Liquid Metal Batteries
November 15–16, 2022, Isaac Newton Institute, Cambridge, UK

Batteries, dynamos, and all that

Frank Stefani

With thanks to

Carolina Duczek, Peter Jüstel, Gerrit Horstmann, Ke Liu, Paolo Personnettaz, Sebastian Röhrborn, Martin Seilmayer, Norbert Weber and Tom Weier

HZDR

HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

How it began (for us…)

Elektrochemische Energiespeicher – Was könnte die Magnetohydrodynamik beitragen?

Tom Weier

27.01.2010

Forschungszentrum Dresden Rossendorf

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DRESDEN concept

HZDR

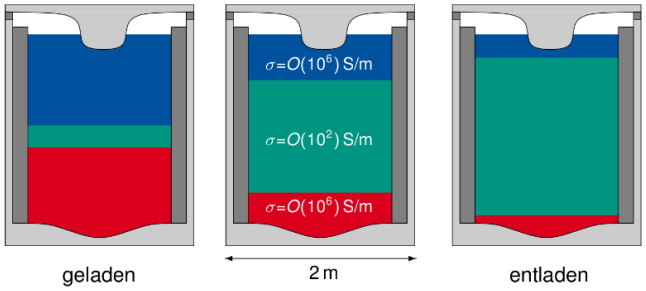
How it began (for us…)

Elektrochemische Energiespeicher – Was könnte die Magnetohydrodynamik beitragen?

Hochtemperaturbatterien

Die „Sadoway-Batterie“

Betriebstemperatur: ∼700°C
Stromdichte: 10. . . 50 kA/m^2



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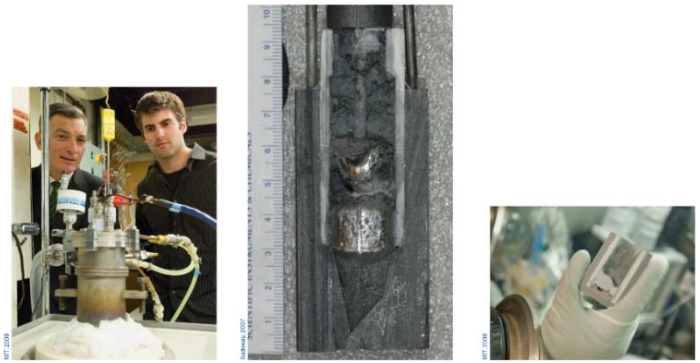
Hochtemperaturbatterien

Die „Sadoway-Batterie“



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DRESDEN concept

HZDR

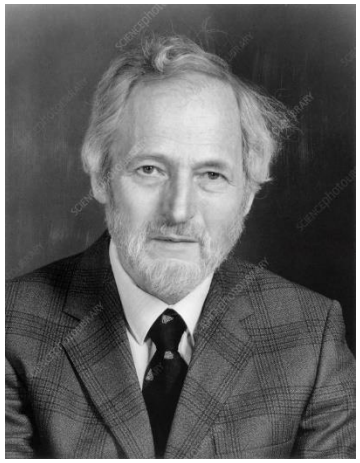
How it began (for us…)

- liquid-liquid interfaces
  fastest in all of electro
  - low activation over
- all-liquid construction
  reliance on solid-state
  - long service life
- all-liquid configuration is self-assembling
  - scalable at low cost

???

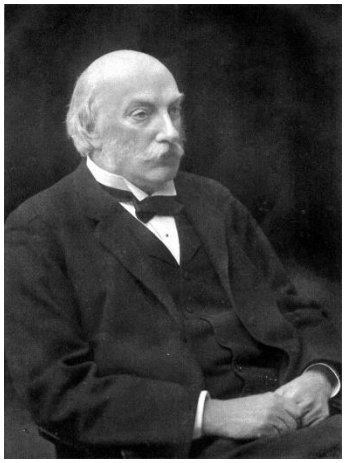
Cambridge

Roger John Tayler



Tayler Instability

Lord Rayleigh



(Rayleigh-Bénard) Convection

Peter Davidson

Electrovortex flow

Gordon Ogilvie

(Tidally forced) waves

Rossendorf

Norbert Weber

Peter Jüstel

Ke Liu

Gerrit Horstmann

Tayler Instability

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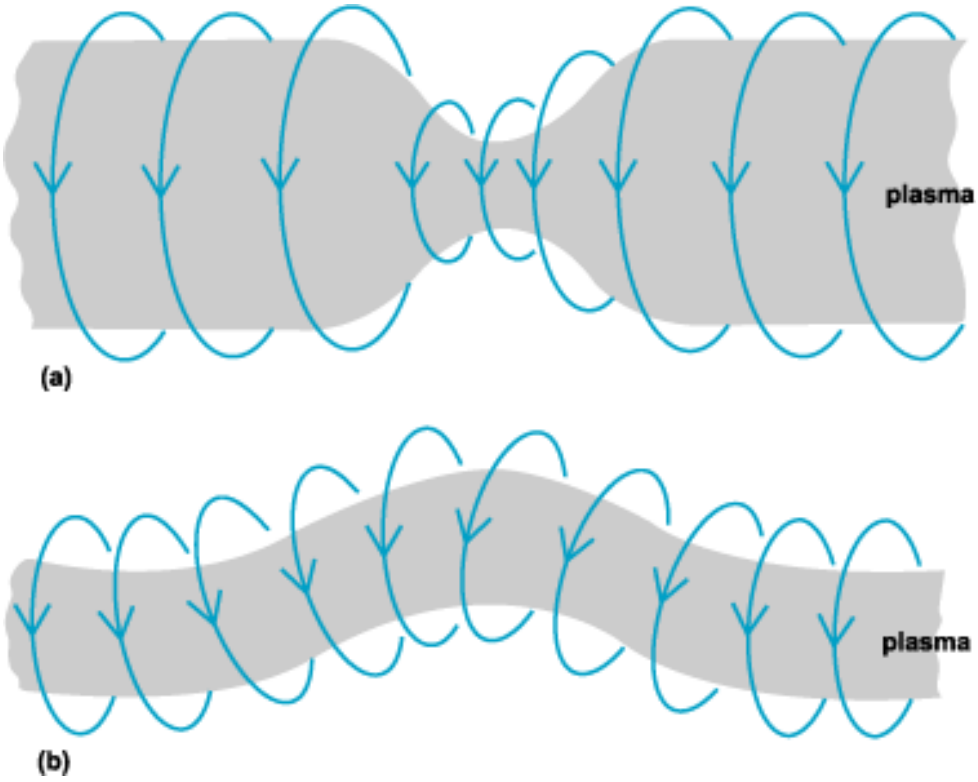
Instabilities in the z-pinch (plasma physics)



Sausage instability m=0: Ideal stability criterion (Michael 1954):

$$\frac{\partial}{\partial r}\left(\frac{B_\varphi(r)}{r}\right)^2 < 0$$

Kink instability m=1: Ideal stability criterion (Tayler 1973):

$$\frac{\partial}{\partial r}\left(r\ B_\varphi^2(r)\right) < 0$$

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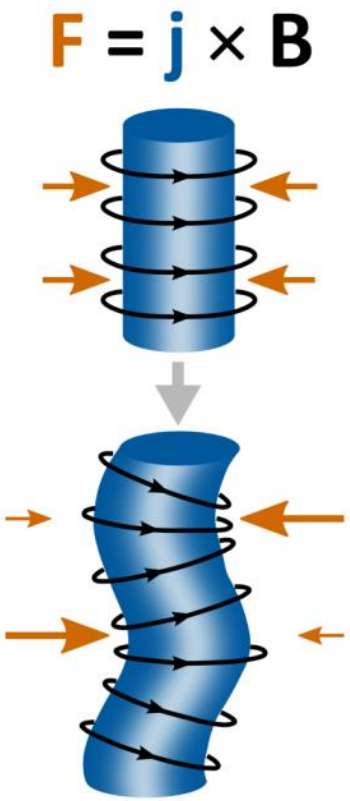
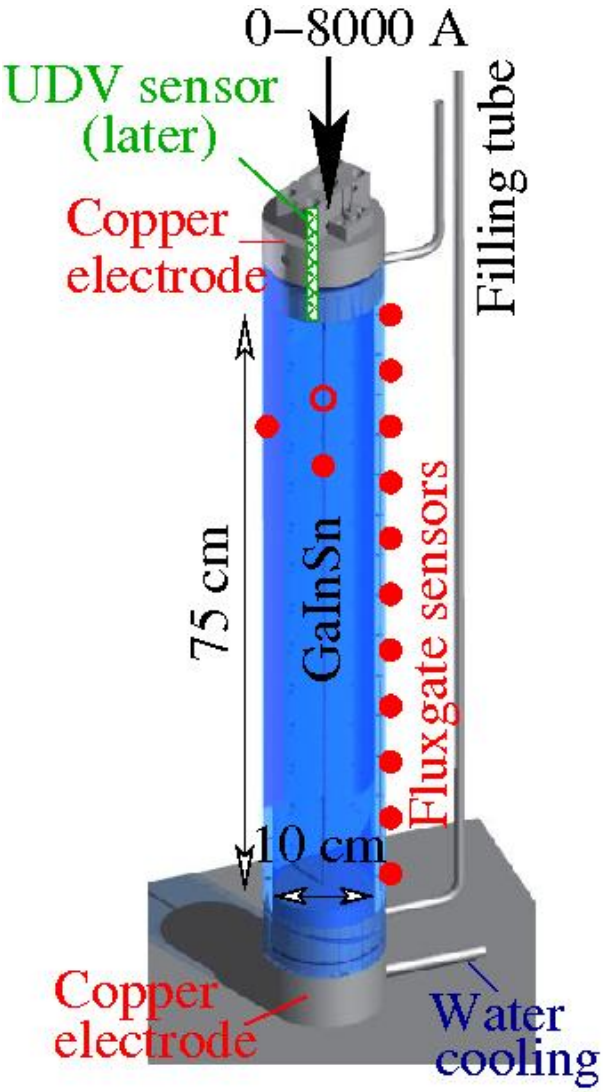
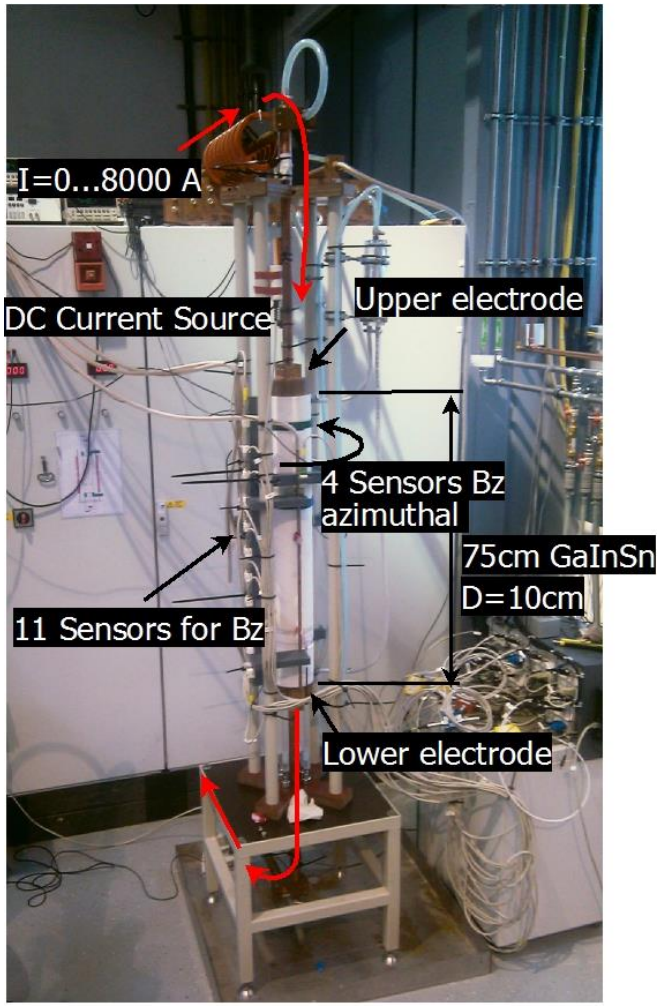
Tayler instability in the liquid metal lab with Pm=$\mu_0 \sigma \nu$ << 1







M. Seilmayer et al.: Phys. Rev. Lett. 108 (2012), 244501

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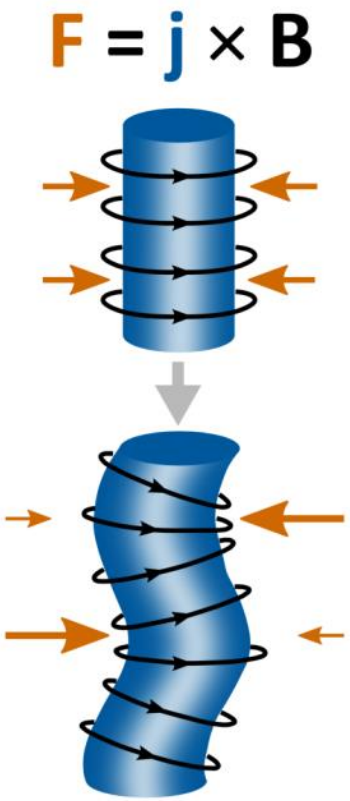
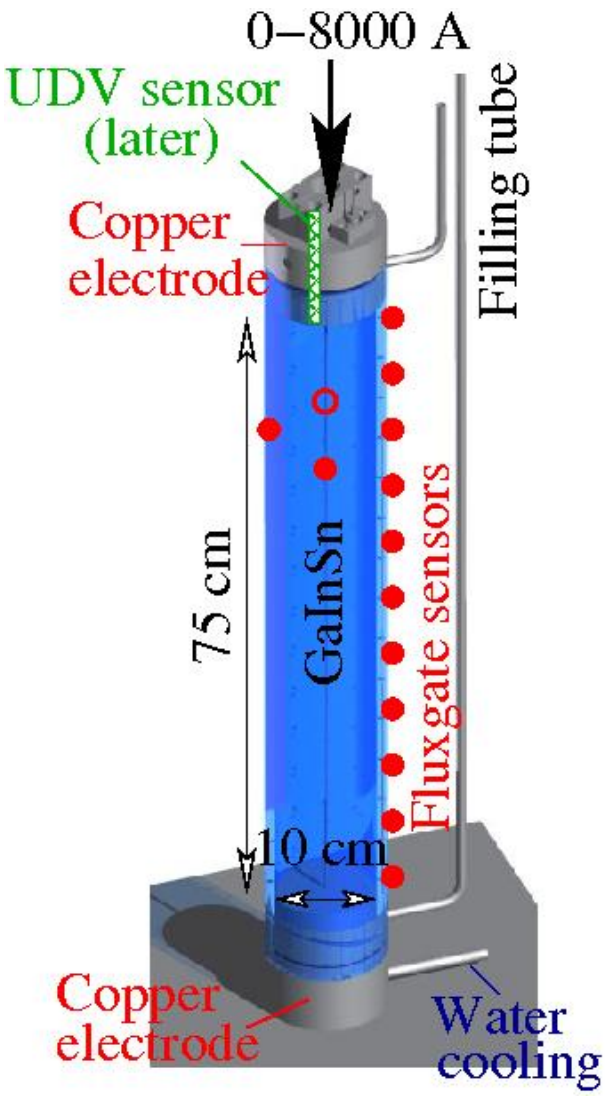
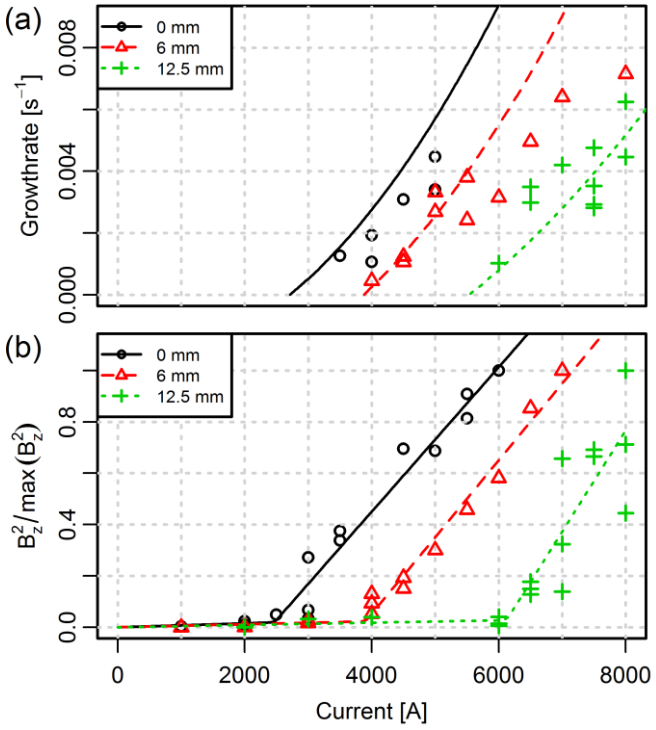
Tayler instability in the liquid metal lab





Good agreement between numerical and experimental growth rates in dependence on the radius of an inner non-conducting rod



M. Seilmayer et al.: Phys. Rev. Lett. 108 (2012), 244501

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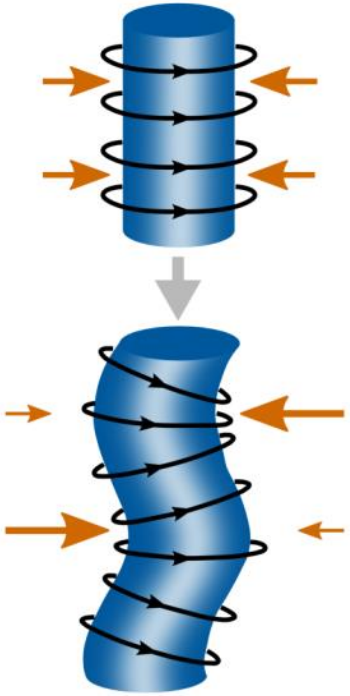
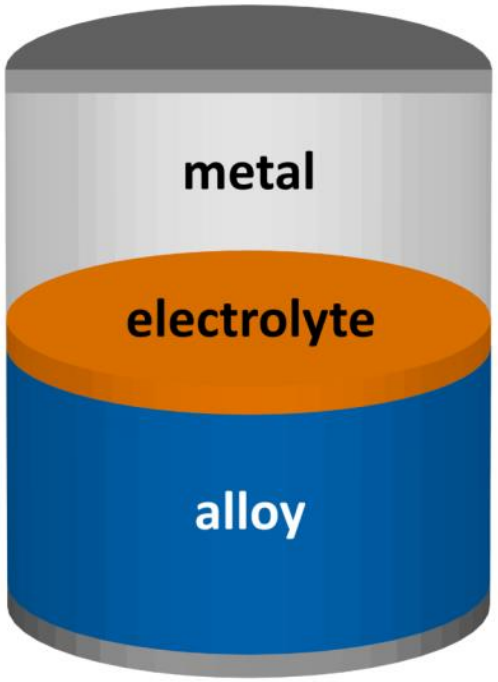
Tayler instability in LMB´s ???

$F = j \times B$





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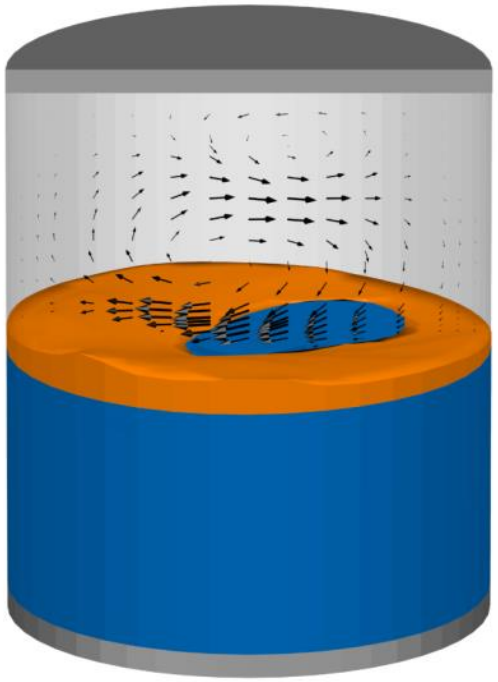

+

stable stratification,
interior at rest

critical current
exceeded: short circuit

???

Tayler instability in LMB´s

Energy Conversion and Management 52 (2011) 2982–2986

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Energy Conversion and Management

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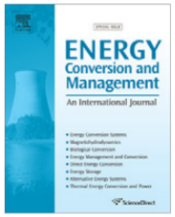

How to circumvent the size limitation of liquid metal batteries due to the Tayler instability

Frank Stefani *, Tom Weier, Thomas Gundrum, Gunter Gerbeth

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ABSTRACT

Recently, a new type of battery has been proposed that relies on the principle of self-assembling of a liquid metalloid positive electrode, a liquid electrolyte, and a liquid metal negative electrode. While this configuration has been claimed to allow arbitrary up-scaling, there is a size limitation of such a system due to a current-driven kink-type instability that is known as the Tayler instability. We characterize this instability in large-scale self-assembled liquid metal batteries and discuss various technical means how it can be avoided.

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How to circumvent the size limitation of LMB´s due to the TI

Ideal MHD → general stability criterion of kink instability with m=1:

$$\frac{\partial}{\partial r}\left(r\, B_\varphi^2(r)\right) < 0$$

In presence of an inner radius r_i, we get a dependence on radius ratio $\eta = r_i/r_o$

$$B_\varphi(r) = \alpha r + \beta/r,$$

$$\alpha = \frac{\mu_0}{2\pi}\,\frac{I_f}{r_o^2}\,\frac{1}{1-\eta^2},$$

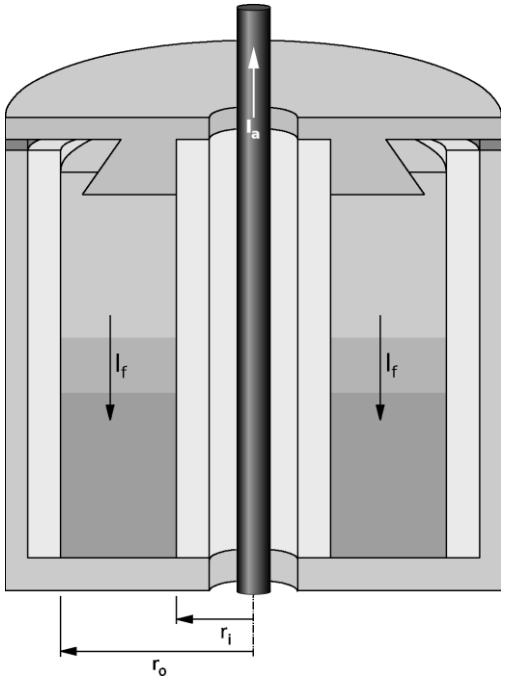
$$\beta = \frac{\mu_0}{2\pi}\left[I_a - I_f\,\frac{\eta^2}{1-\eta^2}\right].$$

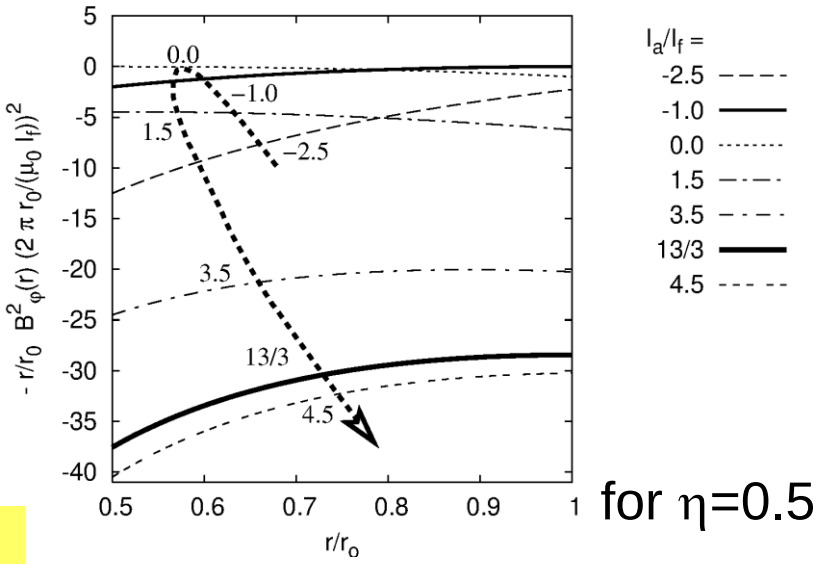
$$\frac{\beta^2}{r^2} - 2\alpha\beta - 3\alpha^2 r^2 > 0.$$

$$I_a < -I_f \quad \text{or}$$

Two solutions for stability:

$$I_a > I_f\,\frac{3+\eta^2}{1-\eta^2}.$$





for η=0.5

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How to circumvent the size limitation of LMB´s due to the TI

