V-KEMS Report

Communities for an Ageing Society

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WARNING: this report contains preliminary findings that have not been peer reviewed. The findings are intended to provoke further study and policy discussion and should not be treated as definitive scientific advice.

Whilst we expect these principles to help others formulate coherent and consistent guidelines, time has prevented any quantitative study of their effectiveness. This could be undertaken, but would require real data and time to build more detailed simulation tools. Thus, we are not able to make specific recommendations from the principles, e.g. we cannot infer that it is safe to do X if you follow principle Y.

Additionally, this report has been assembled in a short time frame, we have made every effort to ensure references and links are present. Where this is not the case, we apologies for the unintentional oversight.

Contents

1 Executive Summary

This report details the progress of a Virtual Study Group (VSG) held between V-KEMS and a group of researchers from across the UK between 09 and 11 March 2022. The study group considered challenges associated with Communities for an Ageing Society. This gave a very broad remit, and it was necessary to narrow the scope significantly in advance of the event. Using the State of Ageing Report in 2020 report produced by the Centre for Ageing Better, organisers and facilitators gave consideration to a number of factors including what was tractable to consider in a 3-day study group. Mindful of the wealth of other activity being undertaken by various groups, efforts were made to look into new/complimentary areas of activity OR build on existing activities.

The Study Group Considered 3 main challenges

- Independence in the Home – Ageing in Place Rural vs City Dwelling
- Prediction – Demand – Hospital Discharge Model
- Prediction – Demand – Long Term Systemic Model

Aging-in-place

Aging-in-place may be desirable for a number of reasons including personal preference. Rural living may pose particular challenges to aging-in-place for many reasons including:

- transport logistics
- care workforce location
- access to health services

We investigated the differences in suitability for aging-in-place in different parts of the UK in relation to these three challenges.

<u>Next Steps/Future Work:</u>

- allow change to weightings for index (perhaps with gui)
- more forward projection and care workforce modelling
- additional transport calculations considering; modelling changes to public transport provision, mobile health services and impact of climate change on transport

Prediction – Demand – Hospital Discharge Model

We considered the short term issues around hospital discharges into social care, identifying the key activities and resource requirements within the system.

The objective: Reduce the number of patients who are medically optimised for discharge but remain in hospital.

Next Steps/Future Work:

- Identify other modelling work in the area and learn from this when building the more complex model.
- Switch coding to Python rather than AnyLogic.
- Build an extended model that splits patients into different categories (e.g mental health, chronic illness, hard-to-place patients).
- Looks at different capacity limits for difficult-to-discharge patients with complex needs. Use different rates, etc for elective and emergency patients.
- Investigate whether earlier and better information sharing about discharge date and care capacity would reduce the number of patients with delayed discharge.
- Examine what would happen with no constraint on social care capacity to help to estimate the true need for social care and identify other potential bottlenecks in the system.
- Try different data sources/locations, e.g. data from NI versus a scenario in Southampton

Prediction – Demand – Long Term Systemic Model

We considered how quantitative modelling could be used to extend a conceptual framework developed by Sarah Wylie from SIBNI. A long term goal is to understand whether a major intervention could reverse this process putting care provision into the hands of a diverse array of community organisations and small businesses. We develop a System Dynamics Model considering the flow of users of the social care system between different states. A mathematical framework is used to investigate the factors driving the consolidation of care provision into the hands of a few large independent companies and the consequences of that for the level of care delivered. Some preliminary work towards analysing the dynamics of the model and extending it to include stochastic demand and budgeting considerations have been carried out.

Next Steps/Future Work:

Progress was made towards adding system dynamics into a long-term Systemic Model. There a number of different areas that would be appropriate for further work.

- In order to address the main hypothesis the key extensions needed for the model are to include a description of the evolution of the number and size of care providers and a more detailed description of budgeting considerations.
- Other extensions of the model which could make it more realistic in describing the diversity of the sector would include:
  - Segmenting the categories E1 to E3 by age and/or care needs (e.g. mental health, chronic disease).
  - Considering other categories e.g. palliative care.

Summary
3 different challenges were considered. This report summarises the background, approaches taken, key findings, results and recommendations for future work. The next step will be to re-engage with stakeholders, identifying their priorities for future work and agreeing routes forward.

2 Virtual Study Group Background

The RAMP continuity programme of Virtual Study Groups has considered a range of challenges. This has included - Safety of Large Events during the covid-19 pandemic, Recovery from the Pandemic: Hospitality and Leisure, This study group considered the challenges associated with an ageing society.

With such a broad scope for consideration, the preparation for this study group considered a range of broad problem areas in consultation with a number of stakeholders.

Drawing on the resources highlighted by the healthy living later network and the *The State of Ageing in 2020* report [1], the VSG facilitators considered a number of working challenge areas in advance of the workshop. These were further refined into 2 broad problem categories for consideration

- Independence in the Home – Rural vs City Dwelling
- Demand Prediction (short term - hospital discharge, long term - dynamic systemic model)

The study group was structured to have a series of background/stakeholder overview presentations on day 1, supplemented with 'drop-in' expertise and advice during the study group.

Background References include

1. https://ageing-better.org.uk/state-of-ageing-20
2. https://www.gov.uk/government/publications/scwg-estimating-the-minimum-level-of-vaccine-coverage-in-care-home-settings-march-2021
3. https://www.gov.uk/government/publications/scwg-what-are-the-appropriate-mitigations-to-deploy-in-care-homes-in-the-context-of-the-post-vaccination-risk-landscape-26-may-2021
4. https://www.gov.uk/government/publications/nihr-the-effectiveness-of-ppe-in-reducing-the-transmission-of-covid-19-in-health-and-social-care-settings-december-2021-update-12-december-2021
5. https://www.gov.uk/government/publications/scwg-chairs-summary-of-role-of-shielding-20-december-2021
6. https://royalsocietypublishing.org/doi/10.1098/rstb.2020.0269

3 Aging in Place

This group considered the differences in suitability for aging-in-place in different parts of the UK.

Aging-in-place is desirable for a number reasons, including personal preference. Rural living can pose particular challenges to aging-in place, including:

- transport logistics
- care workforce location
- access to health services
- digital connectivity
- emergency support

The team proposed the development of an aging-in-place index that would model the suitability of a location for aging-in-place, along with tools to run 'what-if' scenarios. Related indices do exist, but no equivalent for this purpose was identified.

Our initial thoughts followed three strands:

- What factors could be pulled together?
- What is the current picture of access/factors?
- Is a forward projection possible? How will demographics, access and needs change?

3.1 First efforts:

Over the course of the three days, the group progressed on three tasks:

- Data Gathering: what data is available and where? What geographical information/picture do these provide?
- How accessible by public transport are health services and how might change individual-level transport?
- How do we expect the numbers of people requiring care and care workforce availability to change?

