A New Model of Airborne Transmission that Quickly Predicts the Spatiotemporal Infection Risk in Indoor Spaces

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Physics of airborne transmission

1. Virus spreading (turbulent eddy diffusion)
2. Virus transport (advection)
3. Virus source
4. Ventilation
5. Gravitational settling
6. Evaporation
7. Virus deactivation
8. Virus receivers – susceptible people

Wells-Riley models

Assume that the room is fully mixed

Gammaitoni-Nucci (1997) extension:

Source strength

Concentration

Deactivation

$$\frac{\mathrm{d}C}{\mathrm{d}t} = \frac{R}{V} - (\lambda + \beta + \gamma)C$$

Ventilation

Gravitational Settling

Room volume

Buonanno *et al.* 2020;
Miller *et al.* 2020;
Lelieveld *et al.* 2020;
Burridge *et al* 2021;
etc

Extend to an advection-diffusion-reaction equation

- Infectious person a point (stationary) source of virus, constant emission rate of particles
- All particles are the same size, and they carry the same amount of virus
- Droplets are released from the infectious person with zero velocity
- Particles advected with constant velocity
- Turbulent mixing of air
- Droplets quickly evaporate to equilibrium size – neglect evaporation

$$\frac{\partial C}{\partial t} + \nabla \cdot (\boldsymbol{v}C) - \nabla \cdot (K \nabla C) = R\delta(x - x_0)\delta(y - y_0) - (\lambda + \beta + \gamma)C$$

See our preprint @ https://arxiv.org/abs/2012.12267 (only with ventilation now)

Ventilation scenarios

We consider four ventilation scenarios that correspond to different air exchanges per hour (λ)

1. Very poor Ventilation (0.12 h^{-1}): $\lambda = 3.3 \times 10^{-5}$ s^{-1}
2. Poor Ventilation (0.72 h^{-1}): $\lambda = 2 \times 10^{-4}$ s^{-1}
3. ASHRAE-recommended ventilation, pre-pandemic (3 h^{-1}): $\lambda = 8.3 \times 10^{-4}$ s^{-1}
4. ASHRAE-recommended ventilation, post-pandemic (6 h^{-1}): $\lambda = 1.7 \times 10^{-3}$ s^{-1}

Values taken from classroom data

Turbulence: eddy diffusion coefficient

We use the eddy diffusion coefficient

Room volume

$$K = \boxed{\lambda}\left(\frac{V^2}{2N^2}\right)^{\frac{1}{3}}$$

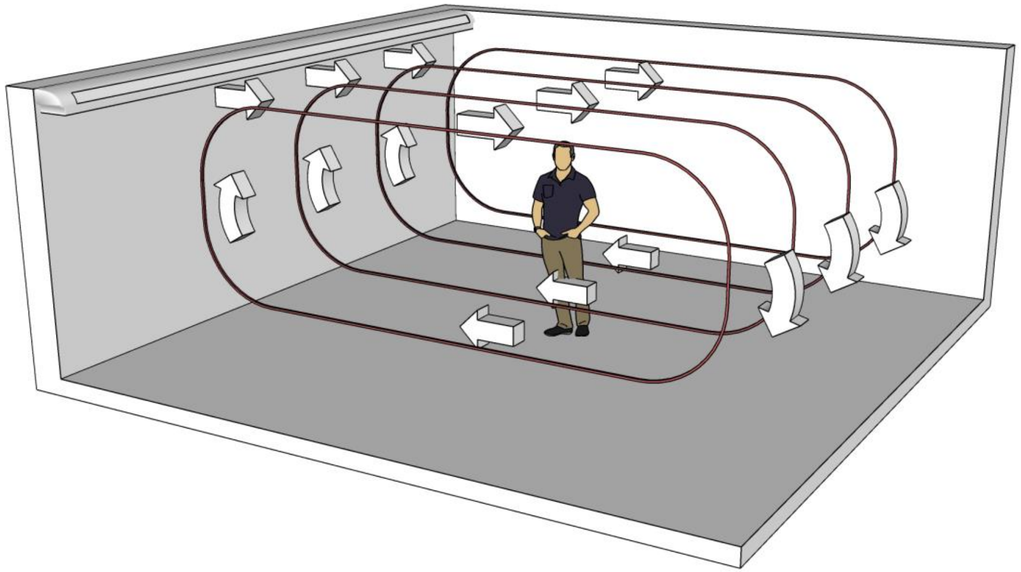
Number of vents (we take $N = 1$)

Note: K scales with λ

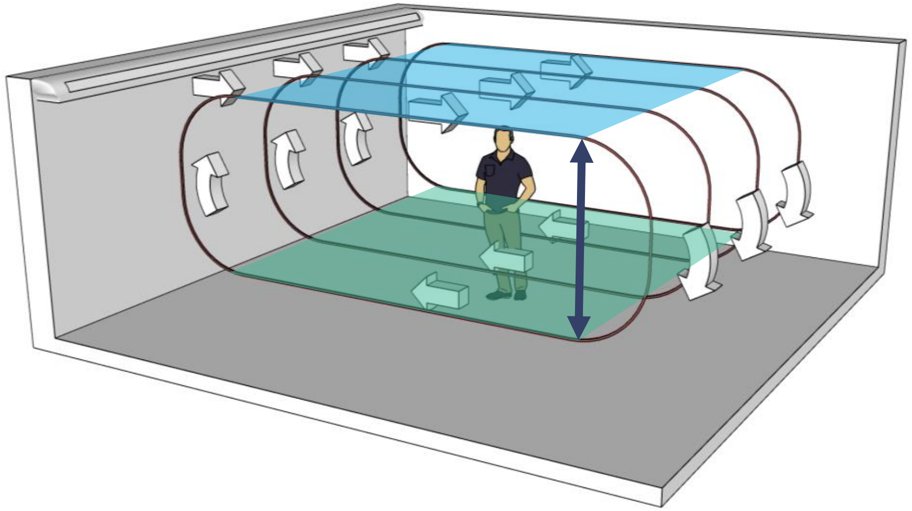
(Foat *et al* 2020)

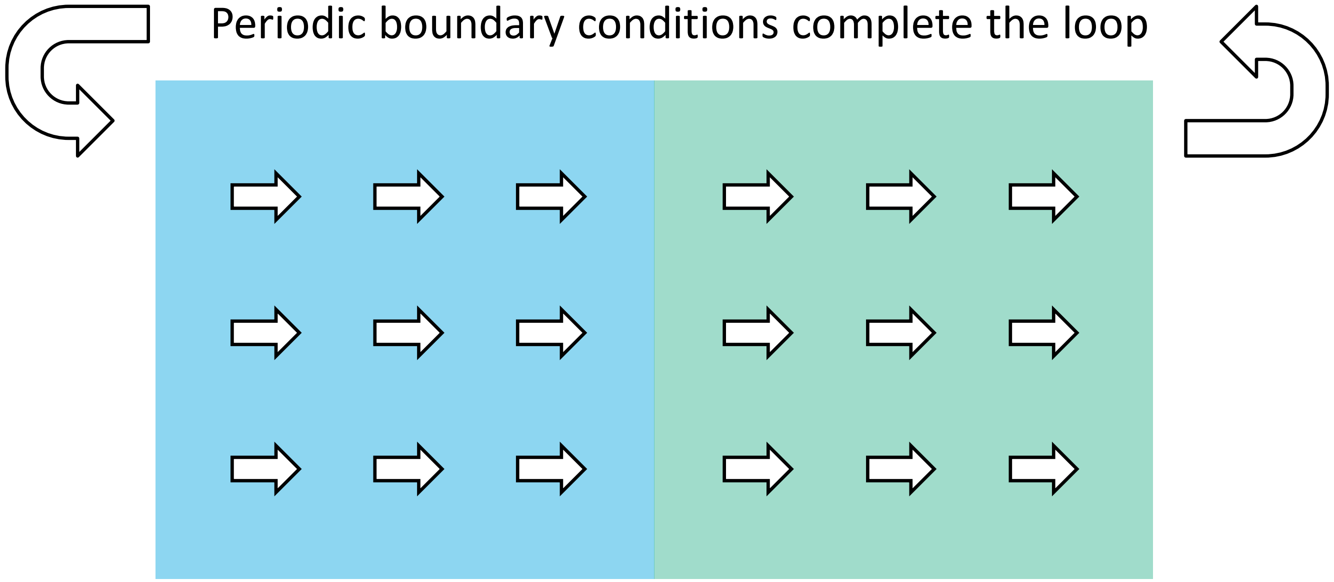
Quasi-3D model – recirculating airflow

Constant velocity



Quasi-3D flattened to 2D





Distance between layers is half the room height (van Hooff *et al* 2013)

