Mathematics related to a model for moving ants

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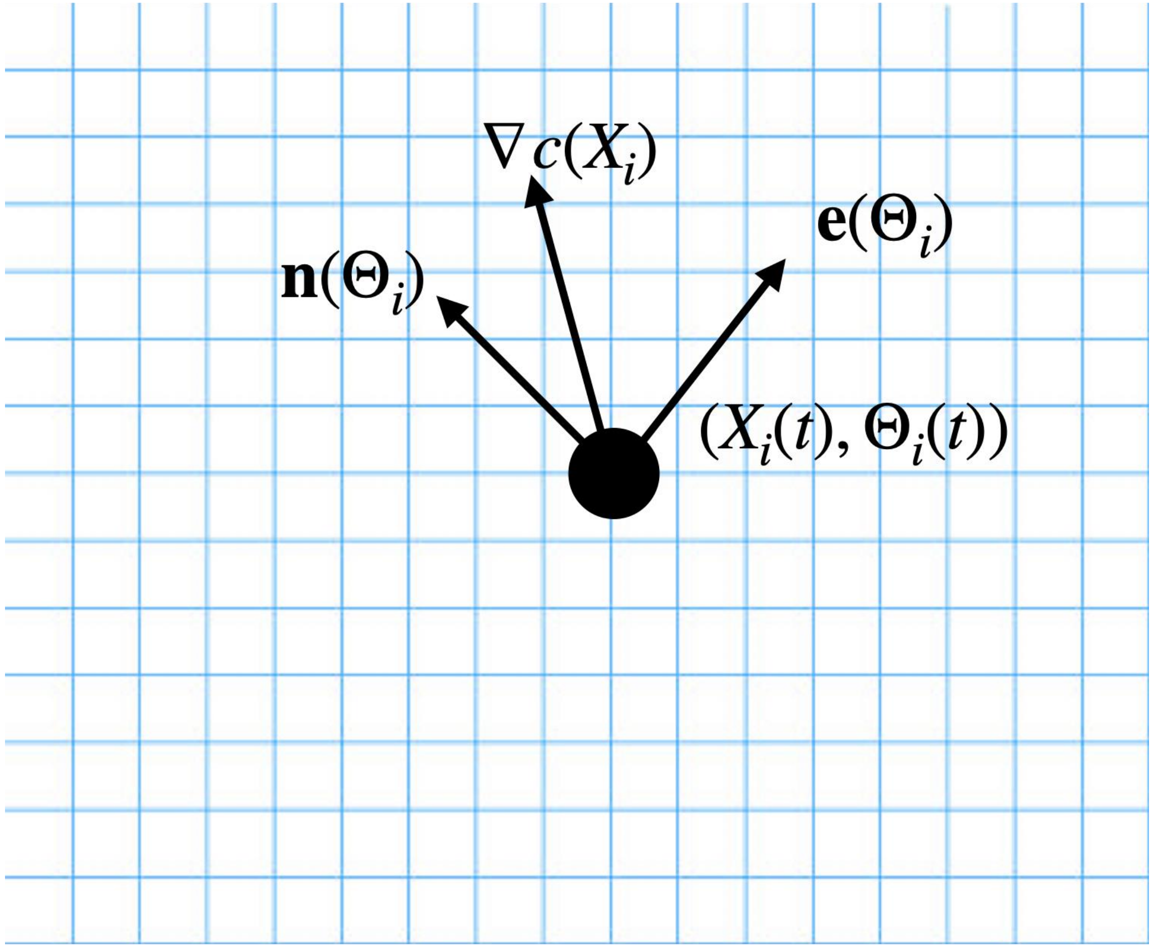
- What do we want to study?
    - What is a minimal mathematical model that captures ants forming lanes or general collective phenomena (also for agents similar to ants (birds, pedestrians, etc.))?
- How to study this?
    - Interacting particle systems
    - The associated mean field limit

- Active interacting particle model
  - Particles that consume energy to maintain a constant speed + an alignment mechanism + some random motion
  - A stochastic particle system (equations of motion):

$$\mathrm{d}X_i = v_0 \mathbf{e}(\Theta_i)\mathrm{d}t + \sqrt{2D_T}\mathrm{d}W_i$$