3.2 Current Picture: What could we use to build an aging-in-place index?

A number of useful data sources were identified, and maps generated based on the data available. These included:

- demographic data
- information on health services locations
- care home places and locations
- locations of care worker in 2011 census

Some example plots we produced:

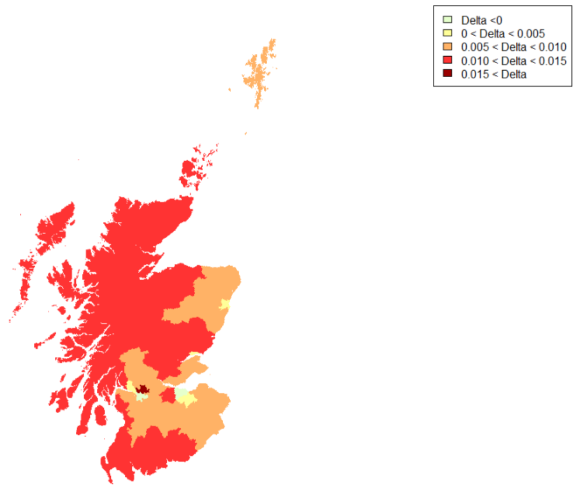

Figure 1: Rate of change in proportion over-80 by local authority in Scotland. Darker red is faster increasing proportion.

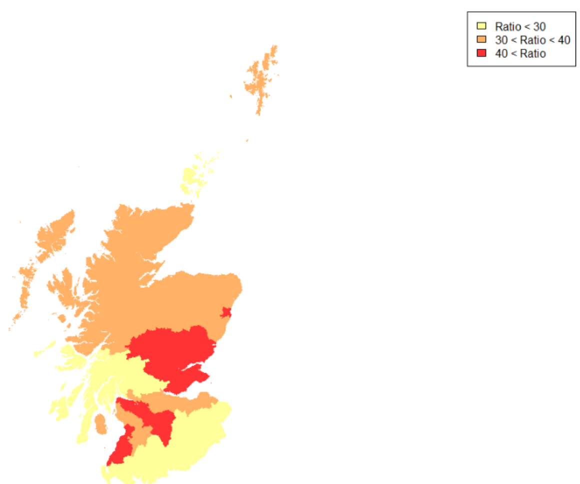

Figure 2: Places in care homes per general population by local authority in Scotland. Darker red is more places available.

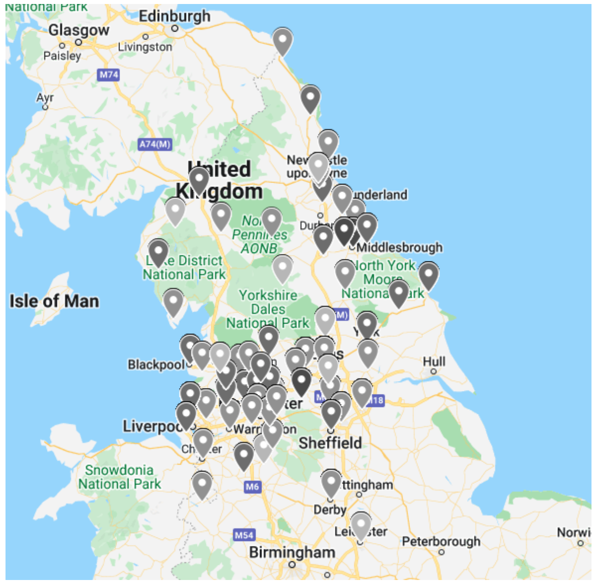

Figure 3: Absolute (approximate) numbers of care workers reported in 2011 census in parts of England - locations chosen for convenience of prototyping. Darker shade of grey is larger number of reported case workers.

There is clearly a wealth of information that could be used to inform an aging-in-place index. This short study highlighted a number of readily available data sources that provide insight and steer. In particular, note the in Scotland more rapidly ageing areas often have fewer car home places - e.g. Aberdeen is aging relatively slowly, but has one of the higher numbers of places in care homes per general population of any local authority in Scotland.

3.3 Transport Based Ratings

The travel time to GP surgeries was considered. Information on GP surgeries is available at a national level (see Figure 4)

For transport based ratings, GP surgery distribution was considered at a local level, using Bicester as an example location. Mapping of journey times from locations to nearest GP using public transport and car journeys was undertaken. For these simulations, it was assumed that patients attended their nearest GP practice. Once completed, a comparable simulation was undertaken for a different location (IPSWICH)

The Figures 5-8 show the results from the work undertaken and give an early indications of the output information possible with this technique.

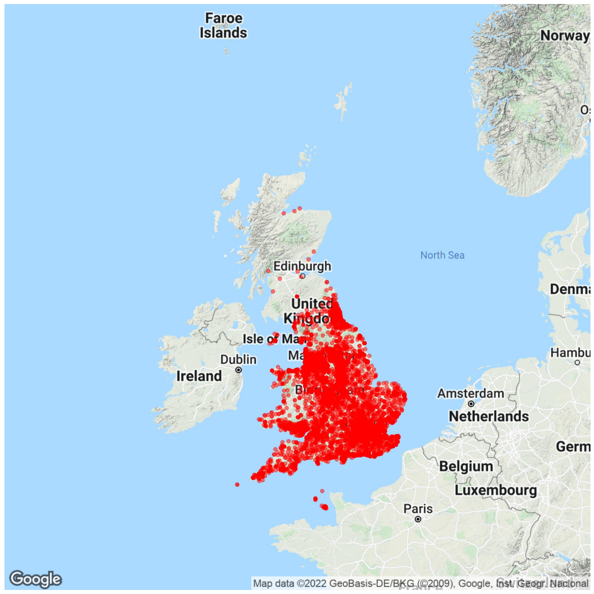

Figure 4: GP surgery locations across England and Wales)

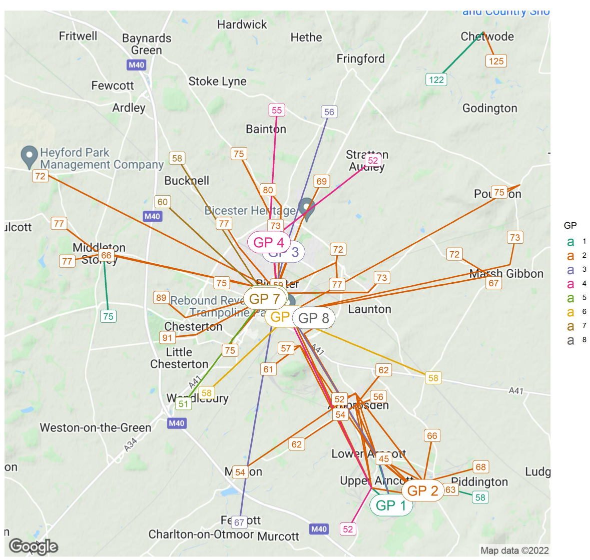

Figure 5: Public Transport Routes and GP Surgeries, Bicester)

