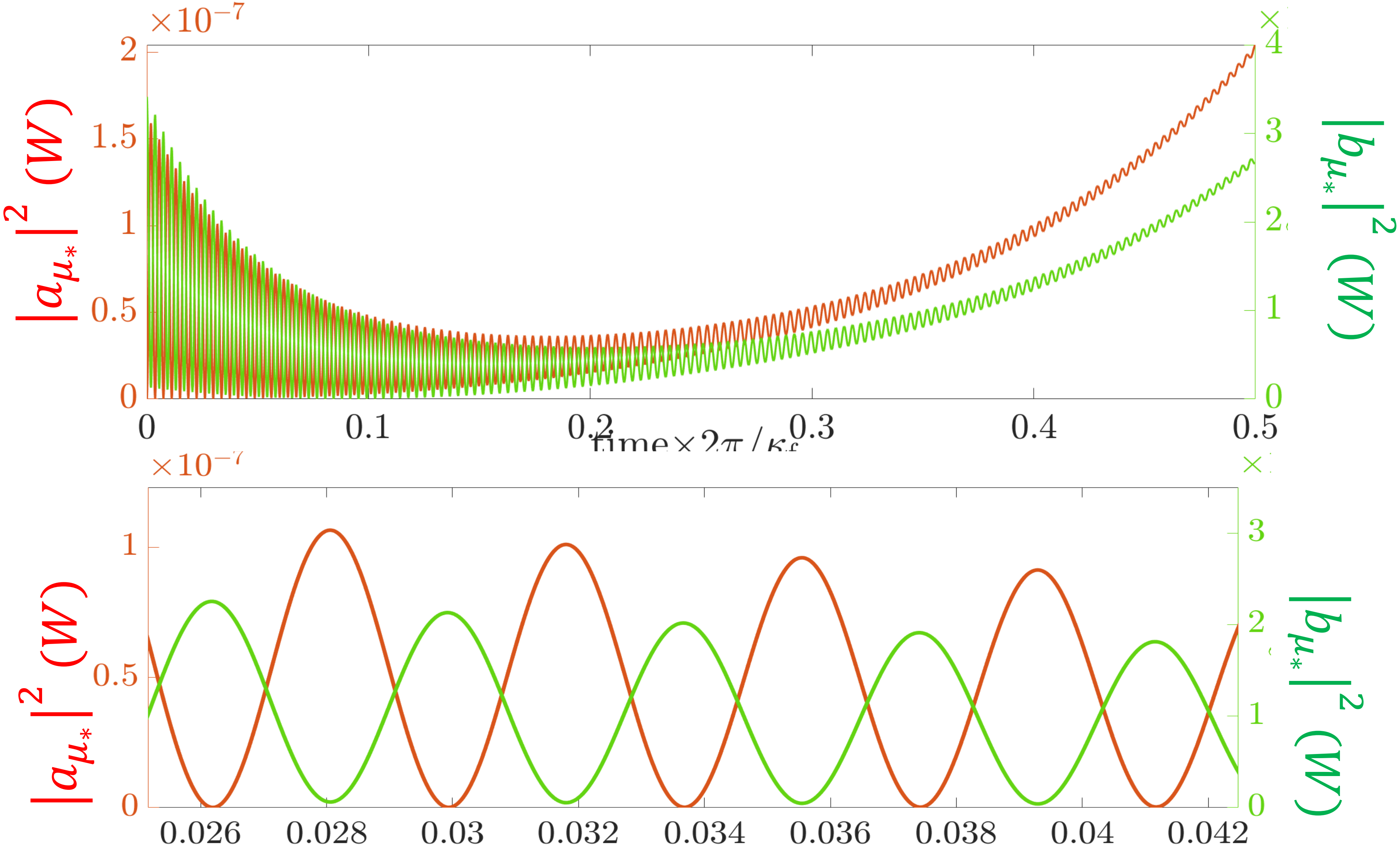


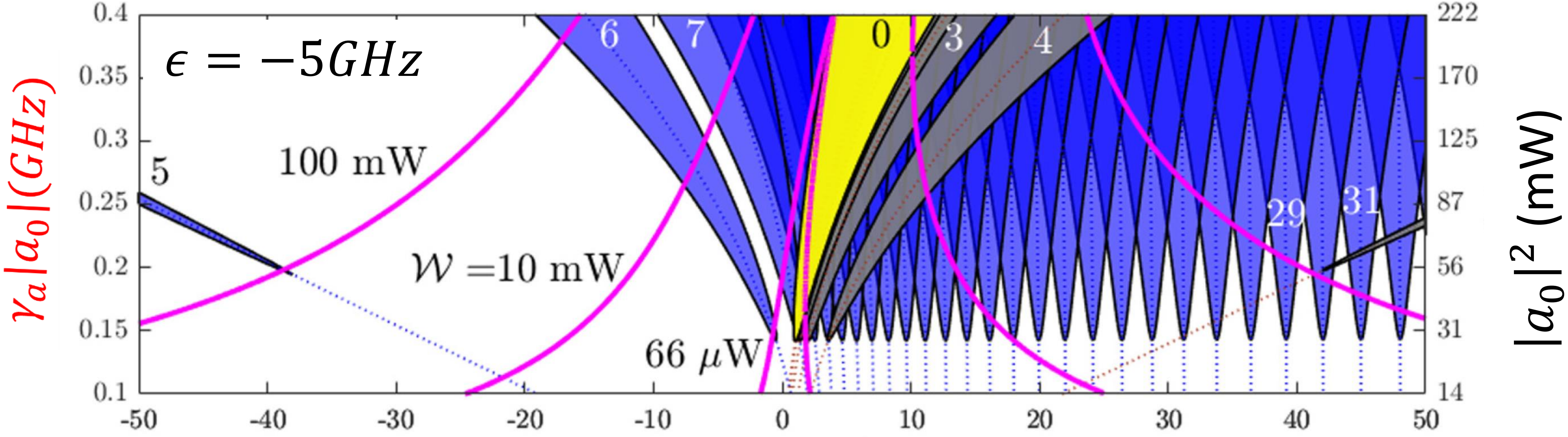
“Rabi” resonator

$$\mu|D_{1a} - D_{1b}| \sim |\varepsilon|$$

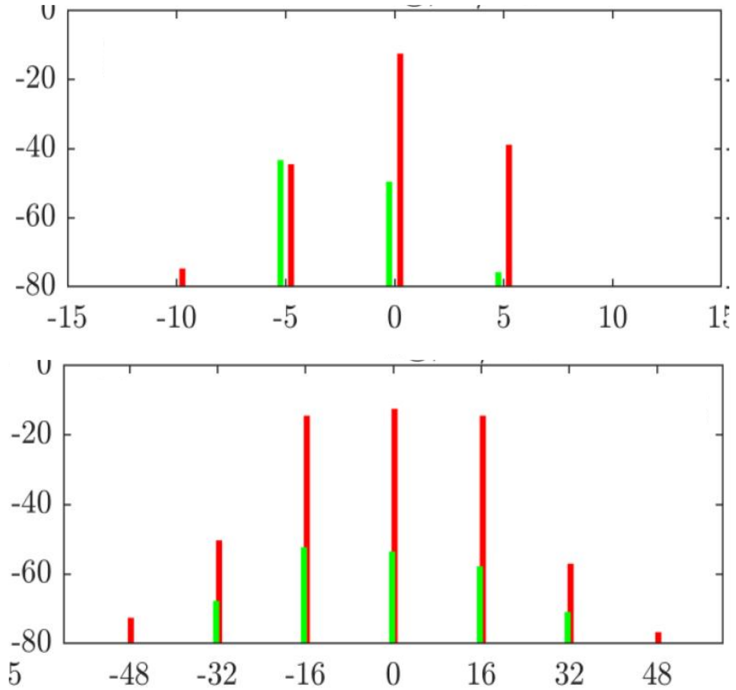


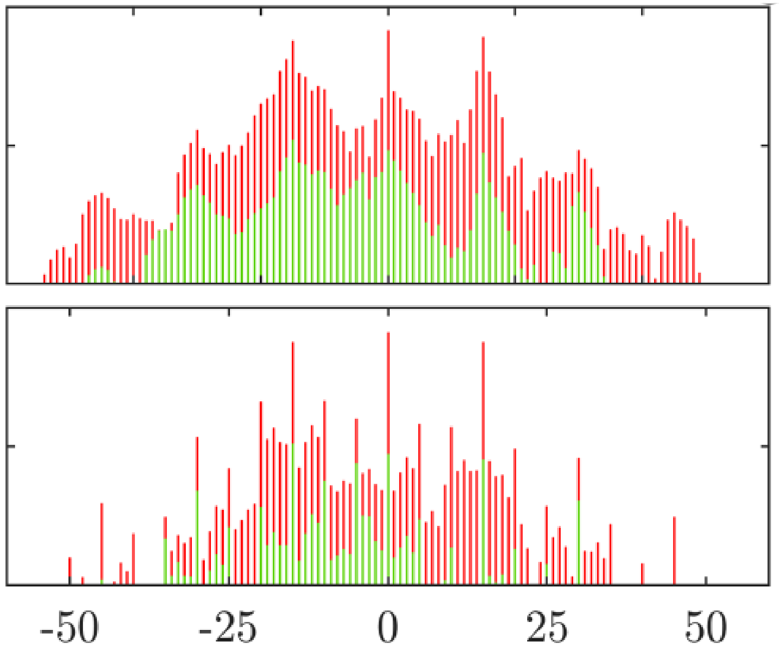