Without current in the rod (non-ideal MHD):

$$\frac{dv_r}{dr}+\frac{v_r}{r}+\frac{im}{r}v_\varphi+ikv_z=0, \tag{A.11}$$

$$\frac{dp}{dr}+i\frac{m}{r}X_2+ikX_3+\left(k^2+\frac{m^2}{r^2}\right)u_r-i\mathrm{Ha}_i^2\frac{m}{r}B_{\varphi,0}b_r+2\mathrm{Ha}_i^2\frac{B_\varphi}{r}b_\varphi=0, \tag{A.12}$$

$$\frac{dX_2}{dr}-\left(k^2+\frac{m^2}{r^2}\right)v_\varphi+2i\frac{m}{r^2}v_r+\frac{\mathrm{Ha}_i^2}{r}\frac{d}{dr}(rB_{\varphi,0})b_r+i\mathrm{Ha}_i^2\times\frac{m}{r}B_{\varphi,0}b_\varphi-i\frac{m}{r}p=0, \tag{A.13}$$

$$\frac{dX_3}{dr}+\frac{X_3}{r}-\left(k^2+\frac{m^2}{r^2}\right)v_z-ikp+i\mathrm{Ha}_i^2\frac{m}{r}B_{\varphi,0}b_z=0, \tag{A.14}$$

$$\frac{db_r}{dr}+\frac{b_r}{r}+\frac{im}{r}b_\varphi+ikb_z=0, \tag{A.15}$$

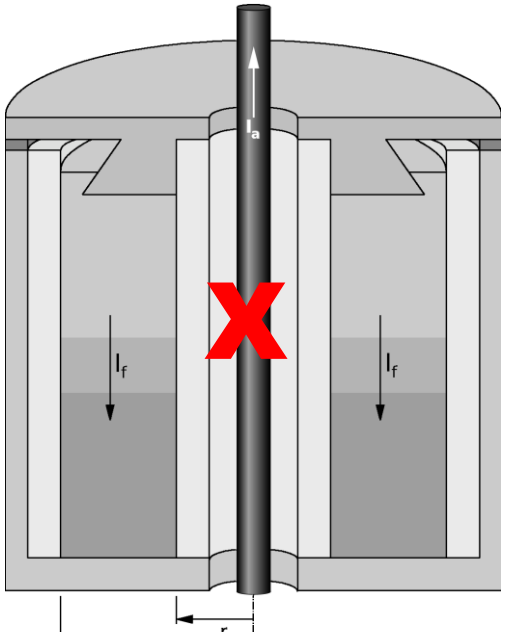
$$\frac{db_z}{dr}-\frac{i}{k}\left(k^2+\frac{m^2}{r^2}\right)b_r+\frac{m}{kr}X_4-\frac{m}{kr}B_{\varphi,0}u_r=0, \tag{A.16}$$

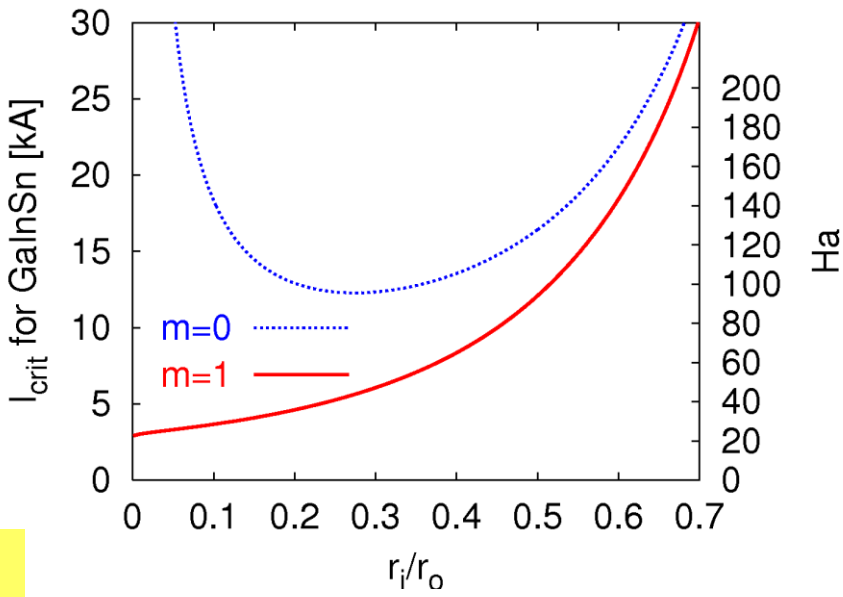
$$\frac{dX_4}{dr}-\left(k^2+\frac{m^2}{r^2}\right)b_\varphi+i\frac{2m}{r^2}b_r-r\frac{d}{dr}\left(\frac{B_{\varphi,0}}{r}\right)v_r+i\frac{m}{r}B_{\varphi,0}v_\varphi=0, \tag{A.17}$$

with $X_2=\frac{dv_\varphi}{dr}+\frac{v_\varphi}{r}$, $X_3=\frac{dv_z}{dr}$, $X_4=\frac{db_\varphi}{dr}+\frac{b_\varphi}{r}$,

and

$$\mathrm{Ha}_i=B_\varphi(r_i)\sqrt{r_i(r_o-r_i)}\sqrt{\sigma/(\rho\nu)},$$
$$=\frac{\mu_0 I_a}{2\pi r_i}\sqrt{r_i(r_o-r_i)}\sqrt{\sigma/(\rho\nu)}.$$





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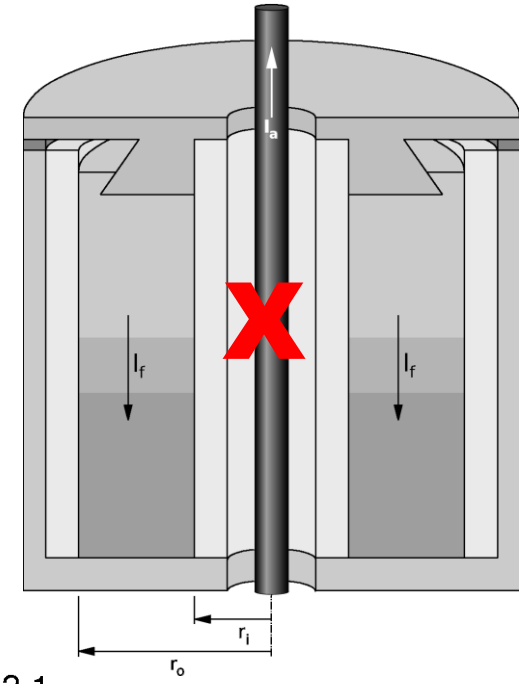

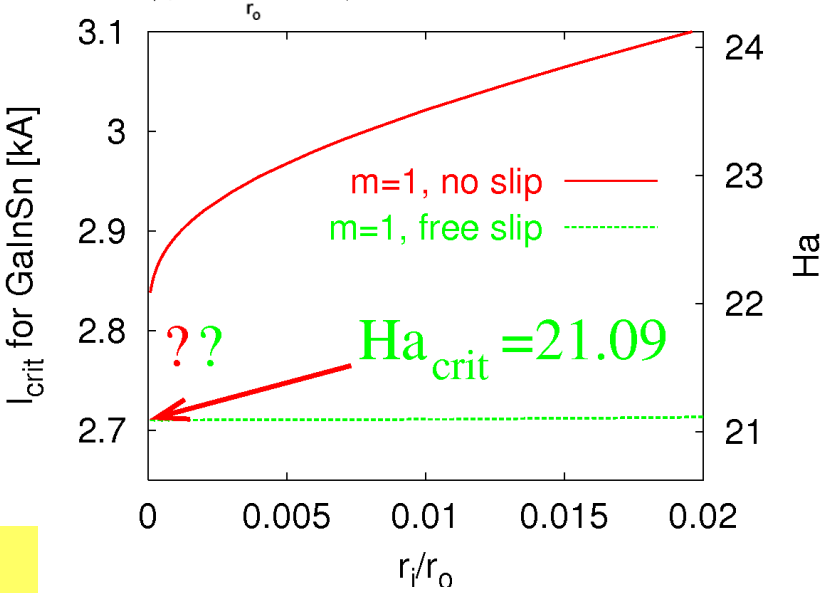

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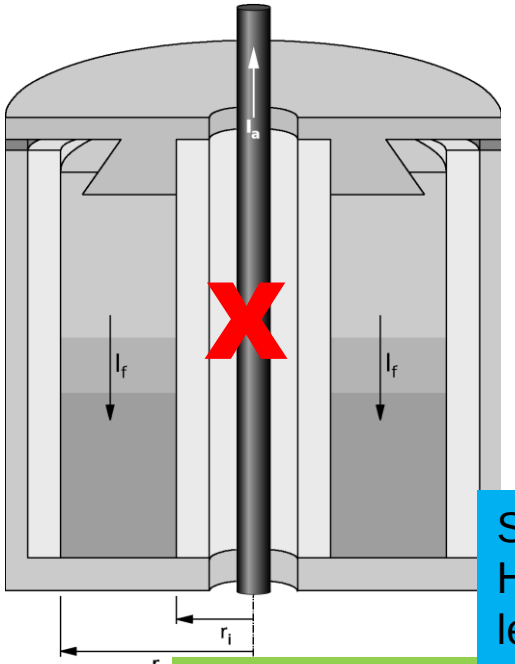
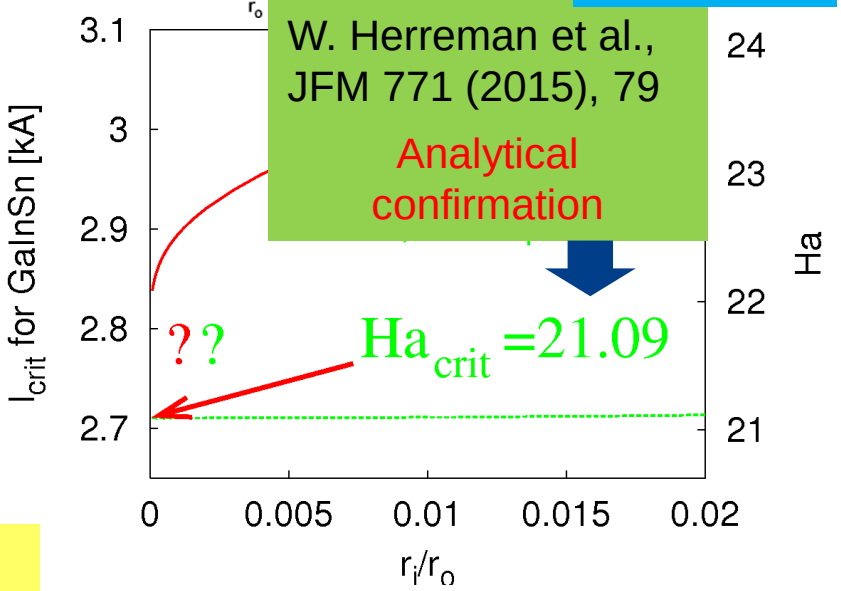


See Wietze Herreman's lecture



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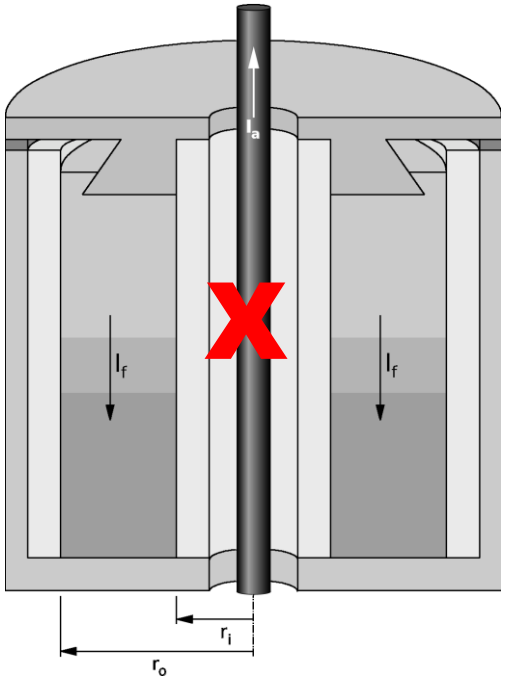
$$\frac{dX_2}{dr} - \left(k^2 + \frac{m^2}{r^2}\right)v_\varphi + 2i\frac{m}{r^2}v_r + \frac{\mathrm{Ha}_i^2}{r}\frac{d}{dr}(rB_{\varphi,0})b_r + i\mathrm{Ha}_i^2 \times \frac{m}{r}B_{\varphi,0}b_\varphi - i\frac{m}{r}p = 0, \tag{A.13}$$

$$\frac{dX_3}{dr} + \frac{X_3}{r} - \left(k^2 + \frac{m^2}{r^2}\right)v_z - ikp + i\mathrm{Ha}_i^2 \frac{m}{r} B_{\varphi,0} b_z = 0, \tag{A.14}$$

$$\frac{db_r}{dr} + \frac{b_r}{r} + \frac{im}{r} b_\varphi + ikb_z = 0, \tag{A.15}$$

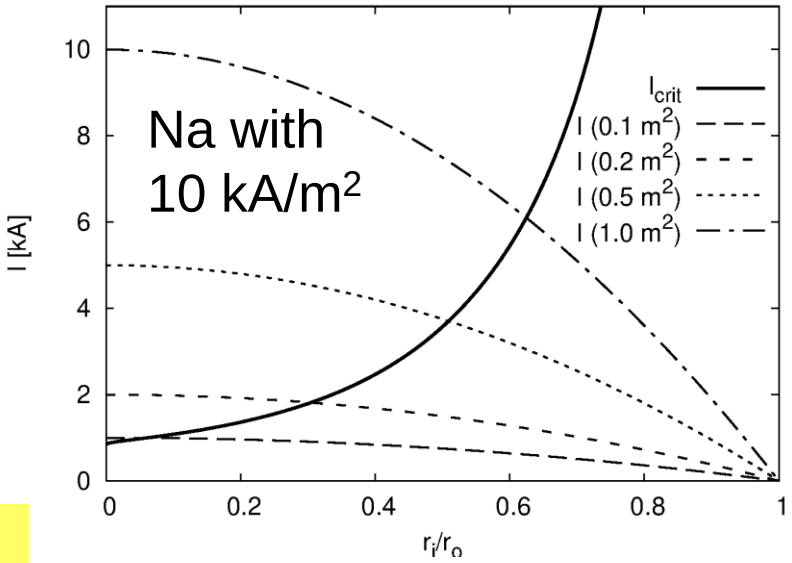


By just widening the inner bore, the critical current can be dramatically increased

with $X_2 = \frac{dv_\varphi}{dr} + \frac{v_\varphi}{r}, \quad X_3 = \frac{dv_z}{dr}, \quad X_4 = \frac{db_\varphi}{dr} + \frac{b_\varphi}{r},$

and

$$\mathrm{Ha}_i = B_\varphi(r_i)\sqrt{r_i(r_o - r_i)}\sqrt{\sigma/(\rho\nu)},$$
$$= \frac{\mu_0 I_a}{2\pi r_i}\sqrt{r_i(r_o - r_i)}\sqrt{\sigma/(\rho\nu)}.$$



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Integro-differential eqn. code (OpenFoam+Poisson+Biot-Savart)

Initialisation: $\mathbf{J}_0, \mathbf{B}_0$

time step: CFL condition with Alfvén velocity

$$\mathbf{f}_L = (\mathbf{J}_0 + \mathbf{j}) \times (\mathbf{B}_0 + \mathbf{b})$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \eta \nabla^2 \mathbf{u} + \mathbf{f}_L$$

$$\mathbf{b}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dV'$$

$$\mathbf{j} = \sigma(-\nabla\varphi + \mathbf{u} \times \mathbf{B})$$

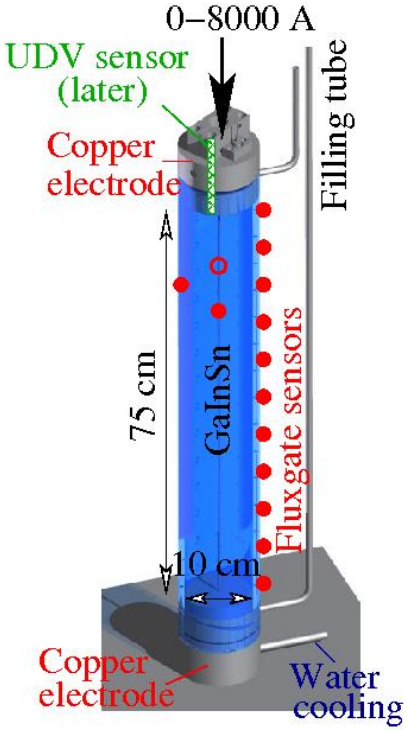
$$\Delta\varphi = \nabla \cdot (\mathbf{u} \times \mathbf{B})$$

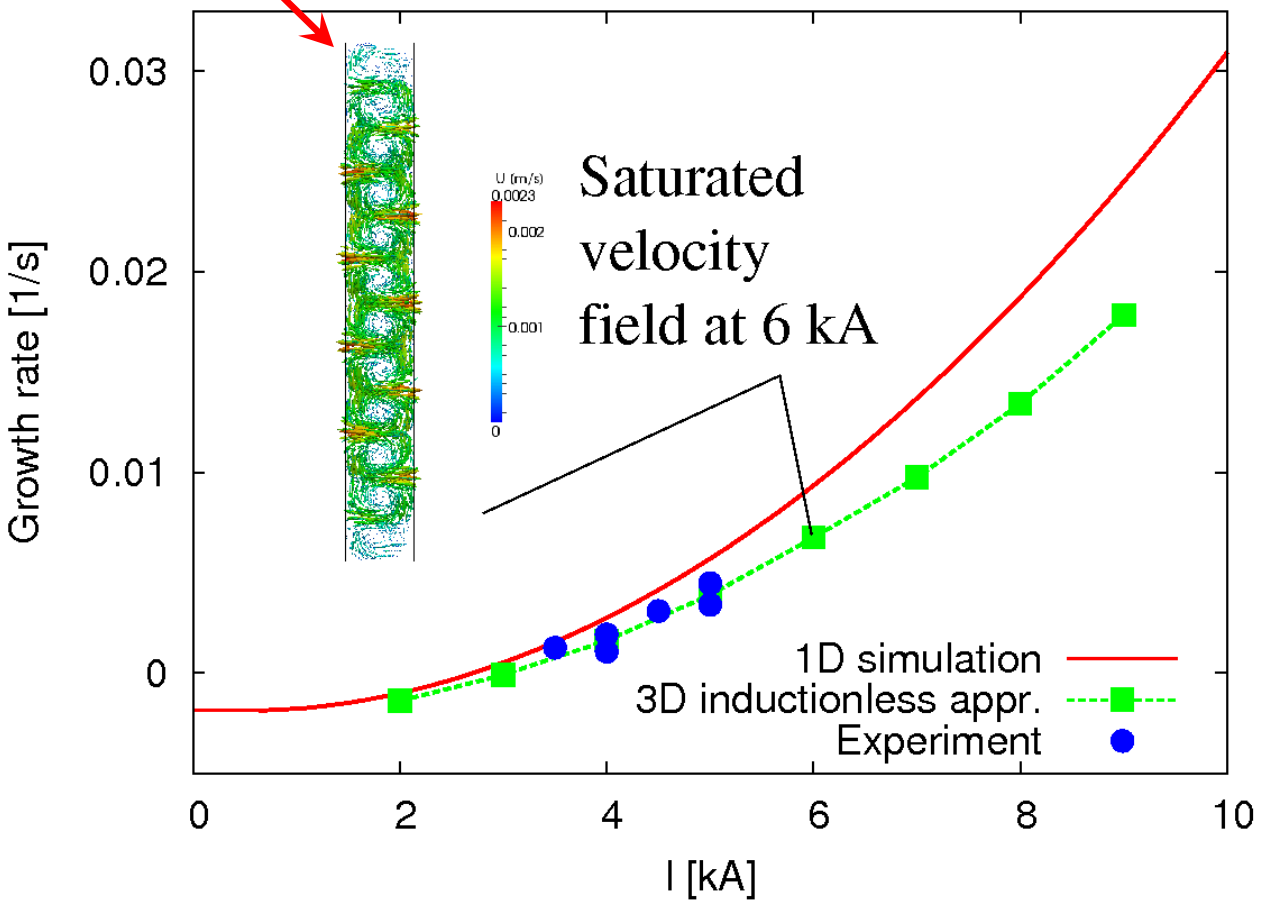
N. Weber et al., New J. Phys. 15 (2013), 043034

Simulation of the TI experiment

Saturation: Re~Ha^2 …also derived in

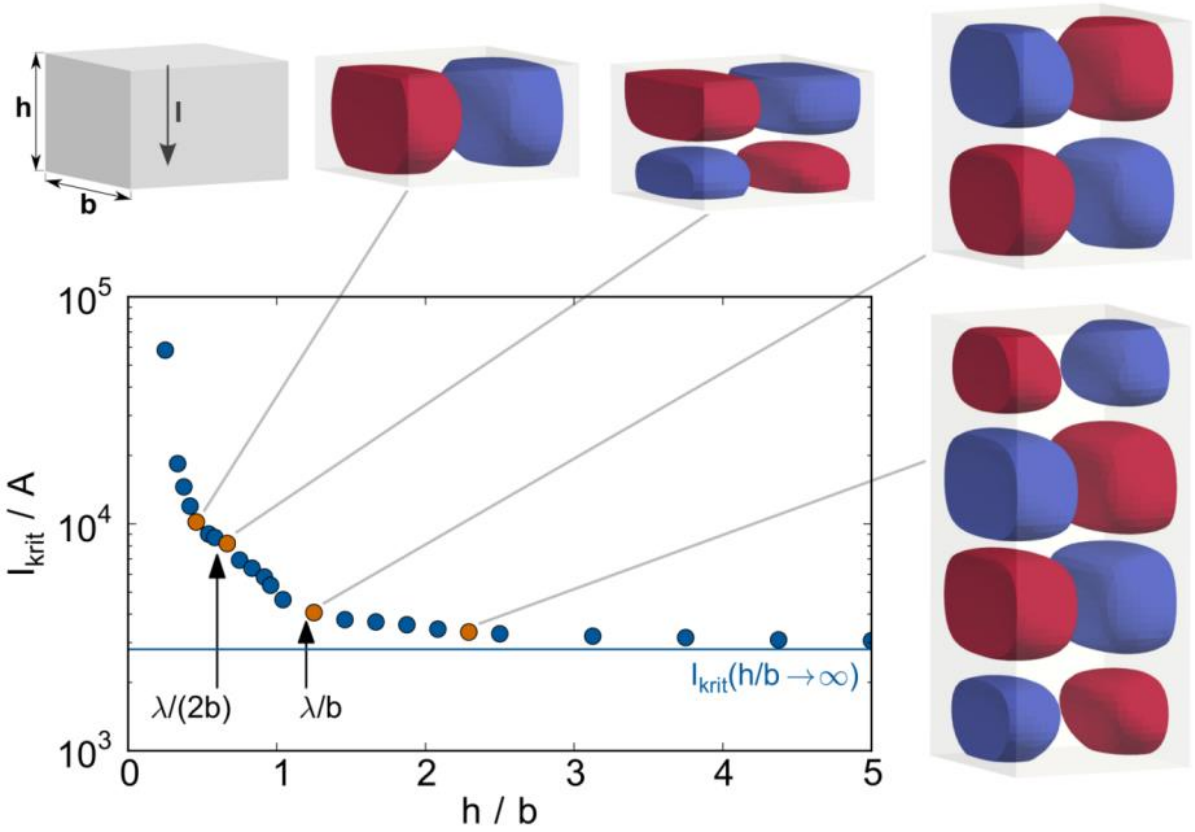
W. Herreman et al., JFM 771 (2015), 79





N. Weber et al., New J. Phys. 15 (2013), 043034

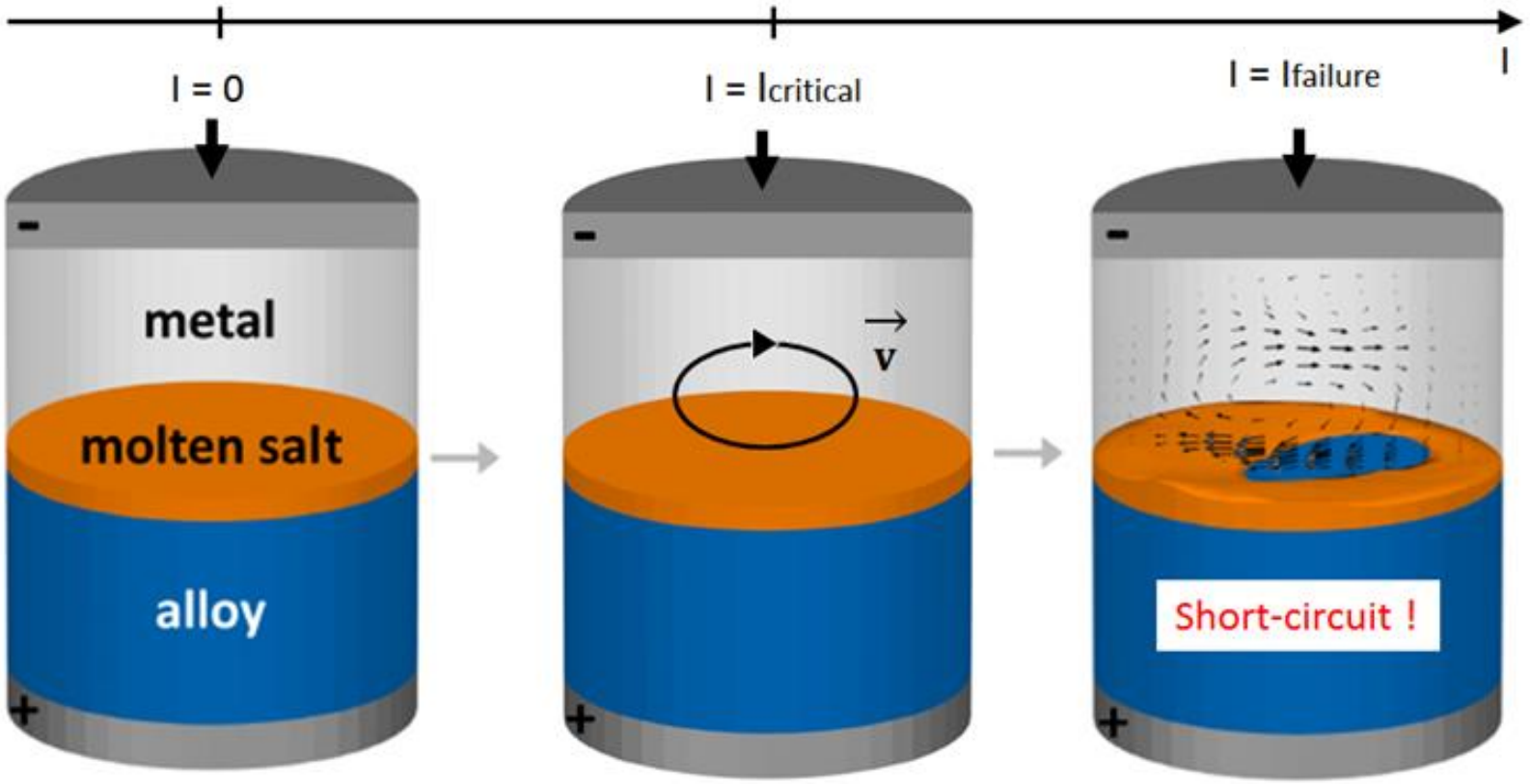
Simulation of TI in a cuboid with different aspect ratios



The critical current increases with decreasing aspect ratio. Plateaus appear at full and half wavelength.