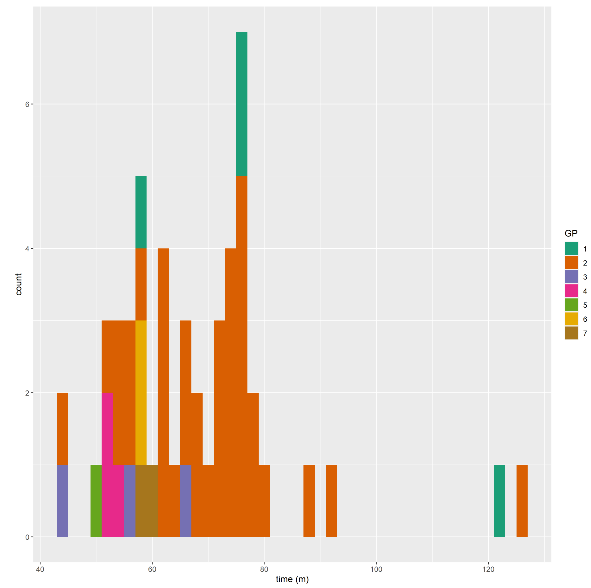

Figure 6: Journey times for public transport GP visits, Bicester)

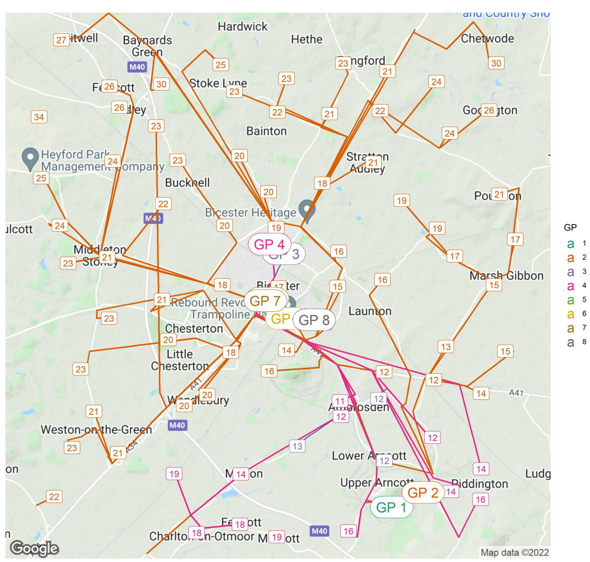

Figure 7: Car Journey Routes and GP Surgeries, Bicester)

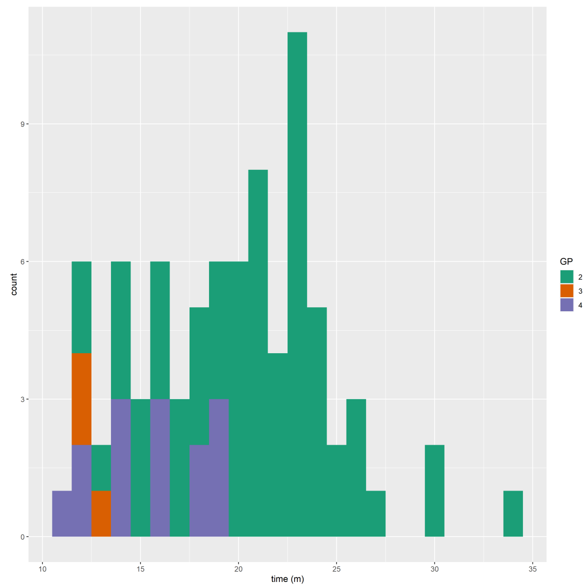

Figure 8: Car Journey Routes and GP Surgeries, Bicester)

3.4 Predicting Locations (MSOAs) with care Shortages

The 2 key tasks undertaken for this activity were estimating how many people need care, AND estimating how many people are available to provide care.

This used the middle layer super output areas (MSOA) data from the 2011 census as its primary feed. The information was split into age groupings. Assumptions included

- The percentage spread of which age group will continue to require care will continue in line with current numbers
- The number of each age group which needed care in each MSOA was used to estimate the number of who would require care in future.

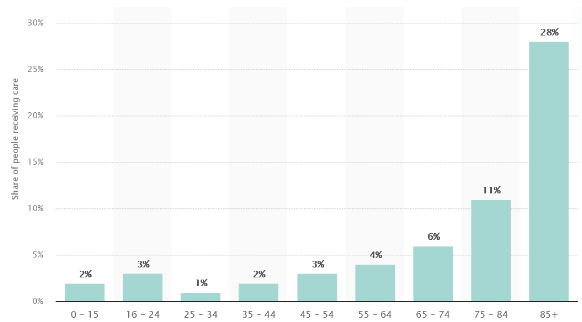

Figure 9: Percentage of People Receiving Care by Age)

To estimate who gives care. data from the 2011 census on how many people were residing in each are who work in health and social care professions was used. An estimate of the percentage of people in each age group who work in caring professions was made using age breakdown and the total number of care givers. This provided an estimate number of care-givers for each MSOA.

Once the estimates of who requires care and who gives care for each MSOA were determined, a concern ration metric was calculated. The concern ratio was the estimated number of care needers/estimated number of care givers. The ratio was calculated for 2015 and 2020 and used to project forward to 2025 and 2030 (a linear relationship was assumed). The change in concern levels between 2015 and 2020 for different location types was considered. These results are shown in Figures 10-12.