The model

$y = w$

$$\frac{\partial C}{\partial y}(x, w, t) = 0$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial x} - K\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) = R\delta(x - x_0)\delta(y - y_0) - (\lambda + \beta + \gamma)C$$

$$C(0, y, t) = C(2l, y, t)$$

$$\frac{\partial C}{\partial t}(0, y, t) = \frac{\partial C}{\partial t}(2l, y, t)$$

$$\frac{\partial C}{\partial y}(x, 0, t) = 0$$

$y = 0$

$x = 0$

$x = 2l$

Analytical solution - and parameters

$$C_{3D}(x,y,t) = \frac{C(x,y,t) + C(2l-x,y,t)}{(h/2)}$$

$$C(x,y,t) = \int_0^t \frac{R}{4\pi K\tau} e^{-(\lambda+\beta+\gamma)\tau} \sum_{m=-\infty}^{\infty} \left(e^{-\frac{(x-v\tau-x_0-2ml)^2}{4K\tau}} + e^{-\frac{(x-v\tau+x_0-2ml)^2}{4K\tau}} \right) \sum_{n=-\infty}^{\infty} \left(e^{-\frac{(y-y_0-2nw)^2}{4K\tau}} + e^{-\frac{(y+y_0-2nw)^2}{4K\tau}} \right) d\tau$$

Rate of particle generation from infectious person:

Breathing:	0.5 particles/s	(Asadi *et al* 2019)
Talking:	5 particles/s	(Asadi *et al* 2019)
Breathing with mask:	0.25 particles/s	(Fischer *et al* 2020)
Talking with mask:	2.5 particles/s	(Fischer *et al* 2020)

Biological deactivation:

$\beta = 1.7 \times 10^{-4}$ s^{-1}

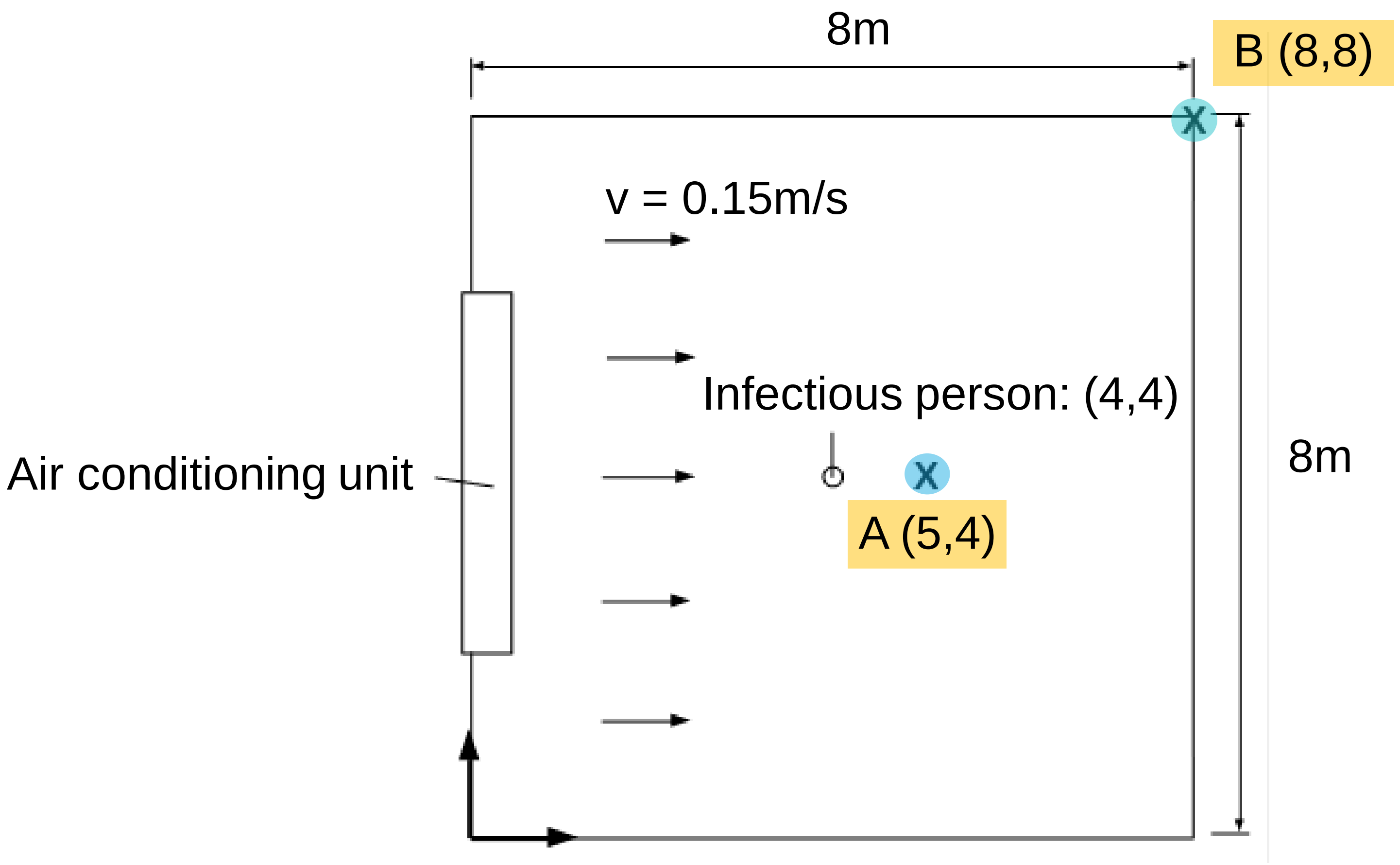
(Doremalen *et al* 2020)

Gravitational settling rate:

$\gamma = 1.1 \times 10^{-4}$s^{-1}

(de Oliveira *et al* 2021)

