Caroline Nore
W. Herreman, L. Wierzchalek, G. M. Horstmann, L. Cappanera,
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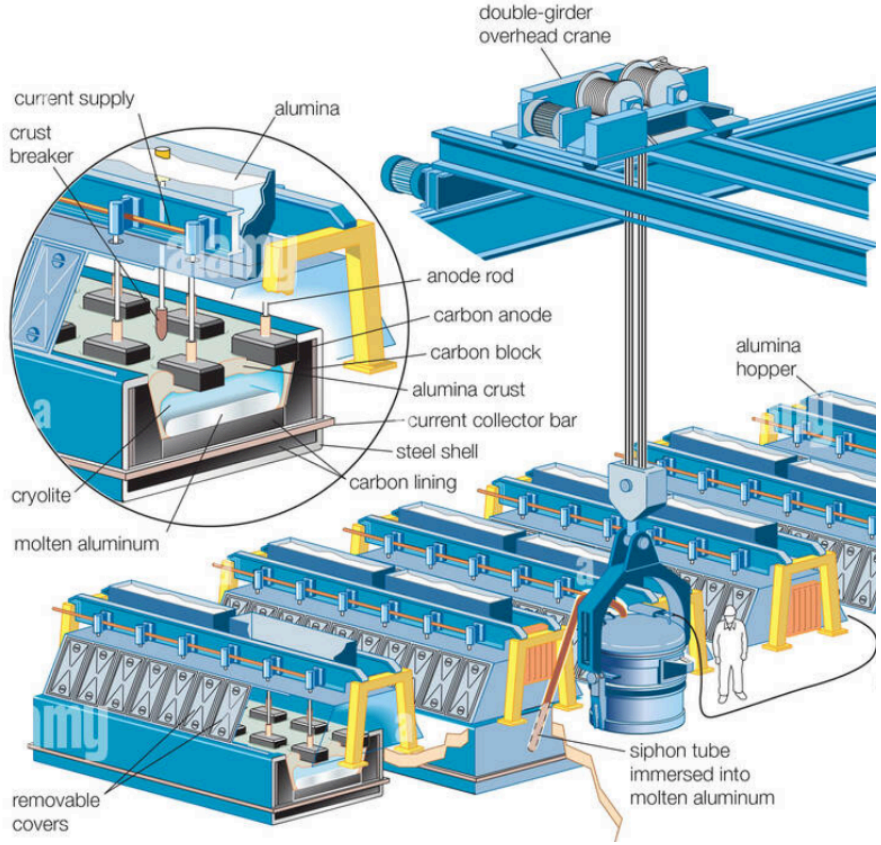
Metal Pad Roll (MPR) Instability in two or three layers of liquid metals

Hall-Héroult reduction cells for Aluminium production subject to MPR instability



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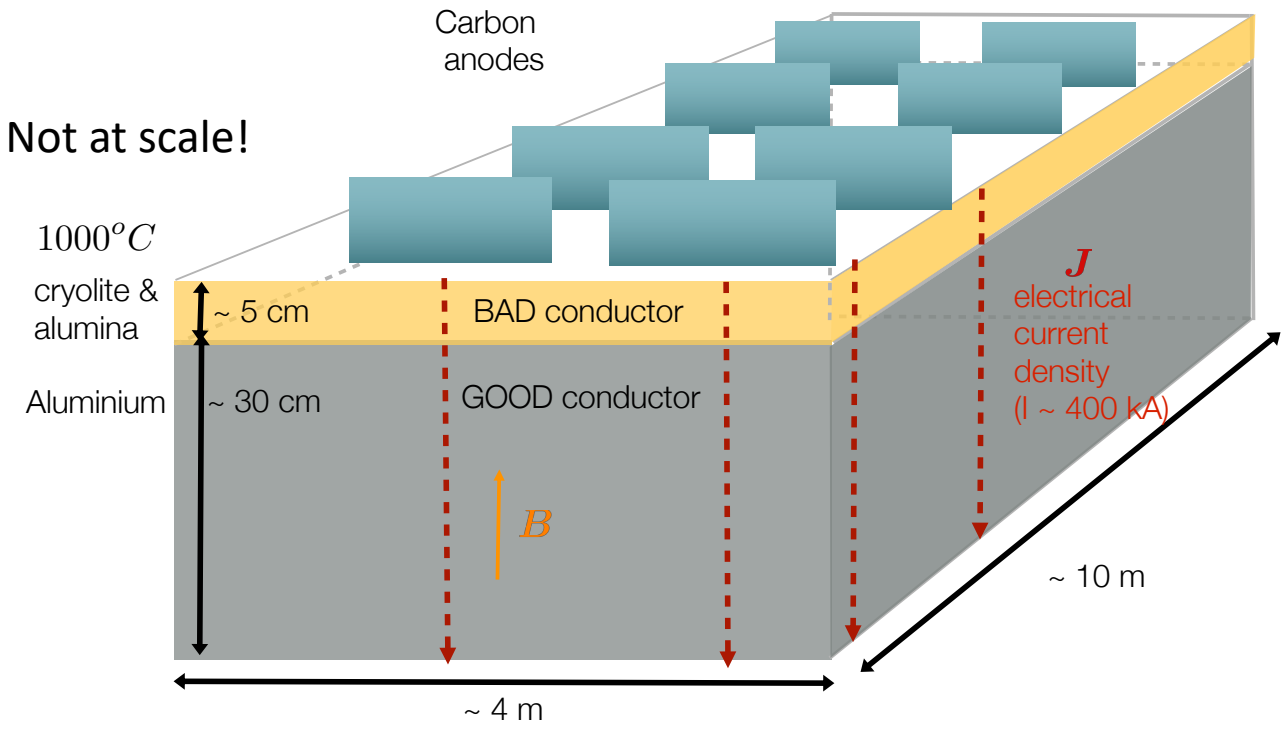

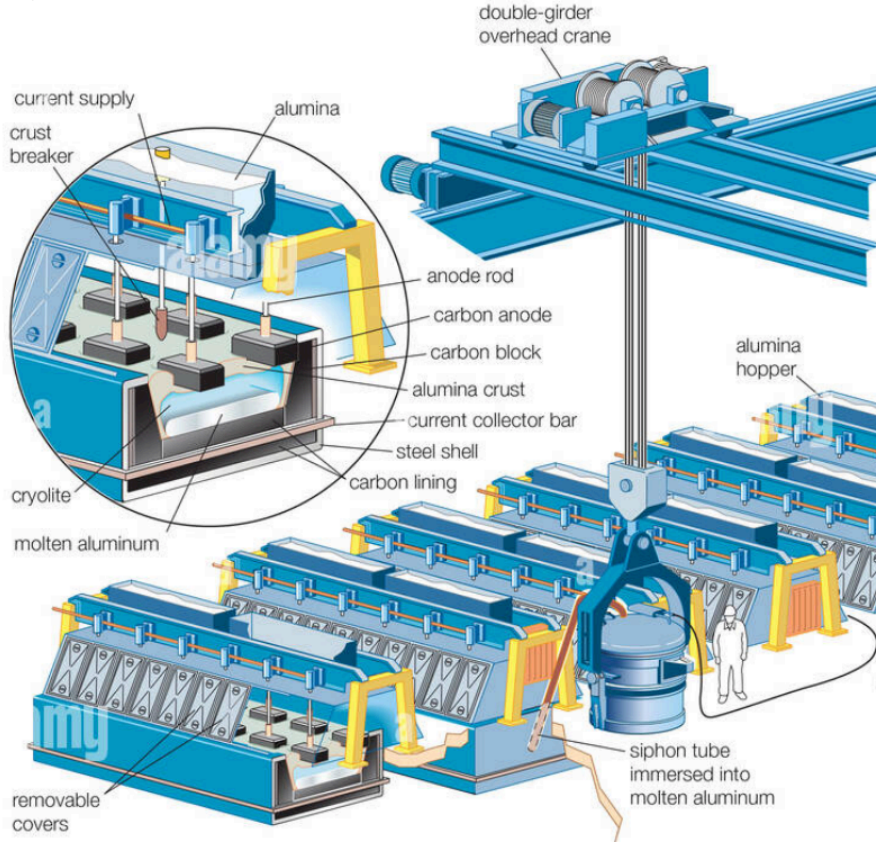

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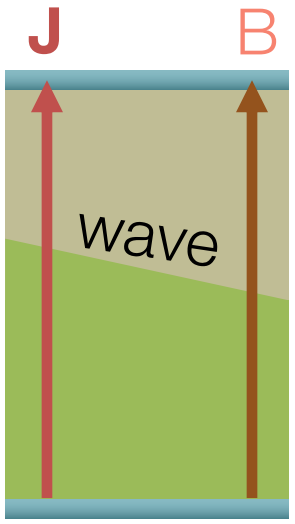
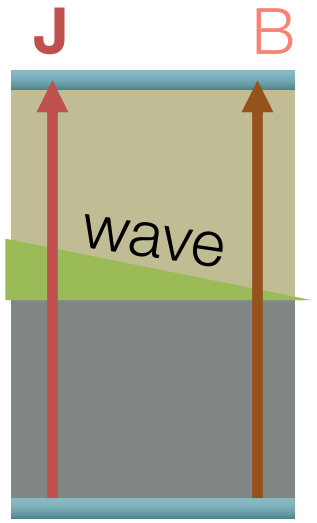