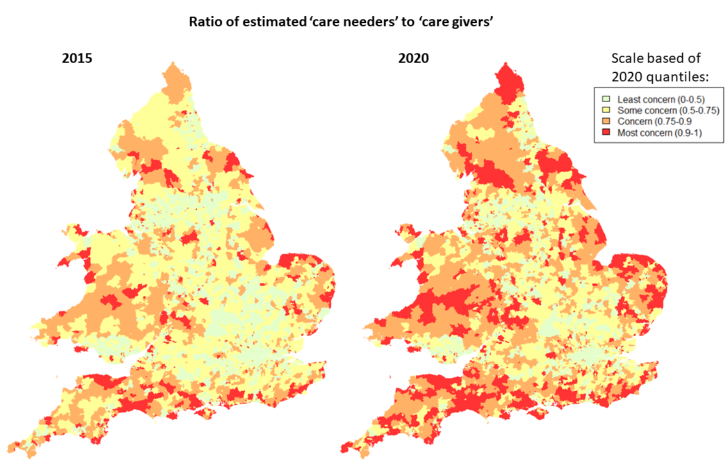

Figure 10: Ratio of estimated 'care needer' to 'care givers'

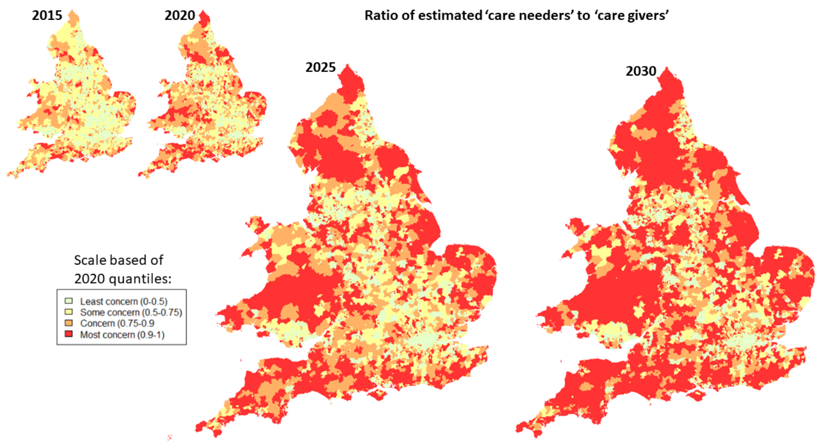

Figure 11: Ratio of estimated 'care needers' to care givers'

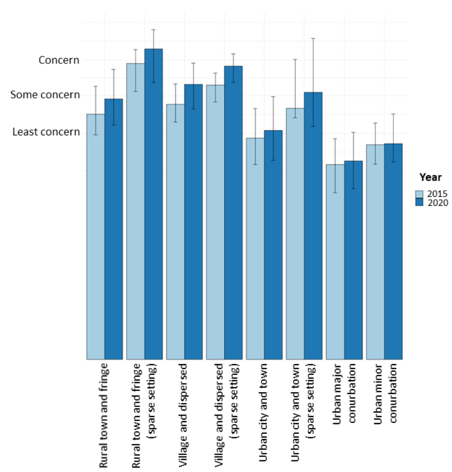

Figure 12: Change in Concern Ratio between 2015 and 2020 by Location Type)

3.5 Summary

Preliminary results indicate that data and information sources for input to an aging-in-place index exist and can provide useful insight. Initial activity has focussed on establishing what information can be extracted from available data sets, understanding the transport calculations which could feed a model and what further inputs are required. Future care requirements and availablity are a key component of this.

3.6 Next Steps

Ultimately, a tool that calculates an aging-in-place index for different areas, and allows some 'what-if' scenario modelling can be envisaged.

Initial activity to achieve this would include:

- more comprehensive data ingestion and use
- allow change to weightings for index (perhaps with GUI?)
- more forward projection and more complex care workforce modelling
- additional transport calculations and alternations modelling (changes to public transport provision, mobile health service, impact of climate change on transport)

3.7 References

1. https://rsnonline.org.uk/rsn-response-to-the-adult-social-care-white-paper
2. https://www.rsnonline.org.uk/ageing-in-place-how-can-we-ensure-rural-residents-have-choice-control-and-support-to-lead-healthy-independent-lives
3. https://livabilityindex.aarp.org/categories/housing
4. https://ageing-better.org.uk/sites/default/files/2021-04/Ageing-in-rural-place.pdf
5. https://livabilityindex.aarp.org/categories/housing
6. https://www.rsnonline.org.uk/ageing-in-place-how-can-we-ensure-rural-residents-have-choice-control-and-support-to-lead-healthy-independent-lives
7. https://www.statista.com/statistics/1233394/distribution-of-care-recipient-by-age-in-the-united-kingdom/
8. https://www.ons.gov.uk/peoplepopulationandcommunity/populationandmigration/populationestimates/datasets/middlesuperoutputareamidyearpopulationestimates
9. https://www.builderonline.com/design/whats-your-neighborhood-score-for-aging-in-place_o
10. https://geoportal.statistics.gov.uk/datasets/rural-urban-classification-2011-of-middle-layer-super-output-areas-in-england-and-wales/about

11. https://www.ageuk.org.uk/globalassets/age-uk/documents/policy-positions/housing-and-homes/ppp_rural_ageing_uk.pdf

4 Prediction - Demand - Hospital Discharge Model

4.1 Background

In this section we consider the short term issues around hospital discharges into social care, identifying the key activities and resource requirements within the system. Work from this section may later feed into the research carried out in the subsequent section which takes a more strategic and long term view of social care.

4.2 Short term objectives and Key Questions

The short term objective of this section of work is to reduce the number of patients who are medically optimised for discharge but remain in hospital, the objective being to reduce the length of stay.

Key questions:

- What are the main causes of delay in getting patients out of hospital and how should funding be focused to best reduce delays?
- Does starting the discharge process earlier reduce the number of patients who are waiting for discharge?
- Does putting in an intermediate care step help with discharge?
- What are the main causes of delay in getting patients out of hospital and how should funding be focused to best reduce delays?
- Do the date and time of discharge affect the delay?

Based on conversations, input and steer, a conceptual model was prepared to describe 'short term' discharge and begin to answer some of the above questions (see Figure 13). This is described below.

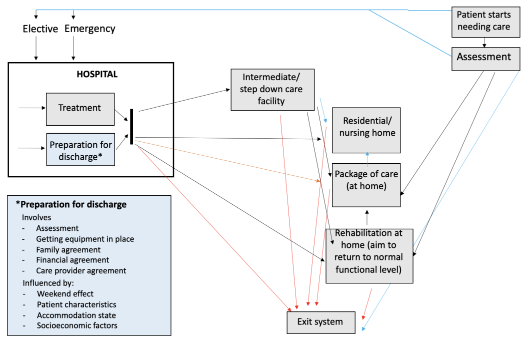

Figure 13: Conceptual 'short term' discharge model

Stage 1: Before entering the hospital

- Patients may enter the care system directly without having been to hospital.
- Following an assessment, they will be directed to either residential care, care at home or they will exit the system (no care offered).

In the simple model we present during the final round up, we have not considered this entry point. Patients only enter the model when they go to hospital.

Stage 2: In the hospital

- Patients are treated during their stay in the hospital.
- It may happen that the patient will either return to their normal level of functioning or end their life here (leaves the system).
- For the patients who are medically optimized for discharge, the discharge arrangements are made during their hospital stay.
- The assessment involves setting up onward care, getting equipment in place, carrying out financial checks and getting agreement on who is paying, determining the patient's functional ability, and liaising with the patient and their family to confirm arrangements. Patients needing care can only leave hospital when this is completed.
- Some patients will leave hospital and go home and have no further care needs.
- There are four discharge pathways (described below). The discharge decision clarifies which discharge pathway is the best option for the patient.