N. Weber et al., New J. Phys. 15 (2013), 043034

When does the battery really fail?



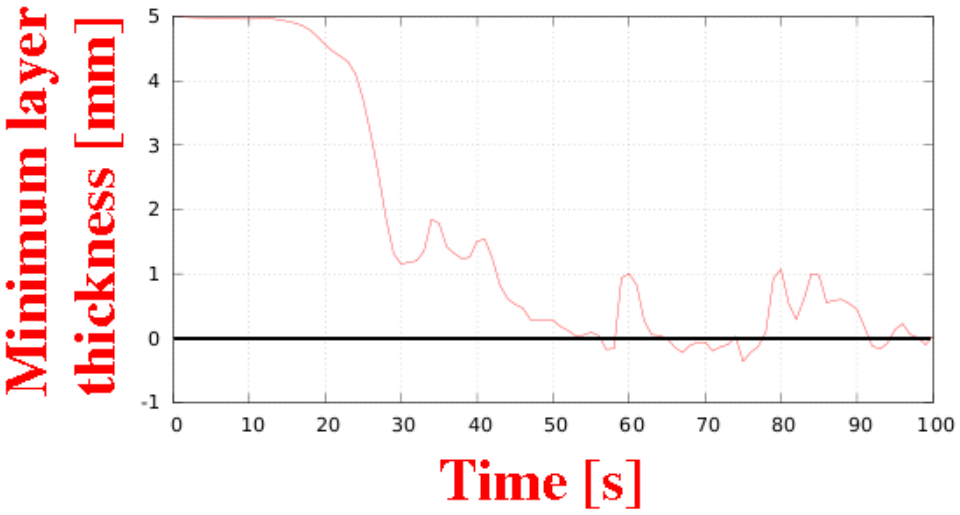
Layer rupture approximately at Richardson number $Ri=E_{pot}/E_{kin} \sim 1$

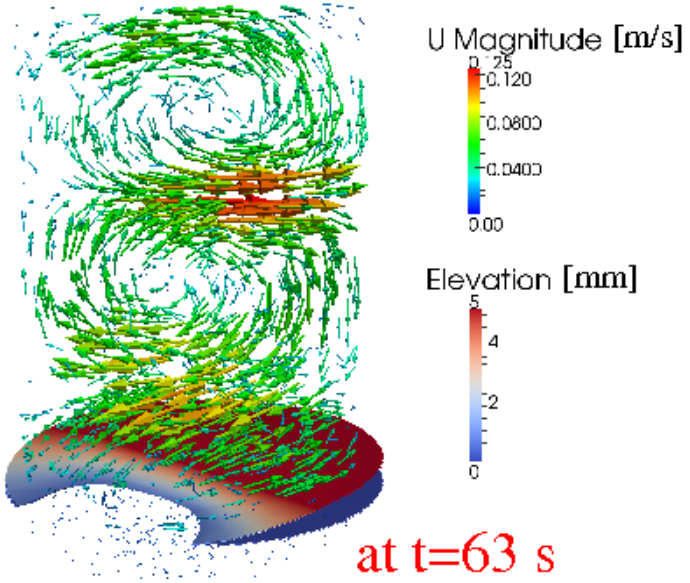
F.S. et al., IOP Conf. Series: Materials Science and Engineering 143 (2016) 012024

Mitglied der Helmholtz-Gemeinschaft

When does the battery really fail? Early simulation with interface

- Magnesium---Salt---Antimon-Battery, 5 mm electrolyte thickness, 50 mm radius (not very realistic!!!)
- Destruction of stable stratification at 30 kA → Short cut!





Leopold Barry: Master thesis (2014)

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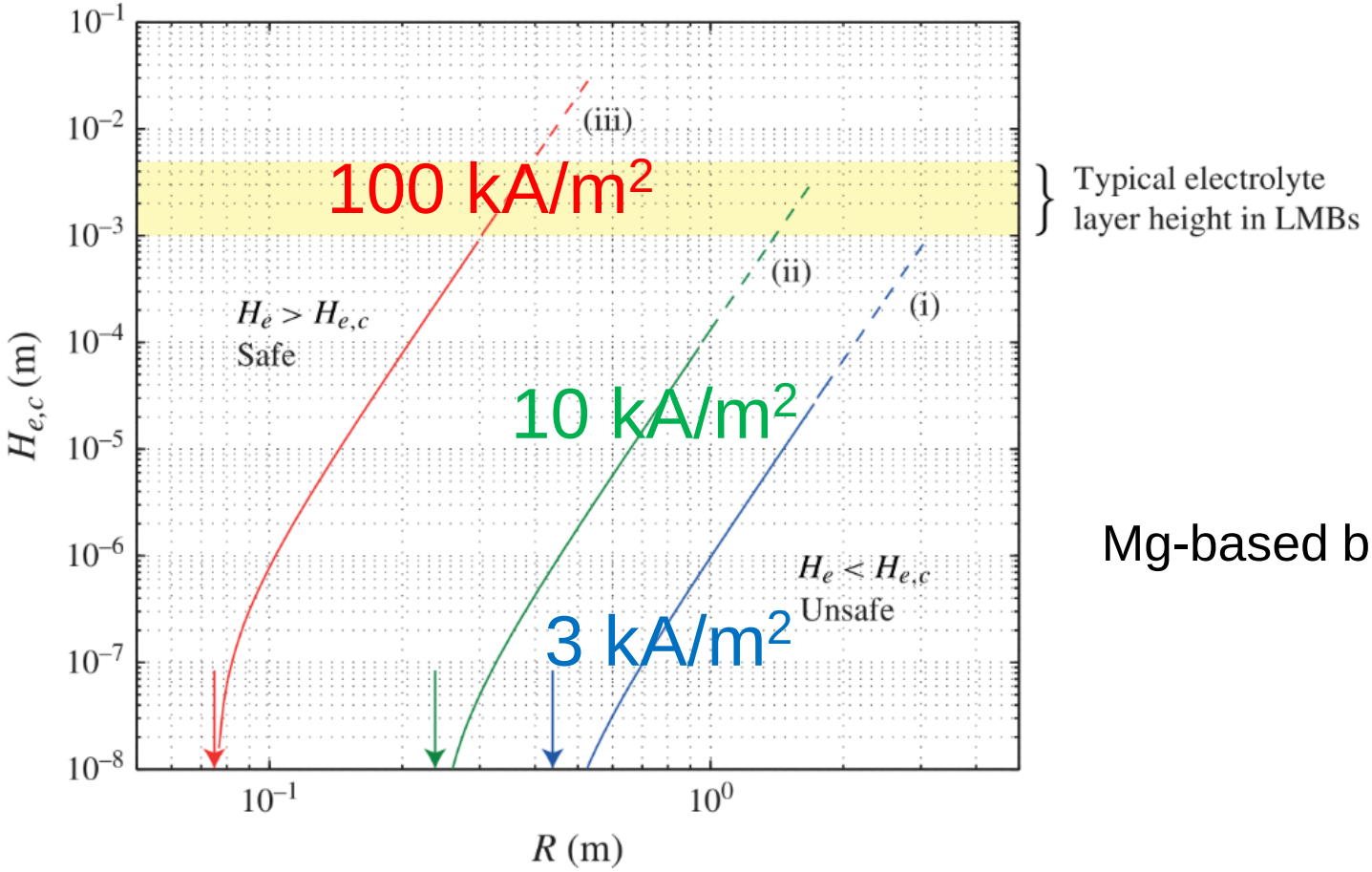
When does the battery really fail? A much better estimate…

Critical height: $$H_{e,c} = \frac{C^2 \gamma_{qs}(Ha)}{32} \frac{\sigma_{top}^2 \mu_0^4 J_0^4 R^6}{\rho_{top}(\rho_e - \rho_{top}) g}.$$

W. Herreman et al., JFM 771 (2015), 79



Mg-based battery

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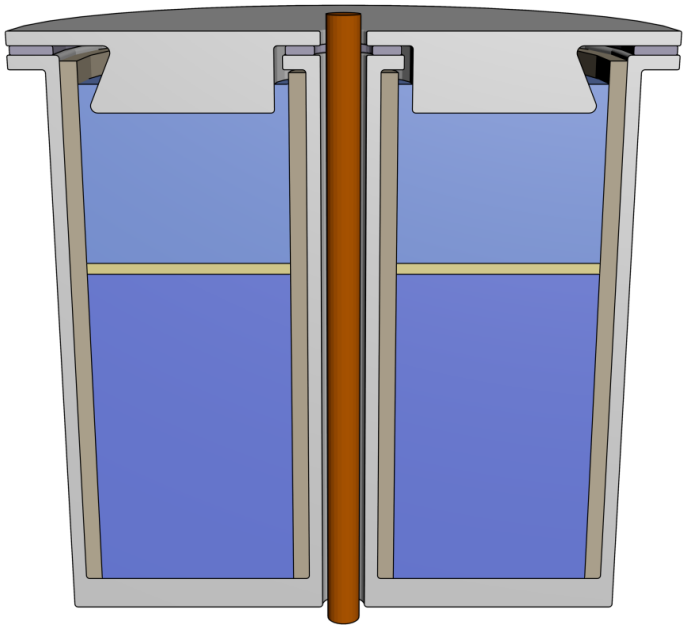
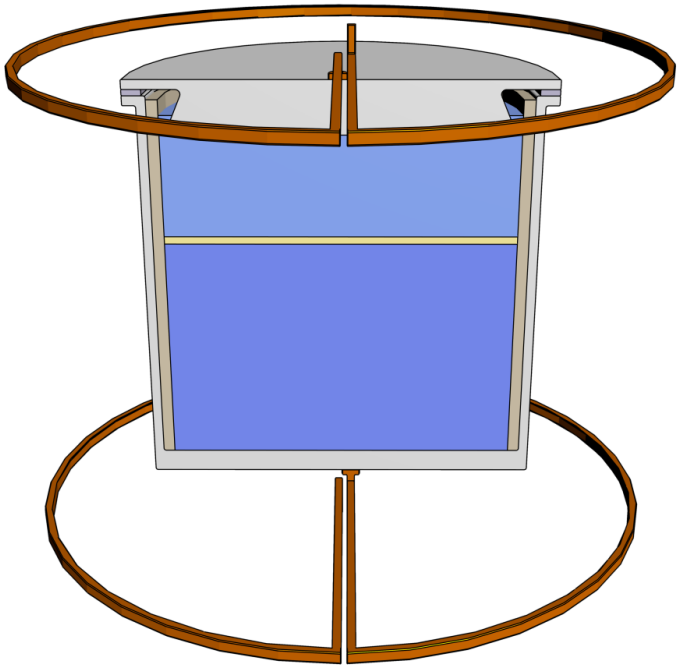
Two technical ways of stabilizing the battery



Return current through the center changes $B_\varphi(r)$

F.S. et al., Energy Conv. Managem. 52 (2011), 2982

US 9,608,296 B2



Applying B_z (Kruskal-Shafranov)

N. Weber et al., J. Power Sources 265 (2014), 166

DE 102013112555 B3

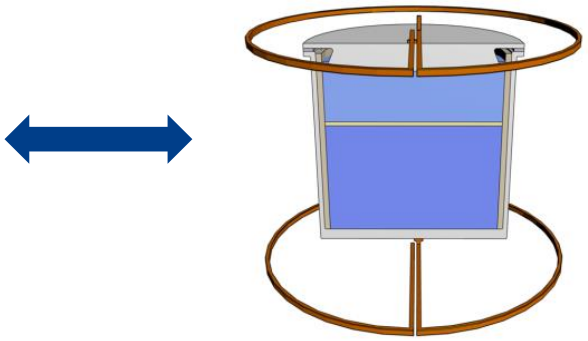
Saturation of TI and mean-field MHD

Ideal fluids, or Pm~1: Two mechanisms contribute to saturation (see also Taylor relaxation in reversed field pinches):

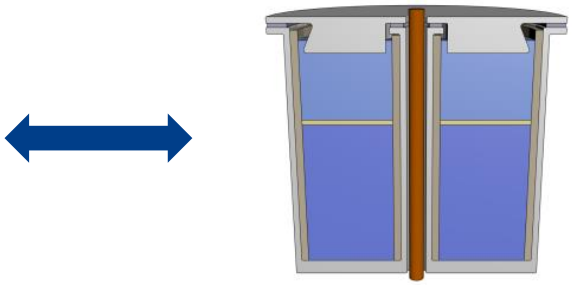
„Natural“ saturation mechanism

Technical stabilization of liquid metal batteries

1. α effect → produces B_z (stabilizes according to the Kruskal-Shafranov limit)



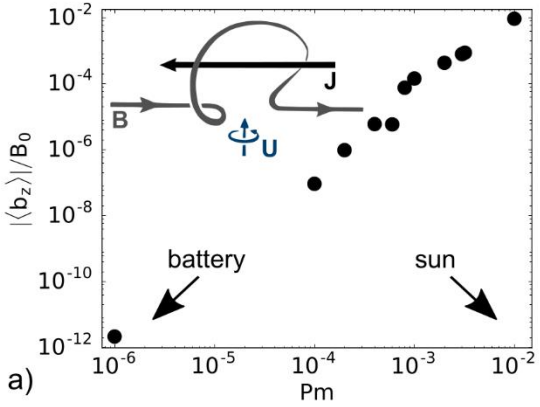
2. Radially dependent β effect → changes $B_\varphi(r)$ (acts like a return current)

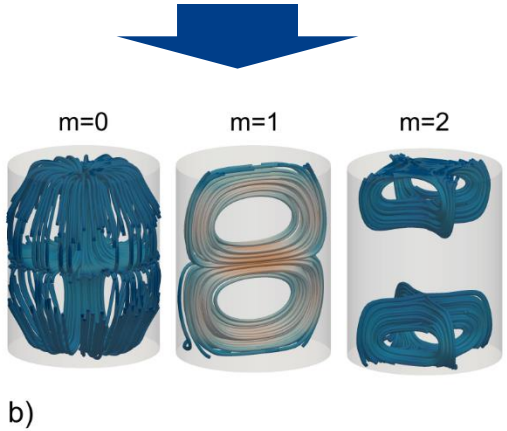


Saturation of the TI at low Pm

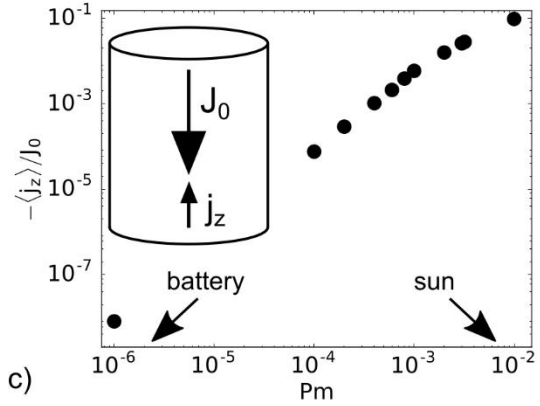
At low Pm, neither the α effect nor the β effect is strong enough to change the magnetic base configuration. The saturation mechanism relies exclusively on the modification of the hydrodynamic base state (nonlinear appearance of m=0 and m=2 flow contributions).

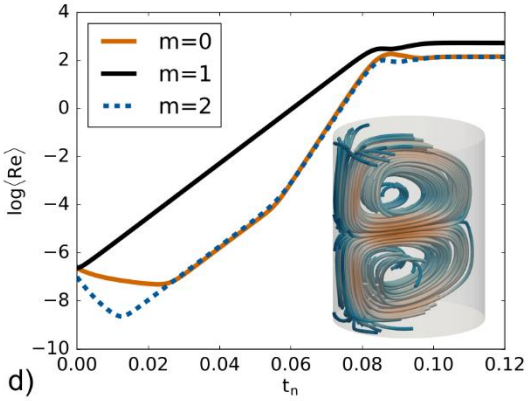
α





β

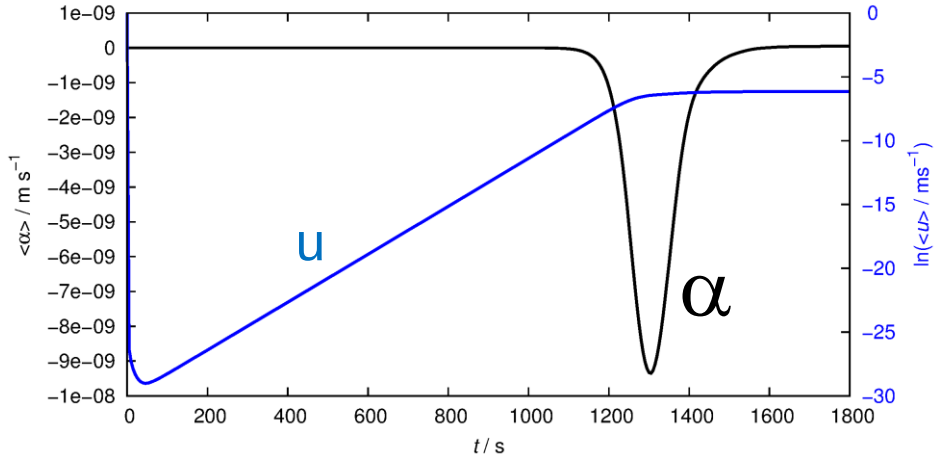


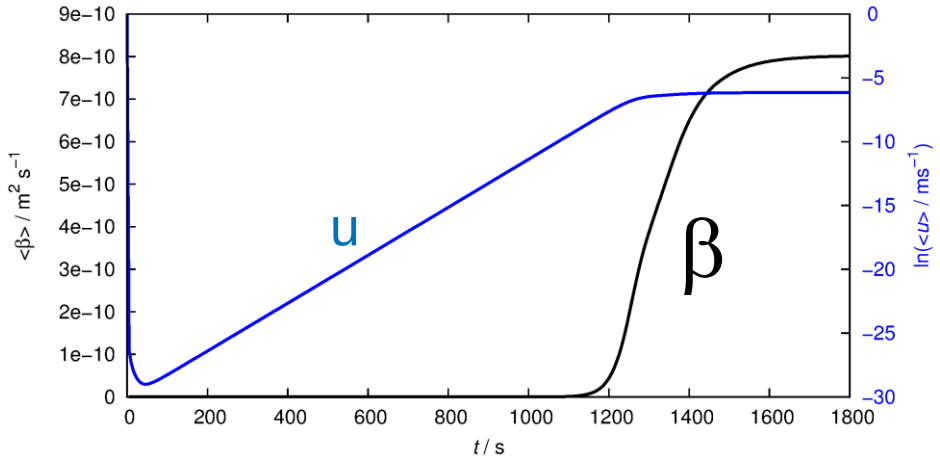


N. Weber et al., New J. Phys. 17 (2015), 113013; T. Weier et al., IOP Conf. Ser.: Mater. Sci. Eng. 228 (2017), 012013

Saturation of the TI at low Pm

For small Ha, α effect appears only in the exponential growth phase and disappears in the saturation regime.





Example: h/d=1.25, Ha=55

Electromotive force of mean-field MHD

$$\mathcal{E} = \alpha \overline{\boldsymbol{B}} - \beta \nabla \times \overline{\boldsymbol{B}}$$

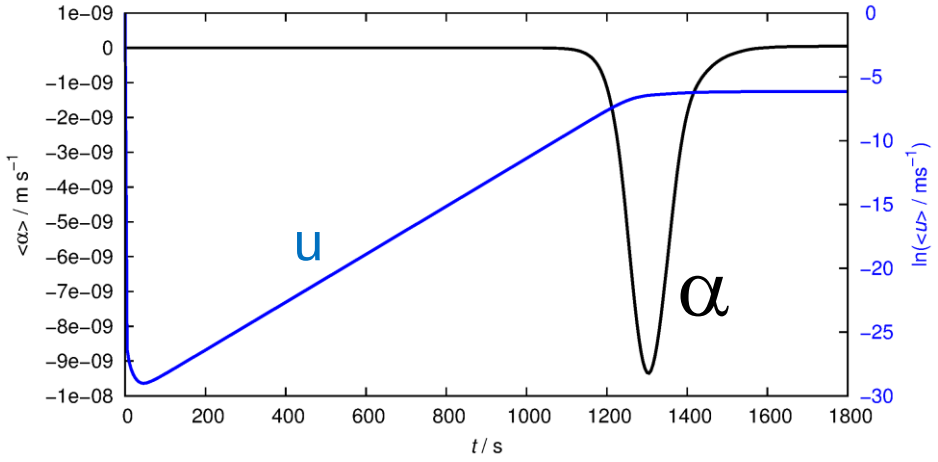
$$\alpha = \overline{(\boldsymbol{u} \times \boldsymbol{b})} \cdot \boldsymbol{B}_0 / B_0^2$$

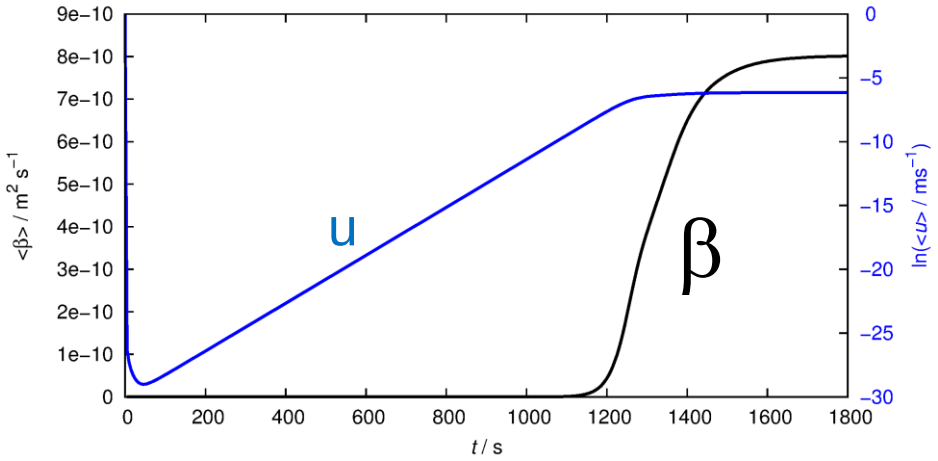
$$\beta = \overline{(\boldsymbol{u} \times \boldsymbol{b})} \cdot \boldsymbol{J}_0 / (\mu_0 J_0^2)$$

Weber et al., New J. Phys. 17 (2015), 113013

Saturation of the TI at low Pm

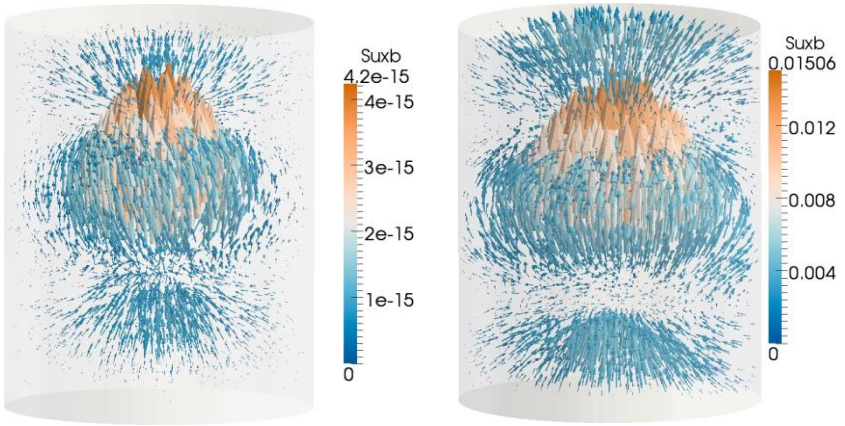
For small Ha, α effect appears only in the exponential growth phase and disappears in the saturation regime.