2 layers



3 layers

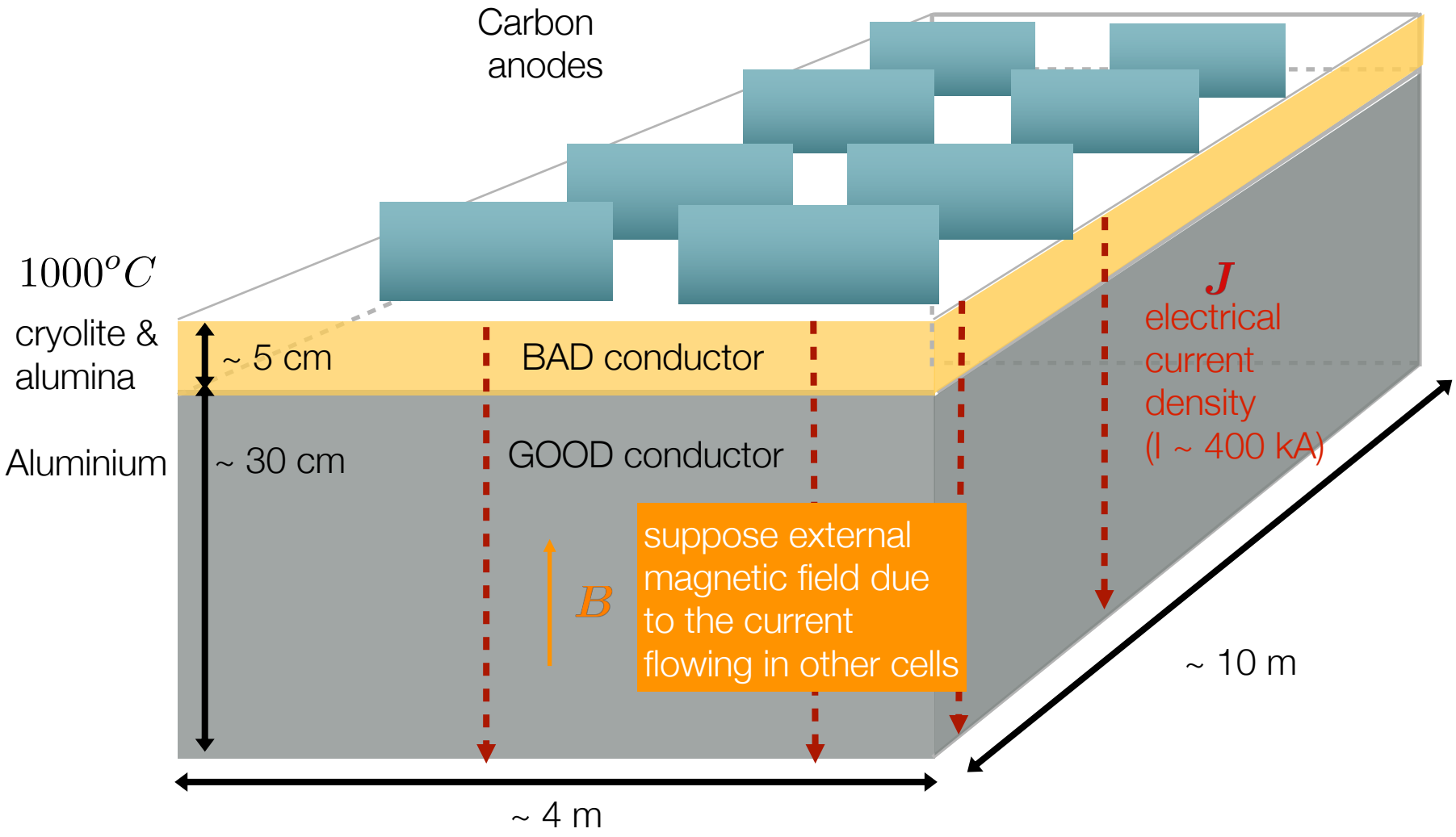
Basic state

- homogeneous current through 2/3 layers
- external magnetic field (vertical B) due to external electrical circuit

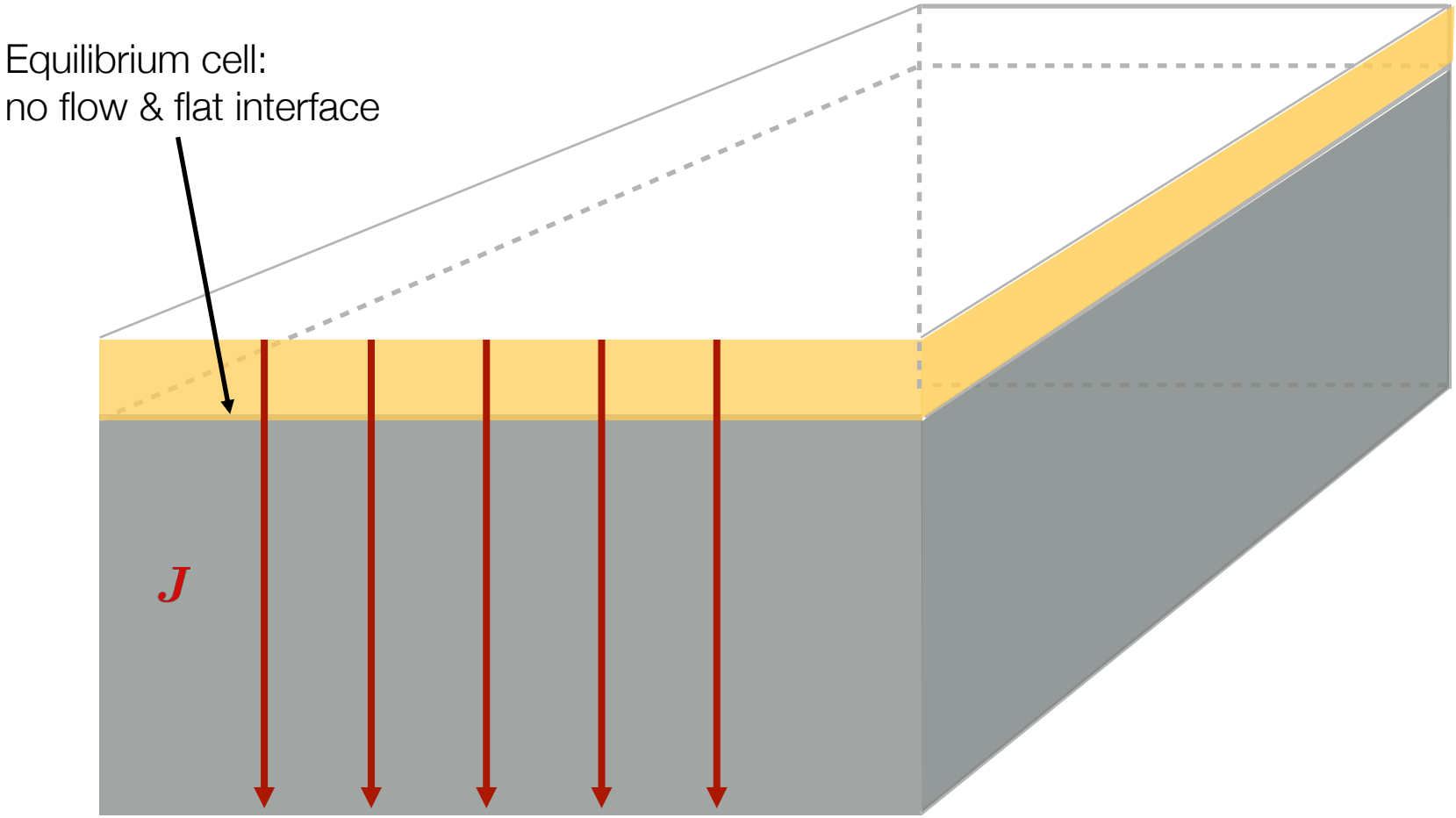
Instability mechanism

- well-known in Al reduction cell, with 2 layers → dangerous short-circuits
- **Sele mechanism** (1977) = best known

Hall Héroult Aluminium reduction cell (from Wietze Herreman)