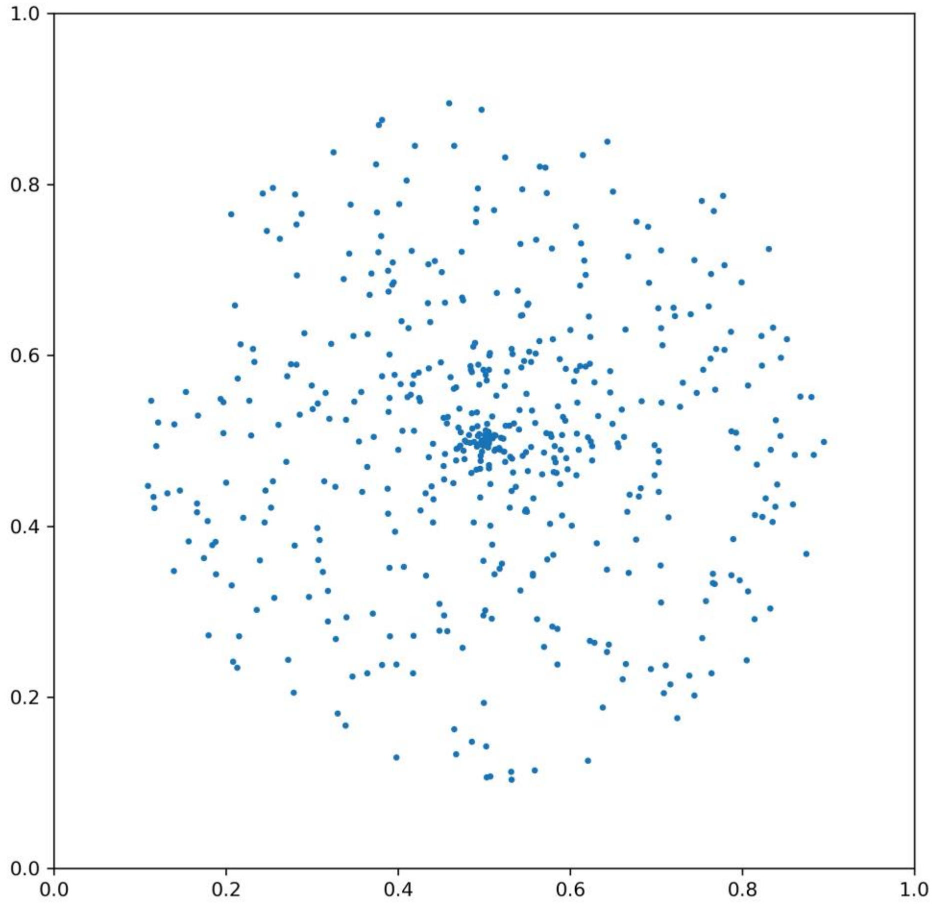
$$\gamma(\mathbf{n}(\Theta_i) \cdot \nabla c(X_i))$$

$$N \sum_{i=1}$$



$$\gamma(\mathbf{n}(\Theta_i) \cdot \nabla c(X_i))$$

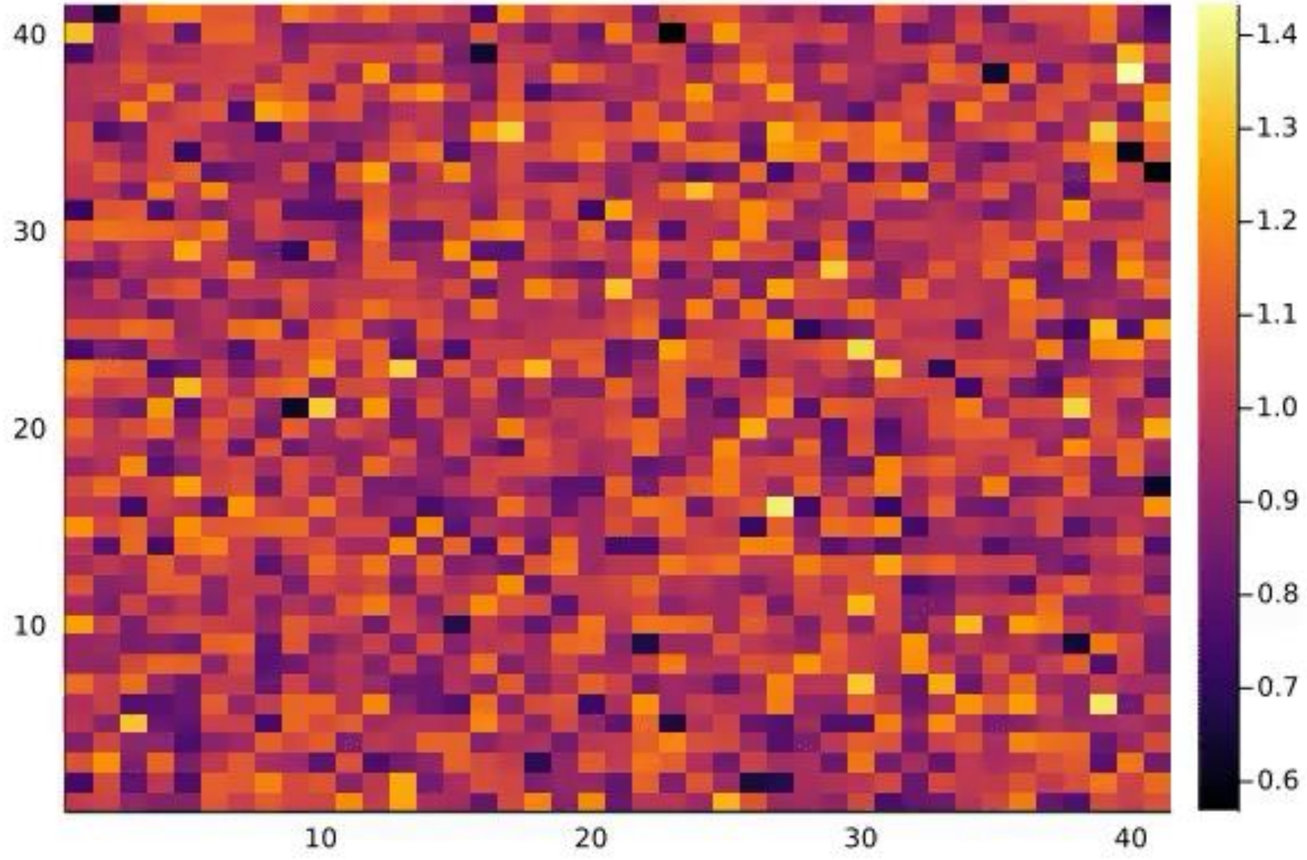
‘Ant spots’