Example: h/d=1.25, Ha=55

Induced current at…

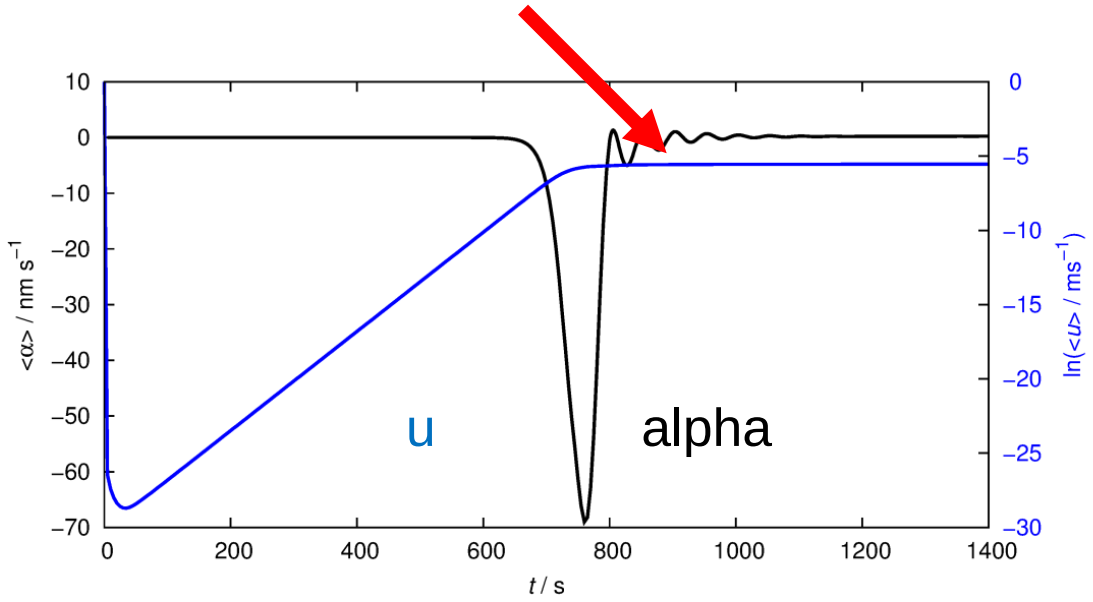


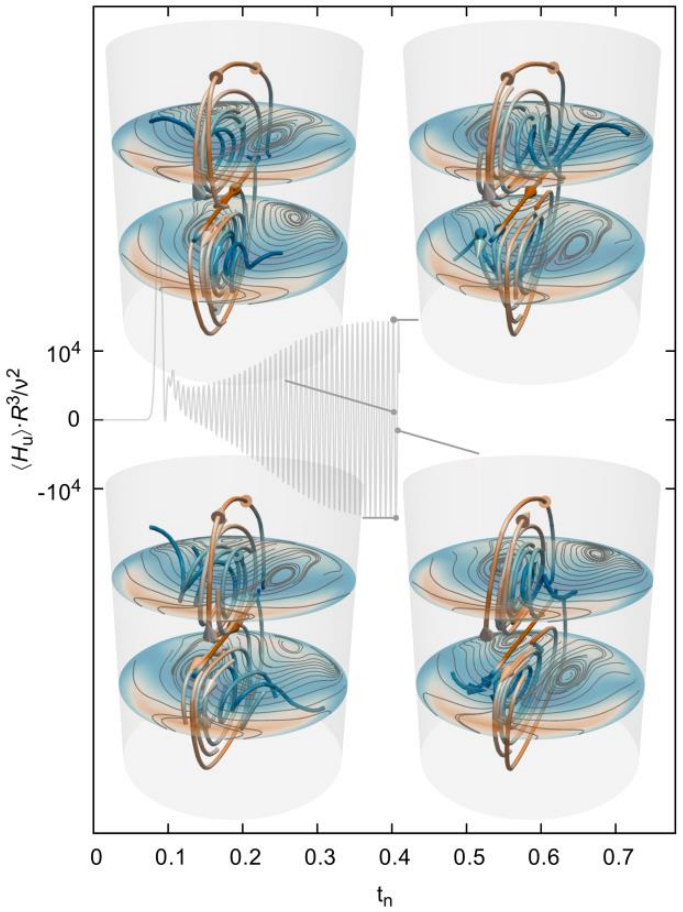
500 s 1800 s

Weber et al., New J. Phys. 17 (2015), 113013

Saturation of the TI at low Pm: What happens at larger Ha?

(Damped) helicity oscillations Ha =70

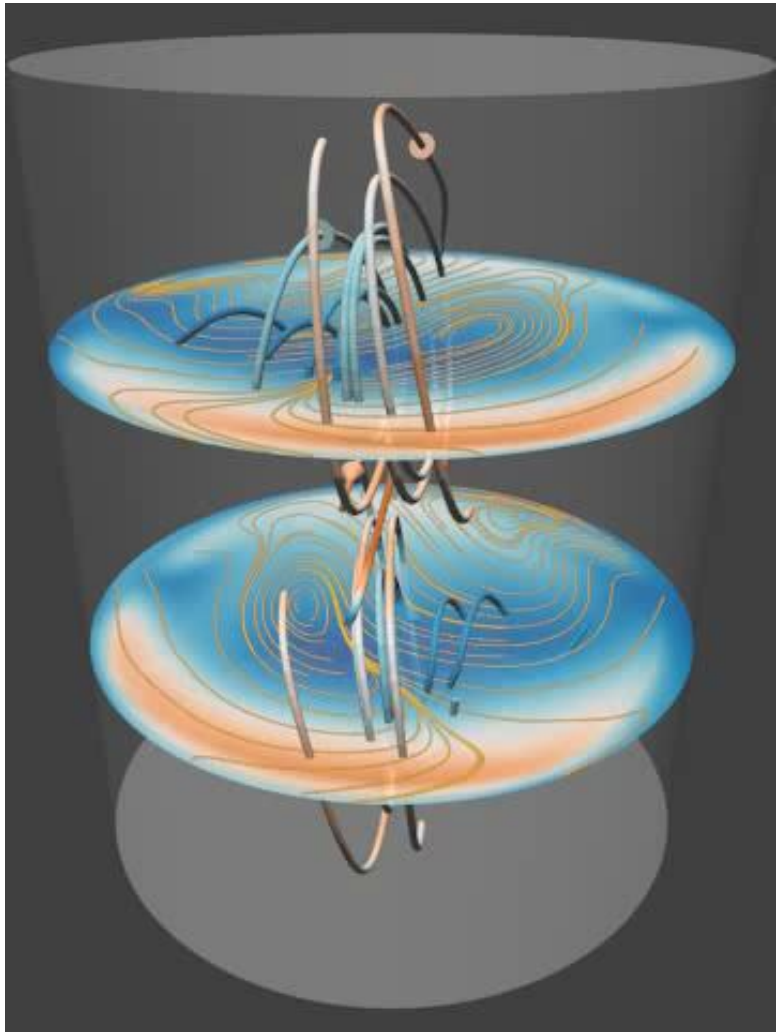




Ha =100

Weber et al., New J. Phys. 17 (2015), 113013

Character of the helicity oscillations at larger Ha



Ha =100
Pm=10^{-6}

Weber et al., New J. Phys. 17 (2015), 113013

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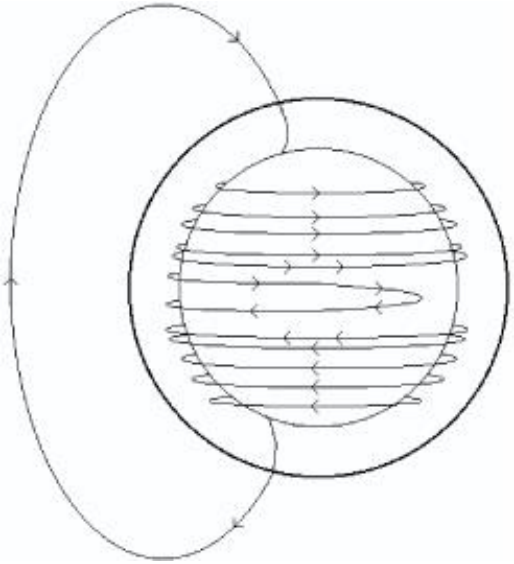
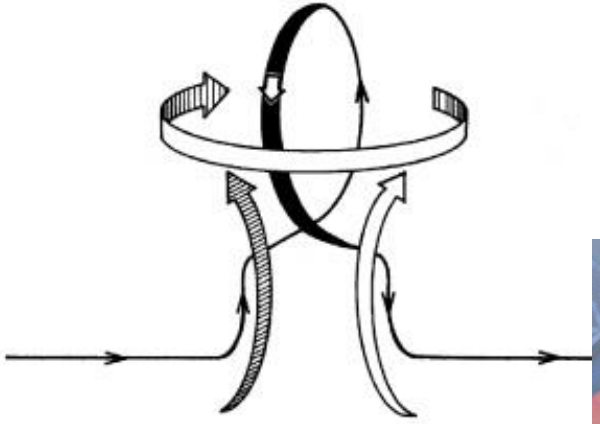
A huge jump to the solar dynamo

Any solar dynamo needs:

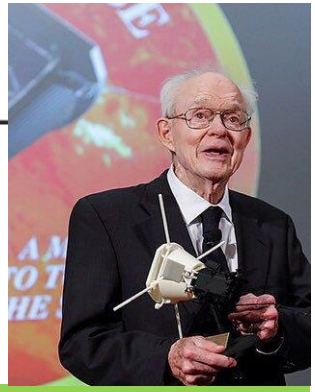
- some Ω effect to wind up toroidal field from poloidal field
- some α effect to regenerate poloidal field from toroidal field



Ω effect



α effect



Parker, Astrophys J. 122, 293 (1955)

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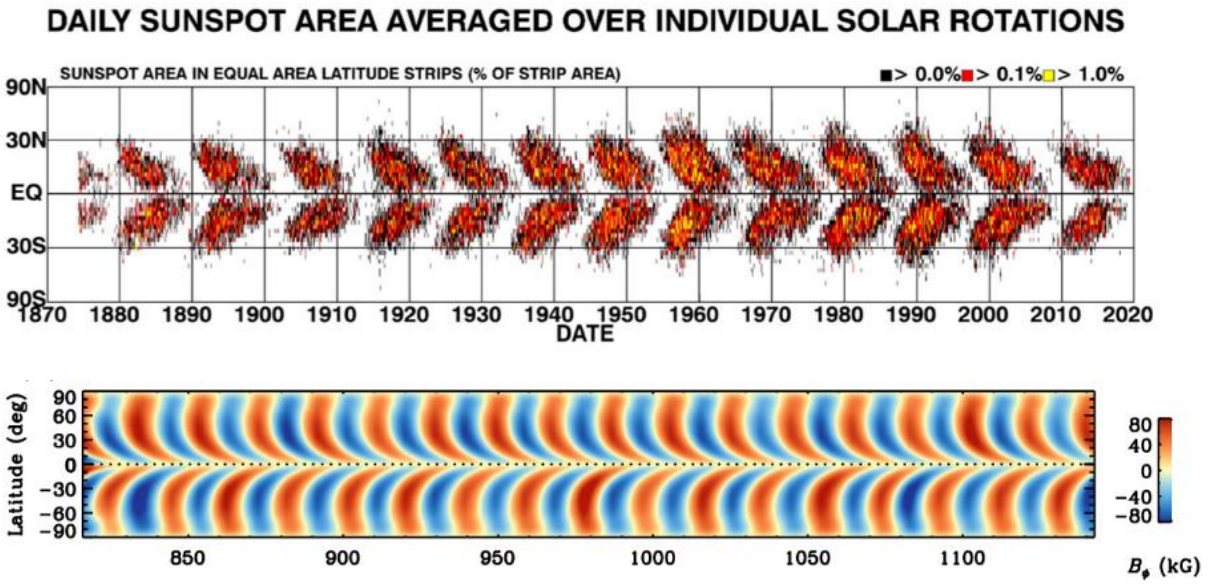
Solar dynamo: Conventional modelling

With appropriate models, e.g. Babcock-Leighton (including meridional circulation), and some parameter fitting, one obtains

- a reasonable period of the Hale cycle (22 years)
- a reasonable shape of the butterfly diagram of sunspots

http://www.solarcyclescience.com/solarcycle.html



Karak, B.B., Miesch, M., ApJ 847 (2017), 69

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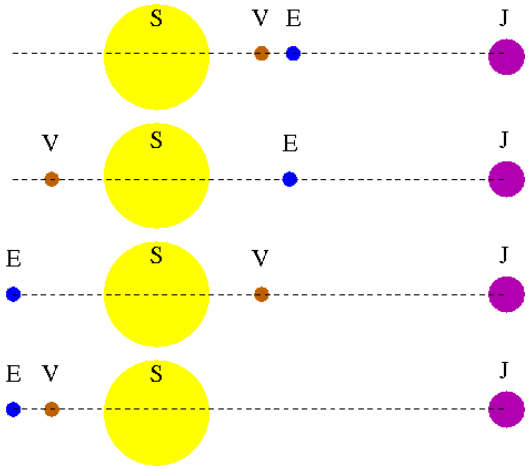
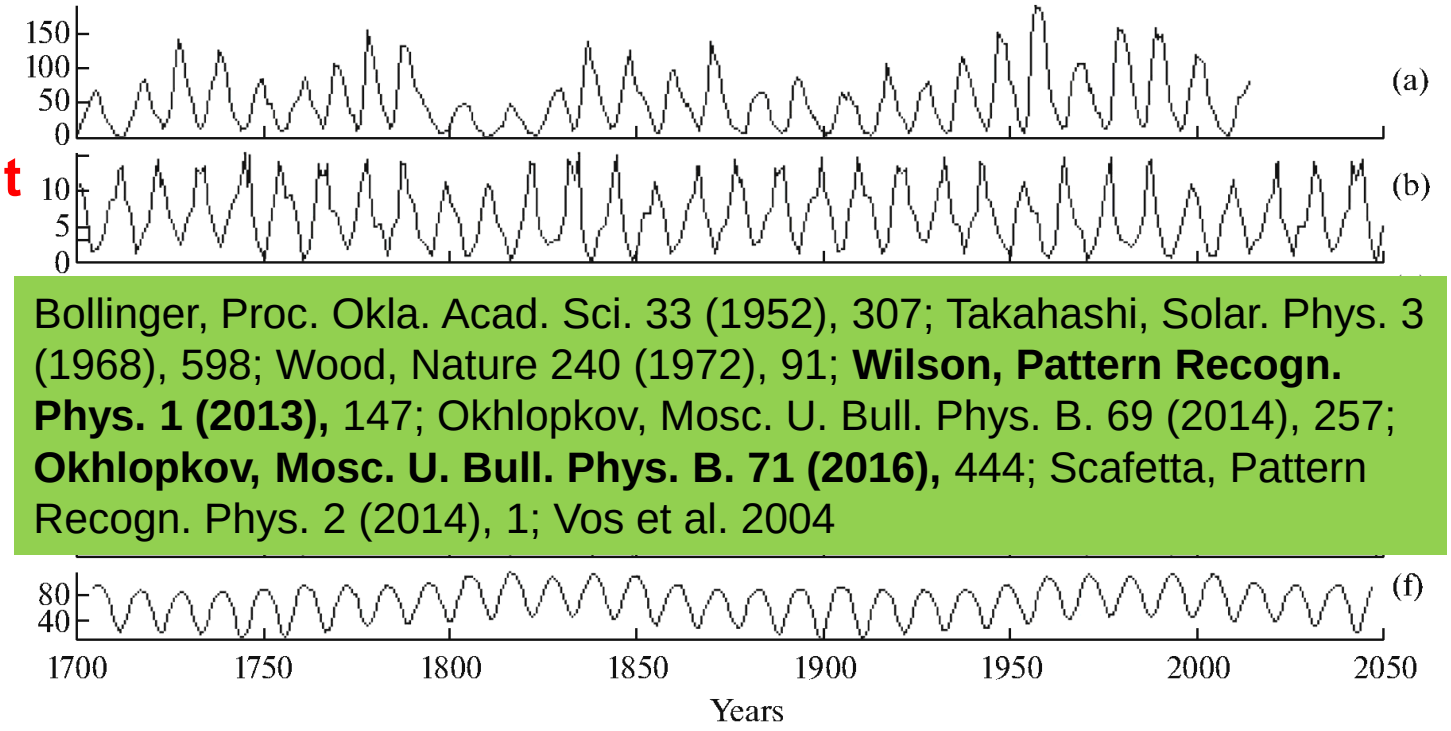
Is there a problem at all? First indication…

Conspicious synchronization of the solar Schwabe cycle with 11.07-yr spring-tide period of tidally dominant Venus-Earth-Jupiter system (despite weak tidal forces)



Sunspots

VEJ alignment index



Bollinger, Proc. Okla. Acad. Sci. 33 (1952), 307; Takahashi, Solar. Phys. 3 (1968), 598; Wood, Nature 240 (1972), 91; **Wilson, Pattern Recogn. Phys. 1 (2013),** 147; Okhlopkov, Mosc. U. Bull. Phys. B. 69 (2014), 257; **Okhlopkov, Mosc. U. Bull. Phys. B. 71 (2016),** 444; Scafetta, Pattern Recogn. Phys. 2 (2014), 1; Vos et al. 2004

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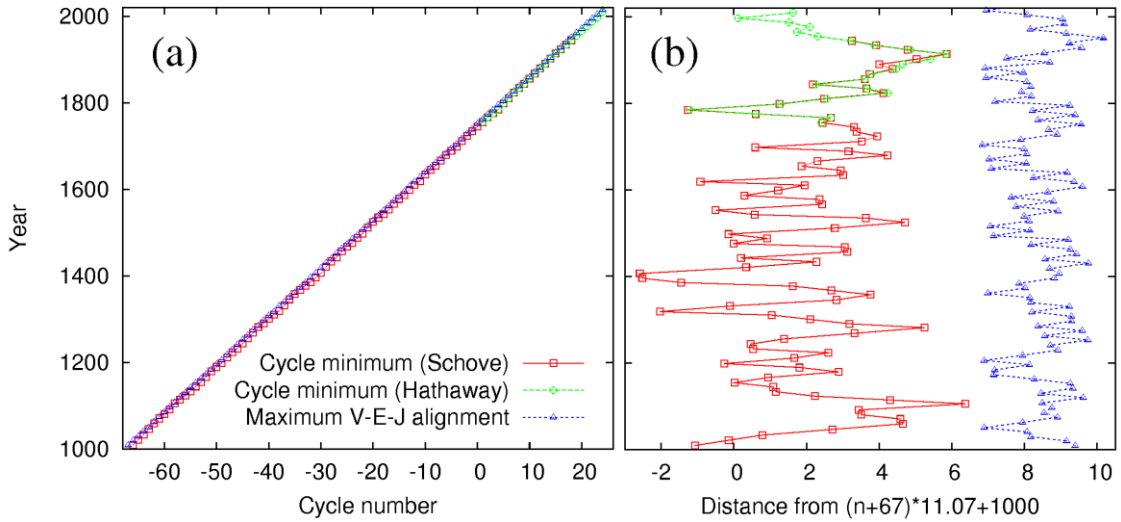
Is there a problem at all? First indication…



Schove, D.J.: J. Geophys. Res. 60 (1955), 127; Hathaway, D.H., Liv. Rev. Sol. Phys. 7 (2010)

Strong indication for a clocked process, in contrast to a random walk process

F. S. et al., Solar Physics 294 (2019)

Old data by Schove (derived from aurorae borealis) are often criticized.

H.-C. Nataf, Solar Physics 297 (2022), 107

However: ^{10}Be and ^{14}C data give similar cycles. Decision requires analysis of more recent data.

F. S. et al., Astron. Nachr. 341 (2020), 600

N. Brehm et al., Nat. Geosci. 14 (2021), 10

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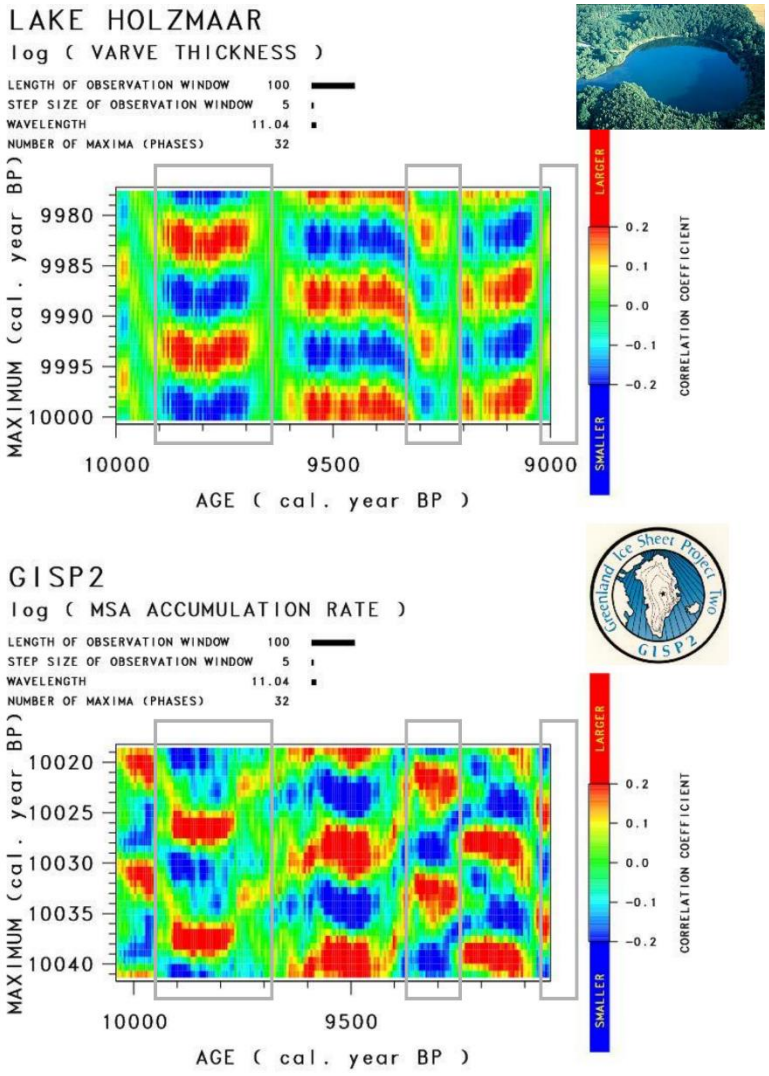
Is there a problem at all? Second indication…

Phase diagrams for algae data from lake Holzmaar und algae-produced Methanesulfonate (MSA) in Greenland ice core GISP2 show 11.04-years cycle with very similar band structures.

Bands are separated by apparent 5.5-years-phase jumps, resulting from nonlinear transfer function (due to optimality condition of algae growth)

Strong evidence for a 11.04-years-cycle that was phase-stable over 1000 years!

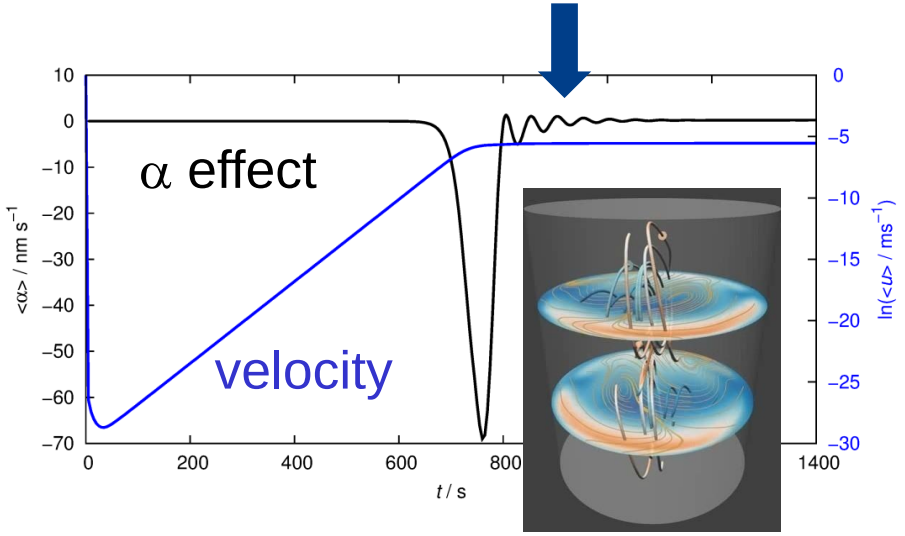


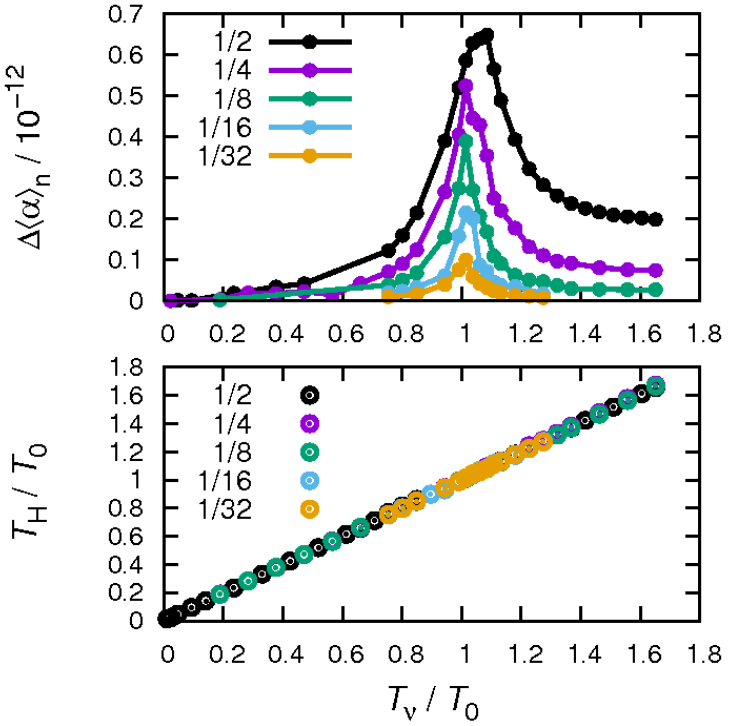
Vos et al., in “Climate in Historical Times: Towards a Synthesis of Holocene Proxy Data and Climate Models”, GKSS School of Environmental Research, p. 293 (2004)

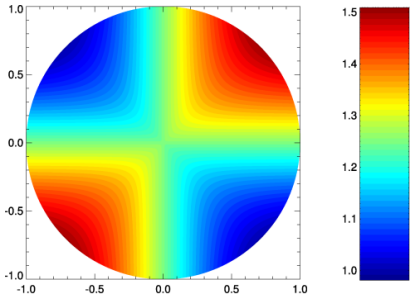
F. S. et al., Astron. Nachr. 341 (2020), 600

Why consider α to oscillate with a period of 11.07 years?