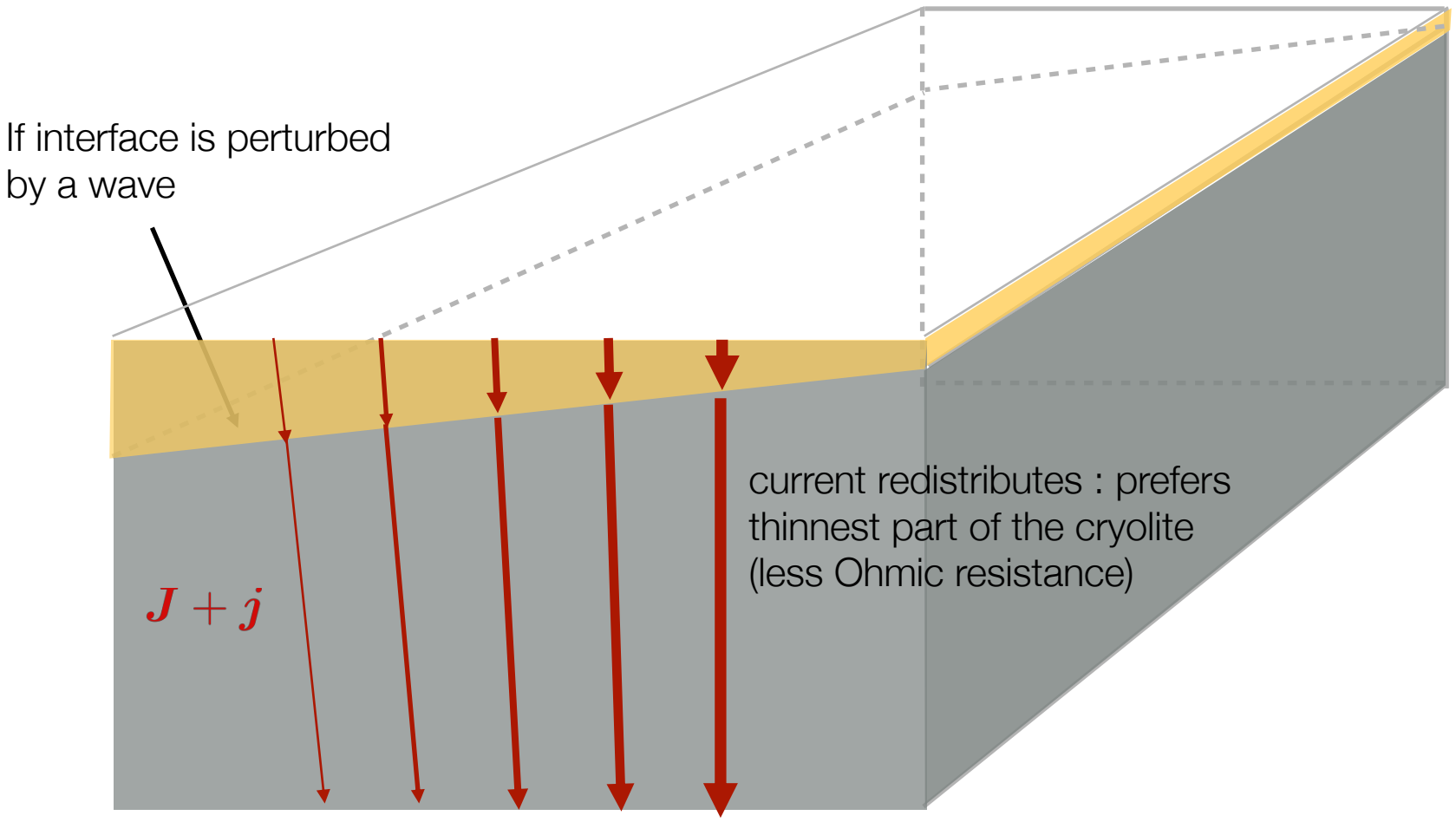
Sele mechanism illustrated



'perfectly homogenous' current density

from Wietze Herreman

Sele mechanism illustrated



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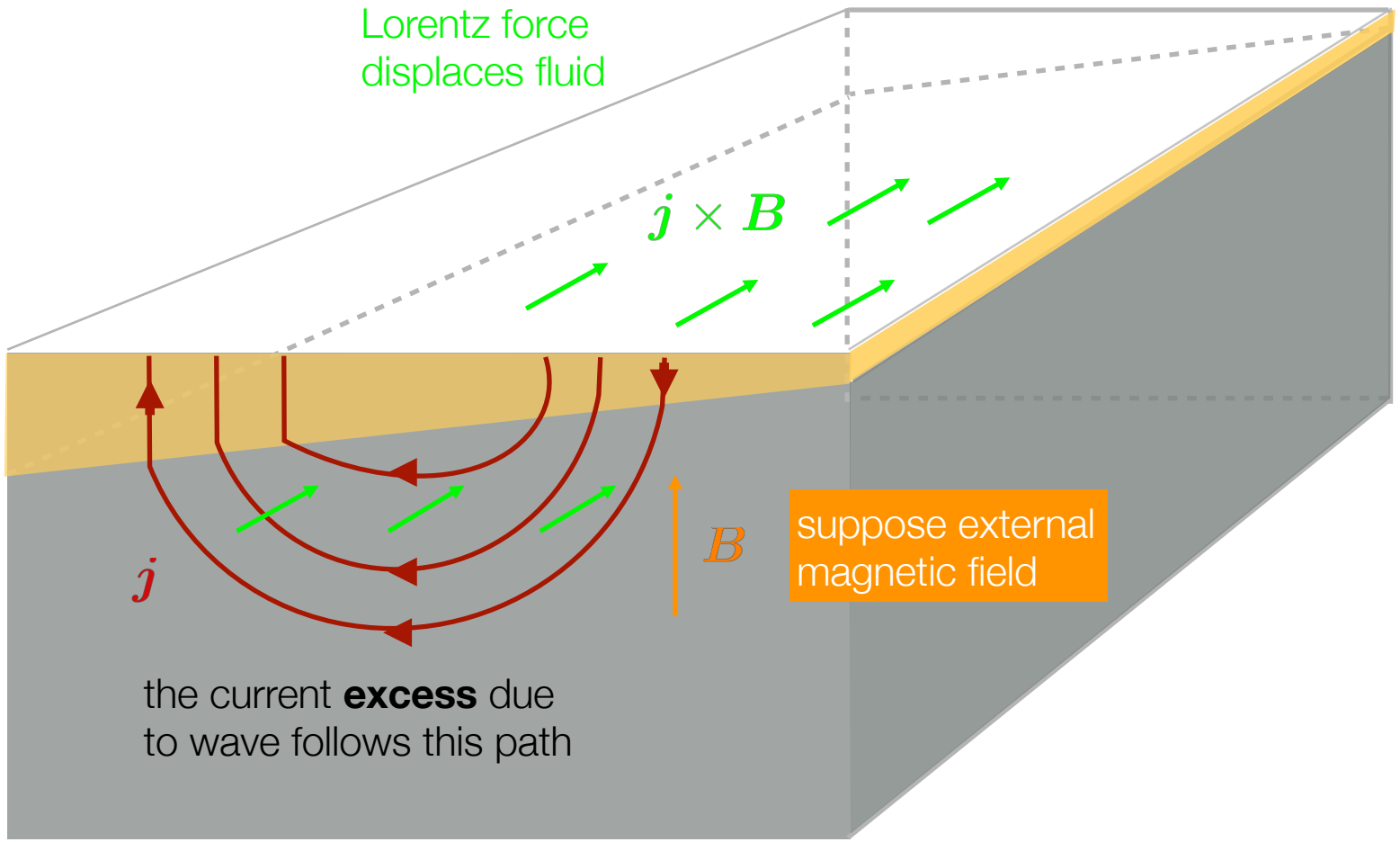
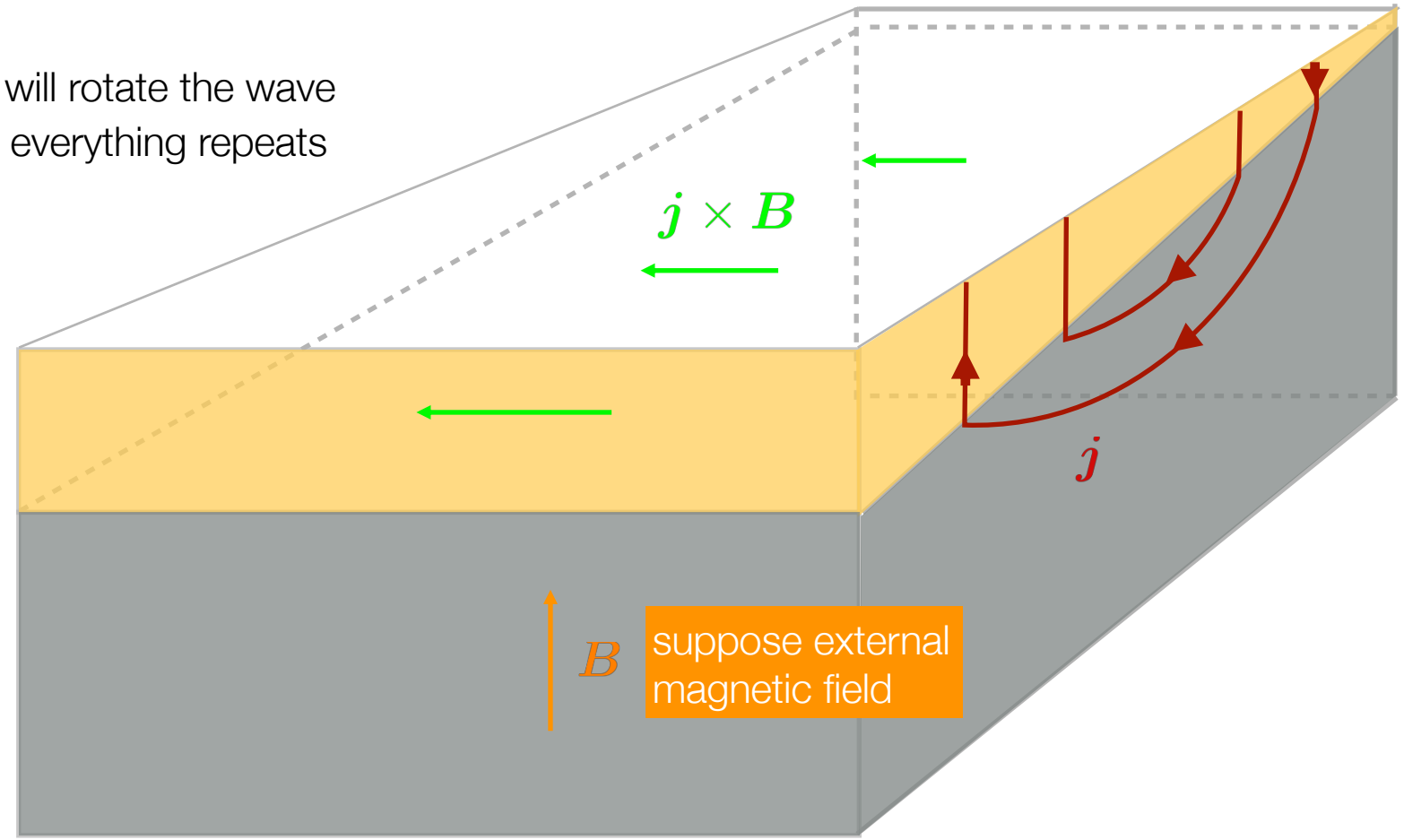
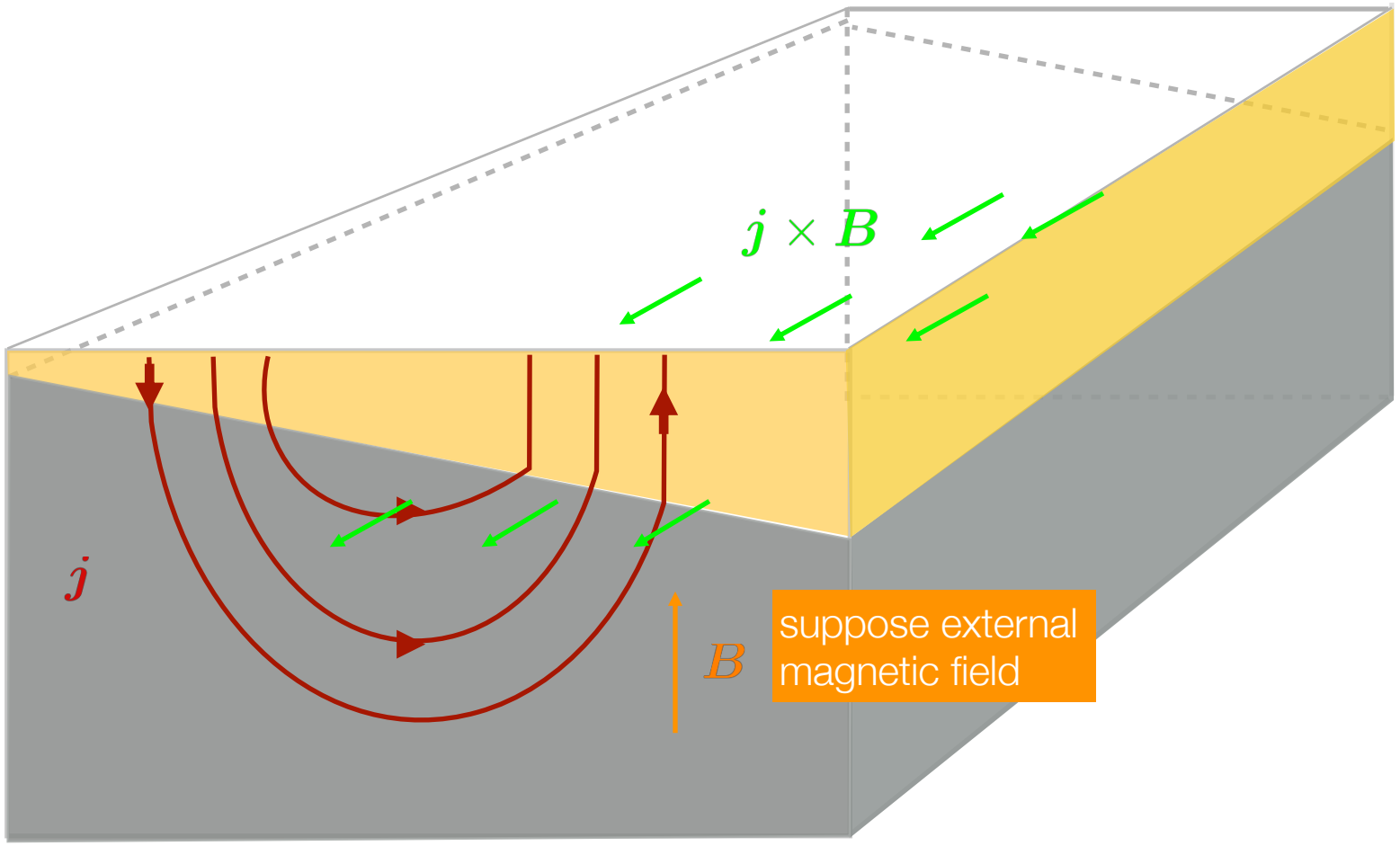