Rabi flops vs parametric gain (Rabi resonator)





δ/κ_a





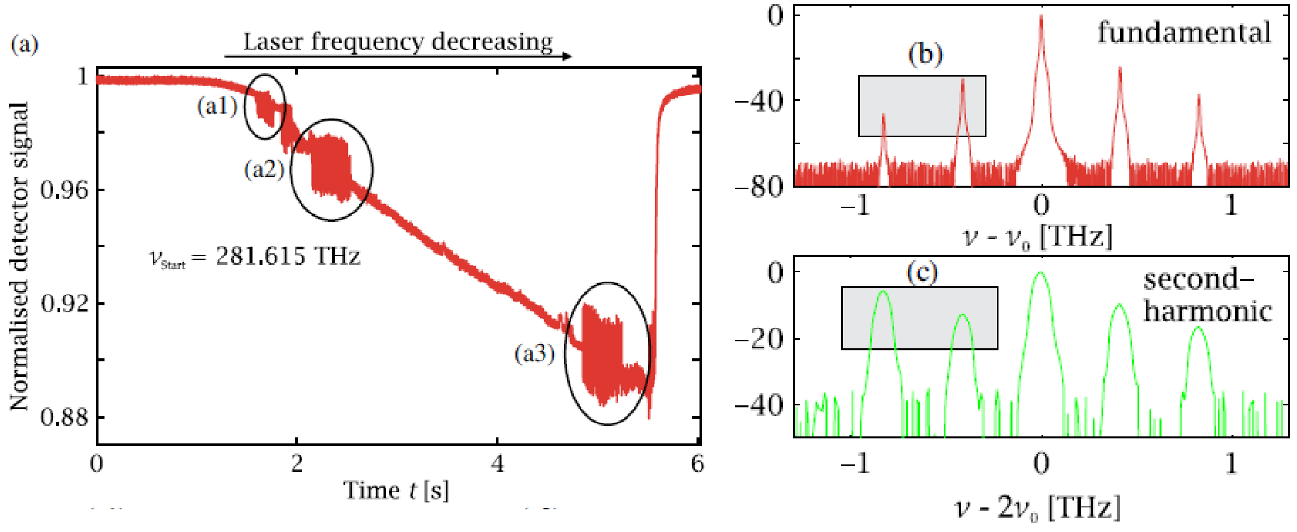
Puzyrev..., PRA 104, 013520 (2021)

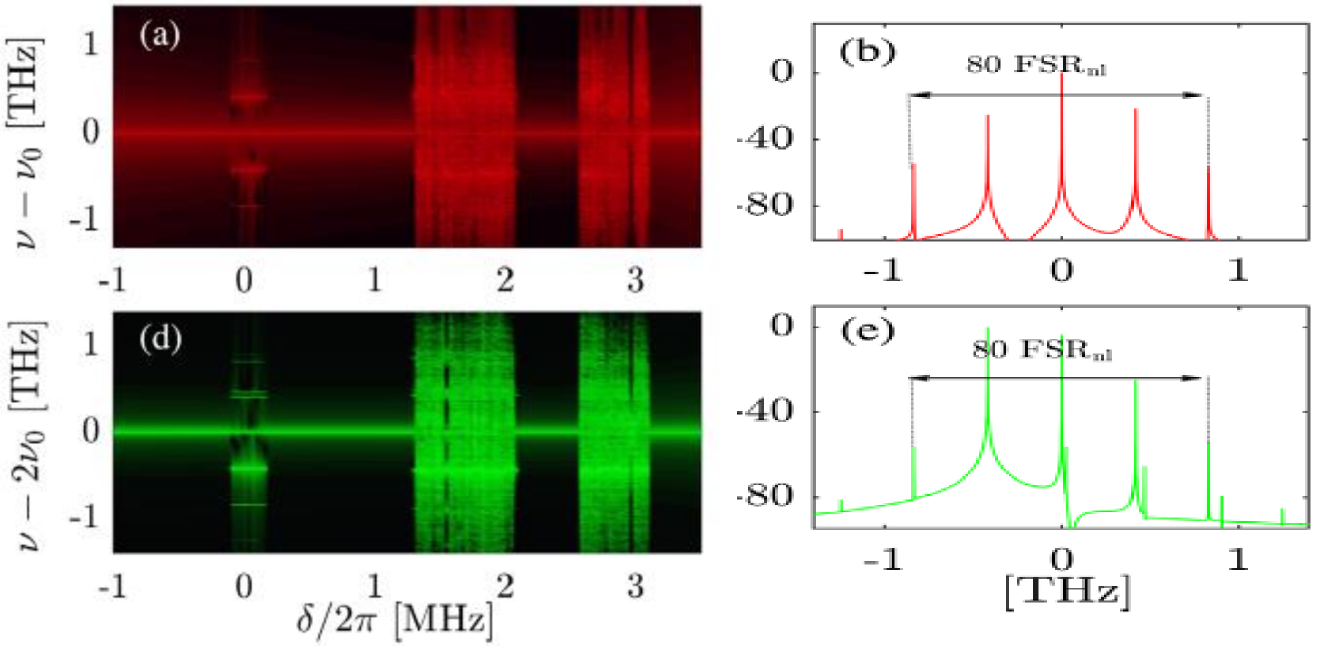
Model to measurements (Freiburg) comparison