How to study the ‘average’ behaviour of this model?

- The mean field limit partial differential equation
- “The probability $f(t, x, \theta)$ of finding a particle at a certain position x and angle θ at time t”
- Then f is a solution of the following PDE for our model

$$
\begin{aligned}
\partial_t f &= \nabla_x \cdot [D_T \nabla_x f - v_0 \mathbf{e}_\theta f] + \partial_\theta [D_R \partial_\theta f - \gamma(\mathbf{n}_\theta \cdot \nabla c) f] \\
0 &= D\Delta c - \alpha c + \eta\rho.
\end{aligned}
$$

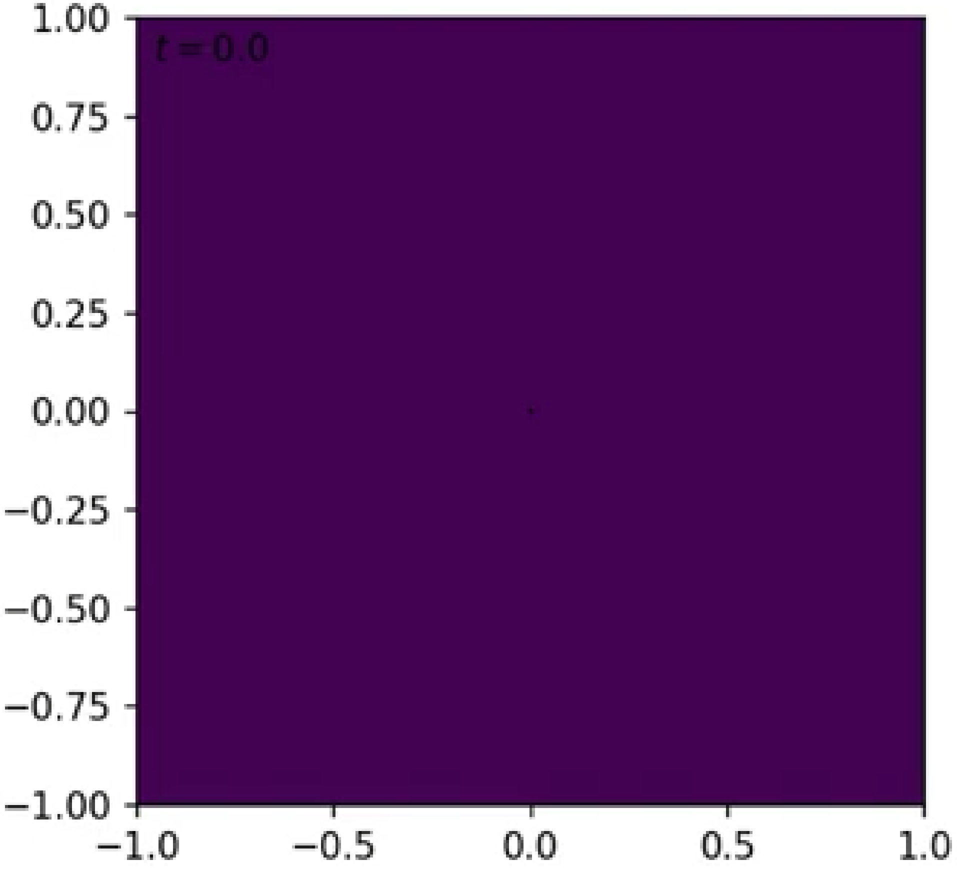


40
30
20
10
10
20
30
40
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6

What can we show mathematically for this PDE?