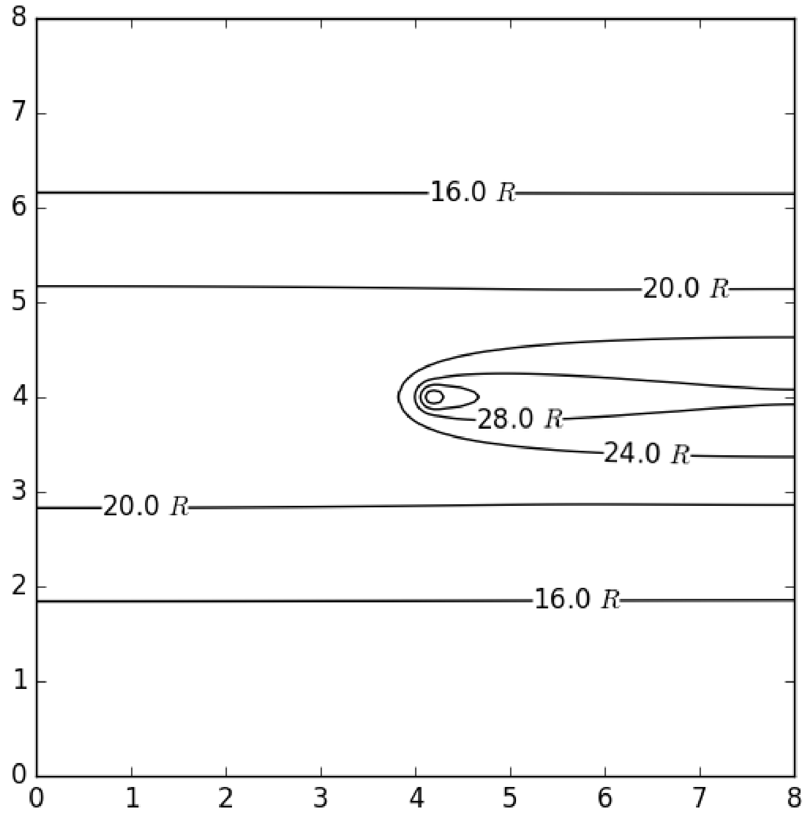
Case study: average-sized classroom



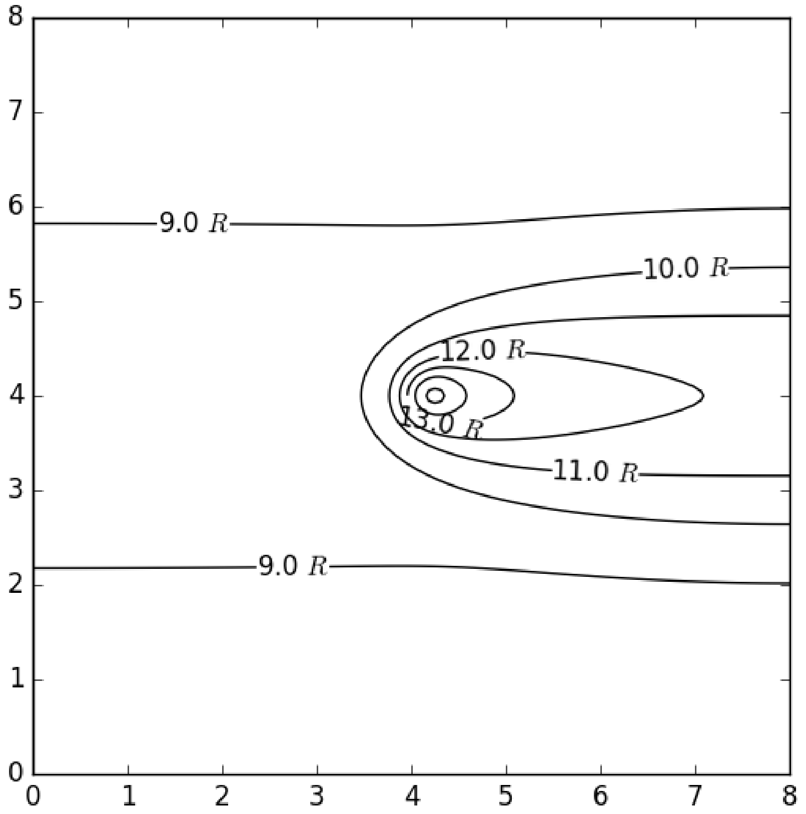
Room height = 3m

Concentration after 1 hour

Poor ventilation



ASHRAE, pre-pandemic



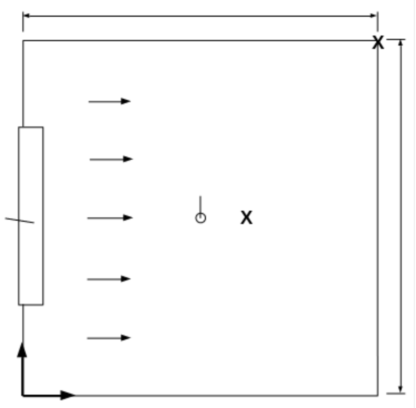
Breathing (mask): $R = 0.25$

Breathing (no mask): $R = 0.5$

Talking (mask): $R = 2.5$

Talking (no mask): $R = 5$

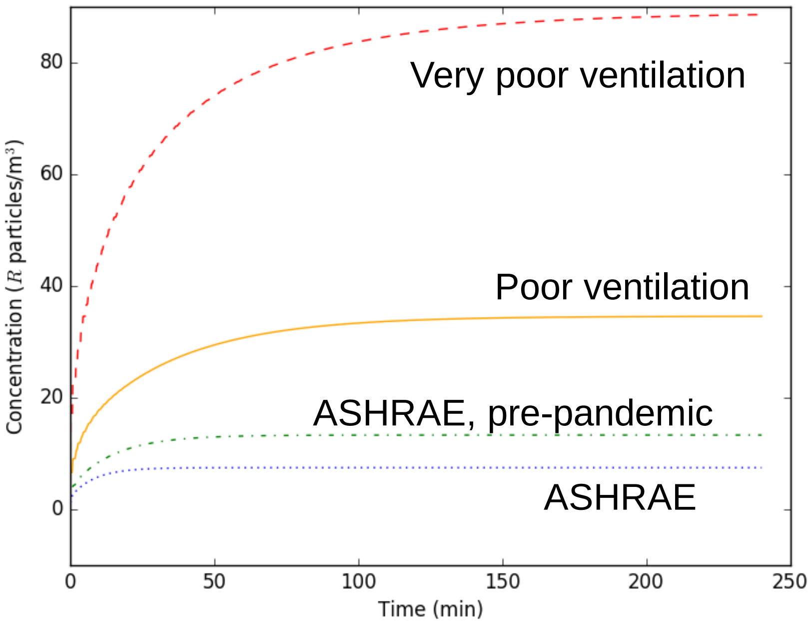
Concentration vs time (any activity type)



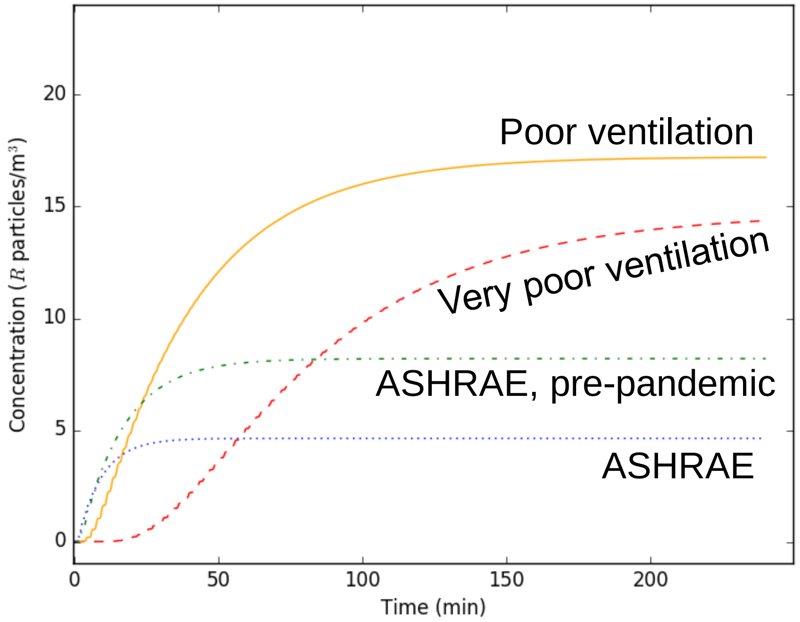
Steady state achieved eventually

Power law for Point A

Point A



Point B



Quantified effect of increasing ventilation

Corner of room: “very poor” better than “poor”?

Spatiotemporal infection risk

Infection risk (Probability of infection):

$$P(x,y,t) = 1 - \mathrm{e}^{-d(x,y,t)k}$$

where the dose inhaled is (adapted from Riley et al 1978 and Vuorinen et al 2020)

$$d(x,y,t) = \int_0^t \rho C_{3D}(x,y,\tau)\,\mathrm{d}\tau$$

What is the median infectious dose that corresponds to 50% infection risk?
Uncertain!

$$0.5 = 1 - \mathrm{e}^{-kd_m}$$

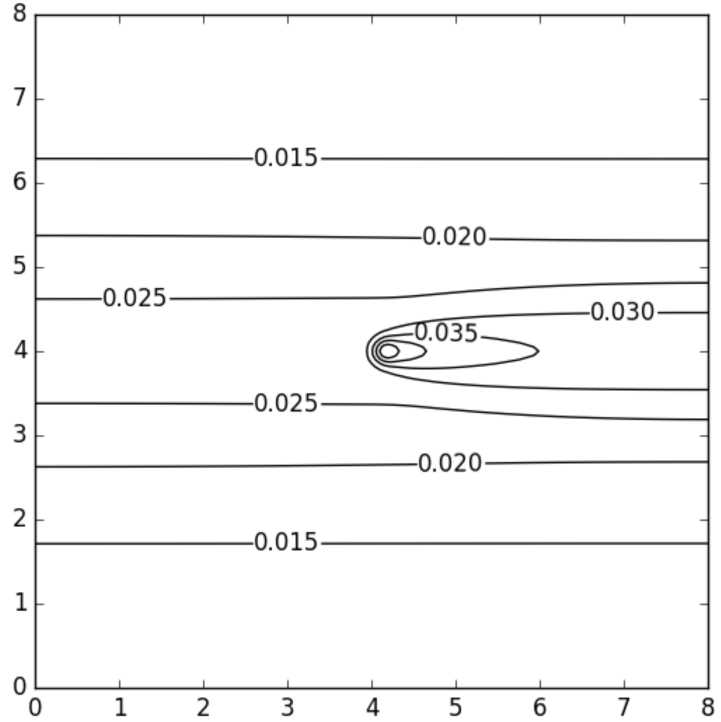
Take $d_m = 100 \text{ particles} \Rightarrow k = 0.0069$ (Burridge et al, 2021)