Szabados ..., PRL 124, 203902 (2020)
Amiune ..., Optics Express 29 (25), 41378 (2022)
Amiune ..., arXiv preprint arXiv:2205.12776 (2022)

$LiNbO_3$

500-1000nm

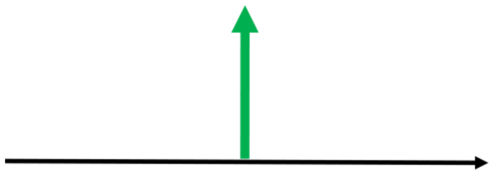




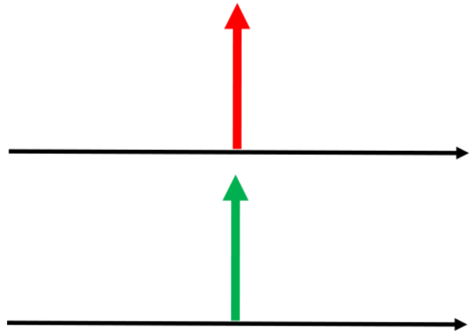
Parametric resonator: OPO or SHG

$$\mu|D_{1a} - D_{1b}| \gg |\varepsilon|$$

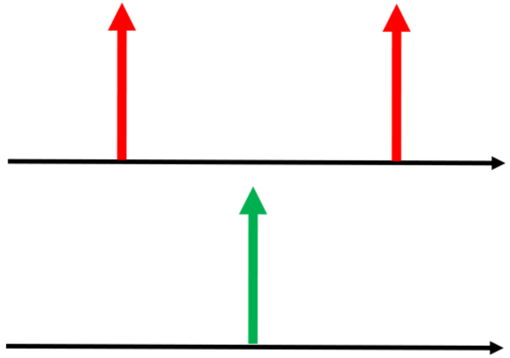
(i) no-OPO state:



(ii) degenerate OPO state:



(iii) non-degenerate OPO states:



$$i\partial_t a_0 = \kappa_a \Delta_{0a} a_0 - \gamma_a b_0 a_0^*,$$

$$i\partial_t a_\mu = \kappa_a \Delta_{\mu a} a_\mu - \gamma_a b_0 a_{-\mu}^*, \;\; \mu \neq 0,$$

$$i\partial_t a_{-\mu} = \kappa_a \Delta_{\mu a} a_{-\mu} - \gamma_a b_0 a_\mu^*,$$

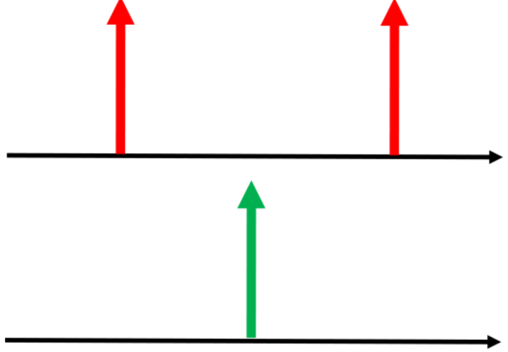
$$i\partial_t b_0 = \kappa_b \Delta_{0b} b_0 + \frac{i\kappa_b}{2}\mathcal{H} - \gamma_b a_0^2 - 2\gamma_b \sum_{\mu_1>0} a_{\mu_1} a_{-\mu_1}$$

$$A = a_\mu e^{i\mu\vartheta + i\omega_\mu t} + a_{-\mu} e^{-i\mu\vartheta + i\omega_{-\mu} t},$$

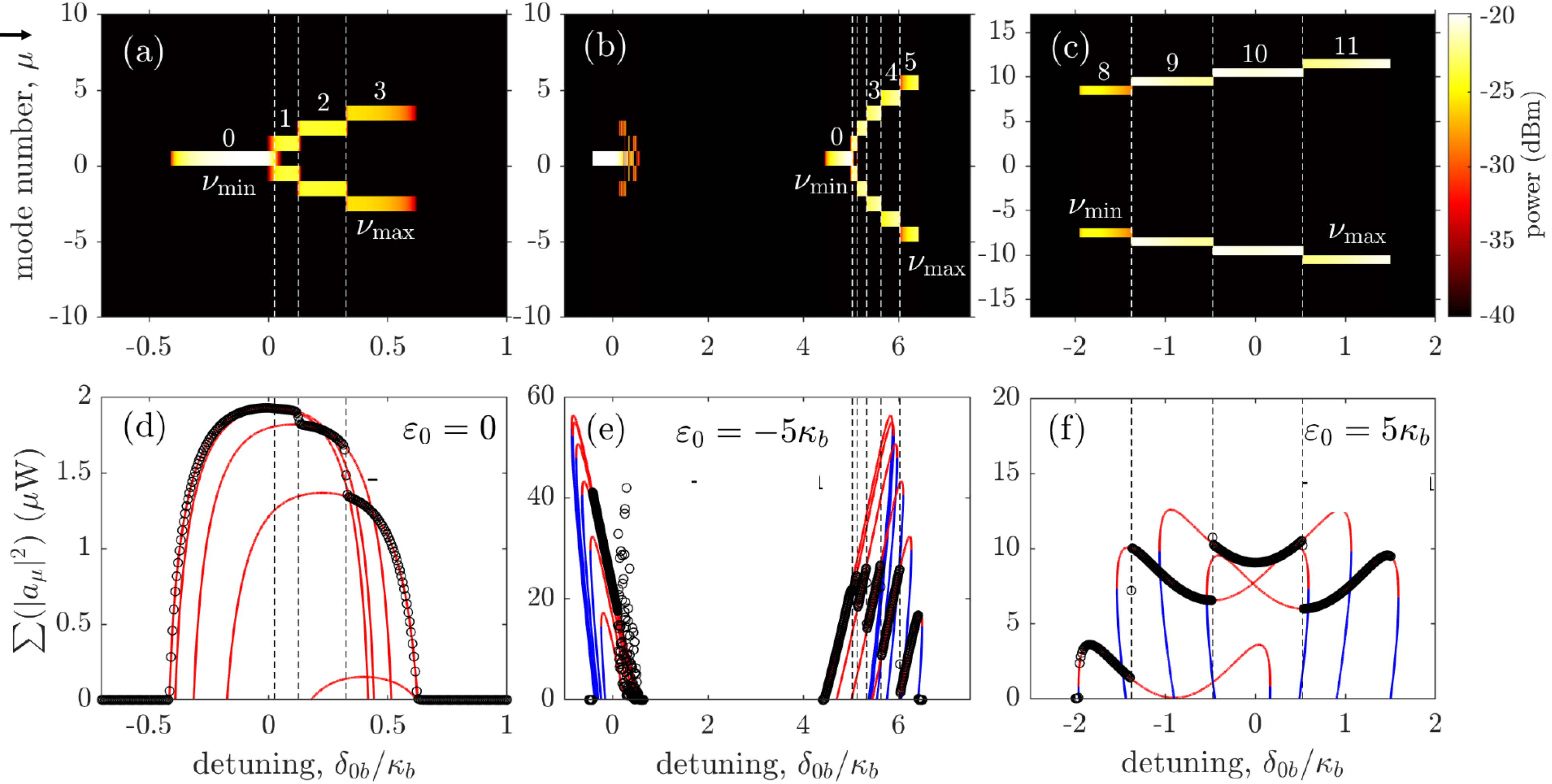
$$B = b_0$$

Non-degenerate OPO states can now be found, and even their linear stability can be analysed explicitly