Current driven Tayler instability (with azimuthal wave number m=1) tends to undergo intrinsic helicity oscillations…







…which can by easily synchronized by tidal (m=2) perturbations (of the VEJ-system)

See later remarks on Rayleigh-Bénard experiment and Rossby waves

N. Weber et al., NJP 17 (2015), 113013; F. S. et al., Solar Phys. 291 (2016), 2197

A simple 1D α–Ω dynamo model including a synchronized α-term

$$\frac{\partial B(\theta,t)}{\partial t} = \omega(\theta,t)\frac{\partial A(\theta,t)}{\partial \theta} + \frac{\partial^2 B(\theta,t)}{\partial \theta^2} - \kappa B^3(\theta,t)$$

$$\frac{\partial A(\theta,t)}{\partial t} = \alpha(\theta,t)B(\theta,t) + \frac{\partial^2 A(\theta,t)}{\partial \theta^2},$$

Externally **„shaken“**

$$\omega(\theta,t) = \omega_0(1 - 0.939 - 0.136\cos^2(\theta) - 0.1457\cos^4(\theta))\sin(\theta),$$

$$\alpha(\theta,t) = \alpha^p(\theta,t) + \alpha^c(\theta,t)$$

$$\alpha^p(\theta,t) = \alpha_0^p \sin(2\pi t/11.07)\mathrm{sgn}(90^\circ - \theta)\frac{B^2(\theta,t)}{(1 + q_\alpha^p B^4(\theta,t))} \text{ for } 55^\circ < \theta < 125^\circ$$

$$\alpha^c(\theta,t) = \alpha_0^c\big(1 + \xi(t)\big)\sin(2\theta)/\big(1 + q_\alpha^c B^2(\theta,t)\big)$$

Internally **„stirred“**

F. S. et al., Solar Physics 294 (2019), 60

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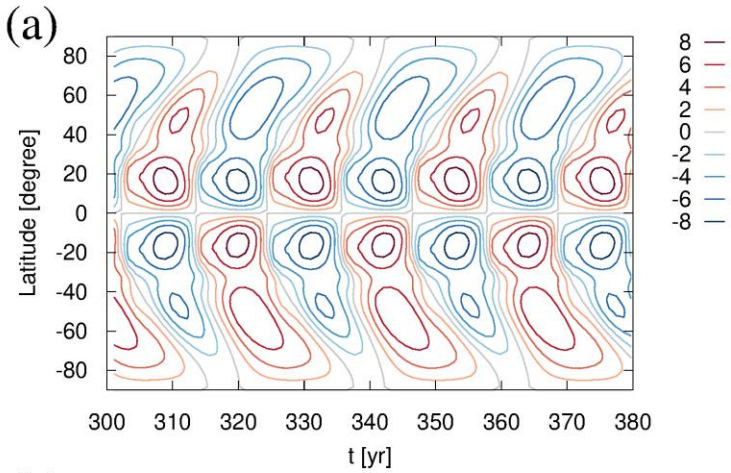
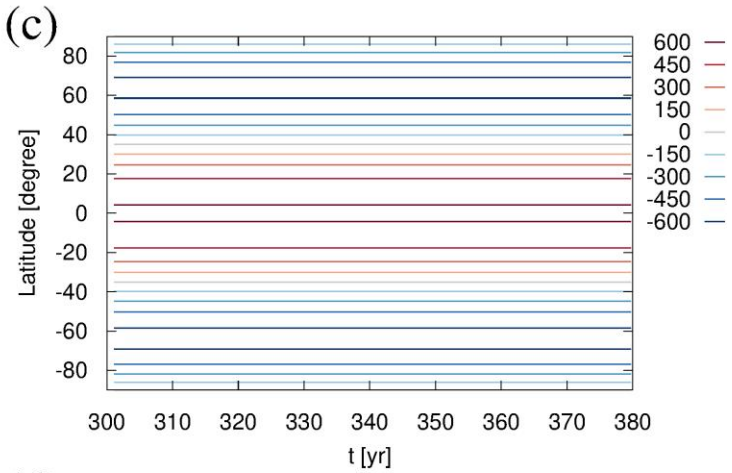
Typical result of an 1D α–Ω dynamo model

While obtaining a clear 22.14-yr period, the butterfly is not very nice yet (since 1D model does not include meridional circulation)

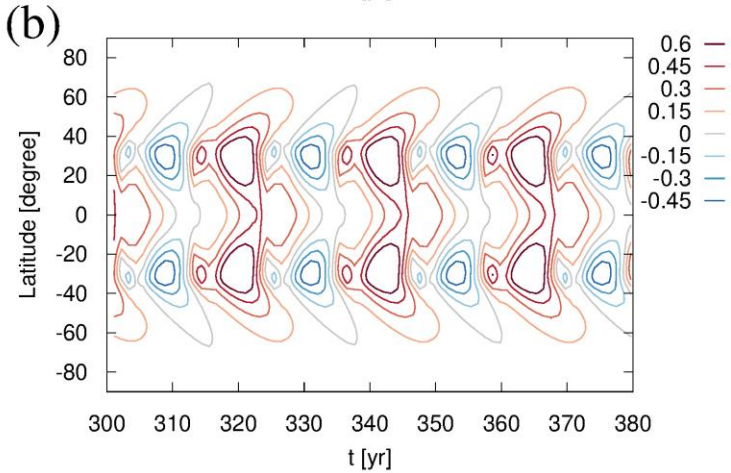
B(θ,t)

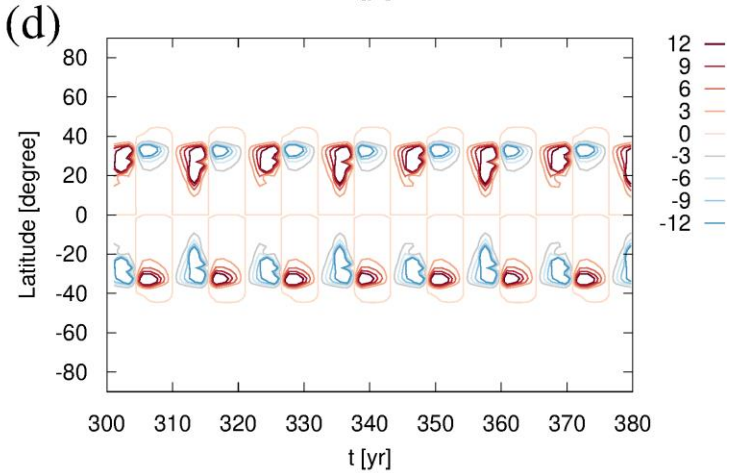




ω(θ,t)

A(θ,t)





α(θ,t)

ω_0=10000, κ=0.2, q_α^p=0.2, α_0^p=100

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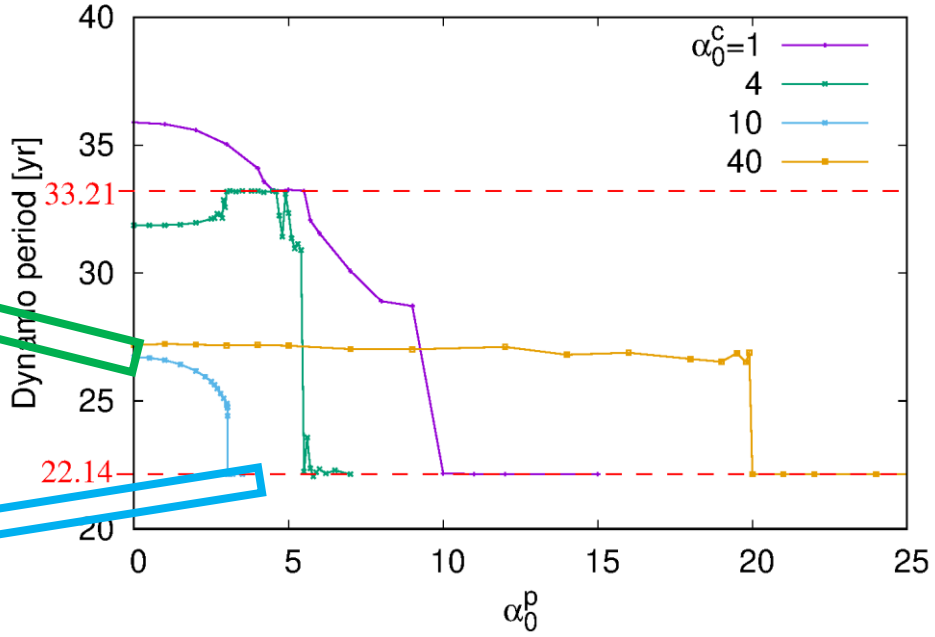
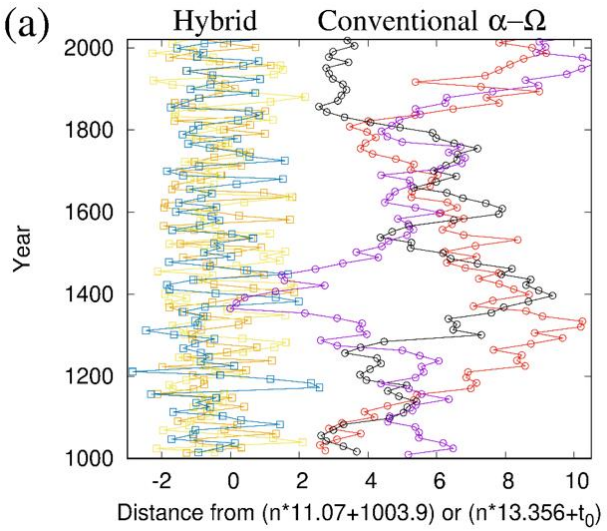
1D-Model with Noise

Conventional α–Ω dynamo yields random walk (RW)

Synchronized (hybrid) model yields clocked process (CP)



(CP) (RW)



A conventional α–Ω dynamo with a "natural" period not too far from 22.14 years is entrained, by parametric resonance, to exactly this period.

F. S. et al., Solar Physics 294 (2019), 60

Mitglied der Helmholtz-Gemeinschaft

More realistic 2D-Model confirms parametric resonance

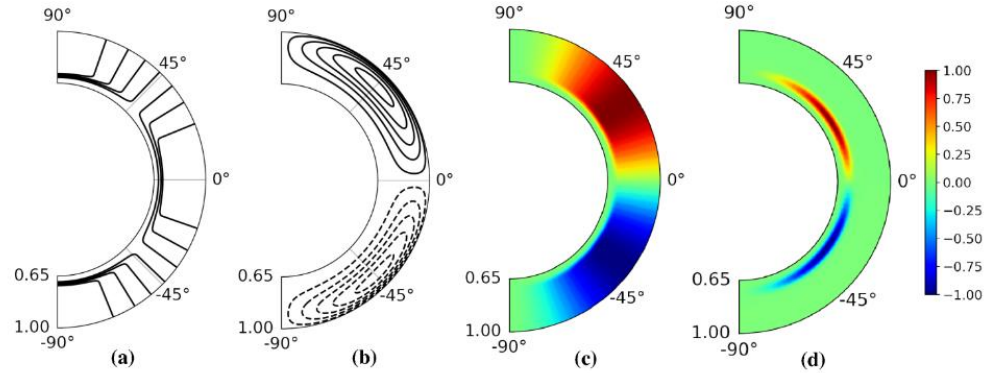
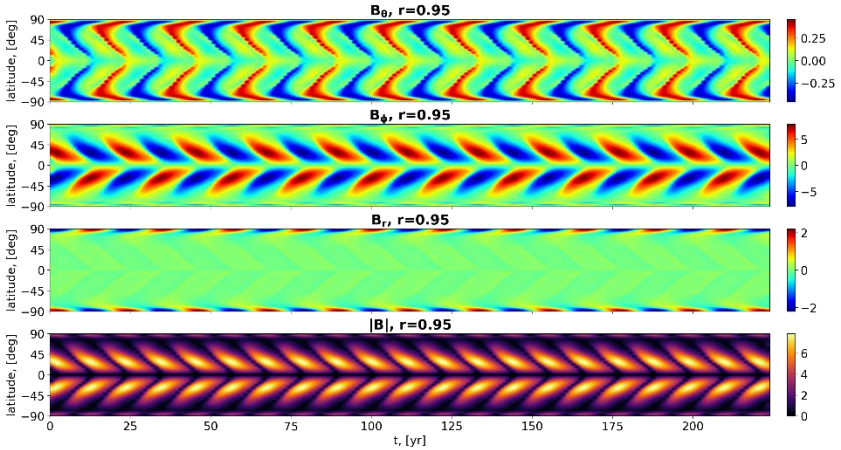
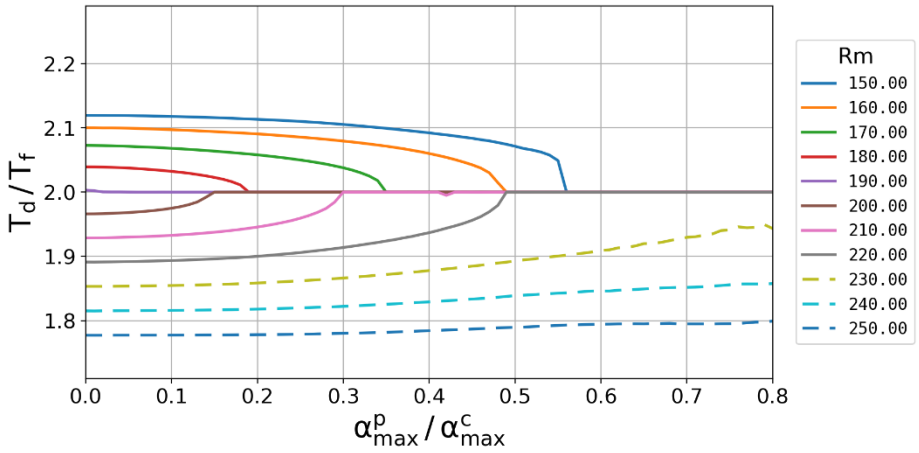


Figure 1. Spatial structures of the main ingredients of the dynamo model in the meridional plane. (a) Isolines of $\Omega(r,\Theta)/\Omega_{\max}$. (b) Streamlines of $\mathbf{u}_p(r,\Theta)$. (c) Constant part of α, taken in the unquenched state: $\alpha^c(r,\Theta)/\alpha^c_{\max}$. (d) Periodic part of α, with the resonance term set to 1: $\alpha^p(r,\Theta)/\alpha^p_{\max}$.





For a reasonable value α_0=1.3 m/s, we need just some dm/s for the synchronized α to entrain the entire dynamo

Can such a value arise from tidal forcing?

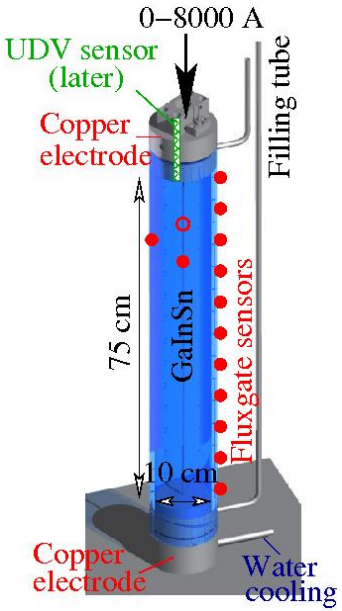
→ Wait until last slide!

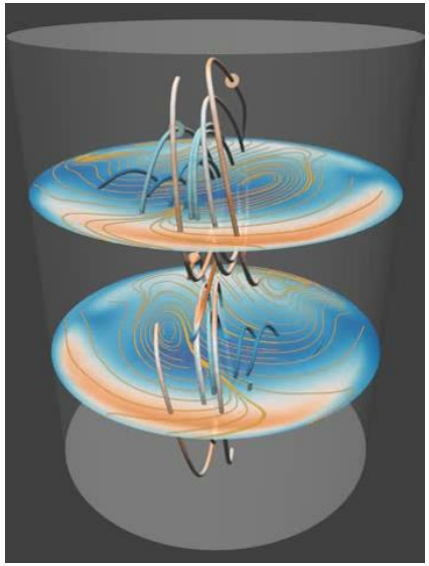
M. Klevs, F.S., in preparation

(Rayleigh-Bénard) Convection

Mitglied der Helmholtz-Gemeinschaft

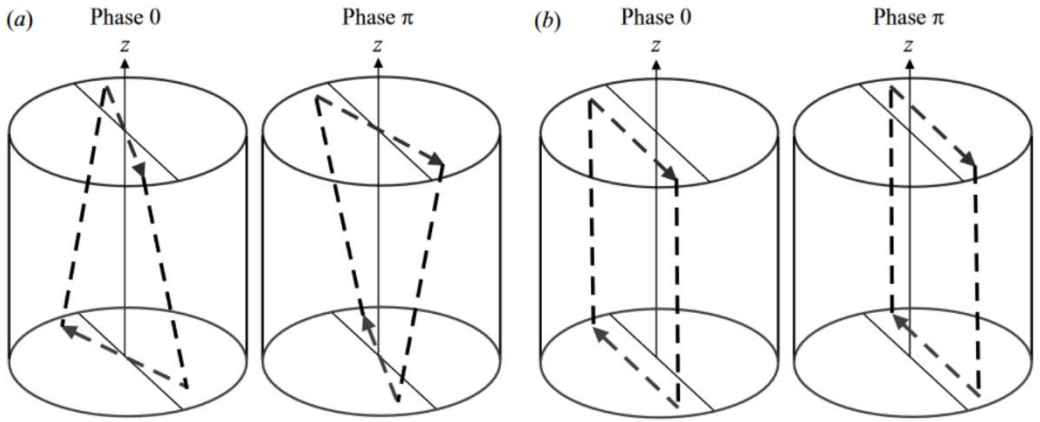
How to realize tidal helicity synchronization in the lab?





Tayler instability: Quite hard to accomplish in experiment

However: A similar sidewise motion, connected with helicity oscillations, occurs as torsional/sloshing motion in Rayleigh-Bénard convection



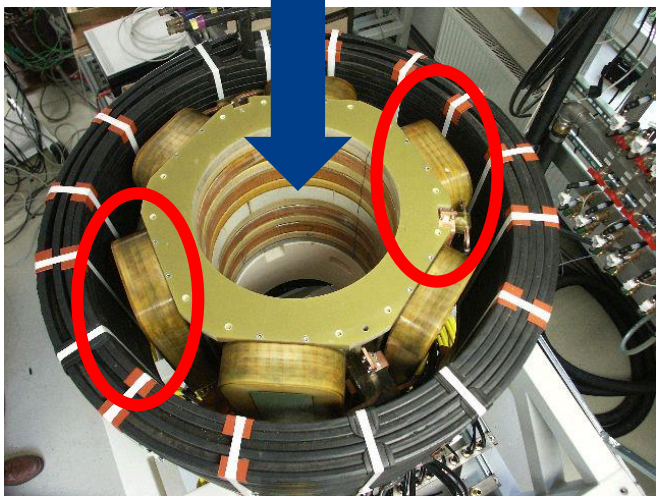
E. Brown, G. Ahlers, J. Fluid Mech. 638 (2009), 383-400

Mitglied der Helmholtz-Gemeinschaft

RB experiment: helicity synchronization with m=2 forcing

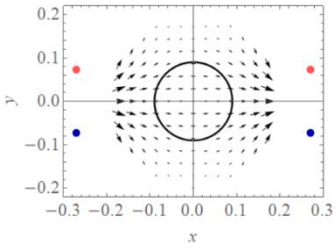
Generic Rayleigh-Bénard experiment to show resonant excitation of helicity by an m=2 perturbation.

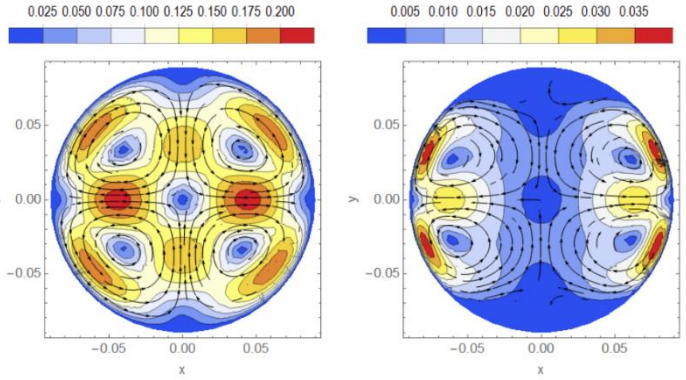
How to realize this: Magnetic forces by currents through 2 coils at opposite positions



Multimag facility at HZDR

Magnetic field





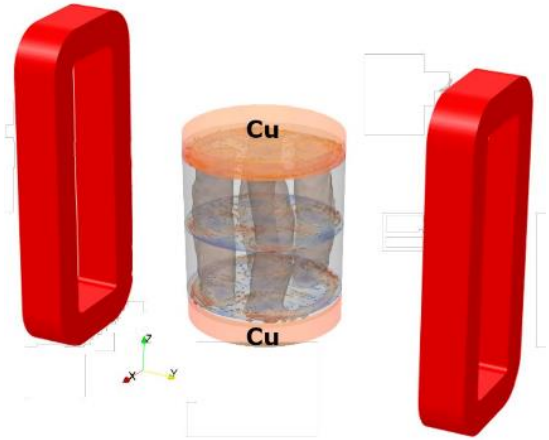
m=2 Velocity at

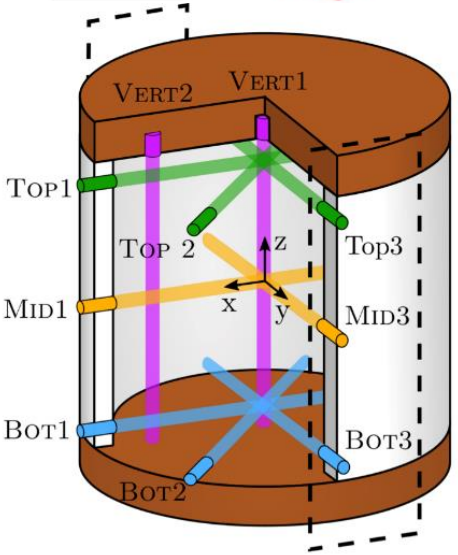
100 Hz **1600 Hz**

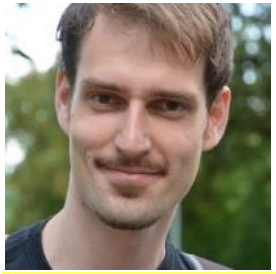
R. Stepanov, F.S.: Magnetohydrodynamics 55 (2019) 207