- Discharge may also be affected by the weekend effect, patient characteristics, accommodation state, and socioeconomic factors. The impact of these effects could be considered in further work.

Stage 3: After Discharge

There are four discharge pathways.

Pathway 1: Rehabilitation at home

- This is a simple discharge
- Patients go to their own homes and return to their normal level of functioning without the need for caregiver support.
- Sometimes, the patients require a new care package or an upgrade to their existing care package. In this case, the patients are moved to another level of the discharge pathway, called a care package at home.
- The patients either return to their normal level of functioning or end their lives here (leave the system).

Pathway 2: Care package (at home)

- Patients who choose this discharge pathway go home but require a care package (e.g., nurse, social worker, bed, wheelchair) placed in their home.
- The patients will either return to their normal level of functioning or end of their lives here (leave the system).
- It is unlikely, but may happen, that the patients will be moved to a residential or nursing home in the case that they are found to need long term care.

Pathway 3: Residential or nursing home

- Patients who follow this discharge route go to either a residential or a nursing home.
- With this route, patients really need special rehabilitation, special treatments, and equipment that cannot be provided at home.
- The majority of patients are likely to end their lives here (leave the system).

Pathway 4: Intermediate/step down care facility

- This is a discharge pathway for patients who need complex care after discharge.
- A small number of patients may return to their own home with/without care support following intermediate care.
- It is unlikely but may happen, that the patients will be transferred to a residential or nursing home to receive long term care.

4.3 Initial Modelling

A prototype model of the discharge process was built in Anylogic and a screenshot is given in Figure 14. This could be used to compare outcomes when the discharge process runs in parallel with treatment versus a situation when the discharge process starts after treatment is complete. Parameters are shown in the Parameter Table). The prototype model is a simplification of the conceptual model (Figure 13).

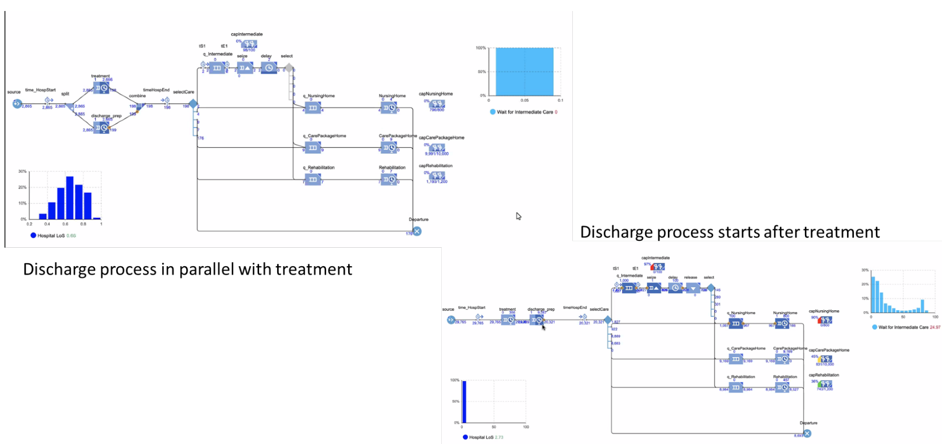

Figure 14: Anylogic Model comparing discharge process in parallel OR after treatment

The model considered the number of patients going through each discharge pathway, and the number of patients who are ready to leave the hospital but waiting in the queue.

Parameter Table			
Parameter Description	**Mean Value**	**Distribution**	**Provenance**
Treatment Duration (Hospital)	7 days	Gamma(α = 3.5, β = 2)	see Figure 13
Preparation for discharge duration (standard)	4 days	triangular(min=3 days, max = 5 days)	DTOC stats Expert opinion
Duration in intermediate care	5 weeks	Triangular (min = 4 weeks, max = 6 weeks)	Expert Opinion
Duration in rehabilitation	5 weeks	Triangular (min = 4 weeks, max = 6 weeks)	Expert Opinion (standard NHS)
Duration in residential/nursing home	2 years	Normal / triangular with a range of around +/1 1 year	BGS website
Duration receiving care at home	9 years	Gamma(α = 9, β = 1	Demographic data gives the life expectancy of someone aged 80 as 9 years. Distribution is a guess! [website]
Duration of assessment for care	6 weeks	Triangular(min = 4 weeks, max = 8 weeks)	Guess / expert opinion.
Arrival Rate	3000 per week	Poisson (3000/7 per day)	Data for University Hospital Southampton suggests around 160k patients per year
Arrival rate patients seeking care		Poisson	

Parameter Table Ctd			
Parameter Description	**Mean Value**	**Distribution**	**Provenance**
% Patients going to each care type (base)	90% leave without rehab/care including those accessing specialist end-of-life services; 6% rehab at home; 1.8% nursing/residential home; 2.2% intermediate/step-down care	N/A	Loosely based on Onen-Dumlu et al. 2022
Patients going to each care type (scenario 1)	90% leave without rehab/care including those accessing specialist end-of-life services; 6% rehab at home; 4% nursing/residential home; 0% intermediate/step-down care	N/A	Designed to experiment with different options: no intermediate care
Patients going to each care type (scenario 2)	90% leave without rehab/care including those accessing specialist end-of-life services; 4% rehab at home; 1.8% nursing/residential home; 4.2% intermediate/step-down care	N/A	Designed to experiment with different options: more intermediate care
Patients need care after rehab vs exit system	17%	N/A	Loosely based on Onen-Dumlu et al. 2022
Capacity of intermediate care	100 (177)	N/A	Pure guess work. Numbers in brackets for this and next few rows are for Onen-Dumlu paper
Capacity of nursing/residential care	800 beds (172)	N/A	Guess: Southampton has 39 care homes and average size of a care home is 20 beds

Parameter Table Ctd			
Parameter Description	**Mean Value**	**Distribution**	**Provenance**
Capacity of care at home	900000(546)	N/A	Useful website Southampton City Council suggests 7000 patients needing care in 2023. The hospital serves a wider population than the city so increased this number.
Capacity of rehab at home	1200	N/A	Guess work based on around 300 per week * 4 weeks

4.4 Next Steps

- Identify other modelling work in the area and learn from this when building the more complex model.
- Switch coding to Python rather than AnyLogic.
- Build an extended model that splits patients into different categories (e.g mental health, chronic illness, hard-to-place patients).
- Looks at different capacity limits for difficult-to-discharge patients with complex needs. Use different rates, etc for elective and emergency patients.
- Investigate whether earlier and better information sharing about discharge date and care capacity would reduce the number of patients with delayed discharge.
- Examine what would happen with no constraint on social care capacity to help to estimate the true need for social care and identify other potential bottlenecks in the system.
- Try different data sources/locations, e.g. data from NI versus a scenario in Southampton.