Puzyrev, DVS, Comms Phys **5**, 138 (2022)

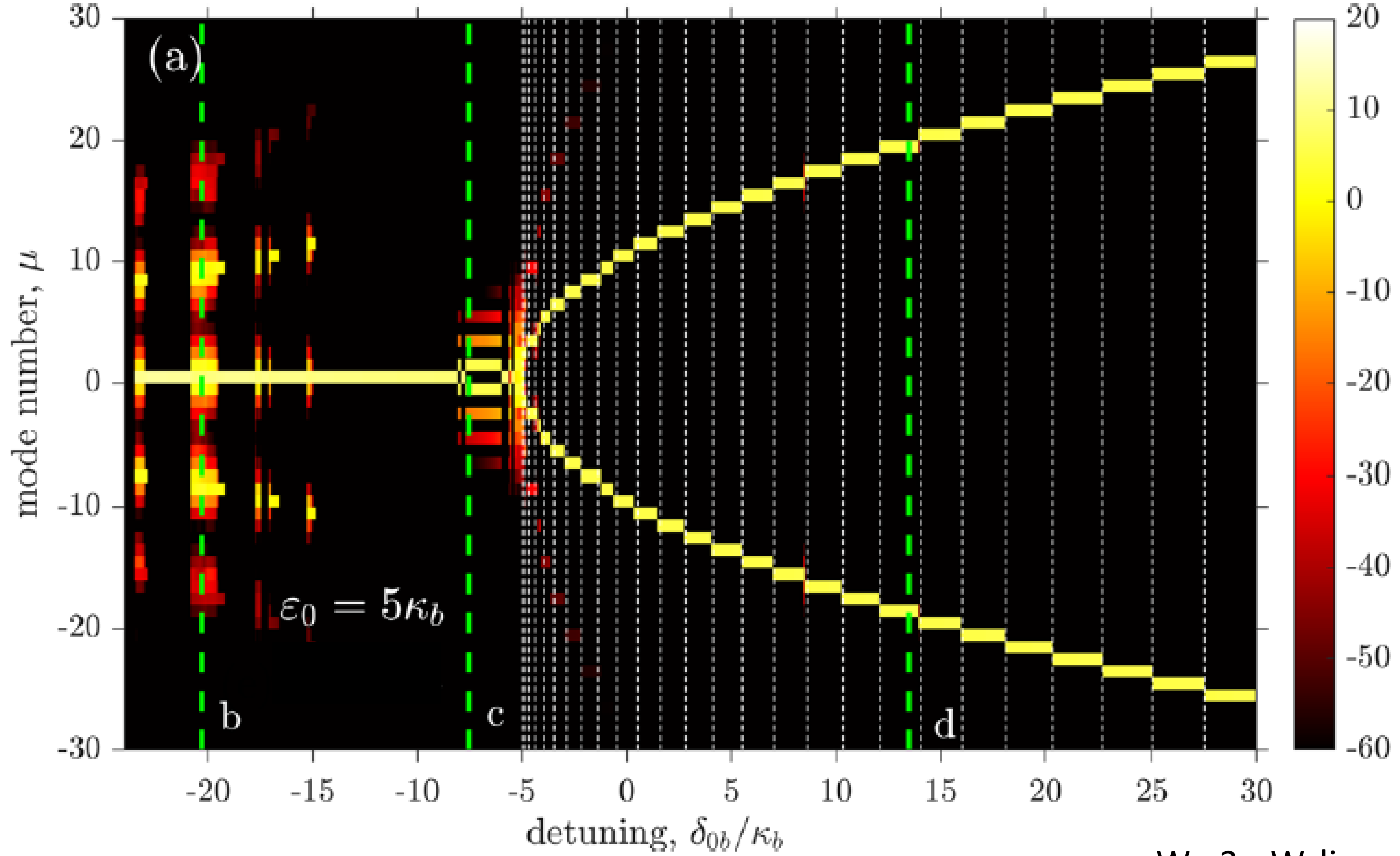


OPO



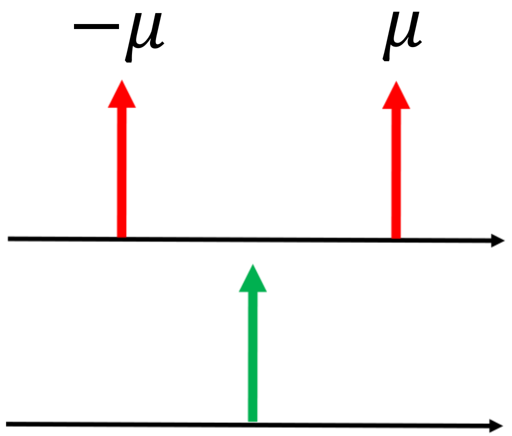
W= 300nW

*Puzyrev, DVS, Comms Phys **5**, 138 (2022)*

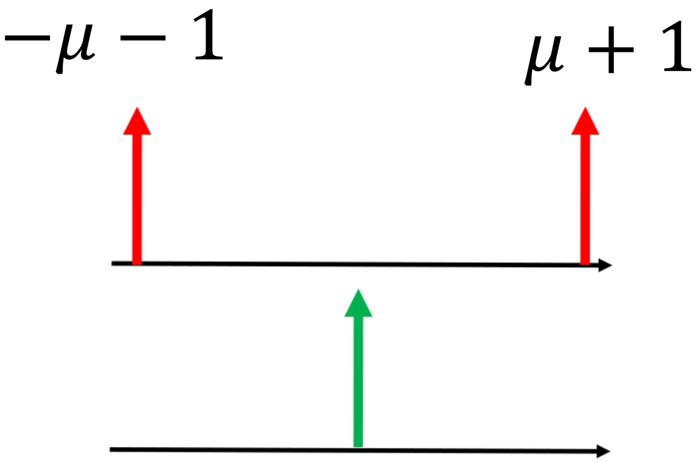


W= 3mW, linewidth 1MHz

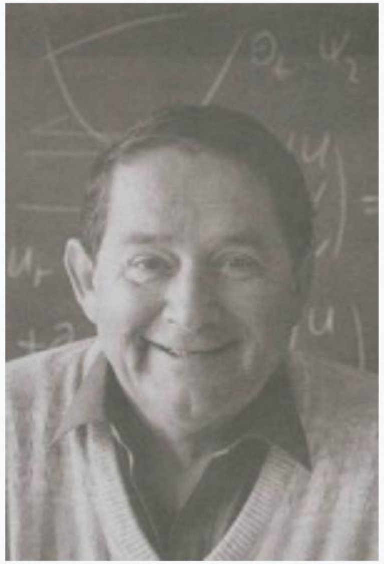
Parametric phase-matching parabola but with steps