RB experiment: helicity synchronization with m=2 forcing

Resonant excitation of helicity oscillation (sloshing) by tide-like forcing



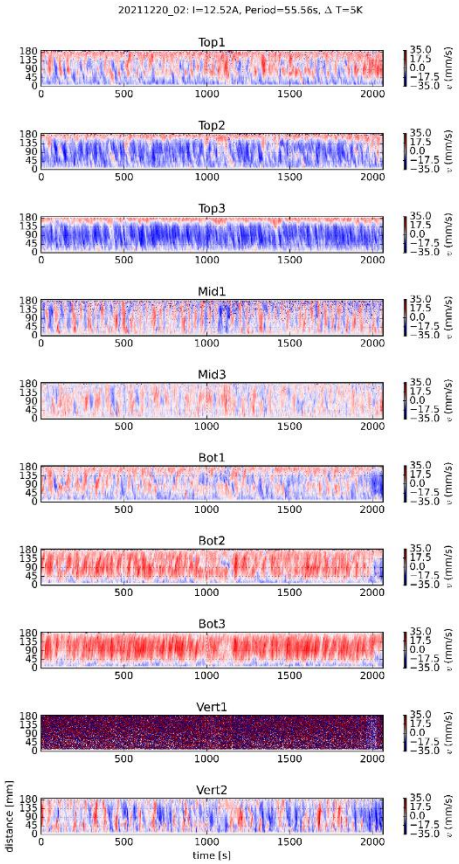


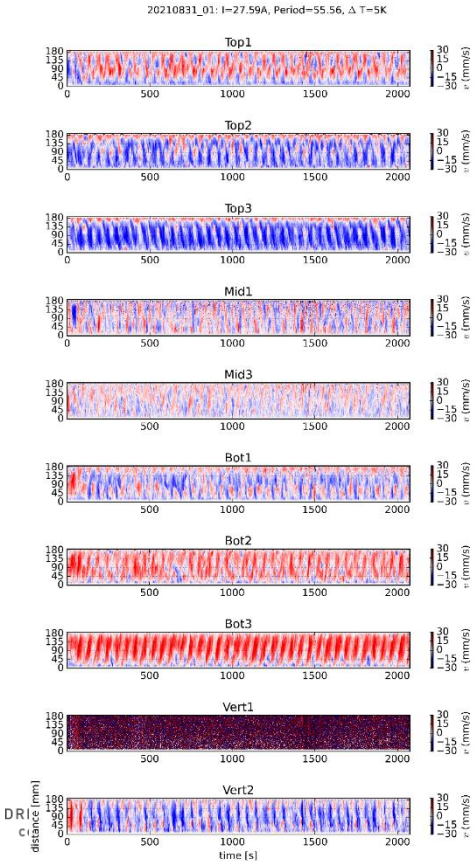


P. Jüstel et al., Phys. Fluids 34 (2022), 104115

12.5 A → **27.5 A**

Helicity synchronisation

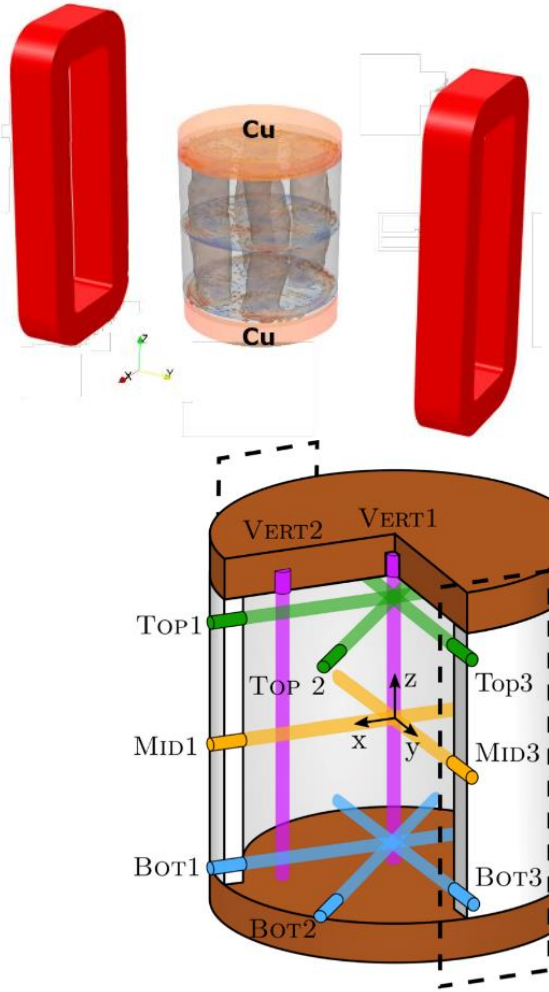


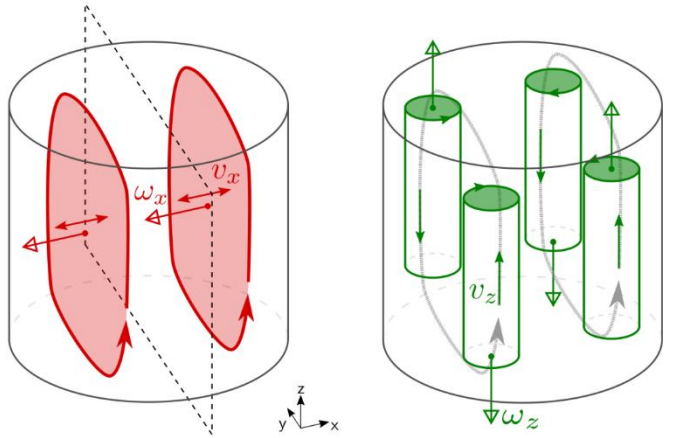


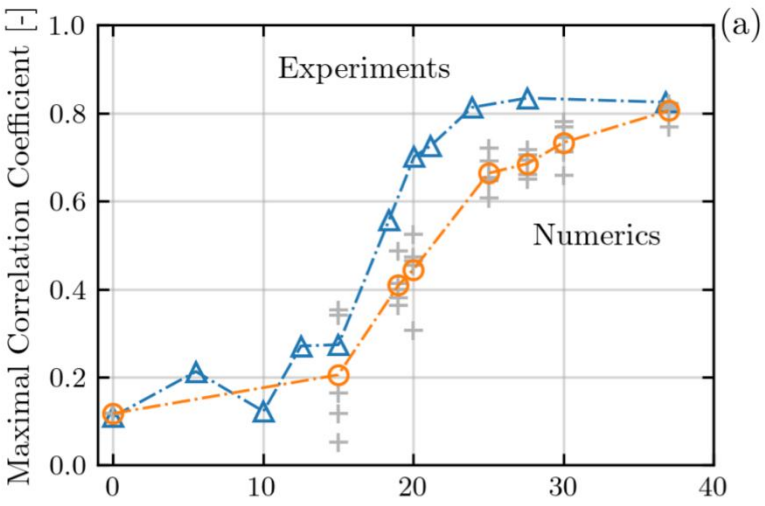
Mitglied der Helmholtz-Gemeinschaft

RB experiment: helicity synchronization with m=2 forcing

Resonant excitation of helicity oscillation (sloshing) by tide-like forcing





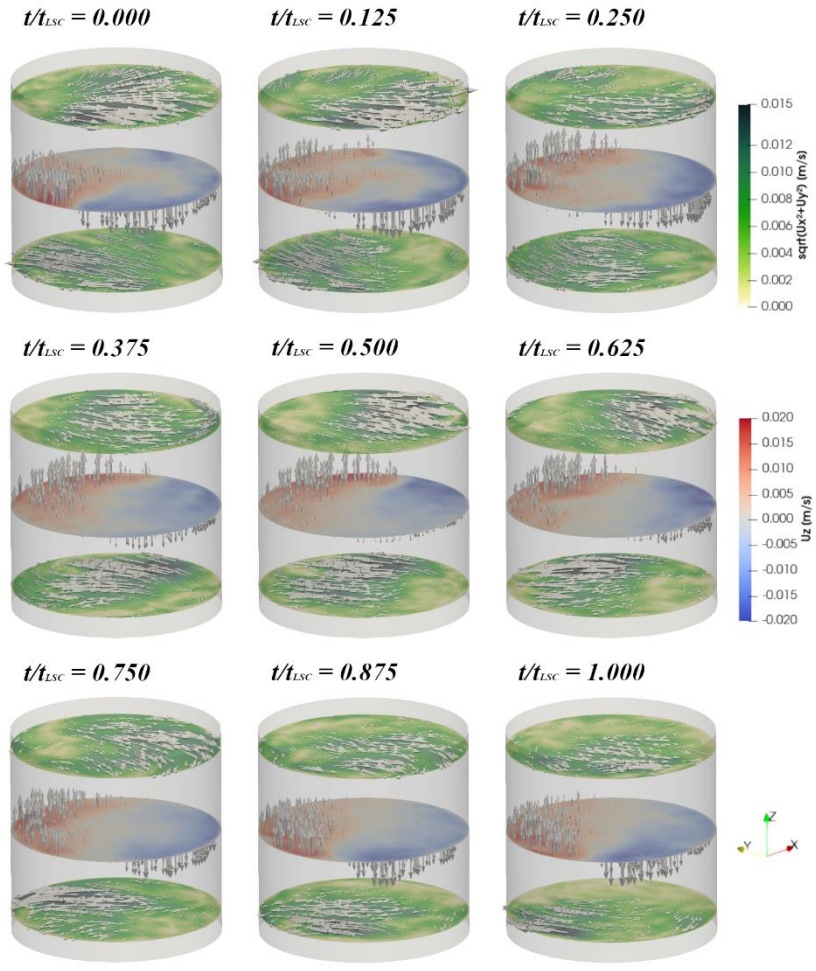


P. Jüstel et al., Phys. Fluids 34 (2022), 104115

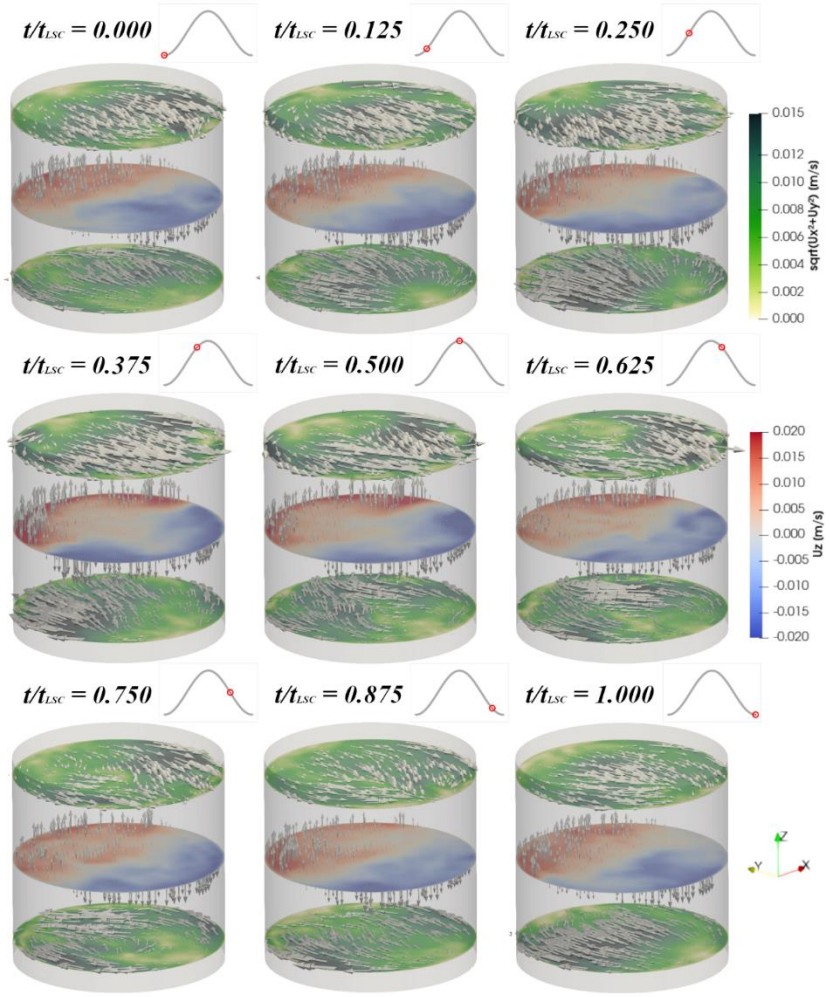
Mitglied der Helmholtz-Gemeinschaft

RB experiment: flow field over one sloshing period

Pure RB without current



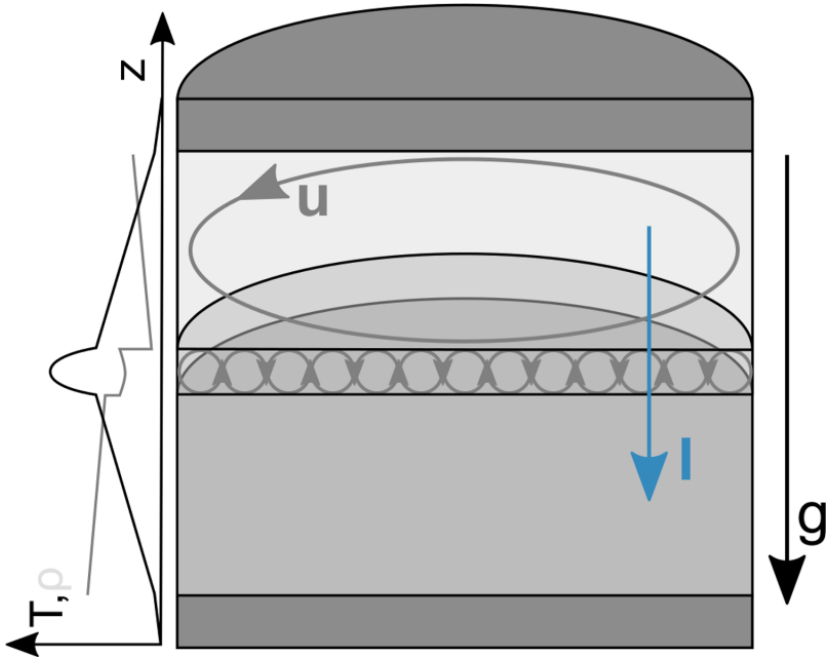
RB + tide-like forcing with 30 A

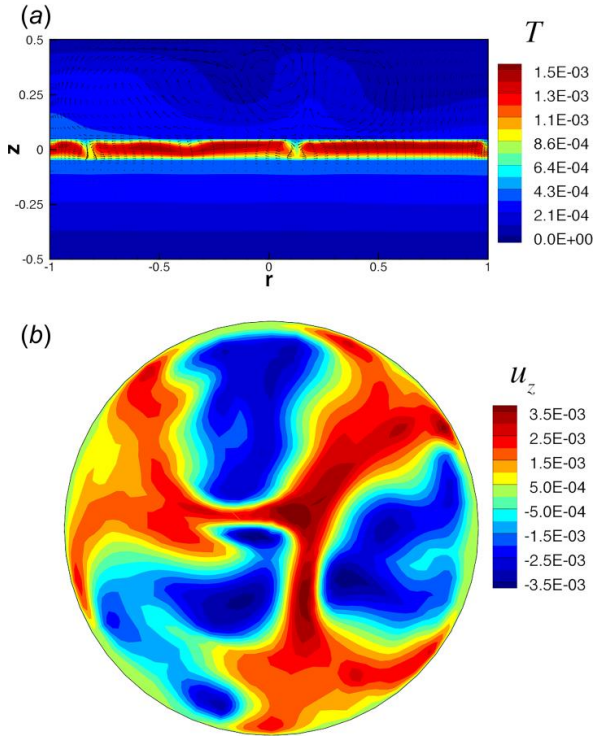


P. Jüstel et al., Phys. Fluids 34 (2022), 104115

Rayleigh-Bénard convection in LMB´s

Thermal convection is most relevant in the electrolyte and the anode





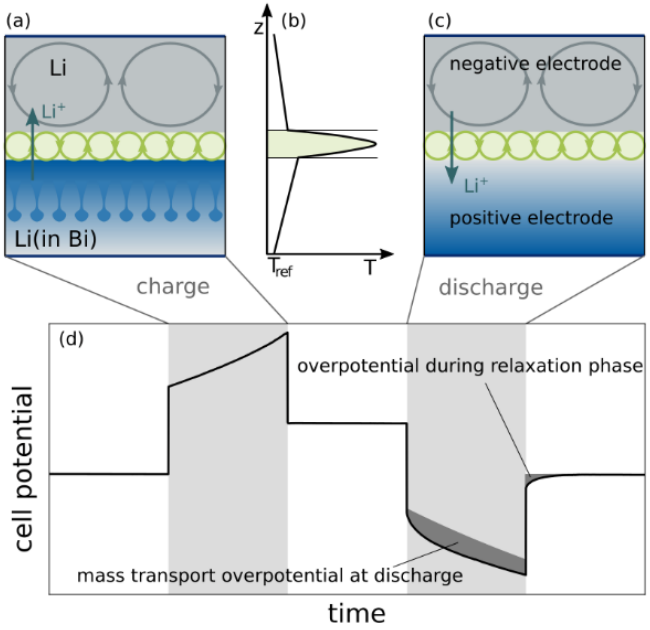
D. Kelley, D. Sadoway, Phys. Fluids 26 (2014), 057102; Y. Shen, O. Zikanov, Theor. Comp. Fluid Dyn. 30 (2016), 275; D.H. Kelley, T. Weier, Appl. Mech. Rev. 70 (2018), 020801

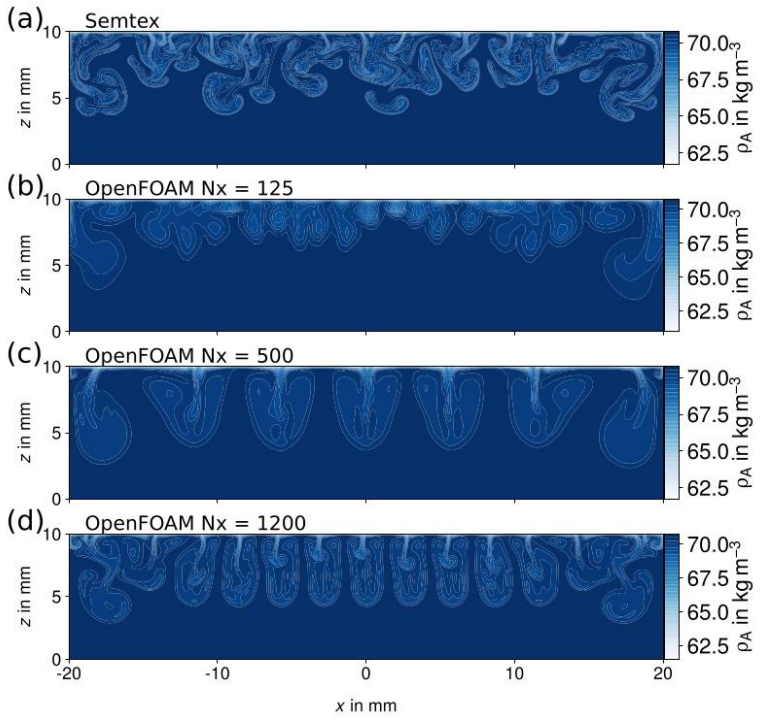
(a) Vertical cross section through the center of the battery, (b) horizontal cross section above the electrolyte. Radius and thickness of the battery are 52 mm, LiCl–KCl electrolyte.

Mitglied der Helmholtz-Gemeinschaft

Coupling of thermal and solutal convection in LMB´s

Solutal convection is most relevant in the electrolyte and the cathode





P. Personnettaz et al., Int. J. Heat Mass. Trans. 188 (2022), 122555

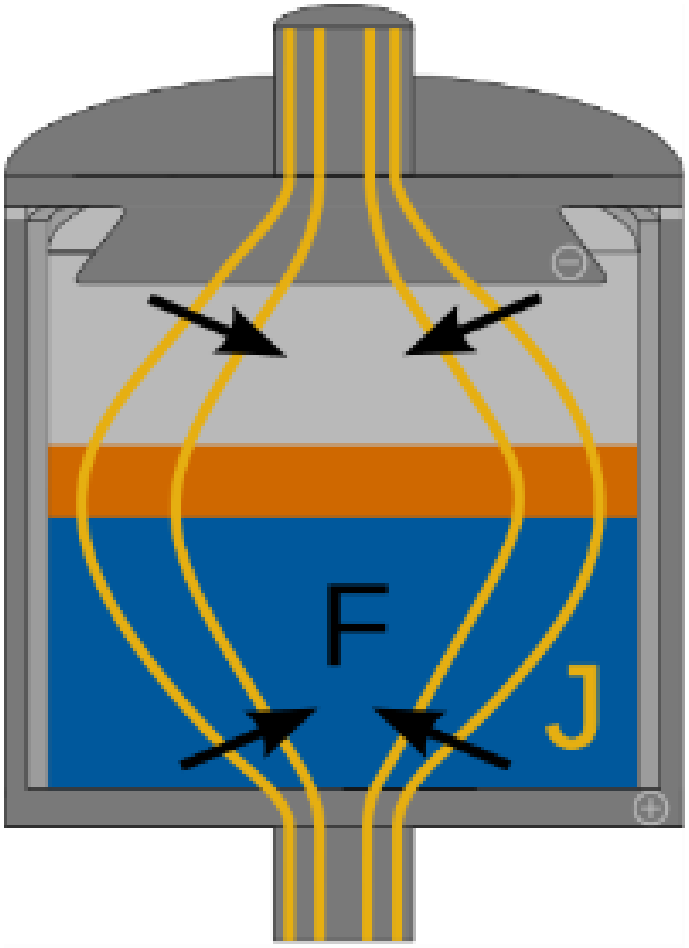
(a) Concentration distribution obtained with Semtex and (b-d) OpenFOAM at different grid resolutions at t = 6.5 s

See Tom Weier's lecture and Carolina Duczek's poster

Electrovortex flow

Mitglied der Helmholtz-Gemeinschaft

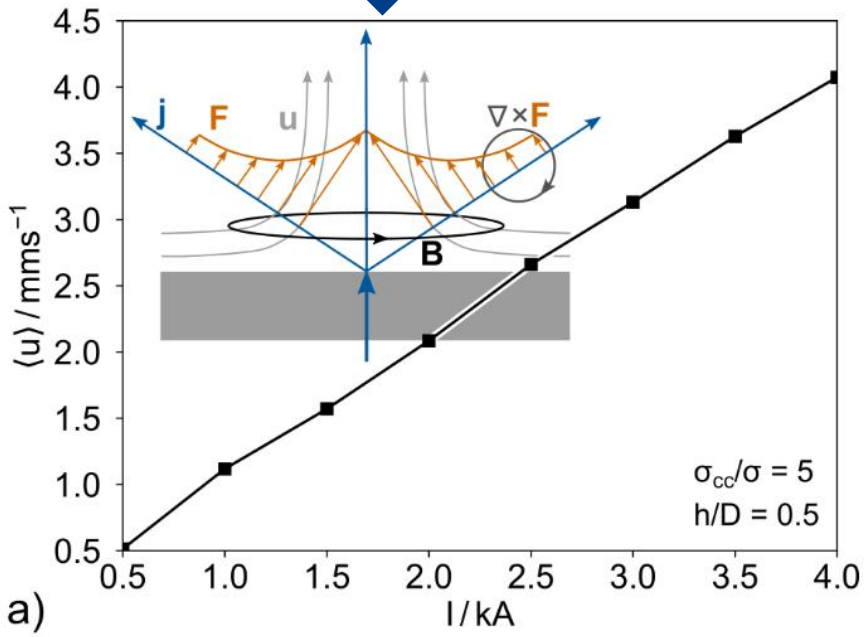
The role of Electrovortex Flows (EVF) in LMB´s



Mitglied der Helmholtz-Gemeinschaft

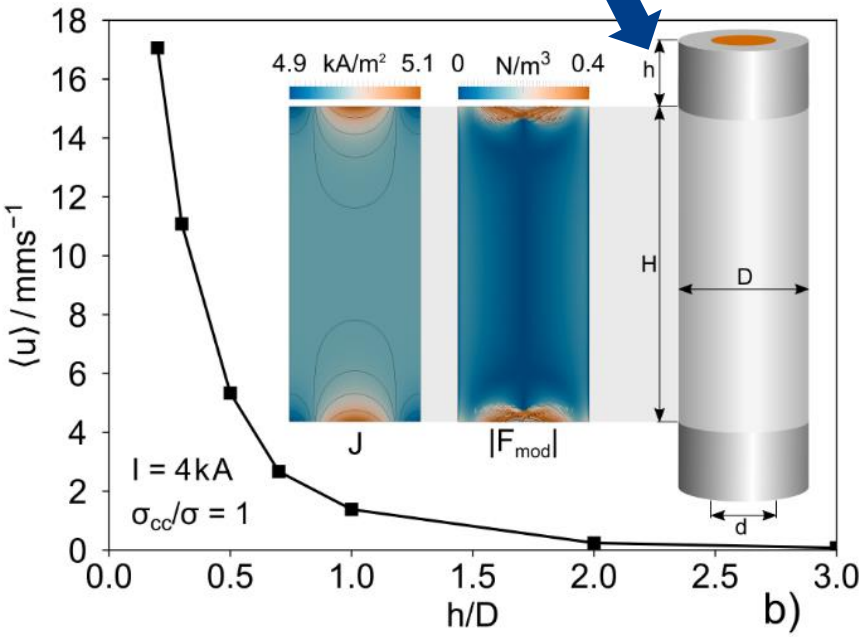
The role of Electrovortex Flows (EVF) in LMB´s