Breathing rate: 8 ℓ/min (Hallett *et al* 2020)

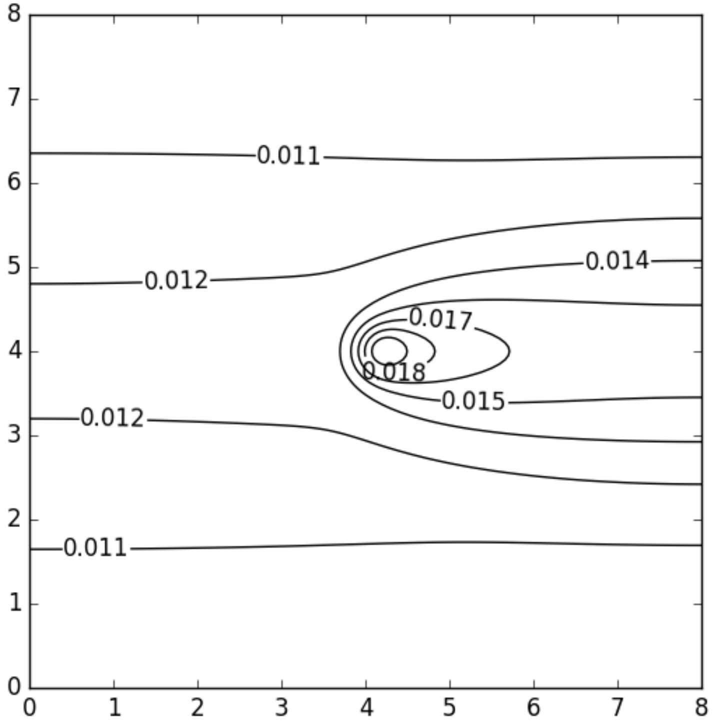
Infection risk maps (at one hour)

One infected person, **Breathing**

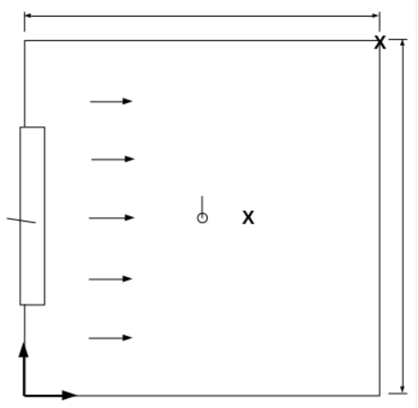
Poor ventilation



ASHRAE, pre-pandemic



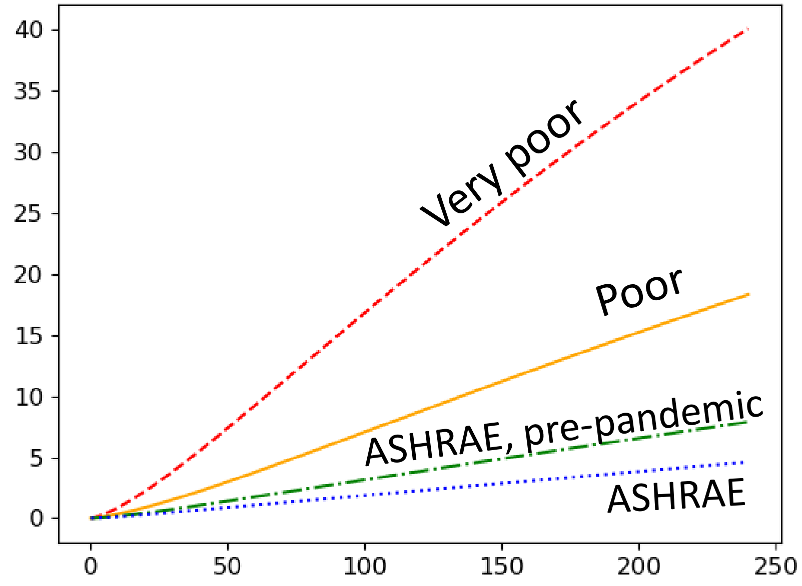
Infection risk vs time (breathing)



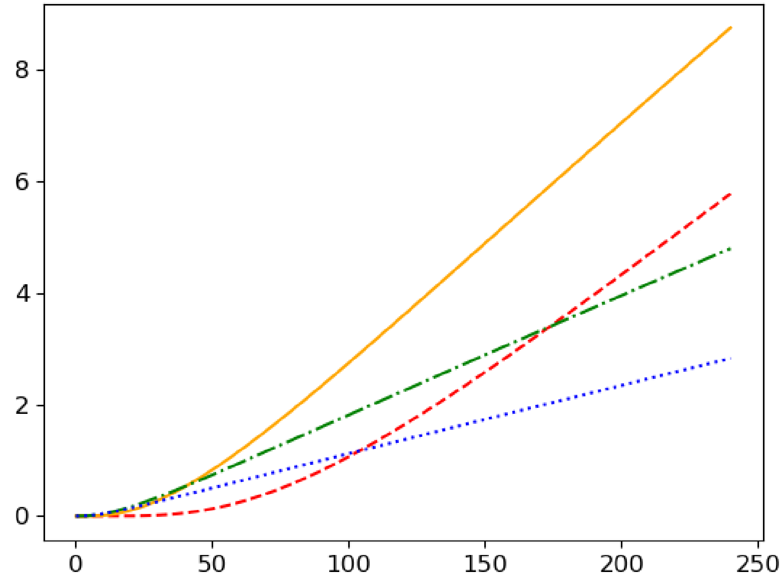
ASHRAE, pre-pandemic

ASHRAE

Point A



Point B



Quantified effect of ventilation on infection risk

“very poor” better than “poor” at some locations?

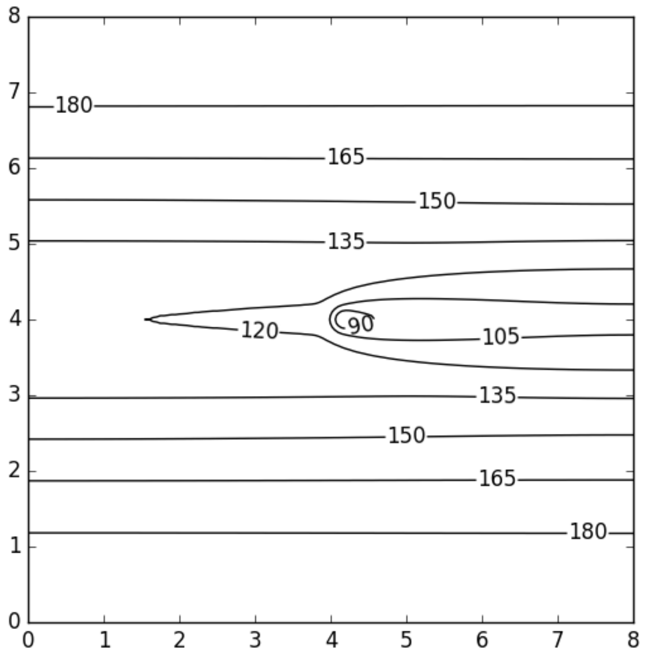
Time to Probable Infection (TTPI) maps

One infected person, at the centre of the room, **talking**

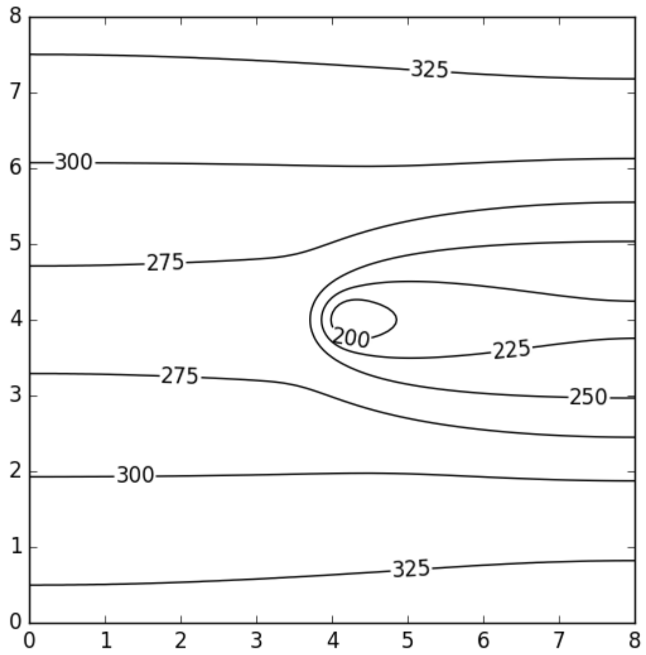
Time to Probable Infection=Time required for the infection risk to reach 50%