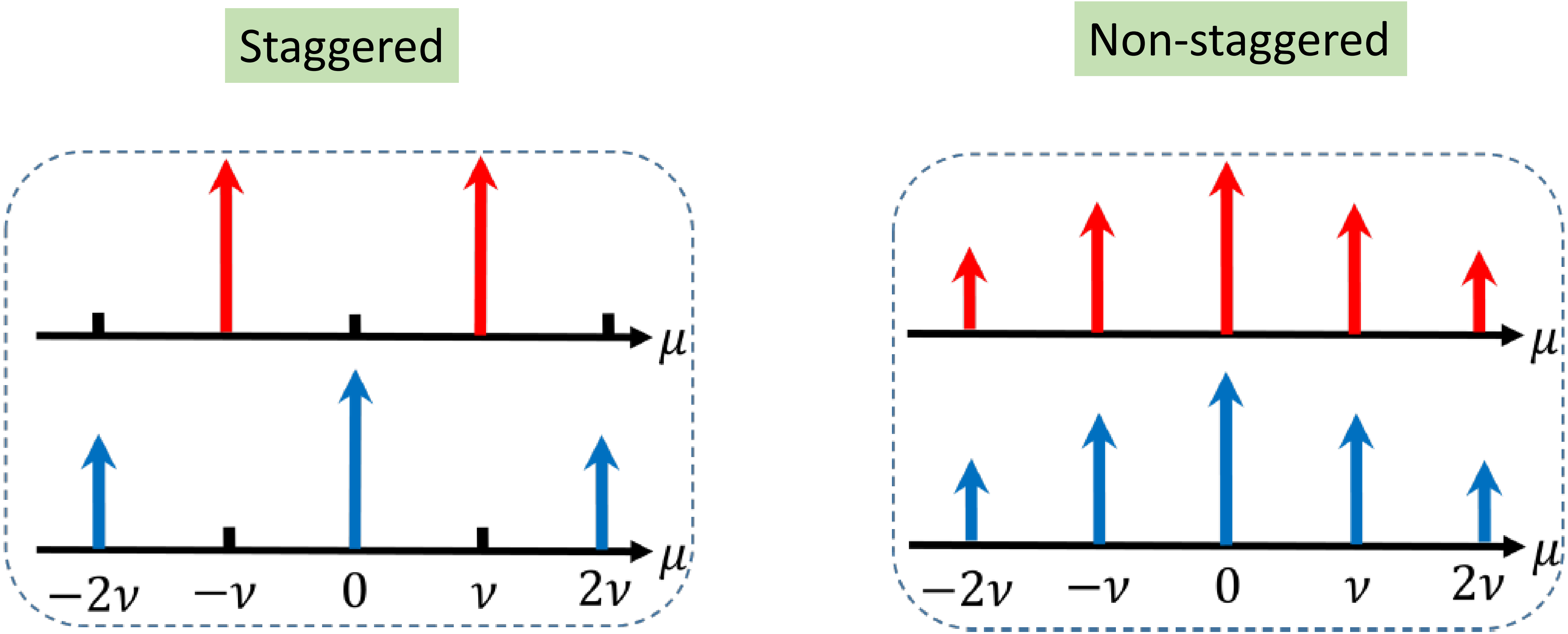
Eckhaus instability



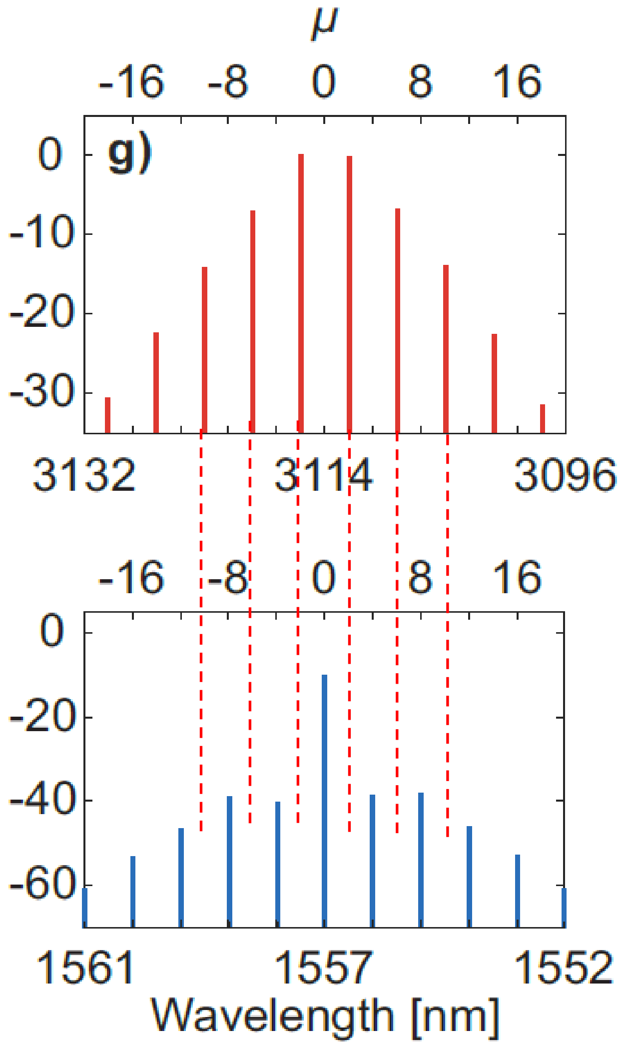
Wiktor Eckhaus

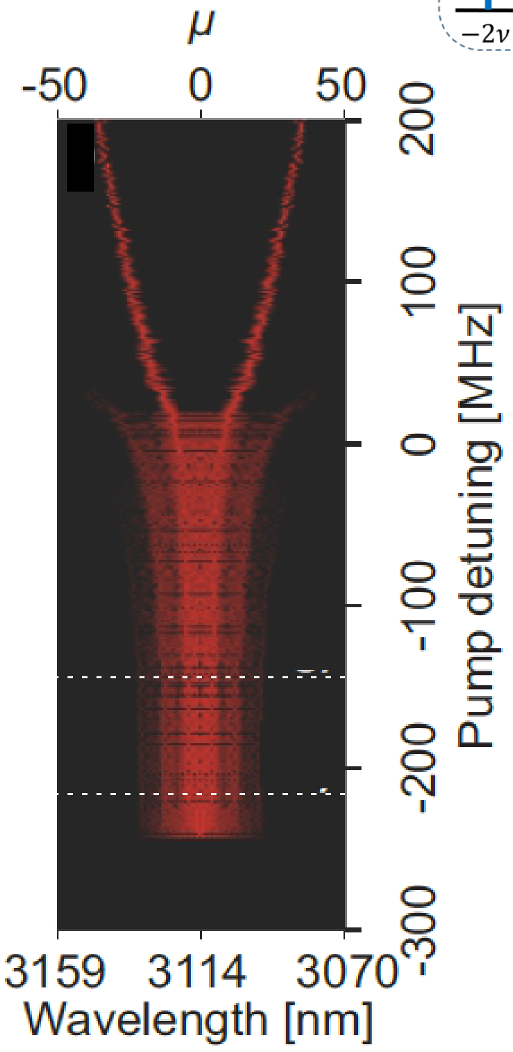


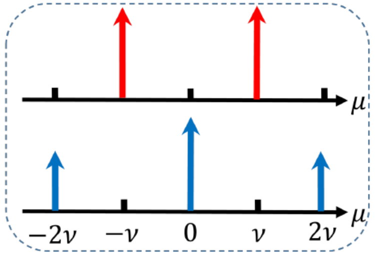
Born	28 June 1930[1] Stanisławów, Poland
Died	1 October 2000 (aged 70) Amstelveen,[2] Netherlands