Sele mechanism illustrated

this will rotate the wave
and everything repeats

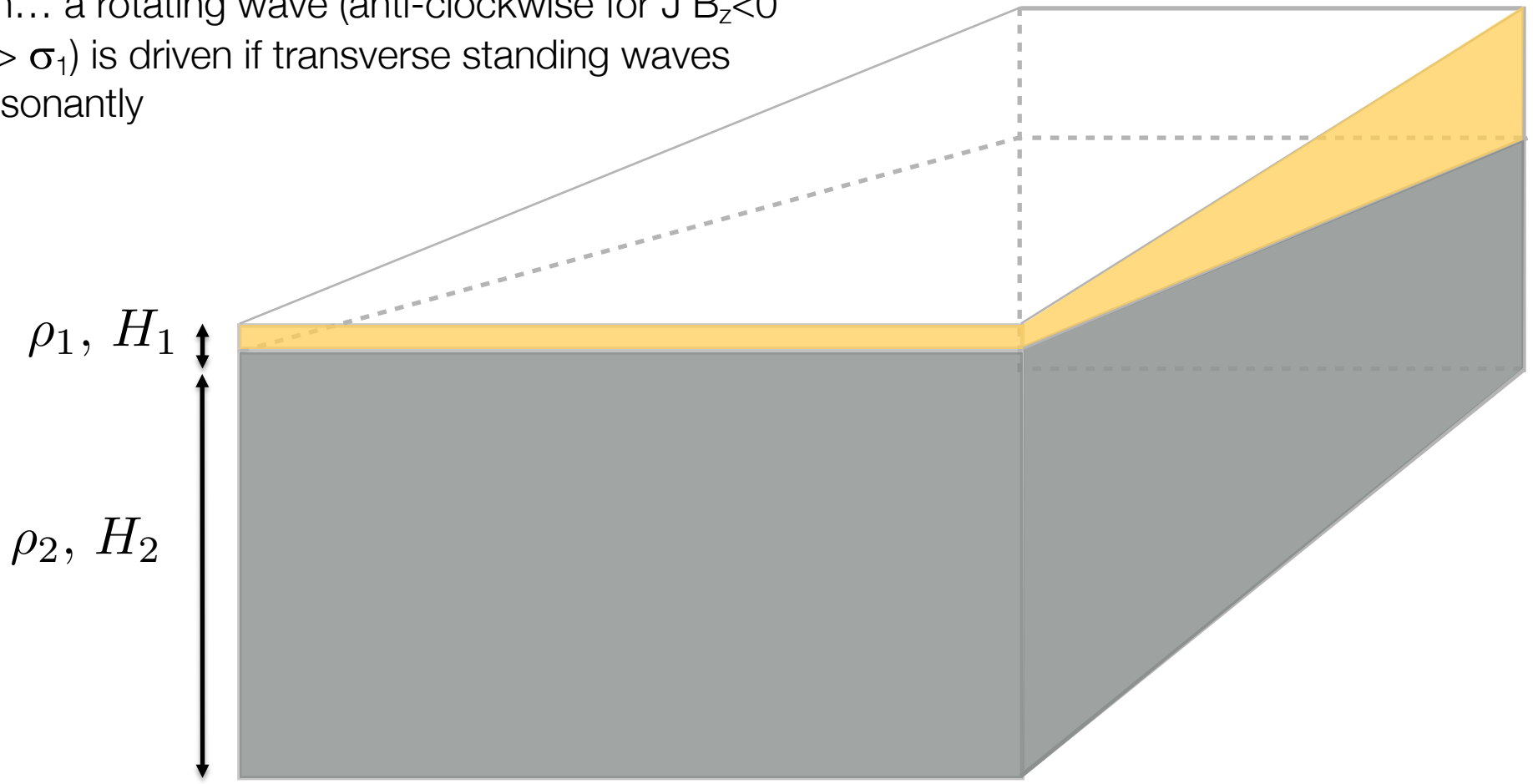


Sele mechanism illustrated



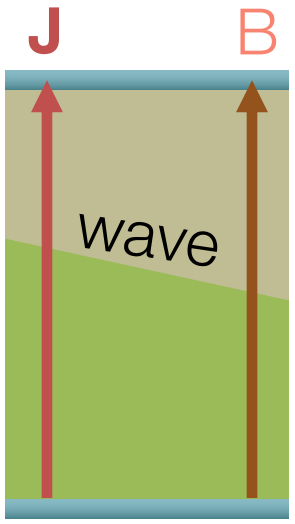
Sele mechanism illustrated

and so on… a rotating wave (anti-clockwise for J B_z<0 and $\sigma_2 >> \sigma_1$) is driven if transverse standing waves couple resonantly

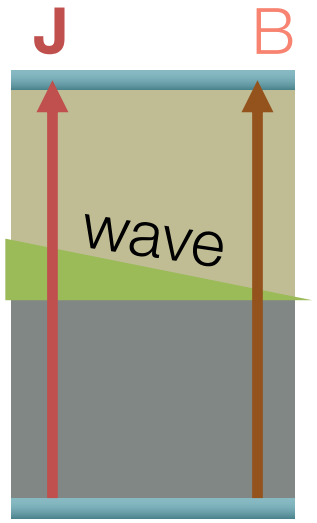


In shallow cells, instability (Sele's criterion) when

$$\beta = \frac{IB_z}{(\rho_2 - \rho_1)gH_1H_2} > \beta_c \quad \text{with} \quad \beta_c = f(\text{geometry}, \text{dissipation})$$



2 layers



3 layers

Metal Pad Roll instability (MPR)

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Instability mechanism

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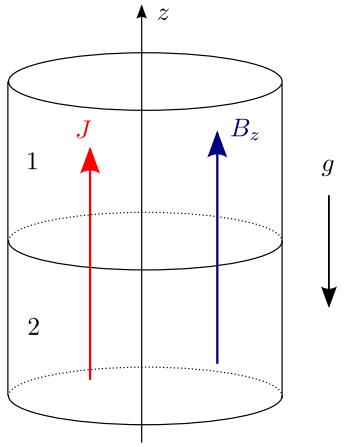
LMB context

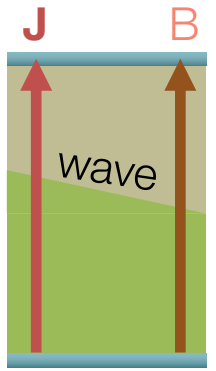
- Zikanov (2015) slab solid model
- Bojarevics & Tucs (2017, 2018) linear stability, shallow cells
- Weber++ (2017) LMB simulation
- Horstmann (2018) LMB simulation + different regimes
- Molokov (2018) linear stability, shallow cells
- etc

Herreman++ (2019) JFM - 2 layers

- Linear stability (Bessel) $e^{\lambda t} e^{i(\theta+\omega t)} J_1(kr)$

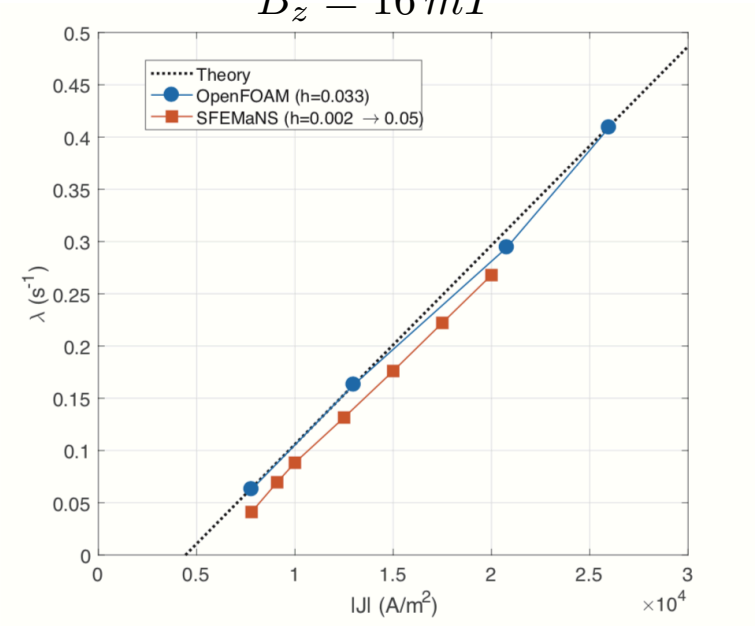
  cylinder, arbitrary fluids, any depth
  with dissipation (viscous and magnetic) $\lambda = \lambda_v + \lambda_{visc} + \lambda_{vv}$



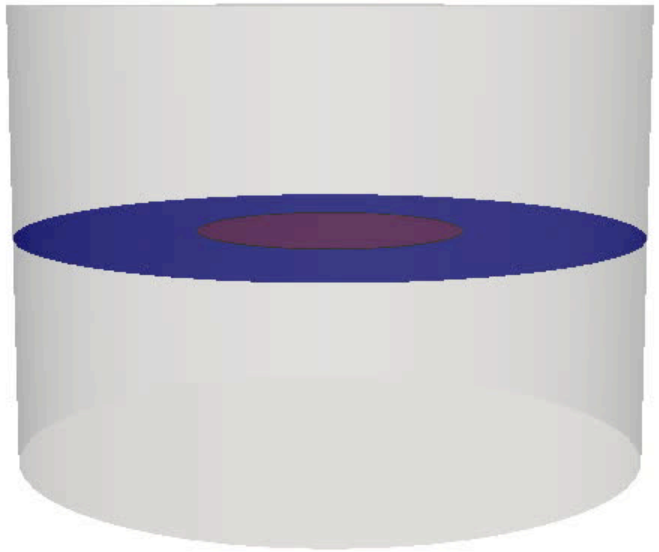


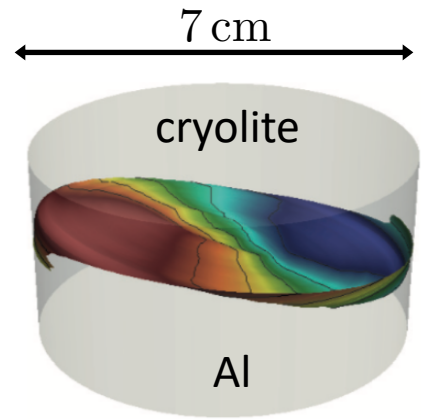
- viscous damping of interfacial waves (exp. validated)
- magnetic damping (num. validated)
- benchmark for SFEMaNS & OpenFOAM (difficult)

$B_z = 16\,mT$



Growth rate comparison for MPR in Hall-Héroult cell





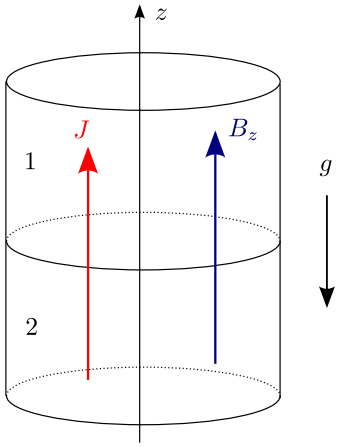
$I = 30\,\mathrm{A},\ B_z = 16\,\mathrm{mT}$

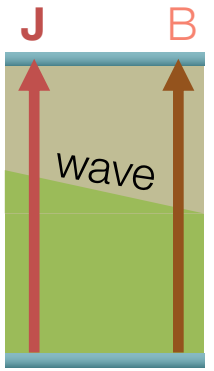
Interface computed with SFEMaNS
(clockwise for $JB_z > 0$ and $\sigma_2 >> \sigma_1$)