Paving the way for recommending
Safe Occupancy Times

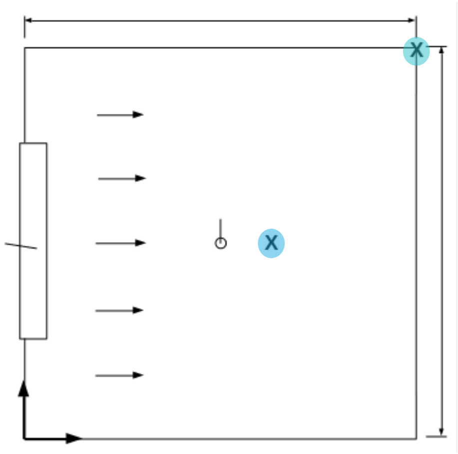
Poor ventilation



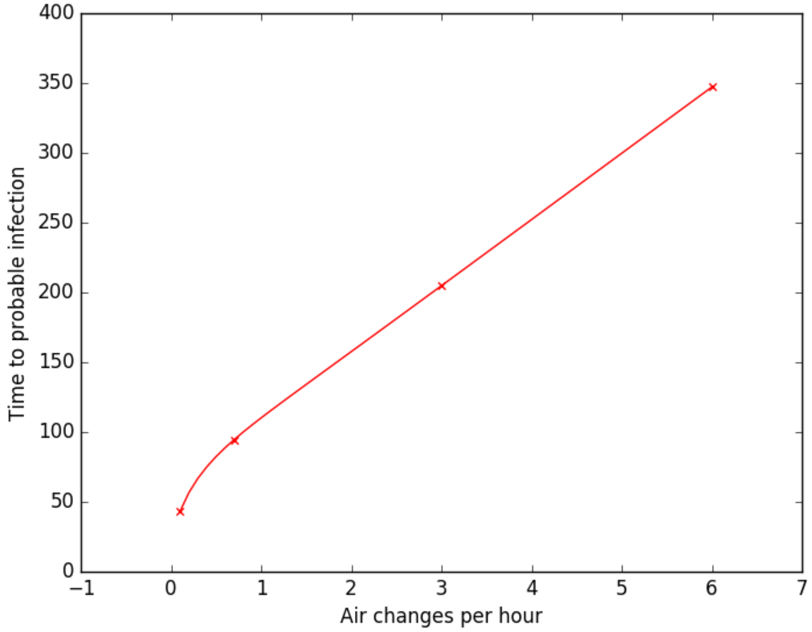
ASHRAE pre-pandemic



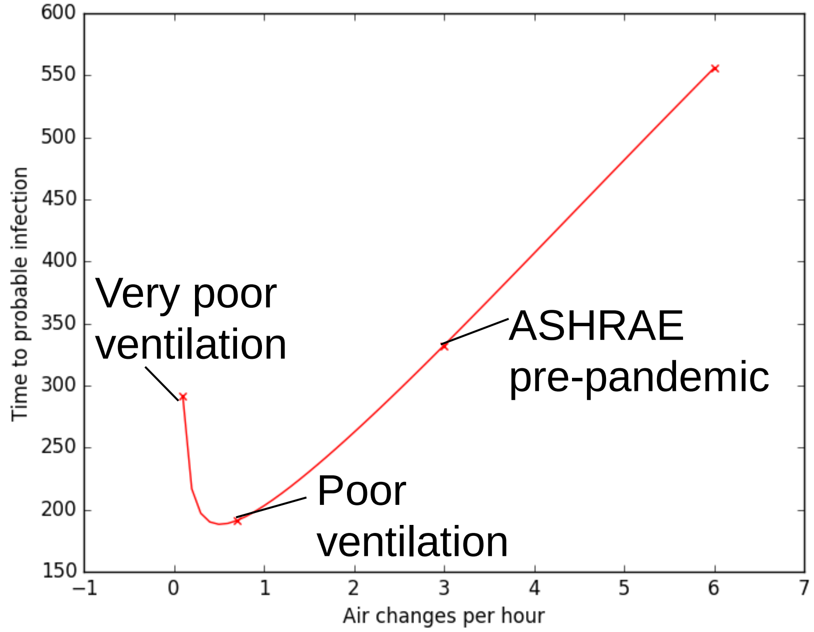
TTPI as air exchanges per hour increase



Point A



Point B



Comparison with data and CFD models

We compared with:

- Hospital air sampling data

Lednicky *et al* 2020; Chia *et al* 2020; Santarpia *et al* 2020

Need a viral load of ~10^11 to make the comparison hold

- CFD simulations of the Guangzhou restaurant superspreader case

Birnir *et al* 2020

Our expected number of infections in the Guangzhou restaurant superspreader case is close to the number of people infected.

Extending the model: particle size distributions

- Aerosols of variable size s.

- The eddy diffusivity K, deactivation rate γ, and deposition rate D all depend on the aerosol size s.

$$\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial x} - K(s)\nabla^2 n = RF(s)\delta(x-x_0)\delta(y-y_0)H(t-t_0) - (\lambda + \gamma(s) + D(s))n$$

BLO model (Johnson et al, 2011)

$D(s) = \beta s^2$ (Stokes' law)

The analytic solution still holds!

Total aerosol concentration:

$$C(x,y,t) = \int_0^\infty n(x,y,s,t)\mathrm{d}s$$

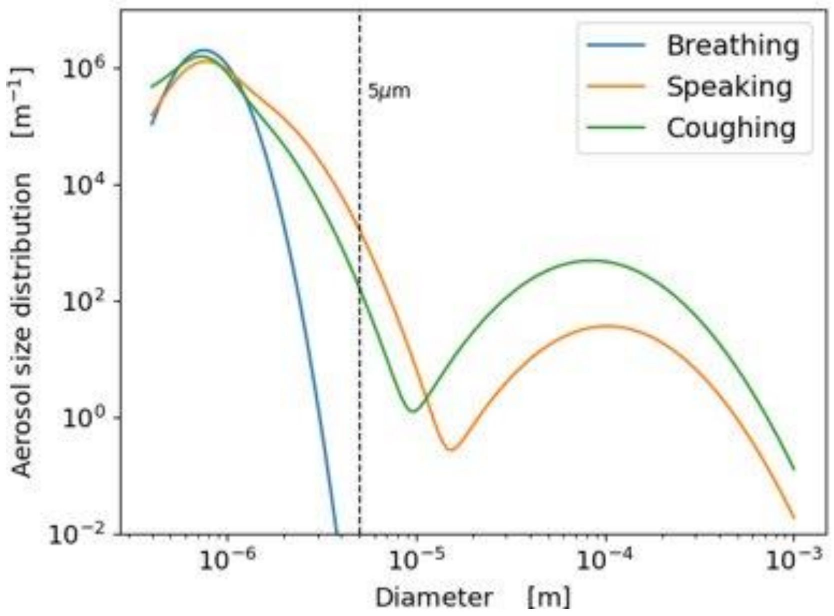
Concentration (BLO model and size-dependent settling)

$$C = \int_0^t \mathcal{I}(\tau) \frac{R}{4\pi K \tau} e^{-\lambda_0 \tau} E(x,y,\tau) \mathrm{d}\tau,$$

$$\mathcal{I}(\tau) = \int_0^\infty F(s) e^{-\beta s^2 \tau} \mathrm{d}s.$$

BLO model (Johnson *et al* 2011)

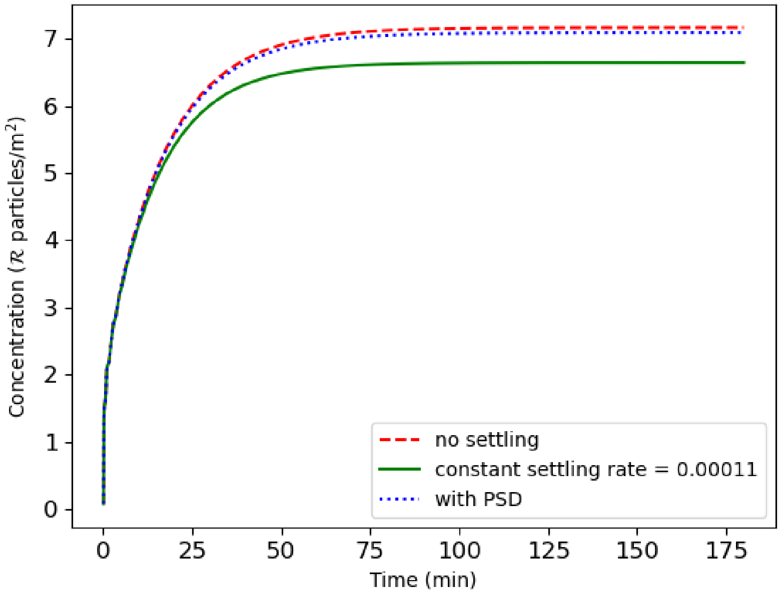
We take the distribution after the particles have evaporated to equilibrium size.