J.A. Shercliff, J. Fluid Mech. 40 (1979) 241



Mean velocity vs. total current

Na-based battery: D = 1 m,
d/D=0.5, H/D=2.4, h is variable



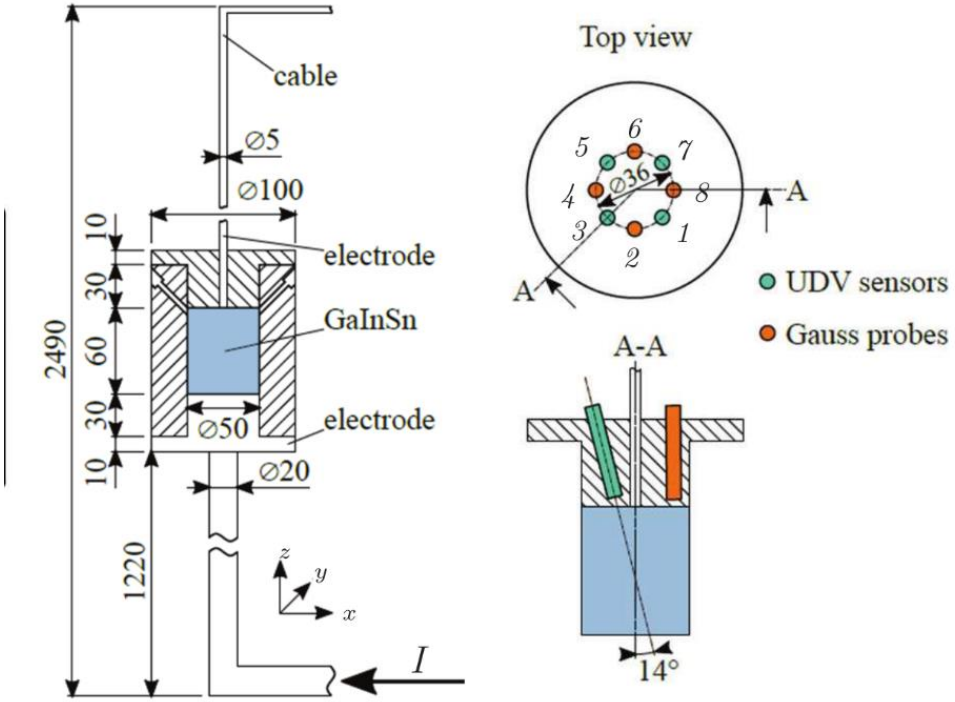
Mean velocity vs. current collector h/D

N. Weber et al., Phys. Fluids 29 (2017), 054101;

T. Weier et al., IOP Conf. Ser.: Mater. Sci. Eng. 228 (2017), 012013

Mitglied der Helmholtz-Gemeinschaft

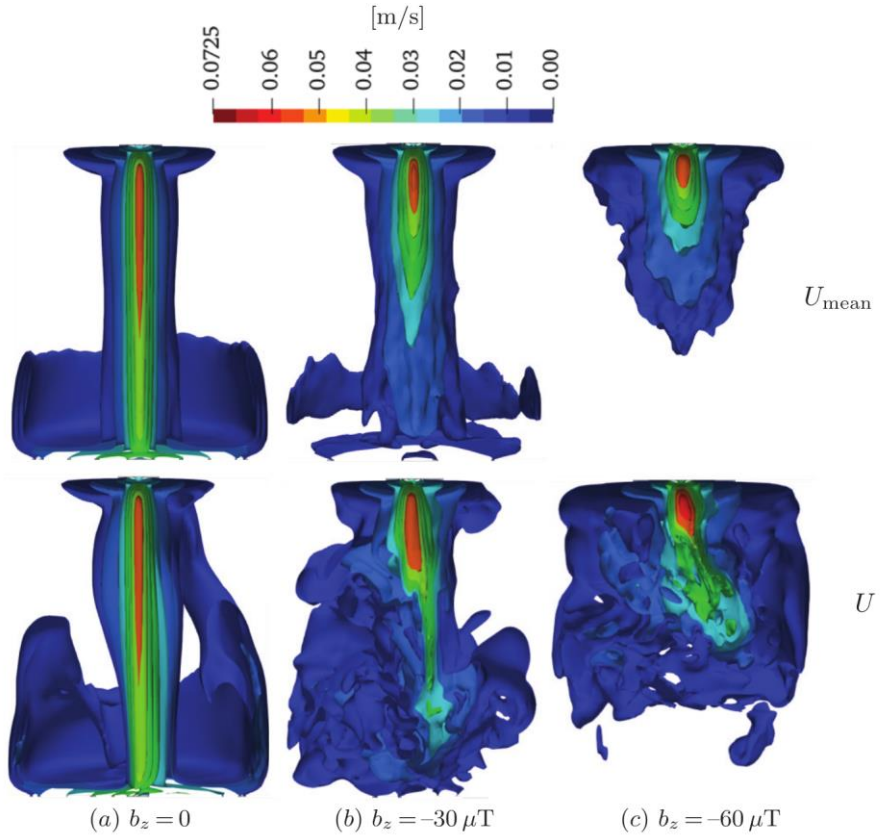
EVF experiment at HZDR with 80 A

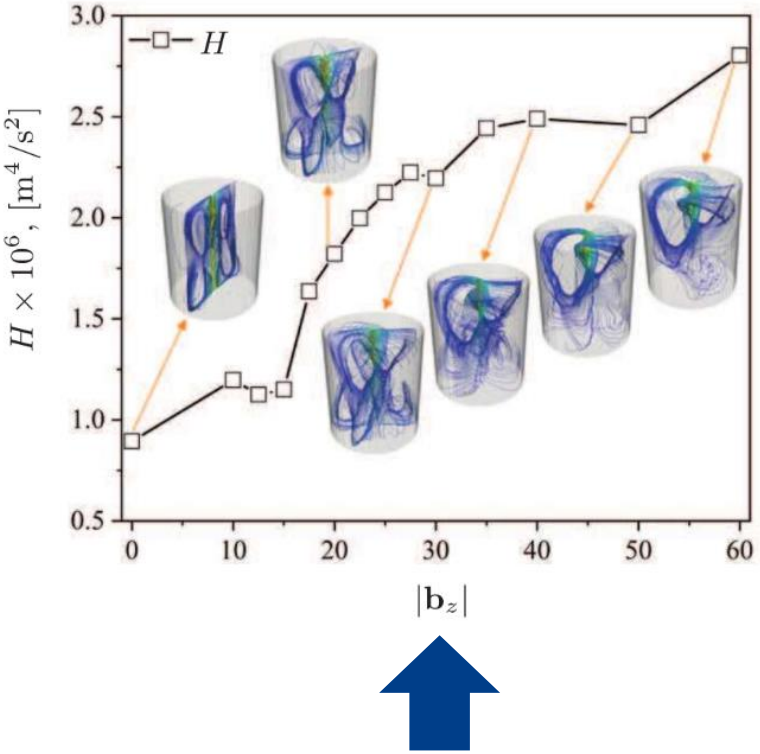


K. Liu et al., Magnetohydrodynamics 56 (2020), 27

Magnetohydrodynamics 57 (2021), 437

EVF experiment at HZDR with 80 A



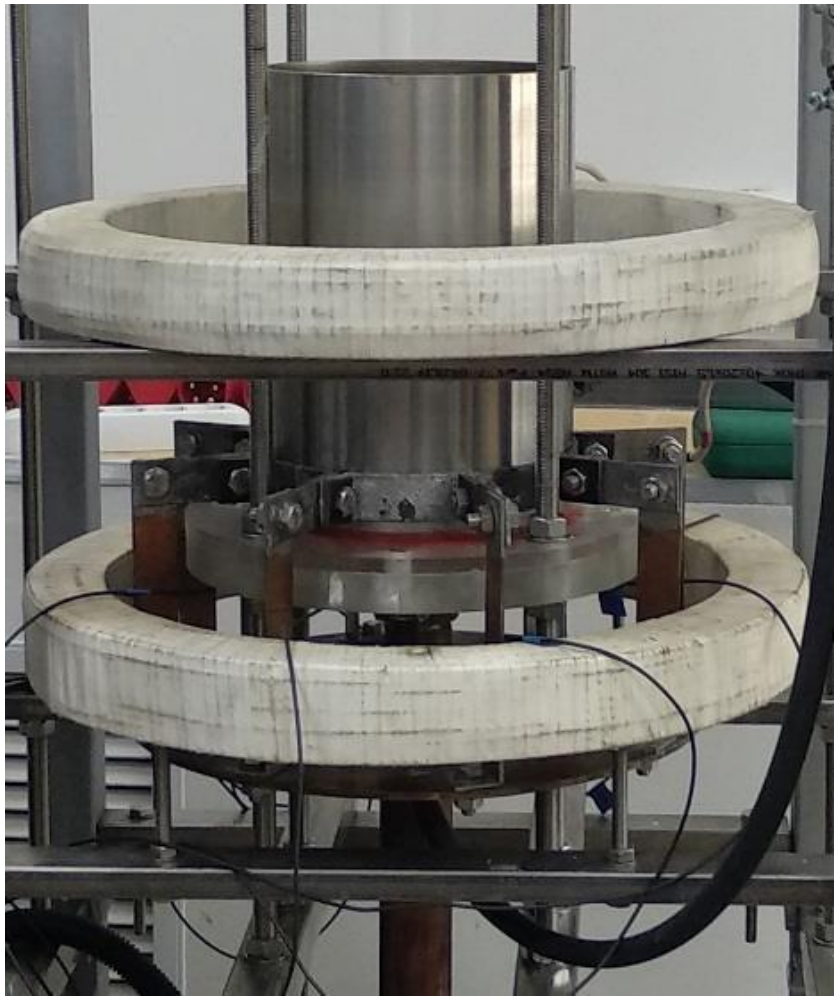


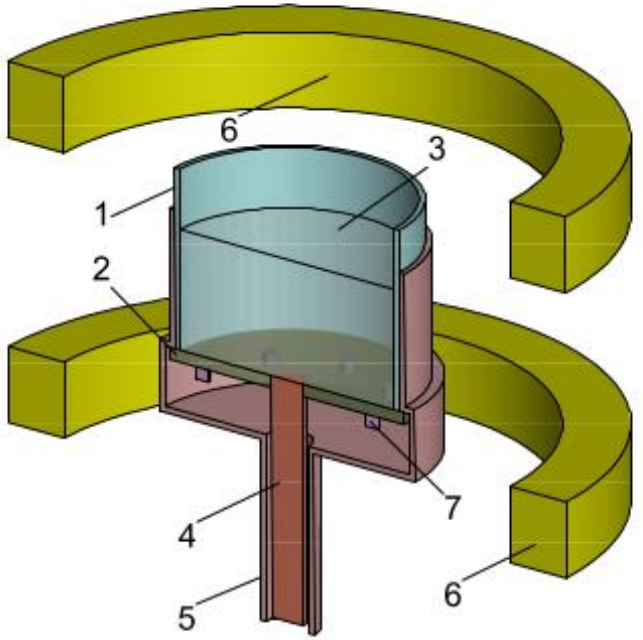
With increasing B_z, central jet deflects and rotates. Emergence of helicity.

K. Liu et al., Magnetohydrodynamics 56 (2020), 27
Magnetohydrodynamics 57 (2021), 437

Mitglied der Helmholtz-Gemeinschaft

EVF experiment in Perm with much stronger current: 1000 A





1 - Wall; 2 - Bottom; 3 – GaZnSn; 4 - Cathode; 5 - Anode; 6 - Helmholtz coil; 7 - UDV transducers;

Diameter=201.4 mm, I=1000 A

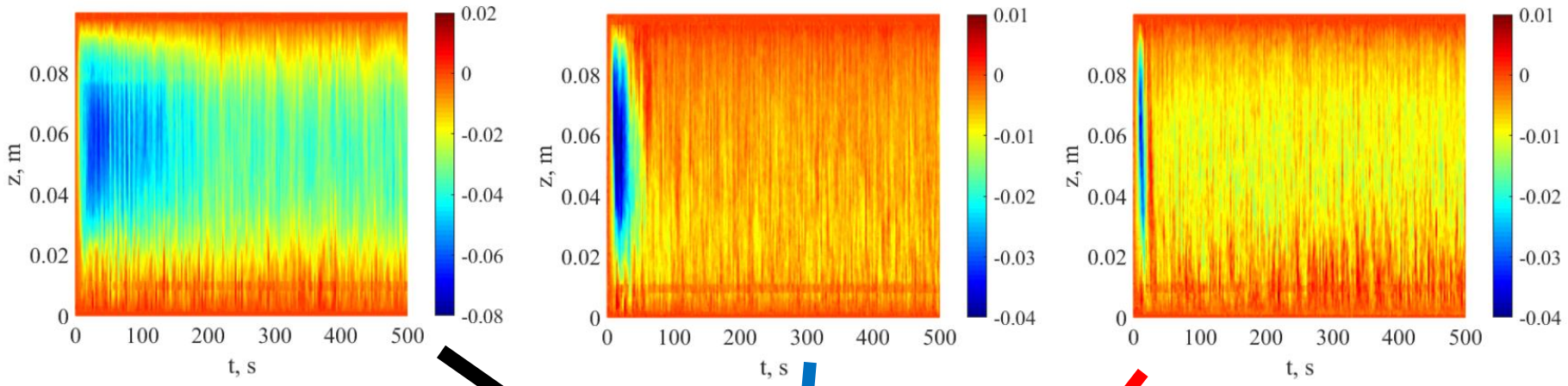
I. Kolesnichenko et al., Phys. Rev. Fluids 5 (2020), 123703

Mitglied der Helmholtz-Gemeinschaft

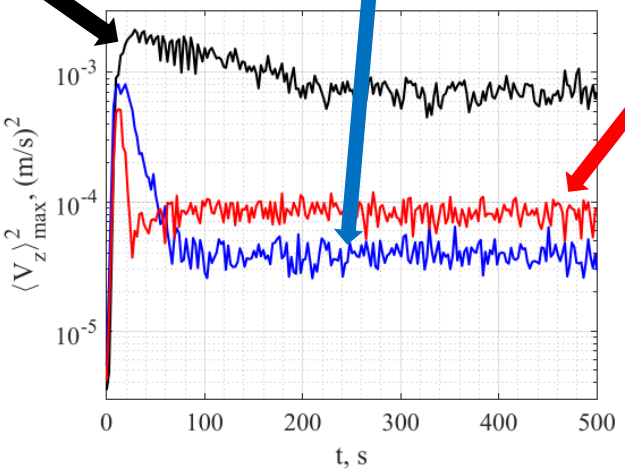
EVF experiment in Perm: Poloidal flow suppression

UDV-measured vertical velocity profile for…

B_{earth} | B_{earth} + 0.1 mT | B_{earth} + 0.5 mT



Maximum (over z) squared velocity



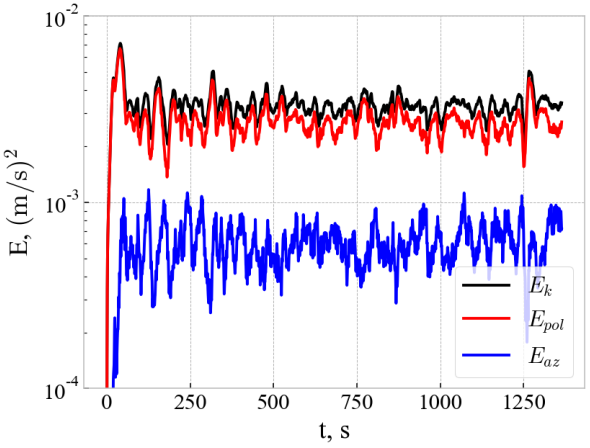
For higher vertical field, poloidal flow is dramatically suppressed

I. Kolesnichenko et al., Phys. Rev. Fluids 5 (2020), 123703

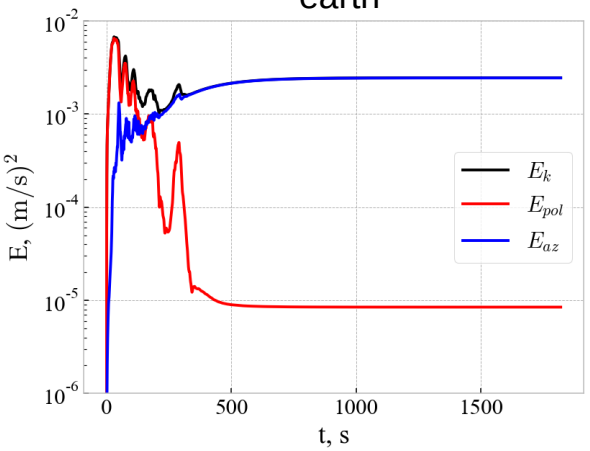
EVF experiment in Perm: Poloidal flow suppression

Numerically simulated flow energies for…

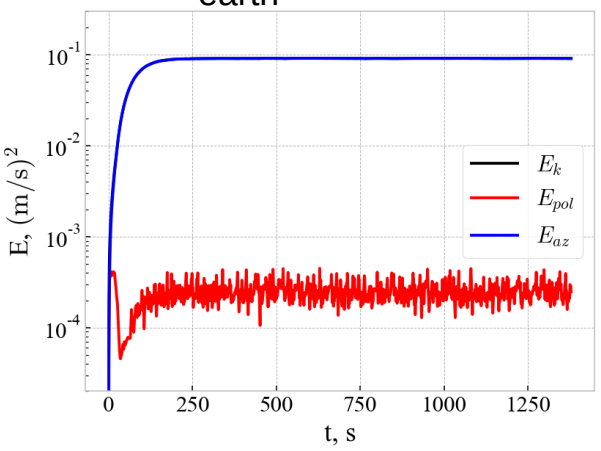
No vertical field

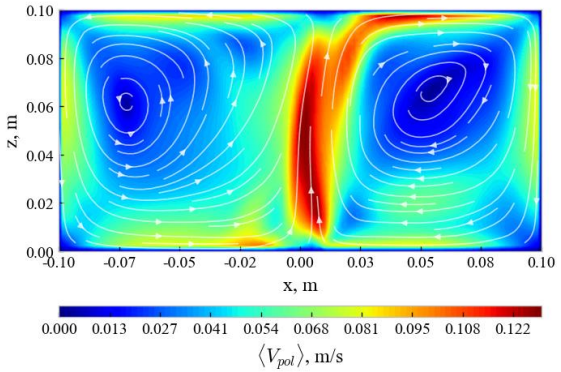


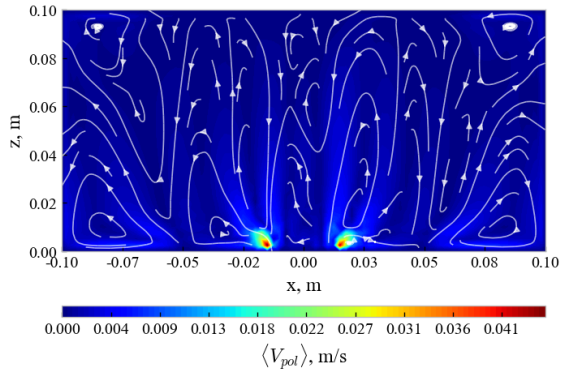
B_{earth}

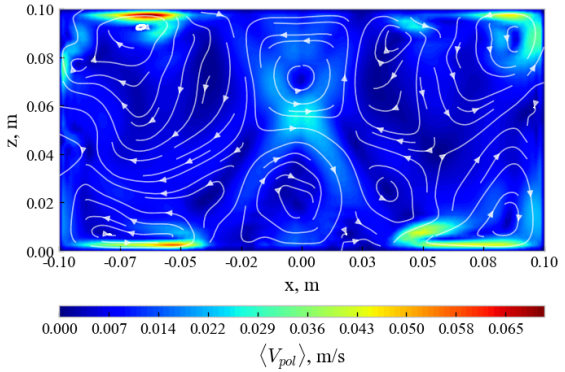


B_{earth} + 1 mT



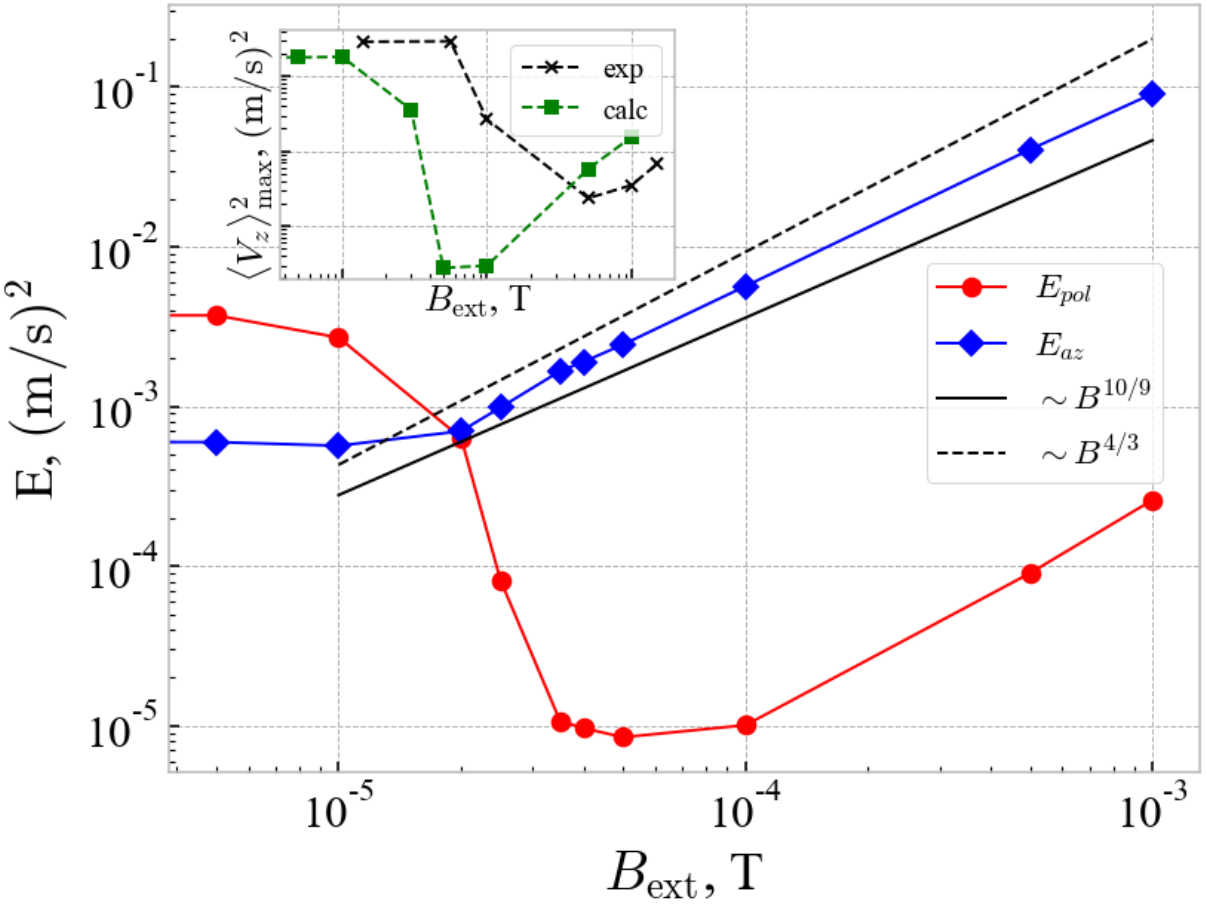






I. Kolesnichenko et al., Phys. Rev. Fluids 5 (2020), 123703

EVF experiment in Perm: Poloidal flow suppression



This confirms the theory of

P. Davidson et al., Eur. J. Mech. B **18** (1999), 693;
W. Herreman et al., Phys. Rev. Fluids 4 (2019), 113702

New results in:

P. Frick et al., JFM 949 (2022), A20

Mitglied der Helmholtz-Gemeinschaft

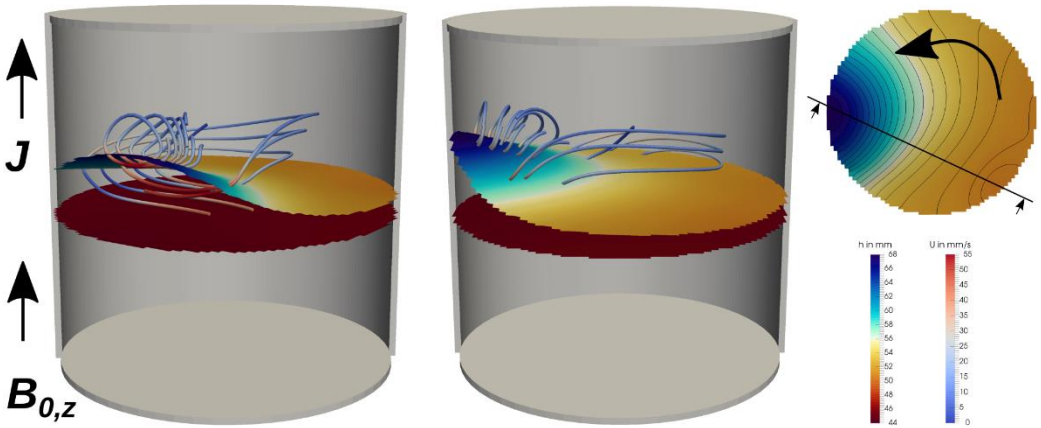
(Tidally forced) waves

Mitglied der Helmholtz-Gemeinschaft

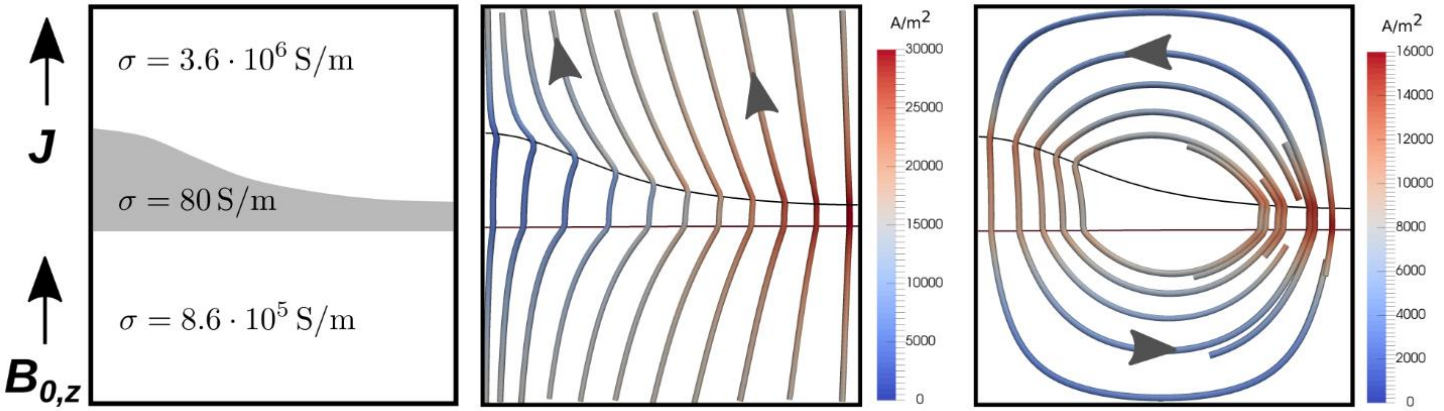
Waves in LMB´s: Usual Metal Pad Roll instability

O. Zikanov, Phys. Rev. E 92 (2015), 063021

N. Weber et al., Phys. Fluids 29 (2017), 054101



Mg||Sb battery:
D=10 cm, h_1=h_3=4.5 cm, h_2 = 1 cm, I = 120 A, B_z = 10 mT



Conductivities Full current Compensation current

Mitglied der Helmholtz-Gemeinschaft

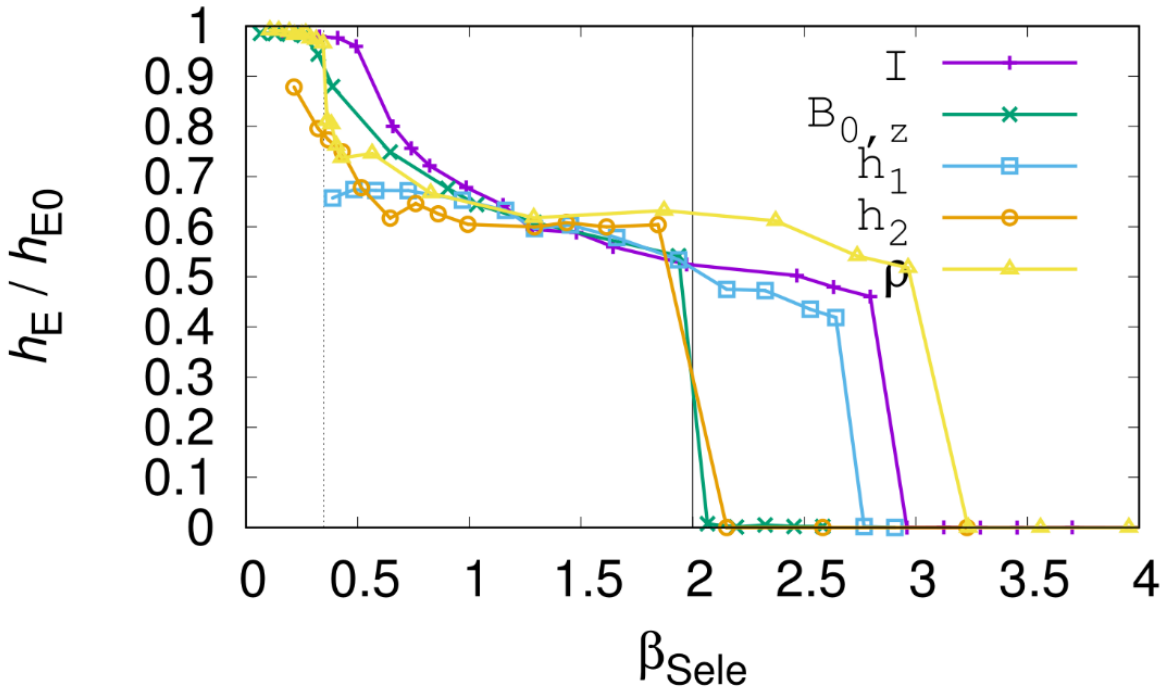
Waves in LMB´s: Usual Metal Pad Roll instability