Nore++ (2021) PRL - 2 layers

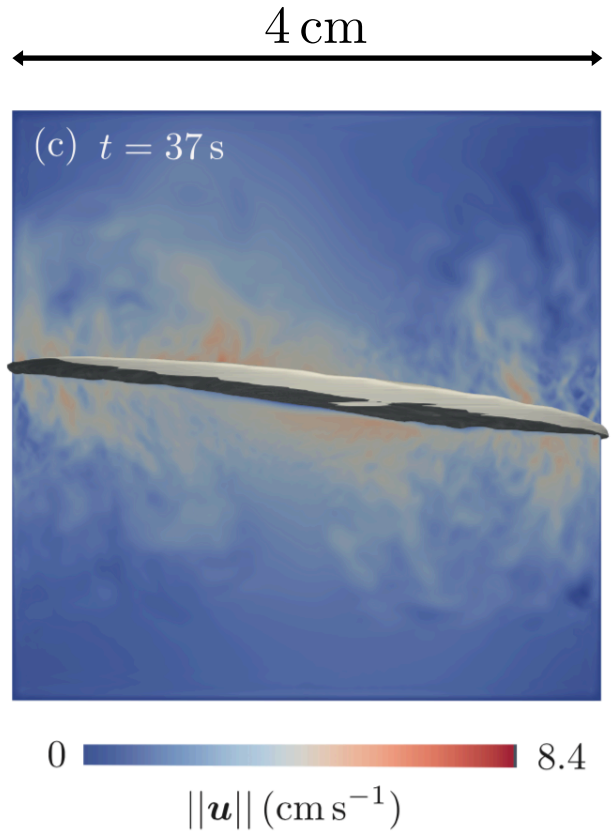
We show that a MPR experiment is feasible at room T: cell with cm dimensions, Ga above Hg or GaInSn, realistic J and B_z

- Numerical application of theoretical model for MPR (Herreman++ 2019)
- Challenging numerical simulations (3D turbulent, high resolution $0.08\,\mathrm{mm}$)





$Re = 15000$

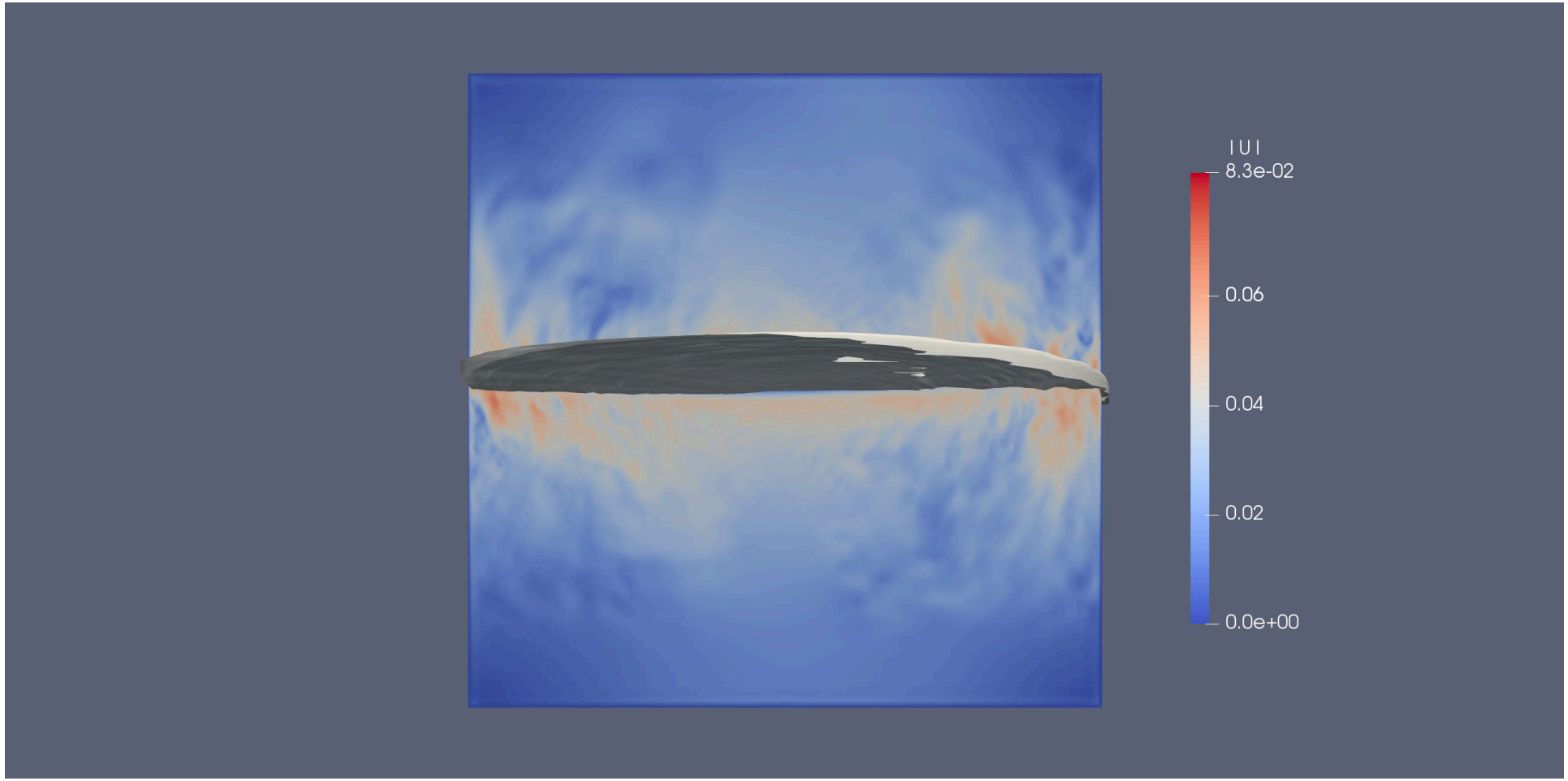


$$I \in [140, 377]\mathrm{A}$$
$$B_z = 15\,\mathrm{mT}$$

→ new multiphase algorithm (L. Cappanera)

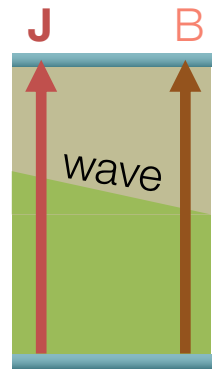
in Hg, $U \sim 8\,\mathrm{cm.s^{-1}} \Rightarrow Re \approx 14000$

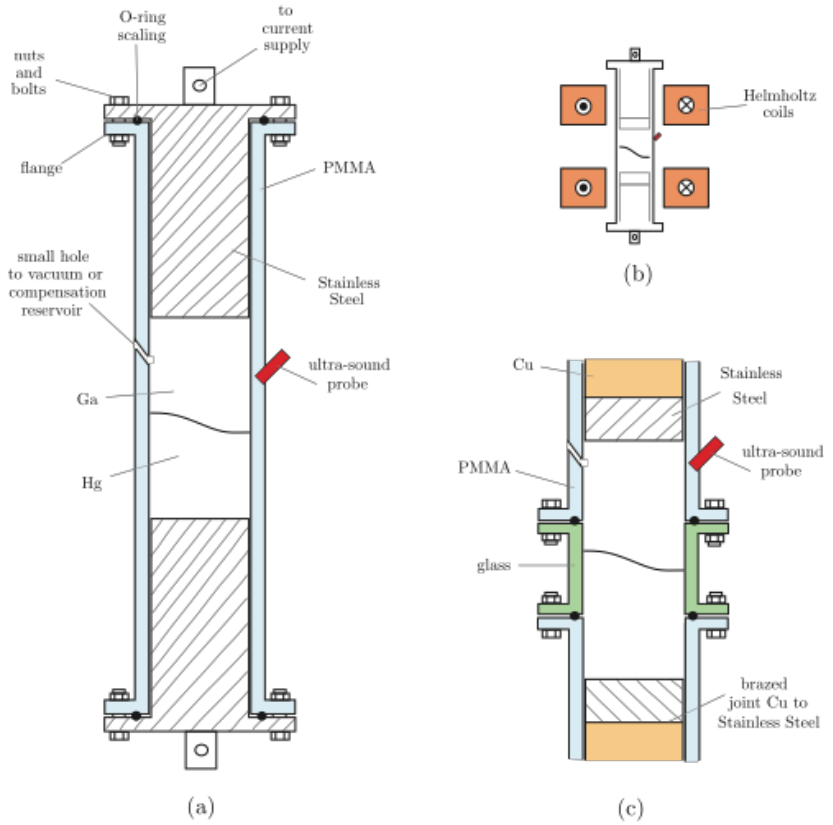
- Challenging numerical simulations (3D turbulent, high resolution $0.08\,\mathrm{mm}$) for Ga above Hg (immiscible pair) $I = 377\,\mathrm{A} \quad B_z = 15\,\mathrm{mT}$



anti-clockwise wave for $JB_z > 0$ and $\sigma_2 < \sigma_1$ (bad conductor on bottom)

- Difficult theoretical calculations lead to ideas for experiments
- Validations with cutting-edge computations
- *Experiments of the Metal Pad Roll instability are feasible at room temperature*





More details in C. Nore, L. Cappanera, J.-L. Guermond, T. Weier, and W. Herreman, PRL 126, 184501, 2021, and its supplementary material

Weber++ (2017) PoF & Magnetohydrodynamics - 3 layers

- Multiphase simulations of Mg-Sb LMB (OpenFOAM)
- MPR is possible in this 3-layer system
- Variation of parameters (J, B, heights, etc)
- pinch of electrolyte

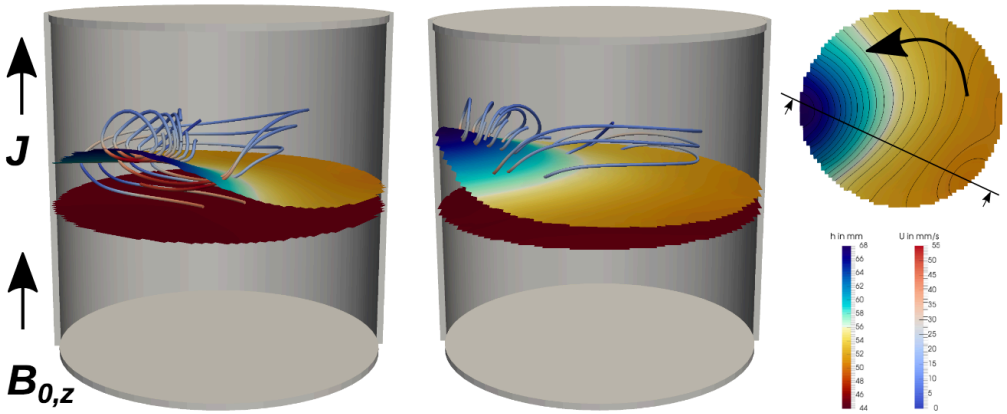
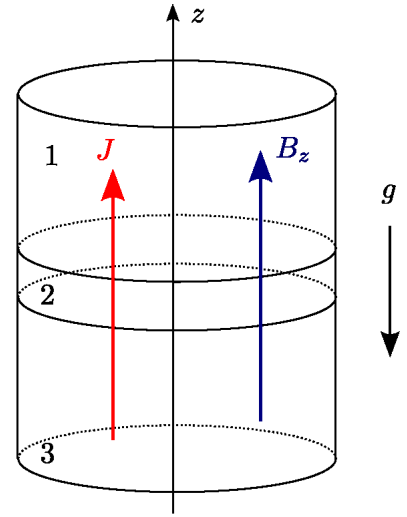


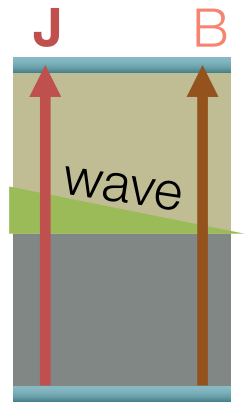
FIG. 5. Streamlines of velocity, surface elevation and direction of rotation for a saturated sloshing instability in a liquid metal battery ($I = 120\,\mathrm{A}$, $B_z = 10\,\mathrm{mT}$, $h_1 = h_3 = 4.5\,\mathrm{cm}$, $h_2 = 1\,\mathrm{cm}$).