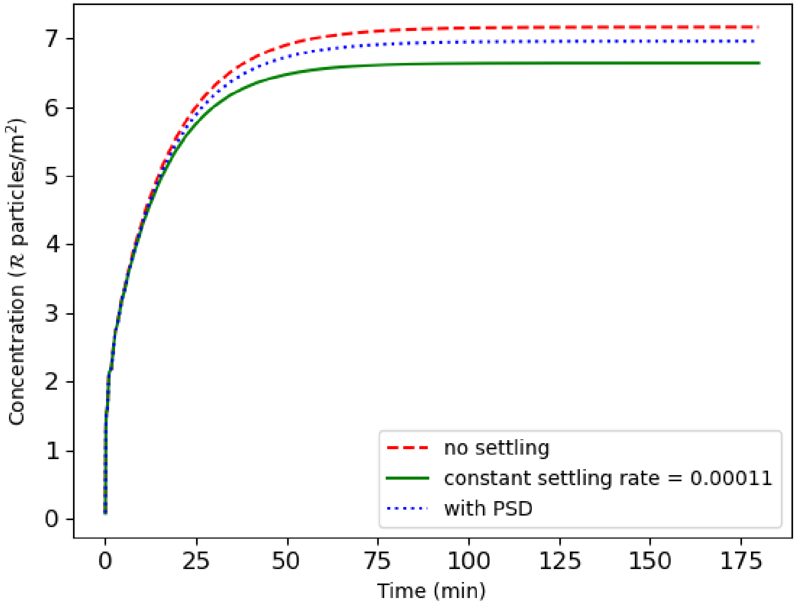
Concentration vs time 1m downstream

Point A
$(x, y) = (5,4)$

Breathing



Talking

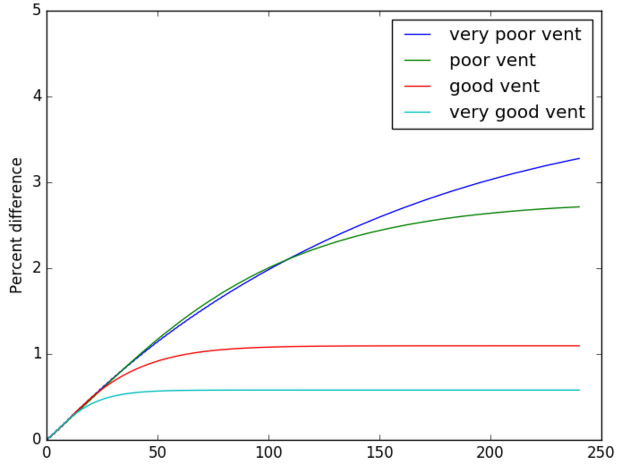


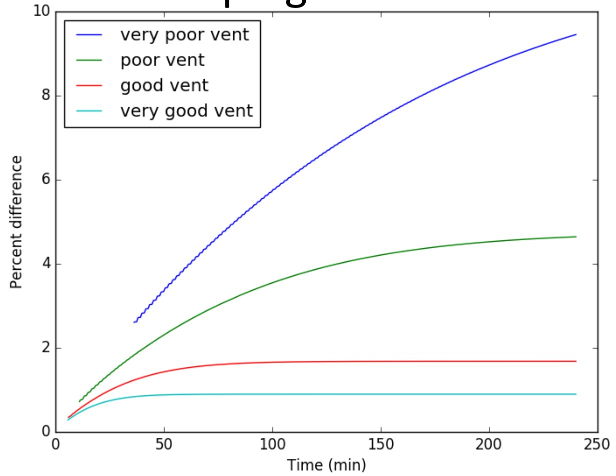
Percentage reduction in the **concentration** when size-depending settling is included

1m downstream of source

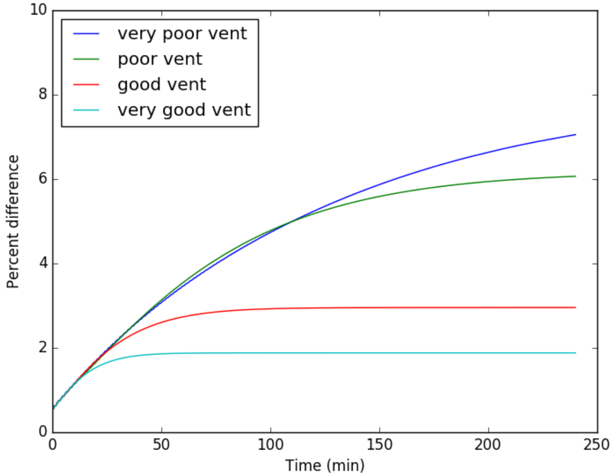
Top right corner

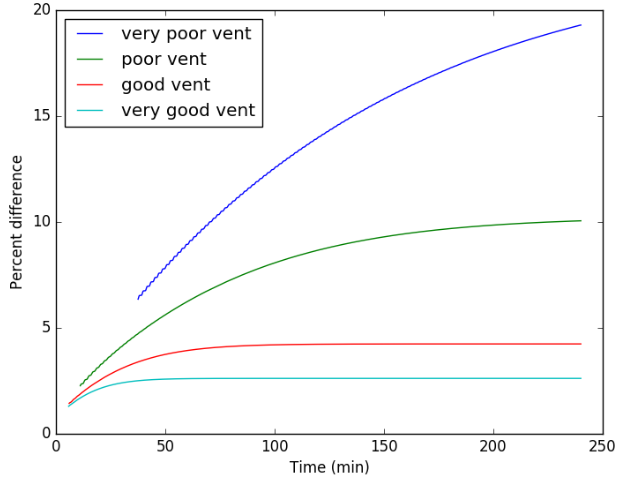
Breathing





Talking





$$\frac{|No\ settling - Settling|}{No\ settling} \times 100$$

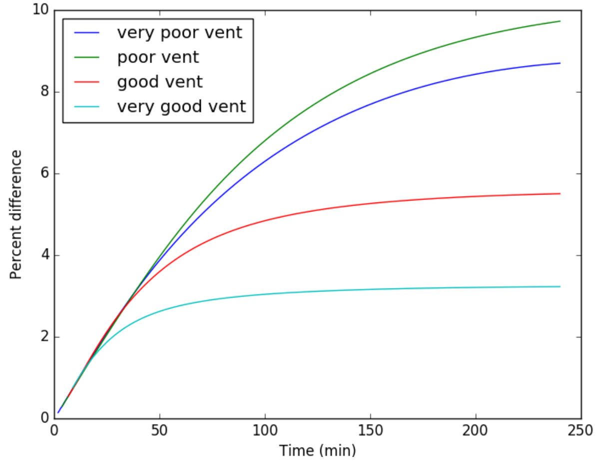
Effect of settling is larger for talking