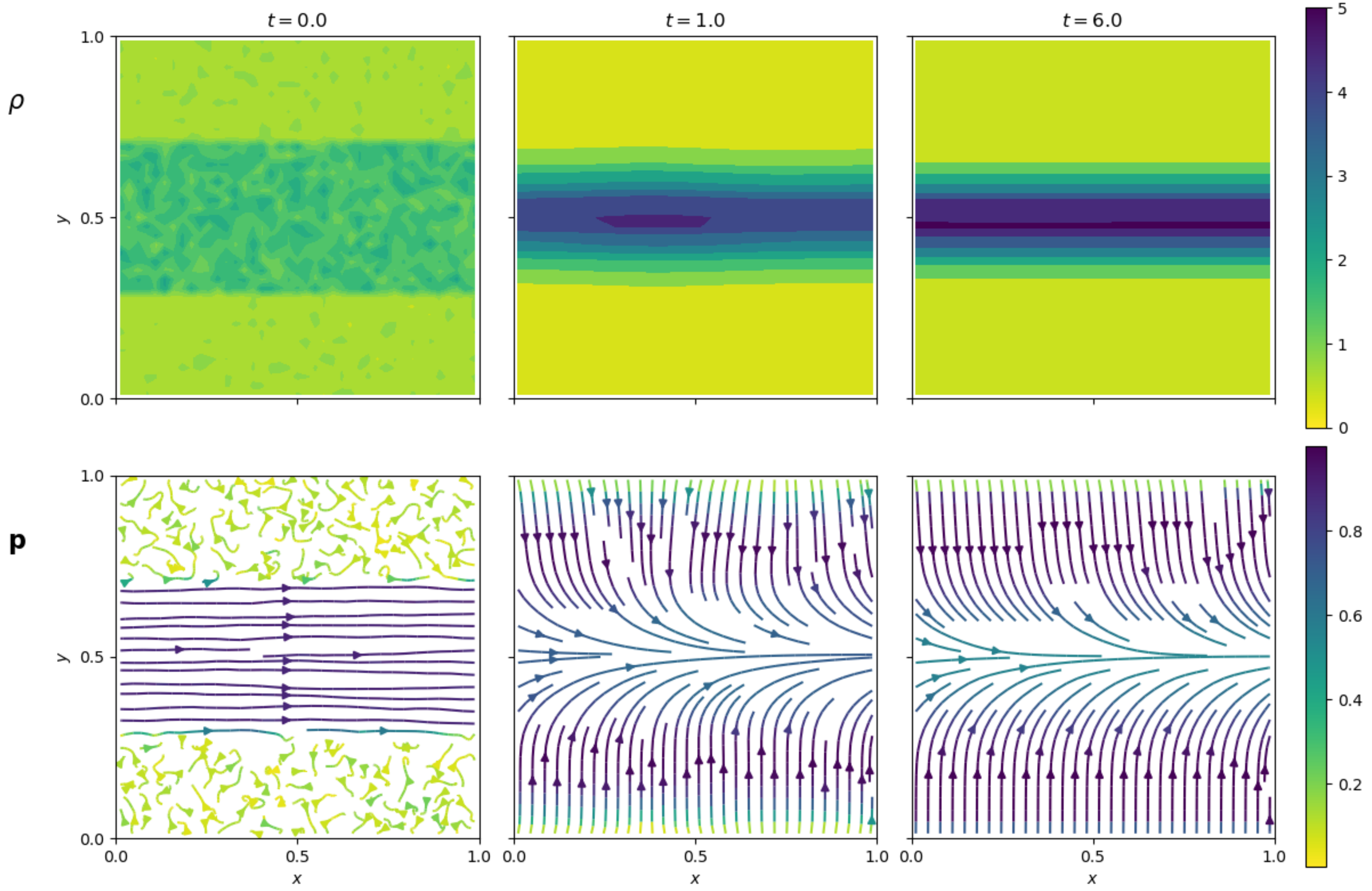
- The model is unstable for strong enough alignment
  - A linear instability exists
- The model is nonlinearly stable around a constant state for small alignment and small initial data
- There is a uniform bound on the solutions, global in time (no blow up)

A more complicated model



$$c(x + \lambda \mathbf{e}_\theta)$$

Simulations support lane formation



Prospects

- Can we show some mathematical results for the more complicated model?
  - Phase transitions, explicit solutions, stability, etc.

Come see my poster!

Thank you!