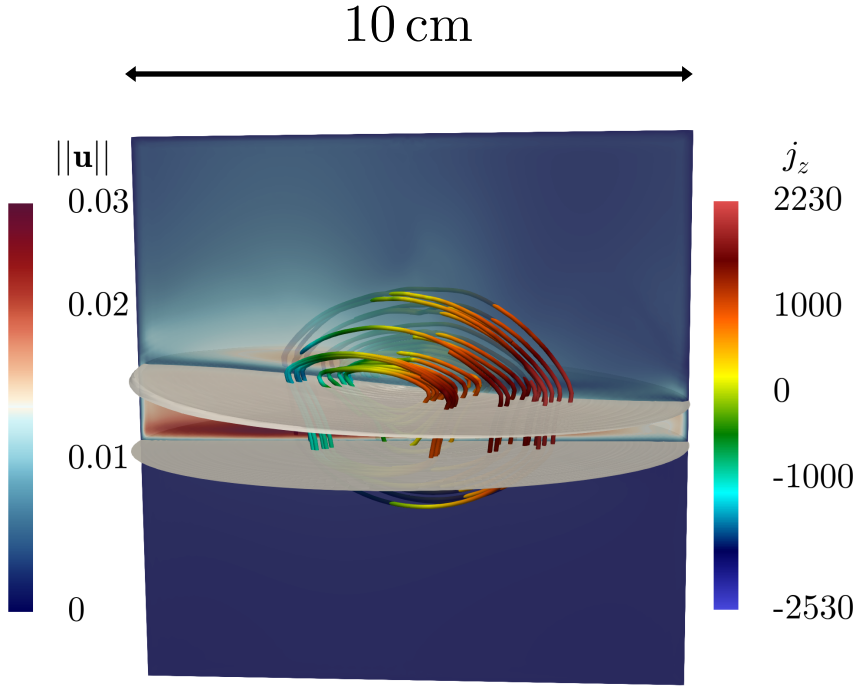
- Mg-Sb LMB redone with SFEMaNS

$$I = 78.5A,\ B_z = 10\,mT\ (J_z = 10^4\ A\,m^{-2})$$

- Sb too heavy to move: act as a 2-layer system







- (Mg)

+ (Sb)

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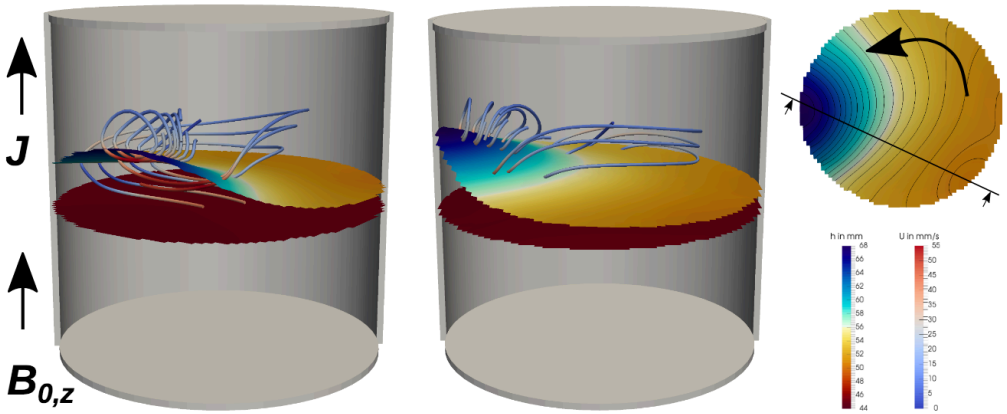

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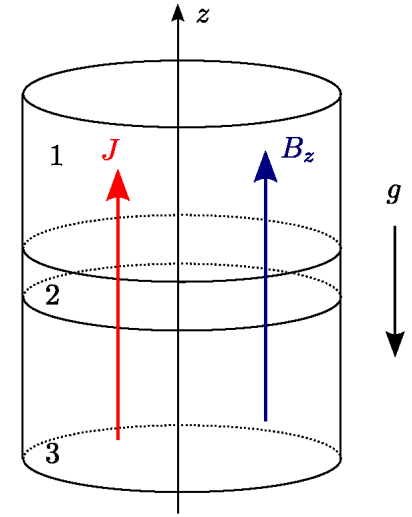
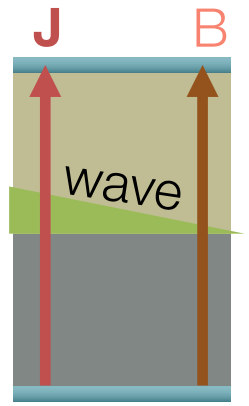
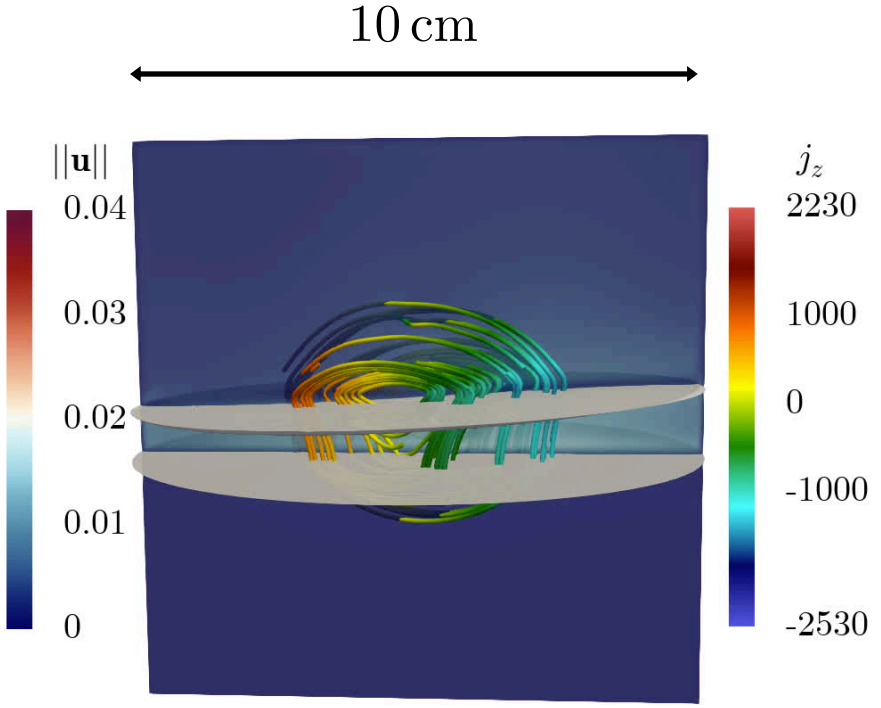






- (Mg)

+ (Sb)

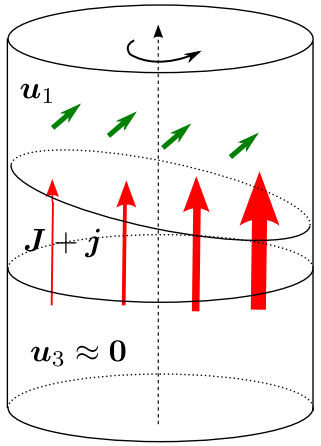
Herreman++ JFM (under revision) - 3 layers

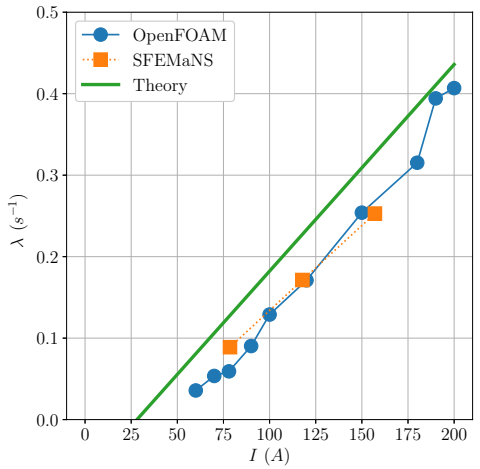
- Linear stability (Bessel) $e^{\lambda t}\, e^{i(\theta+\omega t)}$

  cylinder, arbitrary fluids, any depth $\Delta\rho_{12} = \rho_2 - \rho_1, \quad \Delta\rho_{23} = \rho_3 - \rho_2$

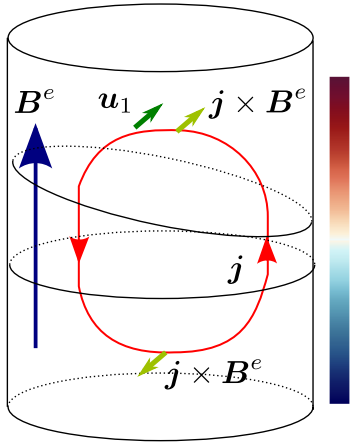
  with dissipation (viscous and magnetic) $\lambda = \lambda_v + \lambda_{visc} + \lambda_{vv}$