Staggered
Non-staggered
μ
μ
μ
μ
−2ν
−ν
0
ν
2ν
−2ν
−ν
0
ν
2ν

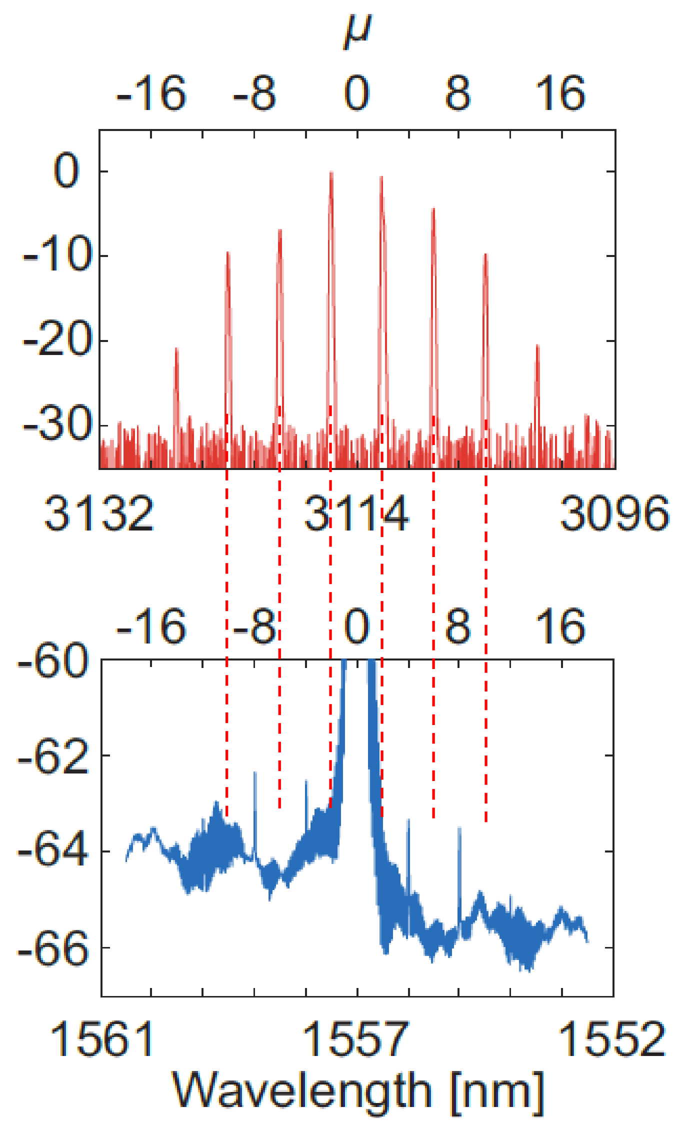
modelling





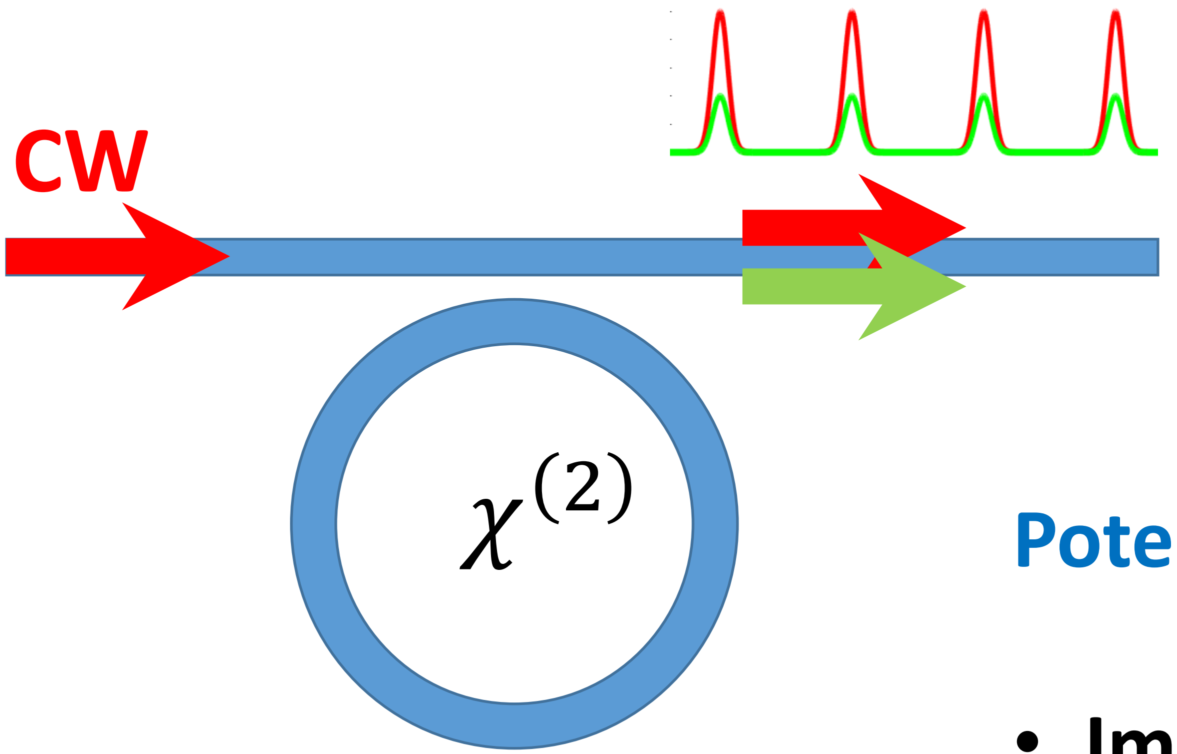


experiment



Puzyrev, DVS, Comms Phys **5**, 138 (2022)
Amiune ..., arXiv preprint arXiv:2205.12776 (2022): CdSiP

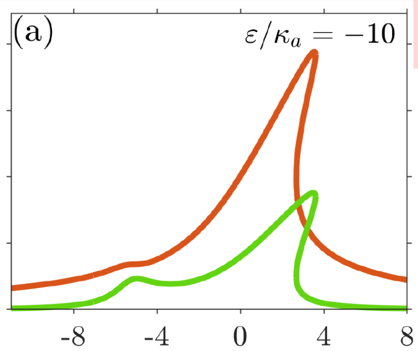
Chi-2 soliton modelocking



Potential benefits:

- **Immediate modelocking across octave**
- **Independence of the dispersion signs**
- **Low excitation thresholds**
- **Near Mid IR coverage**

Dressed-states formalism: Pockels vs cascaded-Kerr



$$\sqrt{\epsilon_\mu^2 + \gamma_a^2 |a_0|^2}$$

Phase-matched or near-phase-matched modes experience Pockels nonlinearity

$$\sqrt{\epsilon_\mu^2 \overset{<<}{+} \underline{\gamma_a^2 |a_0|^2}}$$

$$n = n_0 + n_{\mathrm{P}} |a_0|$$

Modes far from phase matching experience effective (cascaded) Kerr nonlinearity

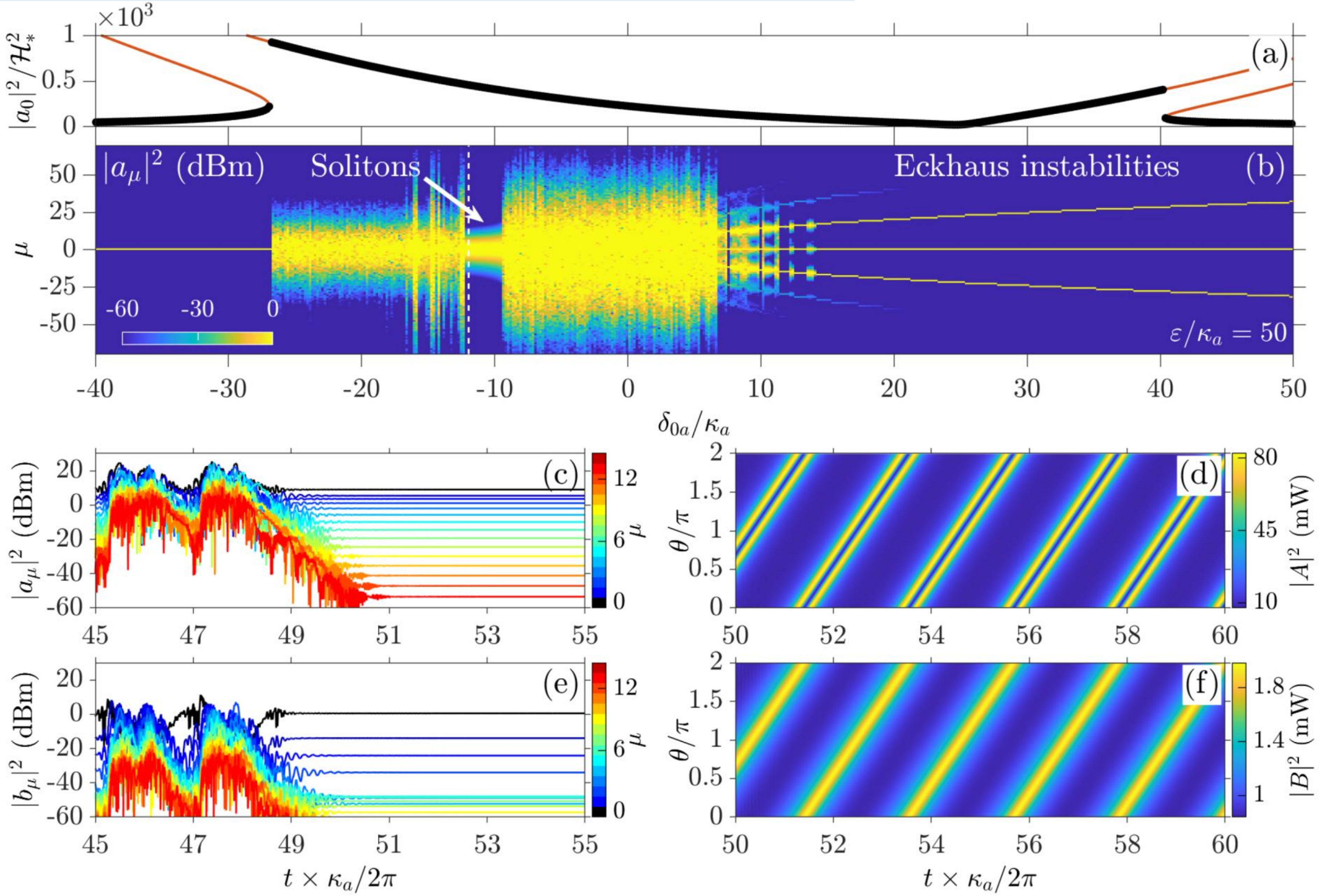
$$\sqrt{\underline{\epsilon_\mu^2} \overset{>>}{+} \gamma_a^2 |a_0|^2}$$

$$n = n_0 + n_{\mathrm{eK}} |a_0|^2$$

Puzyrev..., PRA 104, 013520 (2021)