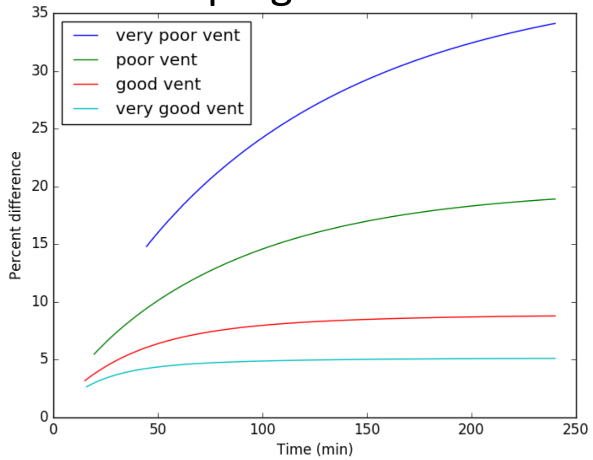
Percentage reduction in **infection risk** when settling is included

1m in front of source

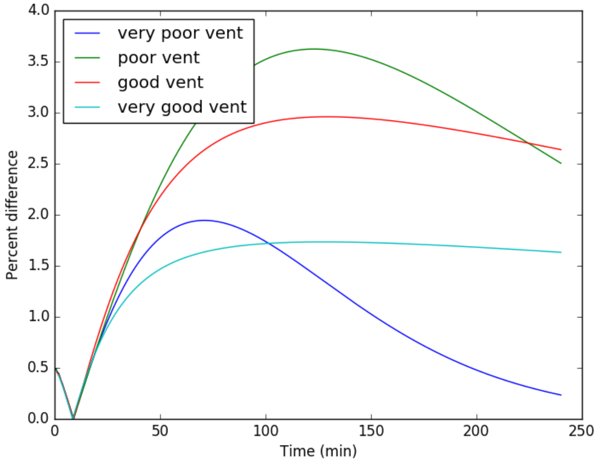
Top right corner

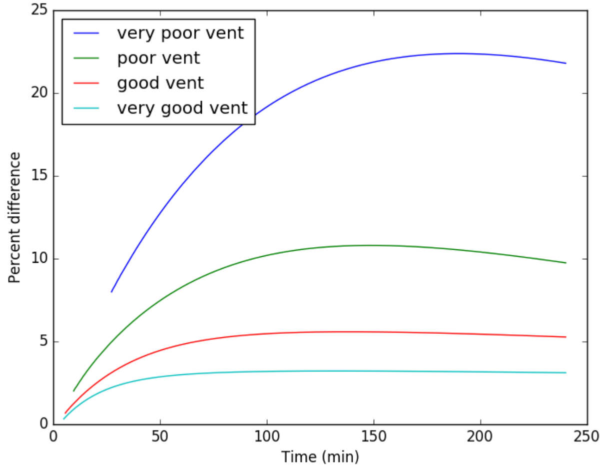
Breathing





Talking





$$\frac{|No\ settling - Settling|}{No\ settling} \times 100$$

Different trend - Infection risk **nonlinearly** dependent on concentration

Can air purifiers make rooms safer?

In theory, cleaning air with HEPA filters of UV radiation can kill COVID-19

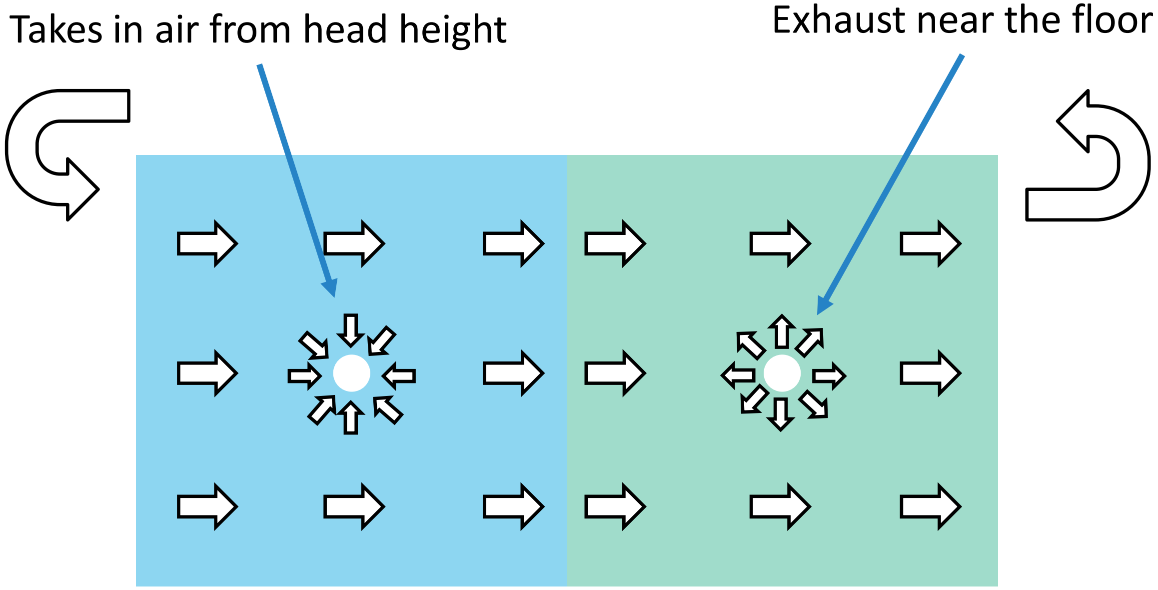
(Christopherson *et al* 2020; Zhao, An & Chen 2020)

The flow produced by the air purifiers complicates matters and can potentially spread the virus further

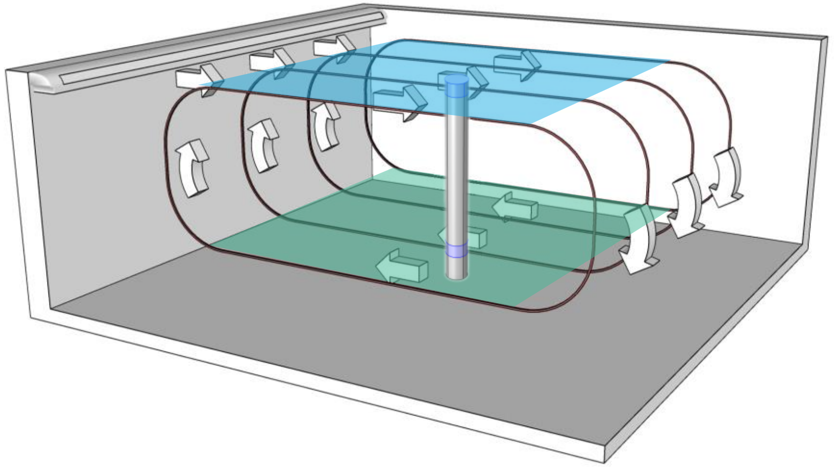
(Elias & Bar-Yam 2020; Ham 2020)

We are incorporating air purifiers into our modelling framework

Air purifiers in the quasi-3D model



Place the purifier in the centre of the room



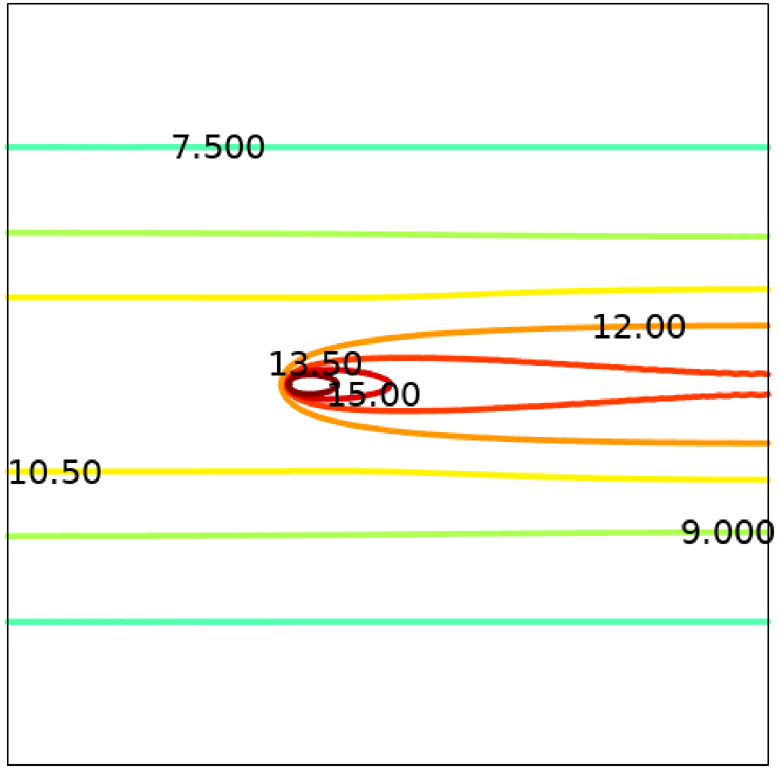
$$\frac{\partial C}{\partial t} + \boxed{\boldsymbol{v}(x,y)}\frac{\partial C}{\partial x} - K\nabla^2 C = R\delta(x-x_0)\delta(y-y_0) - (\lambda+\beta+\gamma)C$$

The airflow is now spatially dependent.
The analytic solution no longer applies.
The problem is solved numerically in COMSOL.