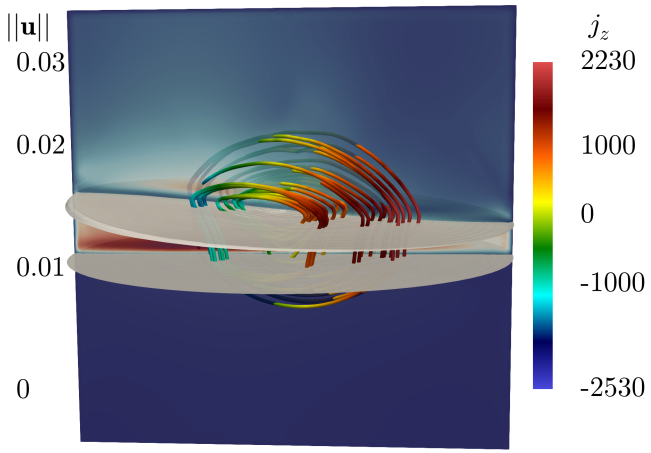
$B_z = 10\,mT$





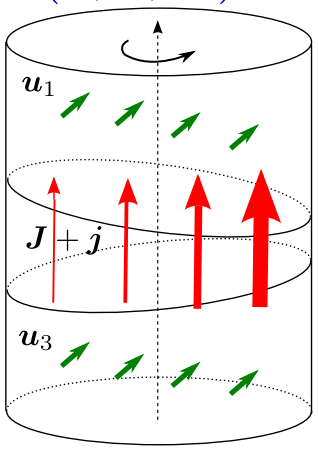
$\Delta\rho_{23} \gg \Delta\rho_{12}$

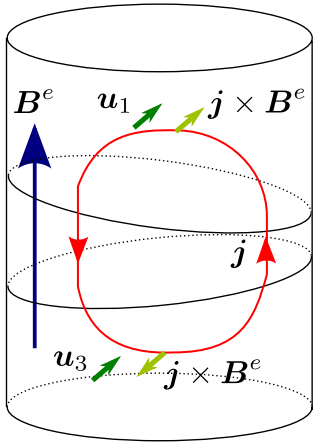




antisymmetric mode
slow frequency ω_-

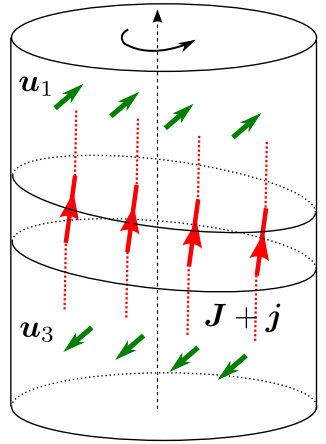
$(1, 1, -)$

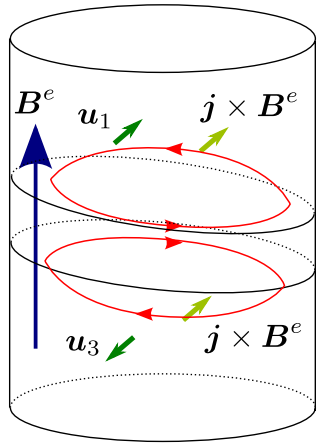




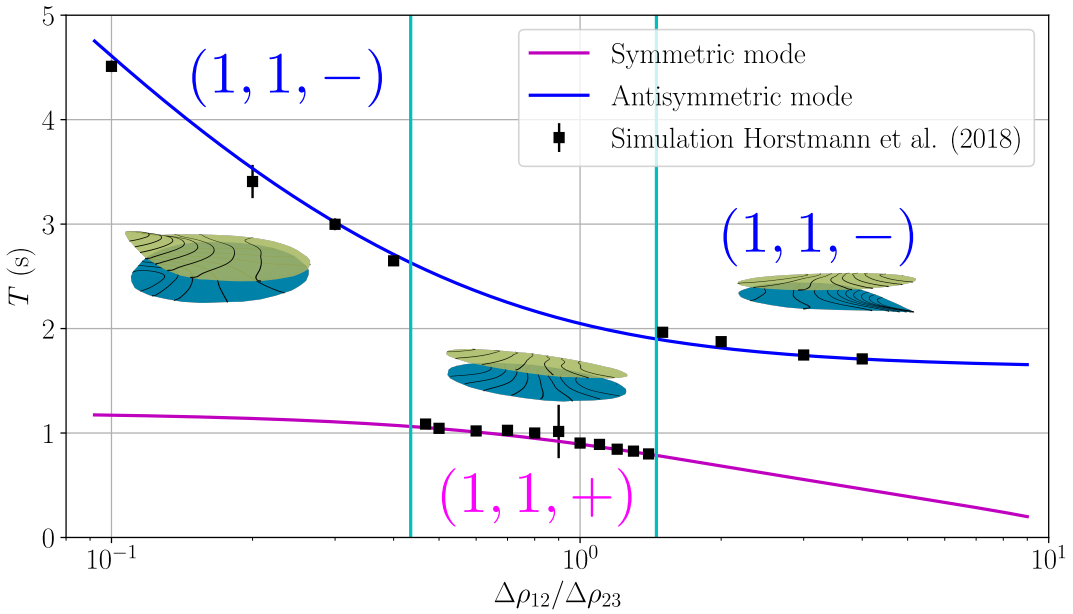
symmetric mode
fast frequency ω_+

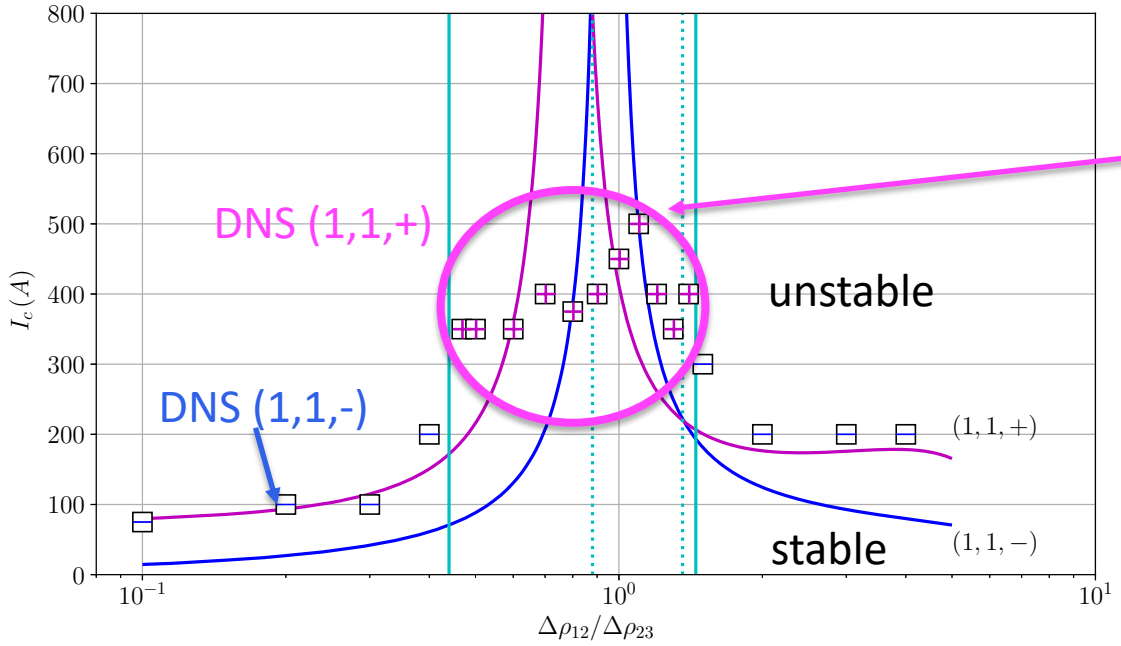
$(1, 1, +)$

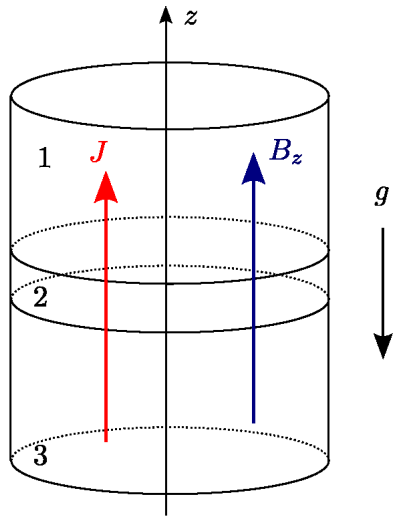




The 3-layer simulations of Horstmann et al. (JFM, 2018) - analysis







$$B_z = 10\,mT$$

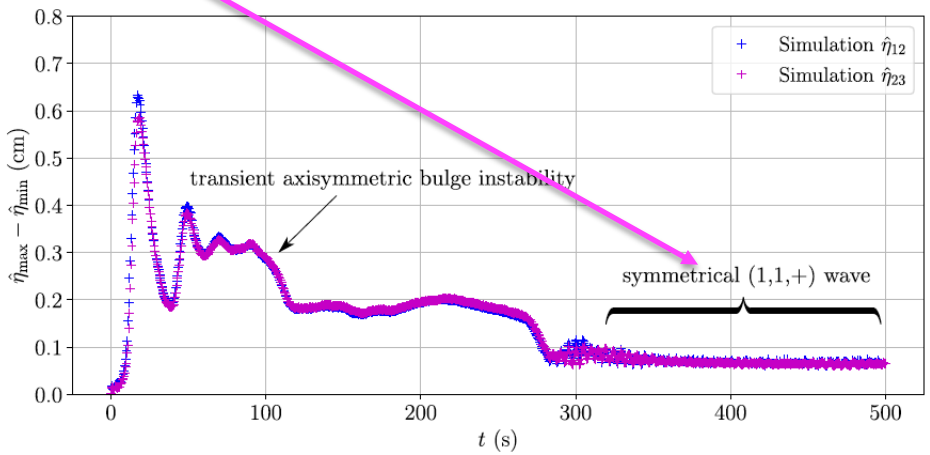
$$(R, H_1, H_2, H_3) = (0.05, 0.045, 0.01, 0.045)\,m$$

$$(\sigma_1, \sigma_2, \sigma_3) = (10^6, 500, 10^6)\,S\,m^{-1}$$

$$(\nu_1, \nu_2, \nu_3) = (6.7, 6.7, 6.7) \times 10^{-7}\,m^2\,s^{-1}$$

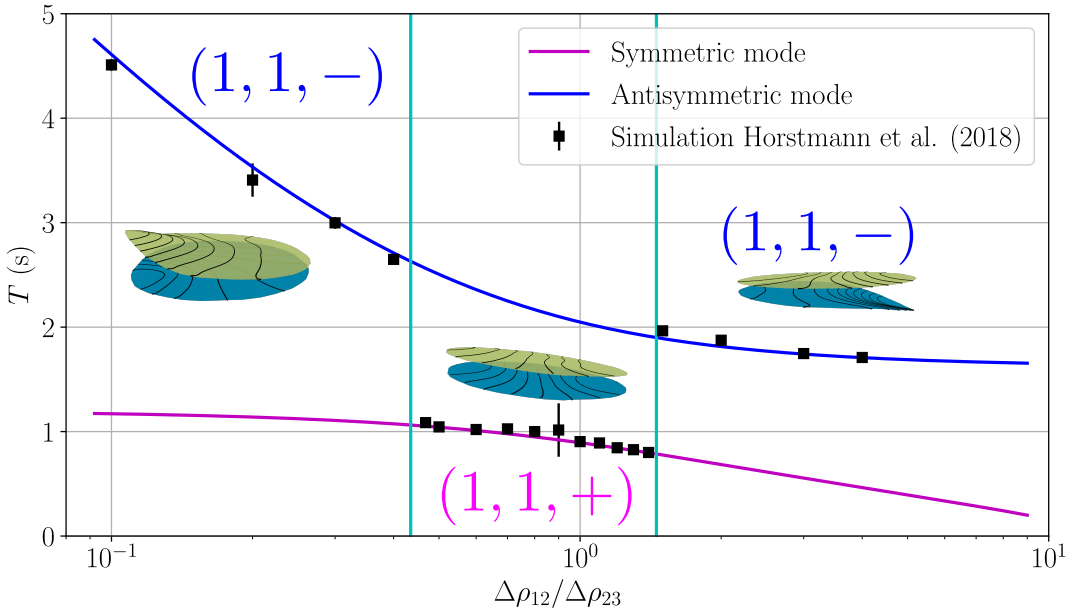
$$(\rho_1, \rho_2, \rho_3) = ([500, 2950], 2500, 3000)\,kg\,m^{-3}$$