4.5 References for Prediction Demand - Discharge AND Systemic Model

1. `https://data.oecd.org/healthcare/length-of-hospital-stay.htm`
2. `https://www.bgs.org.uk/resources/end-of-life-care-in-frailty-care-homes`

3. https://knoema.com/atlas/United-Kingdom/topics/Demographics/Age/Life-expectancy-at-age-80-year
4. https://www.medrxiv.org/content/10.1101/2021.10.25.21265475v1
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6. https://www.england.nhs.uk/statistics/statistical-work-areas/delayed-transfers-of-care/delayed-transfers-of-care-data-2019-20/
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17. https://gss.civilservice.gov.uk/dashboard/tools/adult-social-care-statistics/database.htm

5 Prediction - Demand - Long Term Systemic Model

5.1 Background

In this section we consider how quantitative modelling could be used to extend a conceptual framework developed by Sarah Wylie from SIBNI. This model is aimed at understanding and improving the interactions between the various components of the social care system and the processes driving the consolidation of social care services into the hands of a small number of large independent care providers and the consequences for the quality of care provided. A long term goal of this is to understand whether a major intervention could reverse this process putting care provision into the hands of a diverse array of community organisations and small businesses. The goal of the intervention would be to have as many as possible of the people using social care to receive the gold standard of care, known as self directed support (SDS).

The input provided by Sarah Wylie and is referred to as the SW Hypothesis.

The SW hypothesis, as understood by the group, proposes that:

1. Increases in the number of people requiring social care and the increased severity of their needs has led to a decrease in the per capita funding available and has driven a consolidation of care providers into a small number of large companies. This leads to an reduction in the number of people receiving self directed support due to a lack of diversity of provision.
2. A more diverse social care ecosystem is possible at the same level of cost if the right level of coordination is provided to link up providers with users.

In the sections below we develop a system dynamics model to describe one aspect of the overall system, namely the flow of people between states in which they are: in hospital/in crisis; in adequate social care; and in self directed support, the gold standard of care. To make the model more realistic we convert the model from a set of differential equations to a set of stochastic differential equations in order to incorporate stochastic demand. Finally we consider how to relate the terms in the model to available funding.

5.2 System Dynamics Model

There is a rich interplay of various actors within the system. As a starting point we develop a coarse grained model of this system in which we consider the flow of users of the social care system between different states. These state variables of the system are E_1 the number of people in self directed support, E_2 the number of people in adequate social care, E_3 the number of people in crisis or in hospital. The structure of this model is shown in Figure 15.

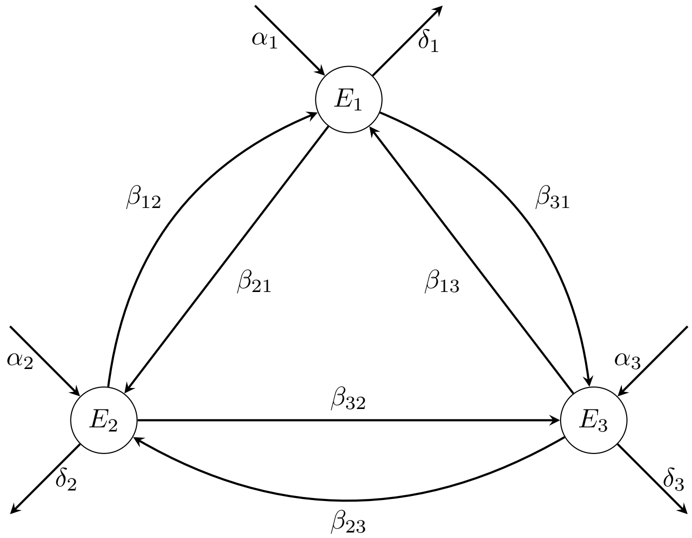

Figure 15: Schematic model system dynamics model.

The equations of the model are

$$\dot{E}_1 = \alpha_1 - \delta_1 E_1 + \beta_{12} E_2 - \beta_{21} E_1 - \beta_{31} E_1 + \beta_{13} E_3 \quad (1)$$

$$\dot{E}_2 = \alpha_2 - \delta_2 E_2 - \beta_{21} E_2 + \beta_{21} E_1 - \beta_{23} E_2 + \beta_{23} E_3 \quad (2)$$

$$\dot{E}_3 = \alpha_3 - \delta_3 E_3 + \beta_{32} E_2 - \beta_{23} E_3 + \beta_{31} E_1 - \beta_{13} E_3 \quad (3)$$

where: α_i, $i = 1, 2, 3$, are the rates at which people enter the social care system in state i; δ_i, $i = 1, 2, 3$, are the per capita rates at which people exit the social care system due to recovery, migration or death; the β_{ij} coefficients, $i, j = 1, 2, 3$, describe the per capita rates at which people transition into state i from state j. This model is related to the Hospital Discharge Model presented in Section 4 in that flows into and out of state 3 are those modelled by the Hospital Discharge Model.

Figure 16 shows a simulation of the model with arbitrary parameters. The equations are set up with the α_i terms including a gradual increase with time. This simulates increasing numbers of people entering the social care system over time.