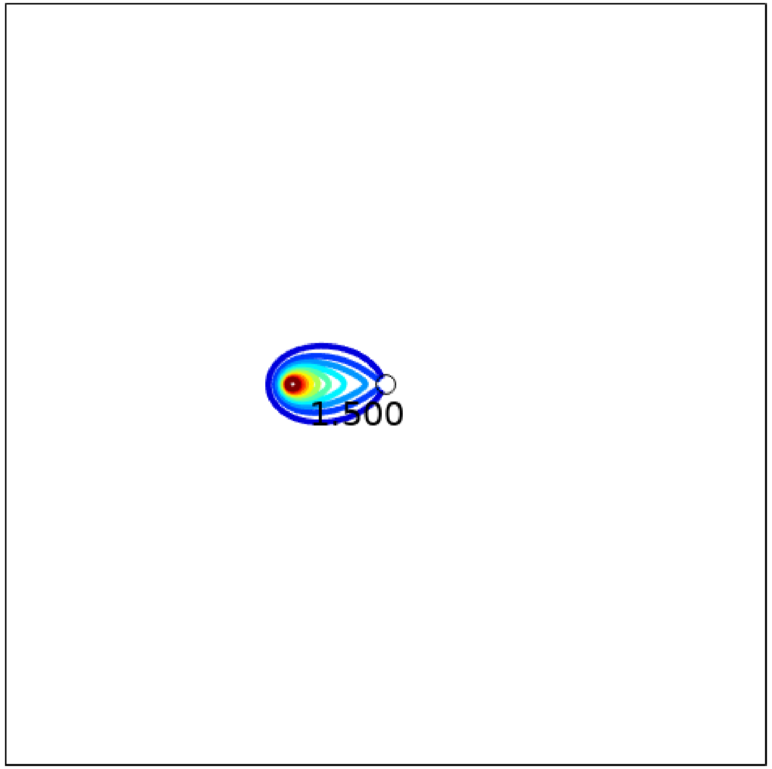
Use constant gravitational settling
(i.e. uniform particle size)

Concentration can be significantly reduced

No purifier



With air purifier

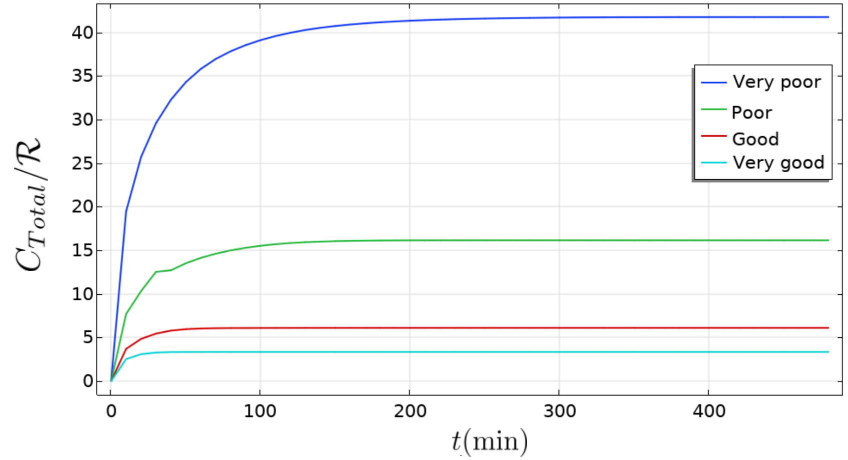


The better the ventilation, the less effective the purifier

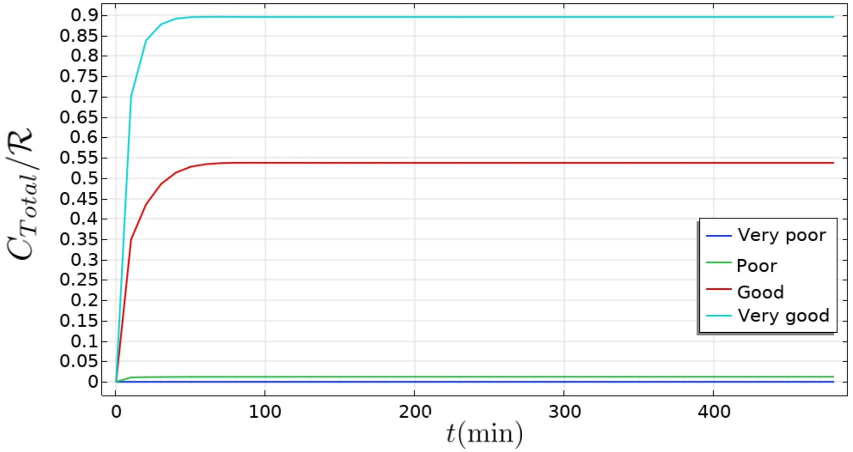
Say there are two people in the room. We do not know who is infected. (any activity type)

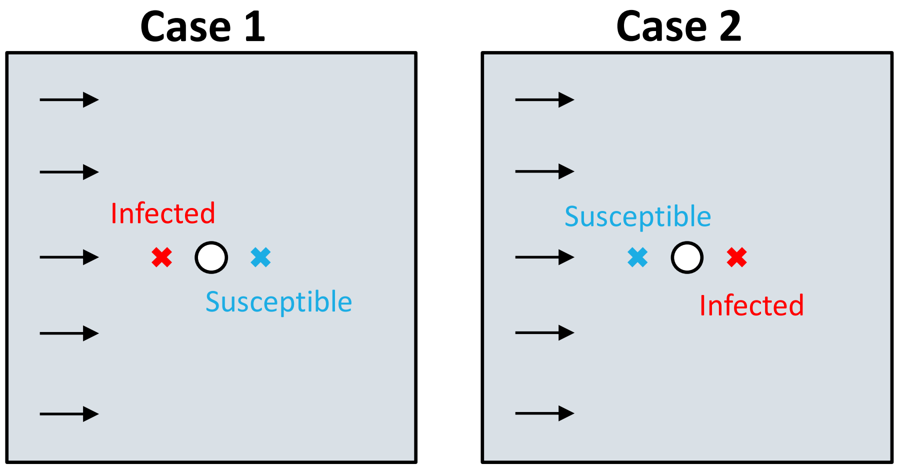
Case 1:

No purifier:



Purifier on:





Relative reduction in concentration		
	Case 1	Case 2
Very poor ventilation	100.00%	99.82%
Poor ventilation	99.92%	98.27%
Pre-pandemic recommended ventilation (Good)	91.18%	44.31%
Post-pandemic recommended ventilation (Very good)	73.24%	34.32%

Air purifiers: current and future work

Questions:

- What is the optimal purifier location when the location of the infected person is unknown? Are there locations/configurations in which air purifiers make matters worse?
- Should model different purifier designs – identify optimal design

Next steps:

- Compare the concentration and infection risk across the whole room.
- Change the locations of the infected person and the purifier.
- Explore different purifier designs, such as placing the inlet and outlet closer
- What is the worst-case scenario?

Couple to agent-based modelling

Mixing

Testing

Interventions

Isolations

Asymptomatic rate

Compliance

Collaboration with TE Woolley, T Dale, J Moore (Cardiff)

http://bit.ly/COVID_model

Summary

- Developed an extension of Wells-Riley type models that gives the spatiotemporal infection risk - can be applied to any location
- The model accounts for different ventilation levels, activity type (breathing/talking), masks, infectious dose, room size, source location, particle effects
- Analytical solution: **fast simulations**
- Incorporated realistic droplet size distributions (analytically) and quantified the reduction to concentration and infection risk
- Incorporating air purifiers (3D flattened to 2D & COMSOL)-flow is a very important - many questions.
- Collaborating with TE Woolley and team to incorporate framework into a holistic decision-making framework