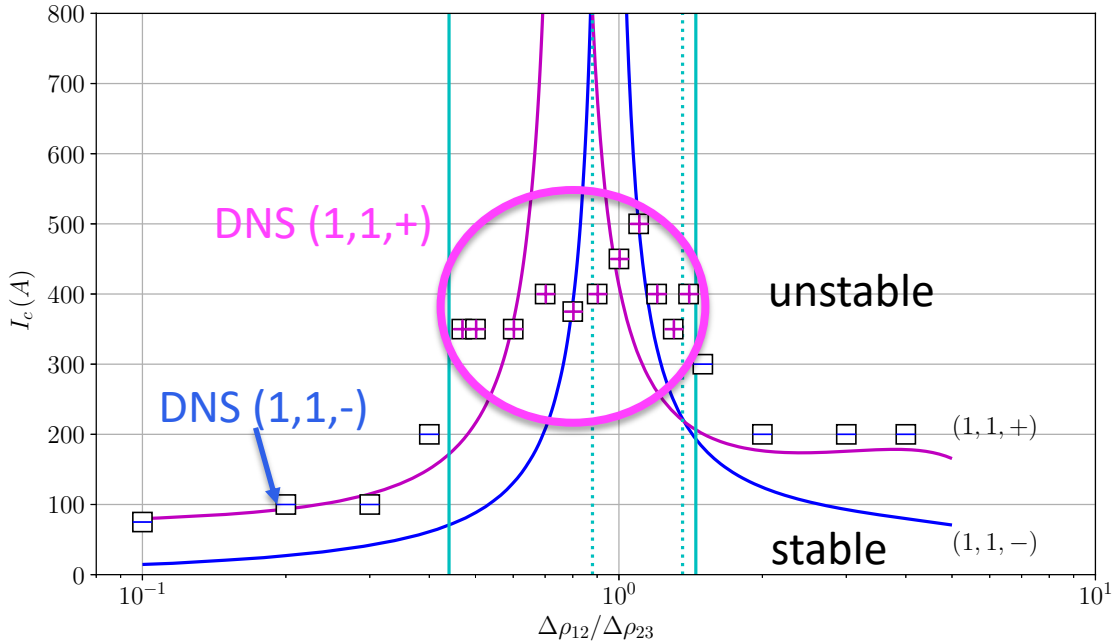
nonlinear behavior at late time (1,1,+)



$$I = 450\,A,\ \Delta\rho_{12}/\Delta\rho_{23} = 1$$

The 3-layer simulations of Horstmann et al. (JFM, 2018) - analysis

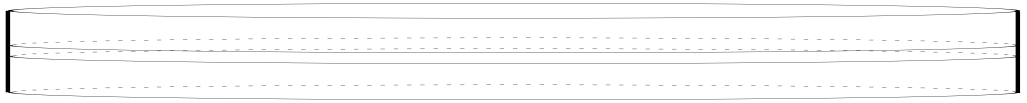




- ratio of density jumps selects the most unstable wave mode
- good agreement for period/ω
- qualitative agreement for nonlinear waves at late time
- symmetric modes as nonlinear feature
- quantitative agreement on growth-rates for I=200A (see paper)

The 10 LMBs of Tucs et al. (JFM, 2018) with 10^5 A

Equivalent cylindrical cell with $R = \sqrt{L_x L_y/\pi} = 3.03\,m$

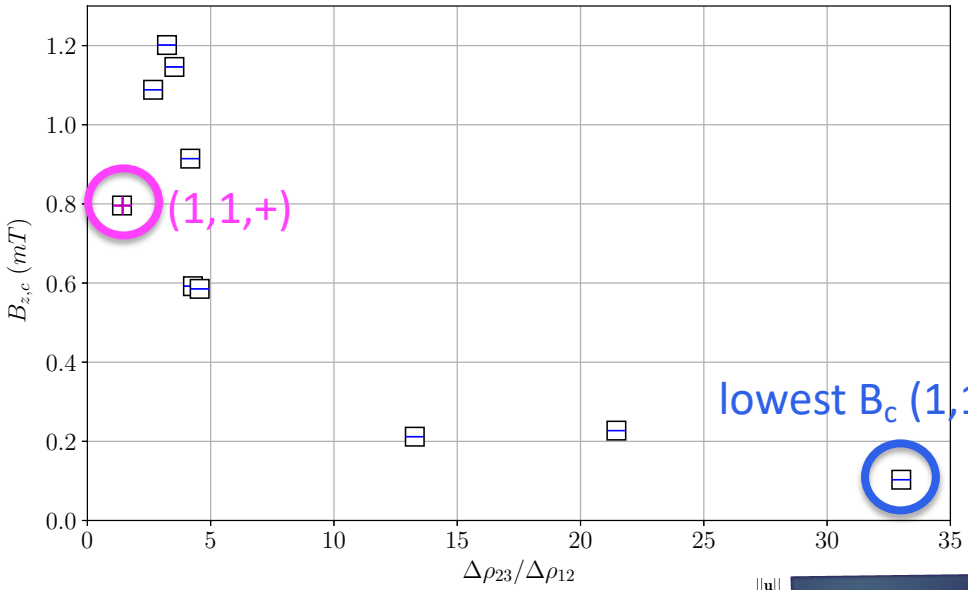


$(H_1, H_2, H_3) = (0.2, 0.04, 0.2)\,m$

# cell	Type	(ρ_1, ρ_2, ρ_3) in (kg m^{-3})	$\Delta\rho_{23}/\Delta\rho_{12}$
1	Li || Te	(489, 2690, 5782)	1.40
2	Na || Sn	(801, 2420, 6740)	2.67
3	Li || Bi	(488, 2690, 9800)	3.23
4	Li || Pb	(488, 2690, 10463)	3.53
5	Na|| Bi	(831, 2549, 9720)	4.17
6	Li || Zn	(488, 1628, 6509])	4.28
7	Li||Sn	(495, 1644, 6877)	4.55
8	Ca || Sb	(1401, 1742, 6270)	13.28
9	Ca || Bi	(1434, 1803, 9720)	21.46
10	Mg || Sb	(1577, 1715, 6270)	33.01

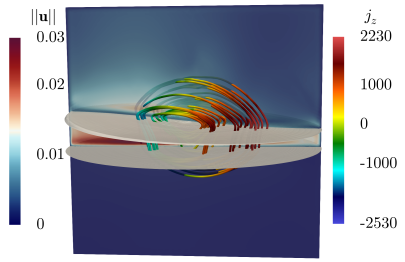
closest to 1

Critical magnetic field B_c for 10^5 A cells



$B_c \in [0.1, 1.2]\,mT$ quite small magnetic fields

- similar to Tucs et al. (2018) results in rectangular cells



The 10 LMBs of Tucs et al. (JFM, 2018) with 10^5 A

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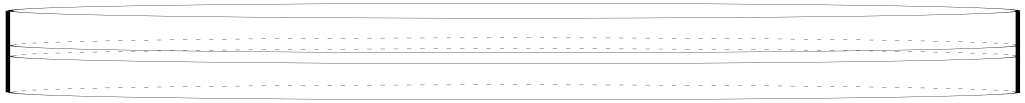

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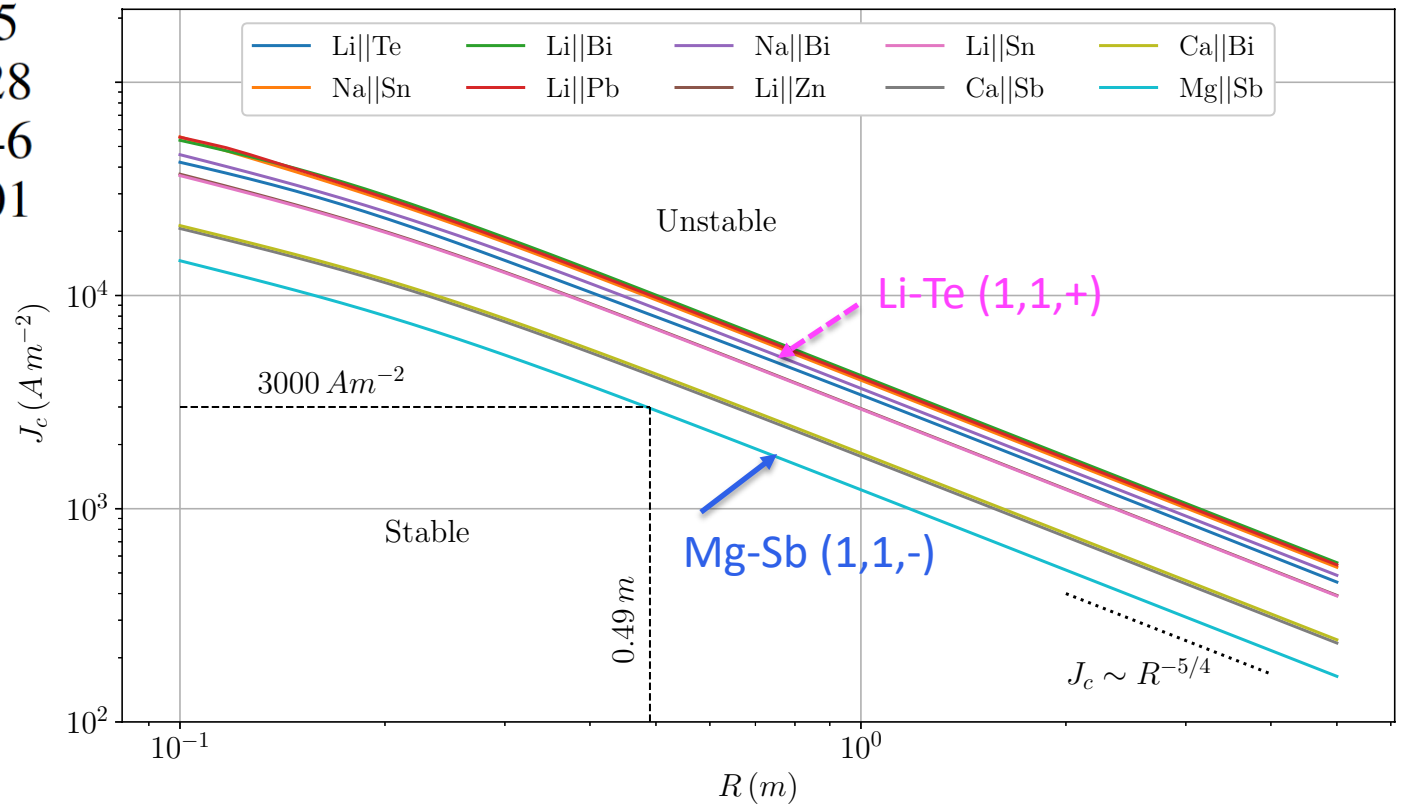
Critical current density J_c for $B_z \approx \mu_0 J R/2$



- Mg-Sb LMB most easily destabilised
- for J=3000 A m^{-2}, LMB stable for R < 0.49m

Conclusion

- Difficult theoretical calculations (2 and 3 layers) lead to ideas for experiments and stability diagrams for future LMBs
- Validations with cutting-edge computations
- *Experiments of the Metal Pad Roll instability (2 layers) are feasible at room temperature*
- *Estimate for manufacturing stable large LMBs wrt MPR (but … not cylindrical, non uniform B_z and other instabilites)*

Acknowledgements

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TEXAS A&M UNIVERSITY

HZDR

HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

UNIVERSITY OF HOUSTON

Thank you for your attention