Cutoff for random walk on the two communities random graph

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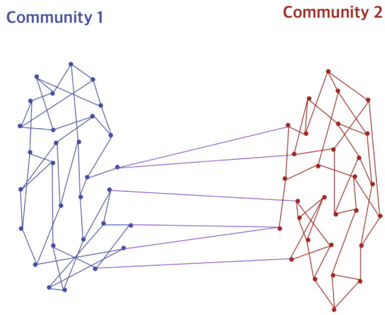
Graphs with community structure

- Community structure: there is a partition of vertices such that vertices in the same group are more likely to be connected than vertices in different groups
- Social networks: acquaintance networks, collaboration networks
- Technological networks: Internet and power grids
- Biological networks: neural networks, food webs, metabolic networks

Two communities graph



- n vertices split into two communities
- their degrees are given
- number of edges across two communities is a fixed function of n
- connect vertices uniformly at random

Mixing

- Simple random walk $(X_t)_{t\geq 1}$ has transition probabilities

$$P(u,v) = \frac{1}{\deg(u)} \mathbb{1}_{\{u\sim v\}}$$

- As $t \to \infty$, X_t converges to the invariant distribution:

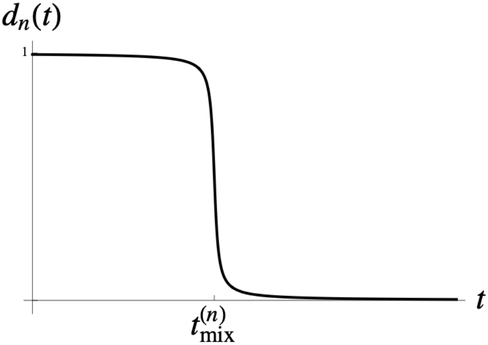
$$\pi(v) = \frac{\deg(v)}{\sum_{u\in V} \deg(u)}$$

- Mixing time is

$$t_{\text{mix}}(\varepsilon) = \inf\{t \geq 0 : d(t) \leq \varepsilon\}$$

where $d(t) = \max_{v\in V} \|P^t(v,\cdot) - \pi\|_{\text{TV}}$.

Cutoff



$t_{\text{mix}}^{(n)}(\varepsilon) \sim t_{\text{mix}}^{(n)}(1-\varepsilon)$ as $n \to \infty$

Main result

- Mixing time is of order $\log(n)$
- If the proportion of edges across the two communities $\gg \frac{1}{\log n}$ the random walk exhibits a cutoff with high probability
- If the proportion is $\ll \frac{1}{\log n}$ then with high probability there is no cutoff
- The change of the mixing behaviour around the threshold $\frac{1}{\log n}$ suggests the changes in the typical geometrical structure of the random graph