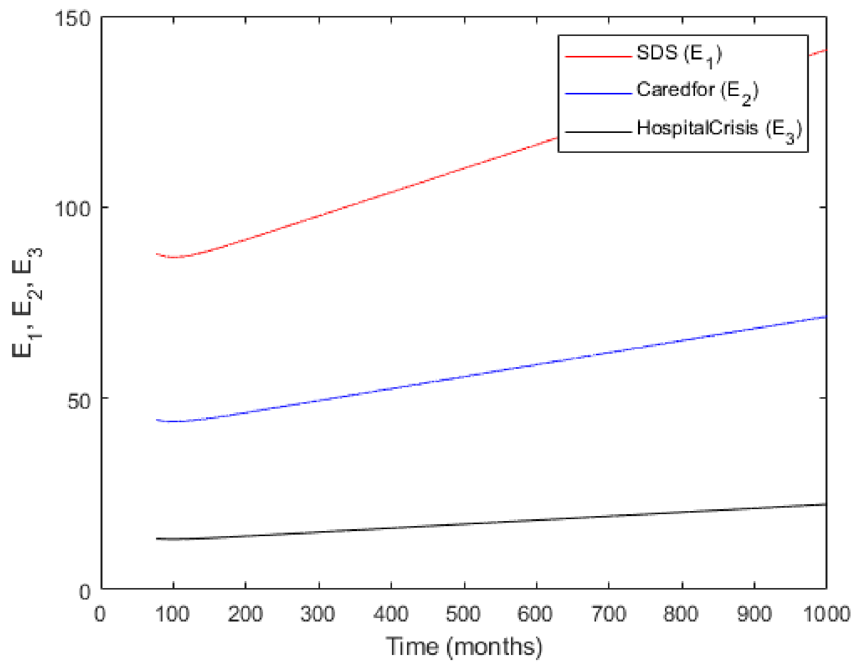

Figure 16: Simulated Results for parameters E_1, E_2 and E_3

5.3 Eigenvalue Analysis

As part of the work of analysing the model some consideration was given to the form of the eigenvalues of a matrix, L consisting of the sum of a negative definite diagonal matrix, $-D$, and an antisymmetric matrix, A. It is relatively easy to show that the real parts of the eigenvalues of such a matrix are negative. To do so consider the eigenvalue equation and take its complex conjugate

$$L\phi = -D\phi + A\phi = \lambda\phi \tag{4}$$

$$L\phi^* = -D\phi^* + A\phi^* = \lambda^*\phi^* \tag{5}$$

Right multiply by ϕ^* and ϕ respectively:

$$-\phi^{*T}D\phi + \phi^{*T}A\phi = \lambda\phi^{*T}\phi \tag{6}$$

$$-\phi^T D\phi^* + \phi^T A\phi^* = \lambda^*\phi^T\phi^*\phi \tag{7}$$

Adding these equations together and using the antisymmetry of A gives:

$$-2\phi^{*T}D\phi = 2\mathcal{R}(\lambda)\phi^{*T}\phi \tag{8}$$

$$\mathcal{R}(\lambda) = -\frac{\phi^{*T}D\phi}{\phi^{*T}\phi} \tag{9}$$

As D is a positive definite diagonal matrix this means that $\mathcal{R}(\lambda) < 0$.

Understanding whether the eigenvalues of L will be real or complex is more complicated. L will either have three real eigenvalues, or complex conjugate pair of eigenvalues and one real eigenvalue. Numerical experiments suggest that the choice will depend on the relative sizes of the matrices D and A. When D is large all eigenvalues will be real, when A is large then there will be a complex conjugate pair of eigenvalues.

Figure 17 shows a numerical experiment in which the absolute values of the eigenvalues of a 3×3 matrix of the form $L = -\lambda D + A$ are calculated. λ parameterises the magnitude of the diagonal part of the matrix. For large values of the matrix three distinct eigenvalues can be seen. This indicates that the eigenvalues are real. At around $\lambda \approx 10$ the absolute values of two eigenvalues become the same showing the formation of a complex conjugate pair. (The apparent crossover at $\lambda \approx 6$ is an artifact of the ordering of eigenvalues by Matlab.)

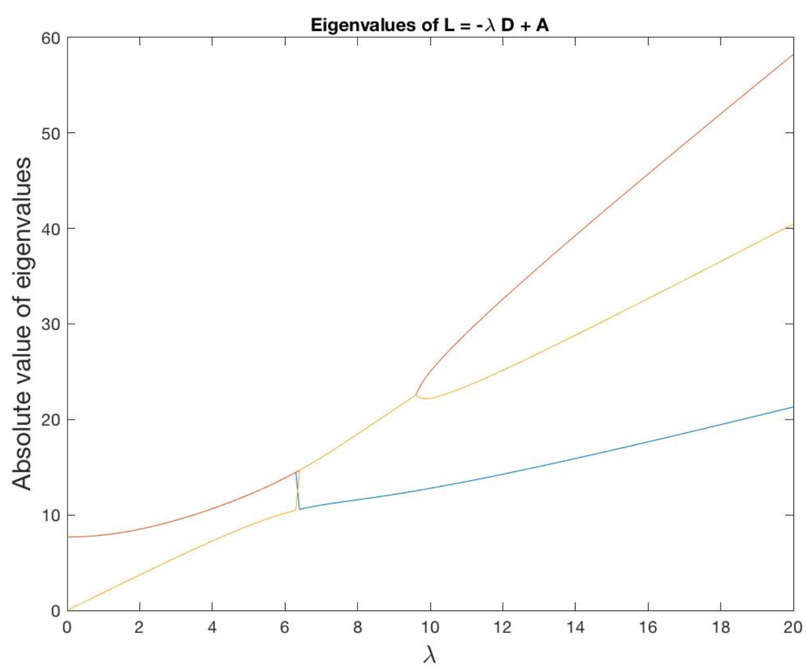

Figure 17: Numerical experiment showing absolute values of the eigenvalues of $-\lambda D + A$. For large values of λ there are three real, distinct eigenvalues. For small values of λ there is one real eigenvalue and a complex conjugate pair of eigenvalues.

The same qualitative behaviour is shown in Figure 18. This shows a numerical experiment in which matrices of the form $-D + \beta_0 A$ are constructed in which β_0 is a parameter and D is a 3×3 diagonal matrix whose elements are drawn from a uniform probability distribution on the interval [0,1]. A is an antisymmetric matrix constructed by constructing a 3×3 random matrix and subtracting its transpose. The elements of the random matrix are drawn from the same distribution as the diagonal elements of D. Figure 18 shows the imaginary part of eigenvalues of $-D + \beta_0 A$ vs β_0. Specifically the imaginary part of the eigenvalue with the largest (most positive) imaginary part is plotted for a large number of random matrices of the form described above. This shows the same qualitative picture as Figure 17, namely that when the dominant contribution to the matrix comes from the diagonal part the eigenvalues are real and when the dominant contribution to the matrix comes from the antisymmetric part complex eigenvalues are present.