T. Sele, Metall. Trans. B 8 (1977), 613

N. Weber et al., Phys. Fluids 29 (2017), 054101

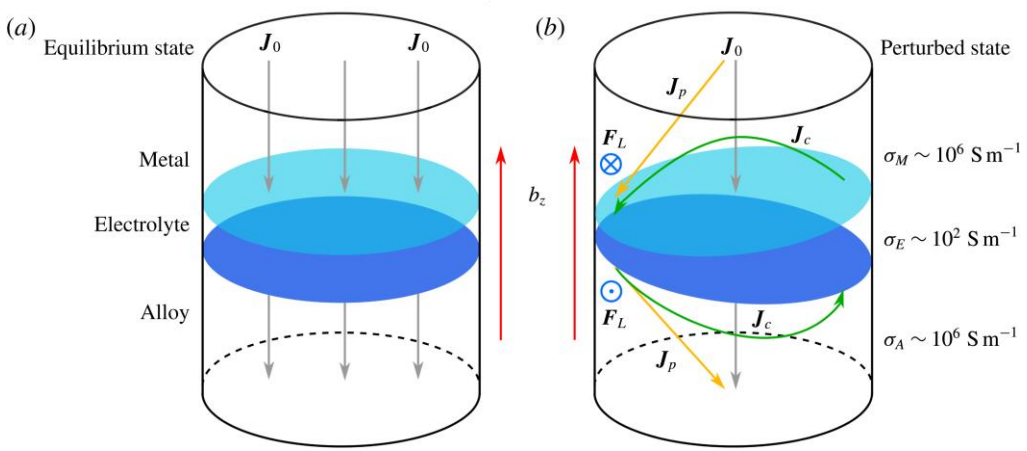
Sele´s criterion works rather well

$$\beta_{\mathrm{Sele}} = \frac{I B_{0,z}}{g(\rho_2 - \rho_1) h_1 h_2}$$



Mitglied der Helmholtz-Gemeinschaft

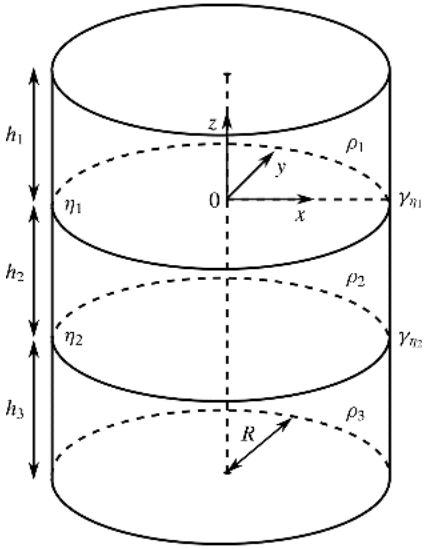
Waves in LMB´s: Metal Pad Roll instability → 3-layer system



N. Weber et al., Phys. Fluids 29 (2017), 054101

G. Horstmann et al., J. Fluid Mech. 845 (2018), 1

See G. Horstmann´s poster



Resulting dispersion relation:

$$a\omega^4 + b\omega^2 + c = 0, \quad \text{with}$$

$$a = \left(\rho_2 \coth\left(\frac{\epsilon_{mn}}{R}h_2\right) + \rho_3 \coth\left(\frac{\epsilon_{mn}}{R}h_3\right)\right)\left(\rho_2 \coth\left(\frac{\epsilon_{mn}}{R}h_2\right) + \rho_1 \coth\left(\frac{\epsilon_{mn}}{R}h_1\right)\right) - \frac{\rho_2^2}{\sinh\left(\frac{\epsilon_{mn}}{R}h_2\right)^2},$$

$$b = -\left((\rho_2 - \rho_1)g\frac{\epsilon_{mn}}{R} + \gamma_{\eta_1}\left(\frac{\epsilon_{mn}}{R}\right)^3\right)\left(\rho_2 \coth\left(\frac{\epsilon_{mn}}{R}h_2\right) + \rho_3 \coth\left(\frac{\epsilon_{mn}}{R}h_3\right)\right) - \left((\rho_3 - \rho_2)g\frac{\epsilon_{mn}}{R} + \gamma_{\eta_2}\left(\frac{\epsilon_{mn}}{R}\right)^3\right)\left(\rho_2 \coth\left(\frac{\epsilon_{mn}}{R}h_2\right) + \rho_1 \coth\left(\frac{\epsilon_{mn}}{R}h_1\right)\right),$$

$$c = \left((\rho_2 - \rho_1)g\frac{\epsilon_{mn}}{R} + \gamma_{\eta_1}\left(\frac{\epsilon_{mn}}{R}\right)^3\right)\left((\rho_3 - \rho_2)g\frac{\epsilon_{mn}}{R} + \gamma_{\eta_2}\left(\frac{\epsilon_{mn}}{R}\right)^3\right),$$

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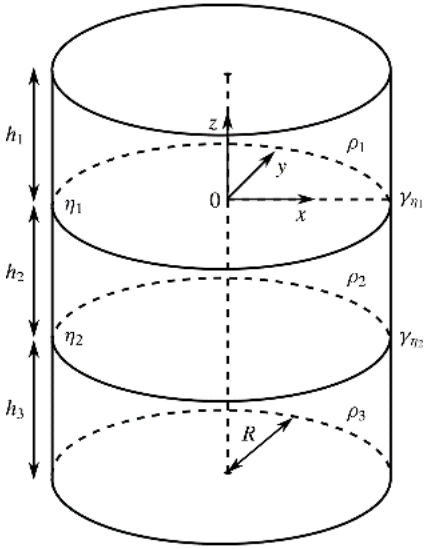
Waves in LMB´s: Metal Pad Roll instability → 3-layer system

Resulting dispersion relation:



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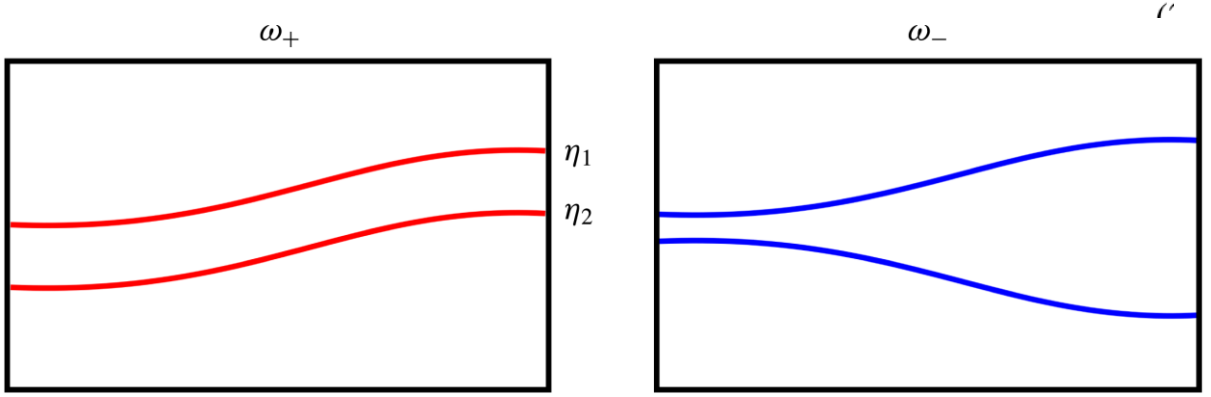
$$c = \left((\rho_2 - \rho_1)g\frac{\epsilon_{mn}}{R} + \gamma_{\eta_1}\left(\frac{\epsilon_{mn}}{R}\right)^3\right)\left((\rho_3 - \rho_2)g\frac{\epsilon_{mn}}{R} + \gamma_{\eta_2}\left(\frac{\epsilon_{mn}}{R}\right)^3\right),$$

Solution:
Fast and slow waves

$$\omega_\pm^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



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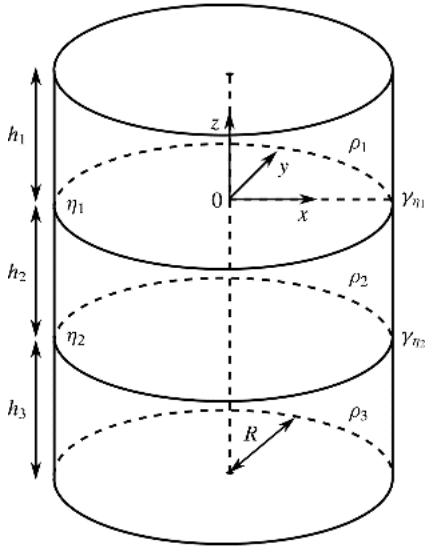
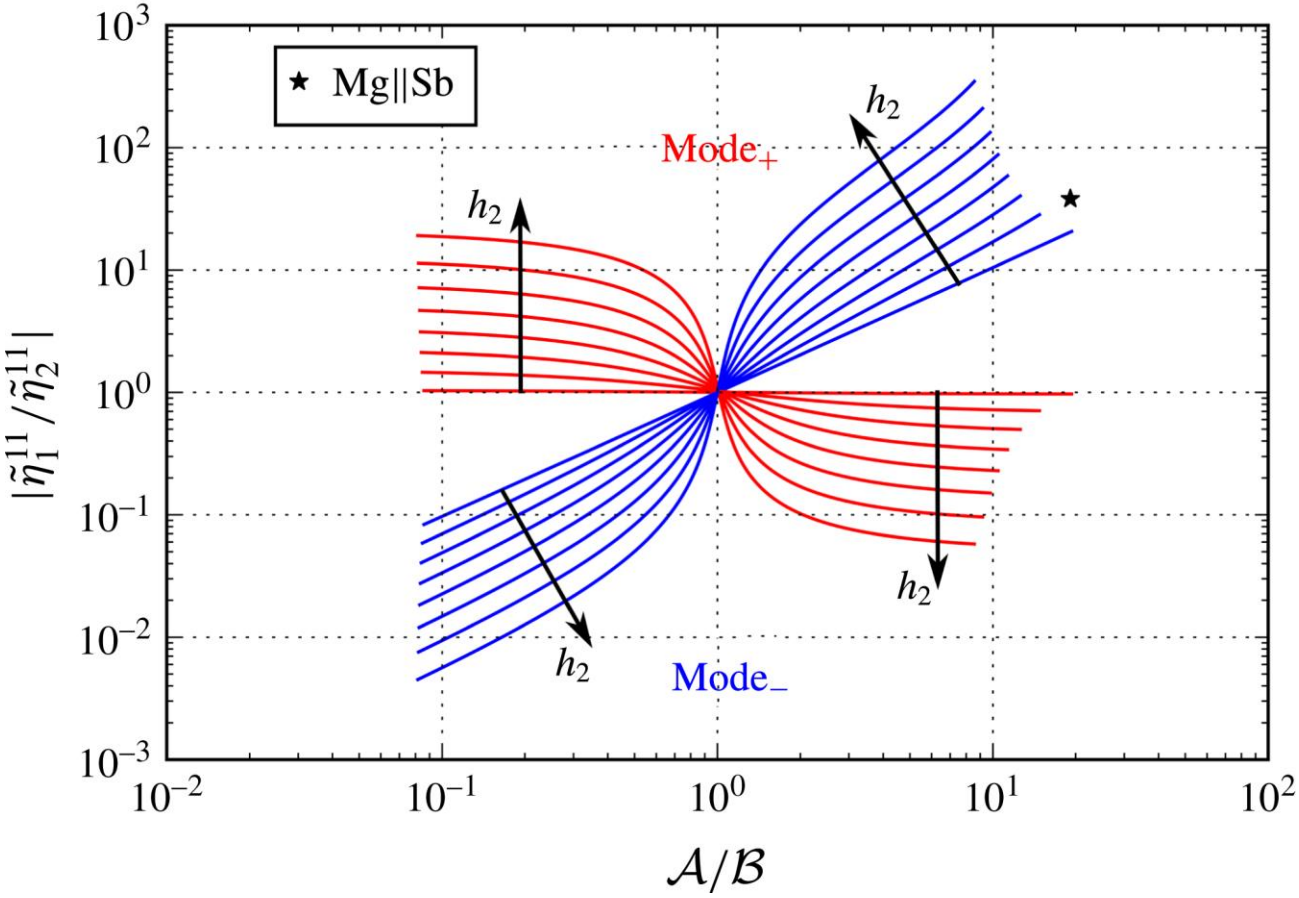
Waves in LMB´s: Metal pad role instability → 3-layer system



$$\frac{\mathcal{A}}{\mathcal{B}} = \frac{\omega_{\eta_2}^2}{\omega_{\eta_1}^2}$$



$$\mathcal{A}_{gravity} = \frac{\rho_3 - \rho_2}{\rho_2 - \rho_1} \quad \text{or} \quad \mathcal{A}_{capillary} = \frac{\gamma_{\eta_2}}{\gamma_{\eta_1}},$$

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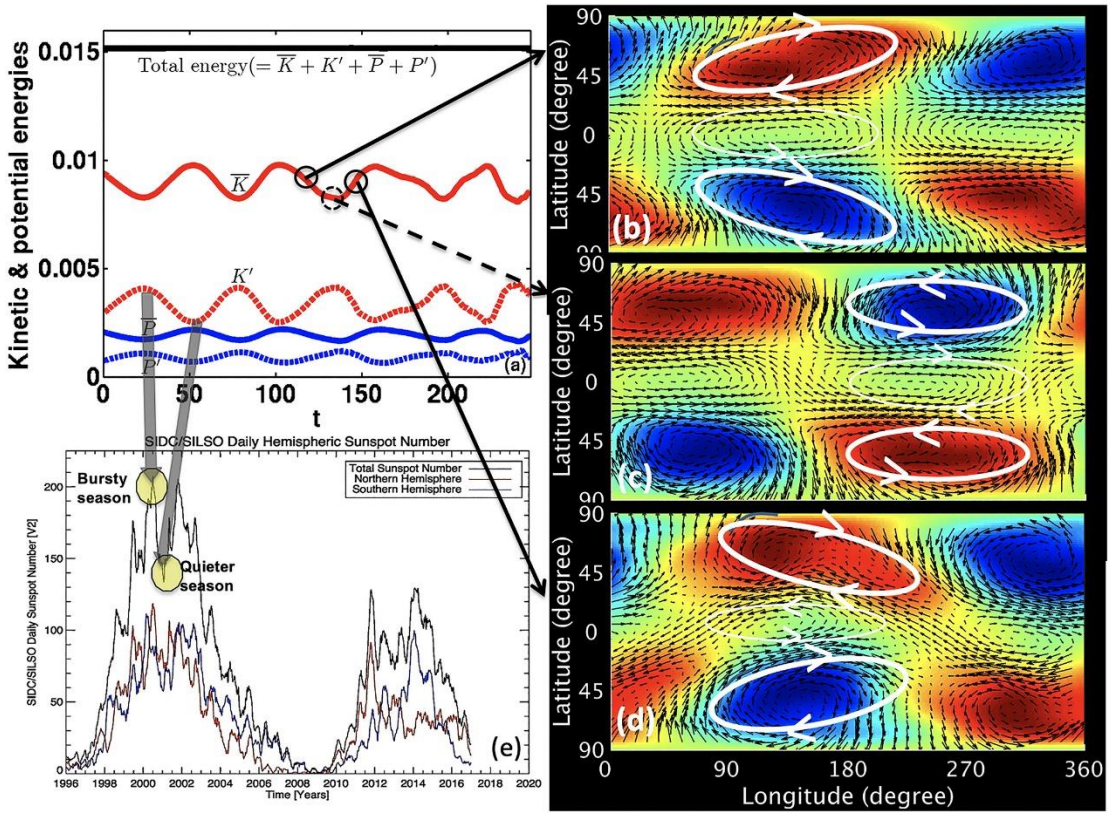
(Magneto-)Rossby waves in the solar tachocline…

…are presently under intense observational and numerical investigation

M. Dikpati, P.A. Gilman, ApJ 551 (2001), 536; M. Dikpati, ApJ 745 (2012), 128; S.W. MicIntosh et al., Nature Comm. 6 (2015), 6492; T. Zaqarashvili, ApJ 856 (2018), 32



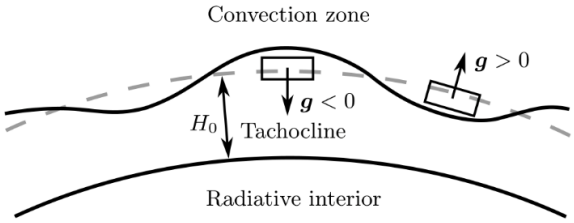
Example:

Tachocline nonlinear oscillations (TNOs)

M. Dikpati, S.W. McIntosh, Space Weather 18 (2020), e2018SW002109

Tidally forced magneto-Rossby waves in the solar tachocline

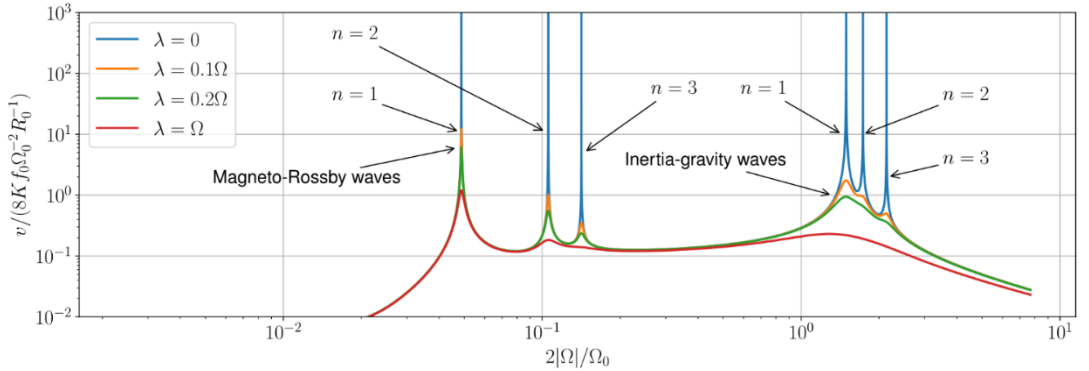
$$\Box_{v_A}^2 v - C_0^2 \Box_{v_A} \Delta v + f_0^2 \frac{\partial^2 v}{\partial t^2} - C_0^2 \beta \frac{\partial}{\partial x}\frac{\partial v}{\partial t} + 2\lambda \frac{\partial}{\partial t}\Box_{v_A} v - \lambda C_0^2 \Delta \frac{\partial v}{\partial t} + \lambda^2 \frac{\partial^2 v}{\partial t^2} = f_0 \frac{\partial}{\partial x}\frac{\partial^2 V}{\partial t^2} - \lambda \frac{\partial}{\partial y}\frac{\partial^2 V}{\partial t^2} - \frac{\partial}{\partial t}\frac{\partial}{\partial y}\Box_{v_A} V$$
$$= \left[f_0\Omega + 2\Omega^2 - \frac{2v_A^2}{R_0^2} + \frac{2f_0\Omega}{R_0} y \right] \frac{4K\Omega}{R_0} \sin\left(\frac{2x}{R_0} - 2\Omega t\right) + \frac{4K\lambda\Omega^2}{R_0} \cos\left(\frac{2x}{R_0} - 2\Omega t\right)$$

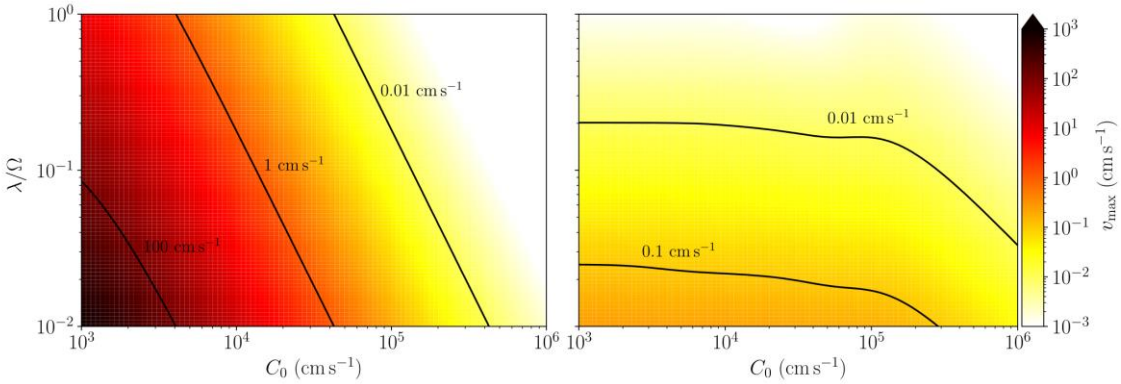


Quasi-analytical calculation of tidal excitation of magneto-Rossby waves in the solar tachocline

→ **Velocities of 1 m/s seem indeed realistic**

As shown above, this could be sufficent for tidal syn-chronization of the dynamo





Horstmann et al., Astrophys. J., in press; arXiv:2208.00644

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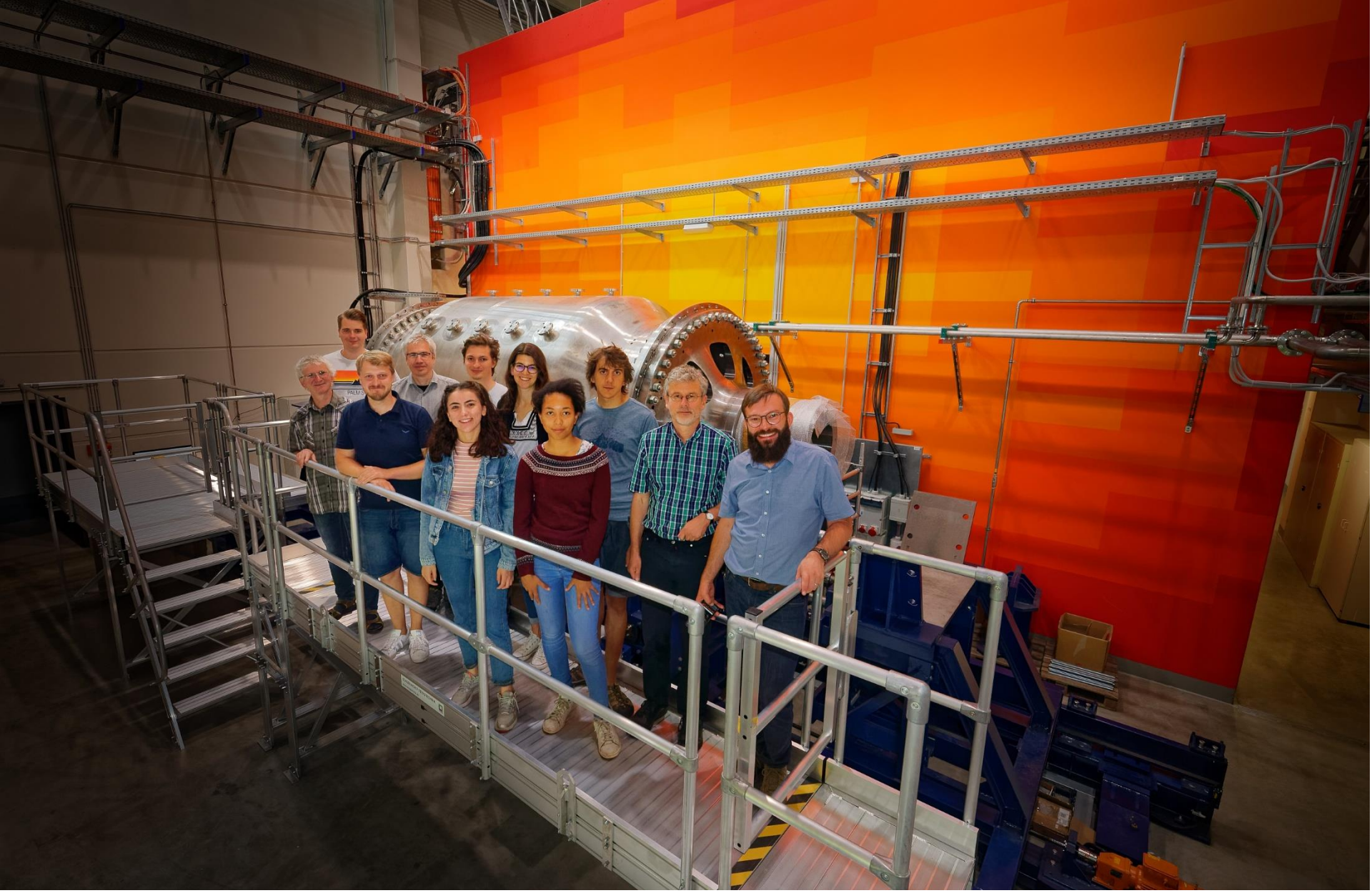
Thank you
for your
attention!



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