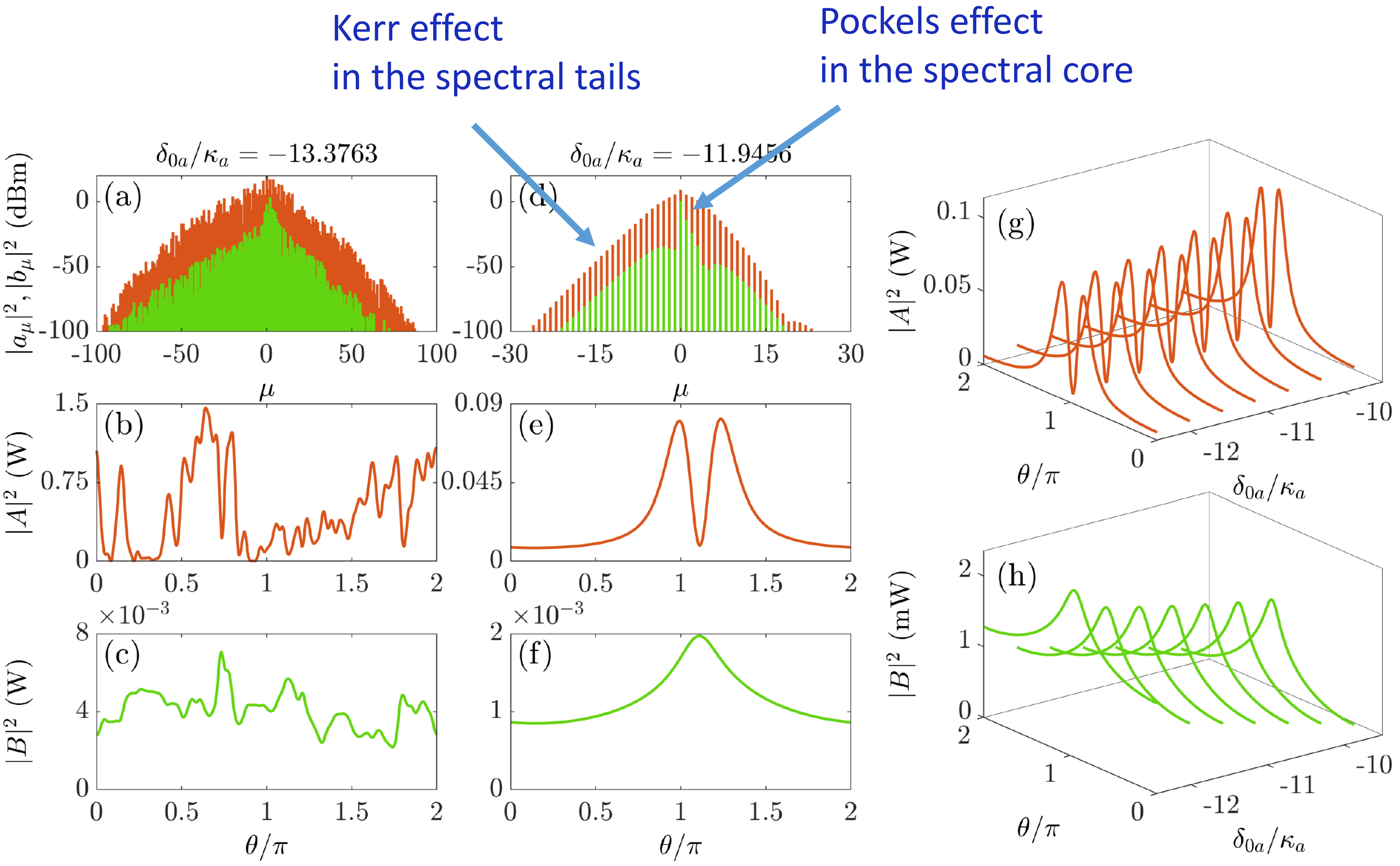
Solitons in "parametric" SHG-resonator

$$\mu|D_{1a} - D_{1b}| \gg |\varepsilon|$$



4mW

Pankratov, DVS (under consideration)



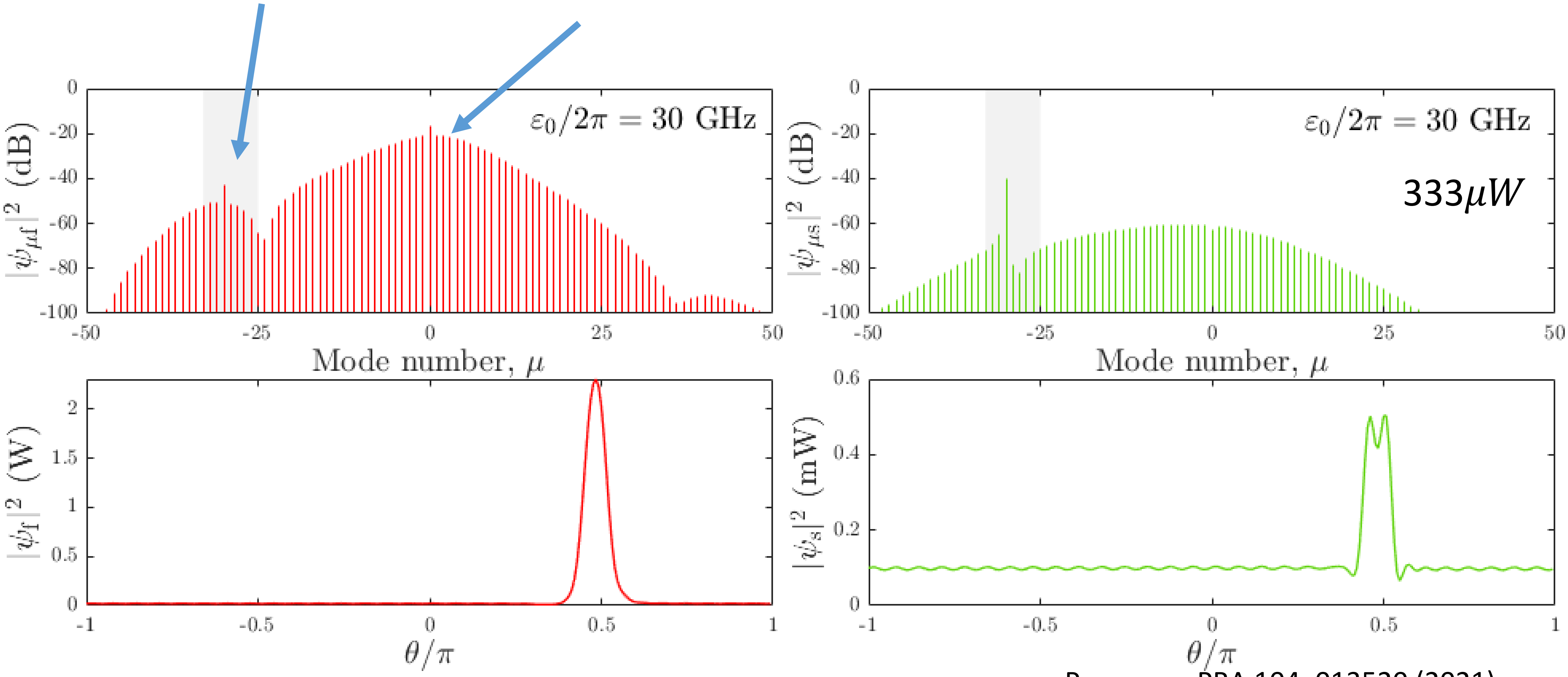
Kerr effect
in the spectral tails
Pockels effect
in the spectral core
$\delta_{0a}/\kappa_a = -13.3763$
$\delta_{0a}/\kappa_a = -11.9456$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
$|a_\mu|^2, |b_\mu|^2$ (dBm)
$|A|^2$ (W)
$|B|^2$ (W)
$|B|^2$ (mW)
μ
θ/π
δ_{0a}/κ_a
$\times 10^{-3}$

SHG solitons in “Rabi” resonator

$$\mu|D_{1a} - D_{1b}| \sim |\varepsilon|$$

Pockels effect
in the spectral tails

Kerr effect
in the spectral core



Puzyrev..., PRA 104, 013520 (2021)

Phase and group velocity matching.
All modes experience Pockels effect.

$$D_{1a} - D_{1b} \approx \epsilon \approx 0$$

$$i\partial_t A = \delta A - \frac{1}{2} D_{2a} \partial_\theta^2 A - \gamma_a B A^* - \frac{i\kappa_a}{2} A$$

$$i\partial_t B = (2\delta - \epsilon) B - \frac{1}{2} D_{2b} \partial_\theta^2 B - \gamma_b A^2 - \frac{i\kappa_b}{2} (B - H)$$

$$A = c\psi(\theta) e^{\frac{i\phi}{2}}$$

$$B = \psi(\theta) e^{i\phi} - iH_0$$

$$\psi = \frac{3\delta}{2\gamma_b} \operatorname{sech}^2\left(\theta \sqrt{\frac{\delta}{2D_{2b}}}\right)$$

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$$i\partial_t A = \delta A - \frac{1}{2} D_2 \partial_\theta^2 A - \gamma |A|^2 A - \frac{i\kappa}{2}(A - H)$$

$$A = \psi(\theta) - iH_0 \qquad \psi = \sqrt{\frac{2\delta}{\gamma}}\, sech\left(\theta \sqrt{\frac{\delta}{2D_2}}\right)$$


Thank you for attention