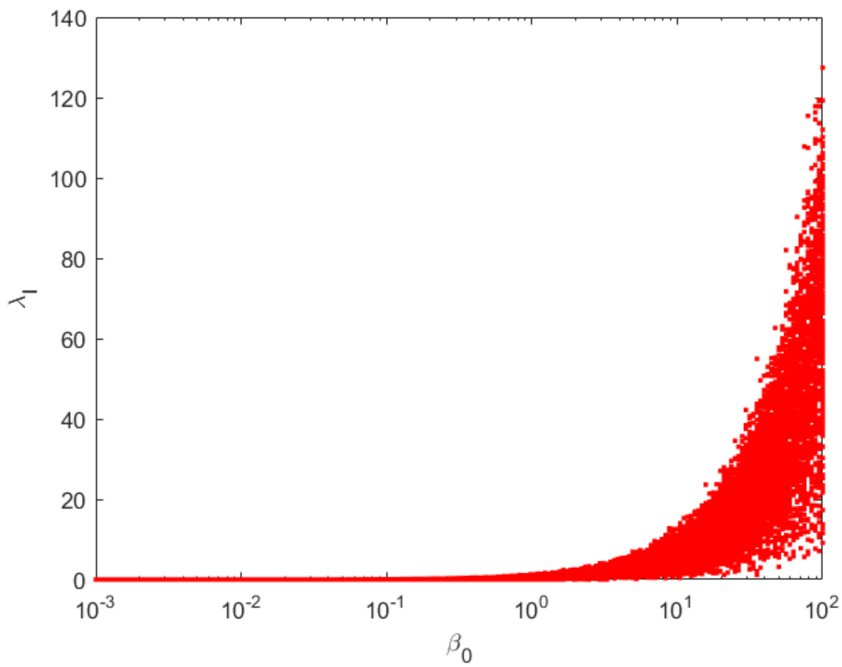

Figure 18: Imaginary components of the eigenvalues of random matrices of the form $-D + \beta + 0A$. For small values of β_0 eigenvalues are real with no imaginary part. For large values of β_0 pairs of complex conjugate eigenvalues are seen.

5.4 Stochastic Demand

To model a more realistic picture in which demand for the social care system is stochastic the model developed above, equations 1 to 3 can be extended by replacing $\alpha_i \mathrm{d}t$ by $\alpha_i \mathrm{d}t + \sigma_i \mathrm{d}W_i$

where W_i is a Wiener process.

$$\mathrm{d}E_1 = \alpha_1 \mathrm{d}t + \sigma_1 \mathrm{d}W_1 + (-\delta_1 E_1 + \beta_{12} E_2 - \beta_{21} E_1 - \beta_{31} E_1 + \beta_{13} E_3)\,\mathrm{d}t \tag{10}$$

$$\mathrm{d}E_2 = \alpha_2 \mathrm{d}t + \sigma_2 \mathrm{d}W_2 + (-\delta_2 E_2 - \beta_{21} E_2 + \beta_{21} E_1 - \beta_{23} E_2 + \beta_{23} E_3)\,\mathrm{d}t \tag{11}$$

$$\mathrm{d}E_3 = \alpha_3 \mathrm{d}t + \sigma_3 \mathrm{d}W_3 + (-\delta_3 E_3 + \beta_{32} E_2 - \beta_{23} E_3 + \beta_{31} E_1 - \beta_{13} E_3)\,\mathrm{d}t \tag{12}$$

A stochastic ODE version of Figure 16 is provided in Figure 19.

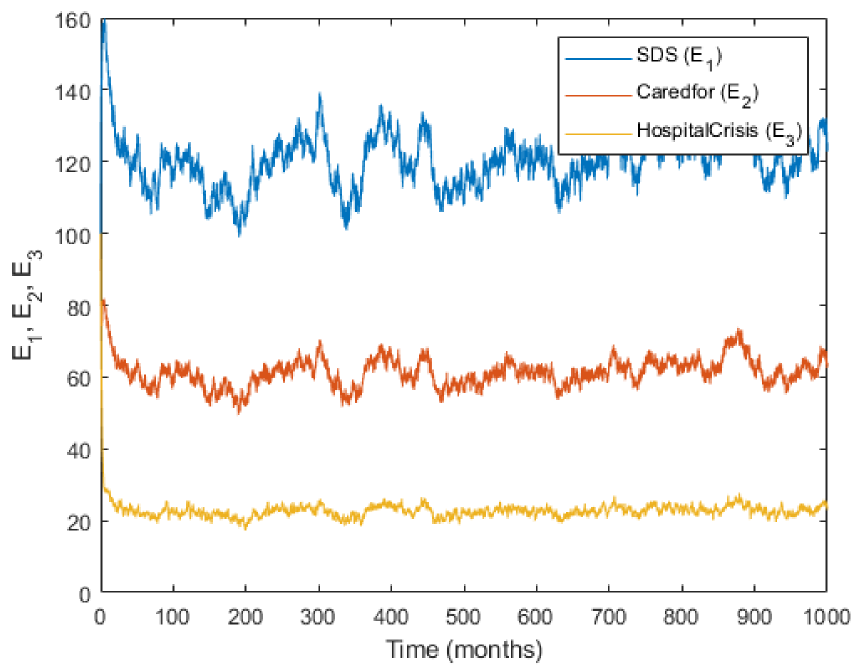

Figure 19: Simulated results for parameters E_1, E_2 and E_3 with stochastic demand.

5.5 Including Financial Considerations

We consider how financial considerations could be coupled into the model. To do so we assume that there is a fixed budget for social and health care of the elderly B_T and that hospital costs are C_3 per person in hospital, i.e. a total of $C_3 E_3$. The remainder of the budget is available for funding social care $B_S = B_T - C_3 E_3$. We anticipate there will be a per capita decrease in this budget as the number of elderly people increases and we want to allocate the budget to maximise E_1/E_2 i.e. the fraction of the population in SDS. Given the available budget B_S and assuming per capita costs of C_1 and C_2 we can set a target for E_1 of T_1 where

$$T_1 = B_S - \frac{(E_1 + E_2)C_2}{C_1 - C_2} \tag{13}$$

$$T_1 = B_T - C_3 E_3 - \frac{(E_1 + E_2)C_2}{C_1 - C_2} \tag{14}$$

The difference $T_1 - E_1$ indicates the degree of underspend and thus is a measure of funding available to promote the standard of care of some social care users to self directed support. This would be incorporated into the modelling framework as an increase in β_{12}, the transfer rate to E_1 from E_2.

5.6 Summary

A very simply dynamical system model the flow of people between different types of care within the social care system has been developed. This provides the beginnings of a mathematical framework which can be used to investigate the factors driving the consolidation of care provision into the hands of a few large independent companies and the consequences of that for the level of care delivered. Some preliminary work towards analysing the dynamics of the model and extending it to include stochastic demand and budgeting considerations have been carried out.

5.7 Next Steps

Progress was made towards adding system dynamics into a long-term Systemic Model. There are a number of different areas that would be appropriate for further work. In order to address the main hypothesis the key extensions needed for the model are to include a description of the evolution of the number and size of care providers and a more detailed description of budgeting considerations.

Other extensions of the model which could make it more realistic in describing the diversity of the sector would include

- Segmenting the categories E_1, E_2 and E_3 by age and/or care needs (e.g. mental health, chronic disease)
- Considering other categories e.g. palliative care.

5.8 References

See Section 4.5

6 Concluding Remarks

3 different challenges were considered. This report summarises the background, approaches taken, key findings, results and recommendations for future work. The next step will be to re-engage with stakeholders, identifying their priorities for future work and agreeing routes forward.

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