Structure Preserving and Structure Exploiting Methods in Simulation, Optimal Control and Motion Planning

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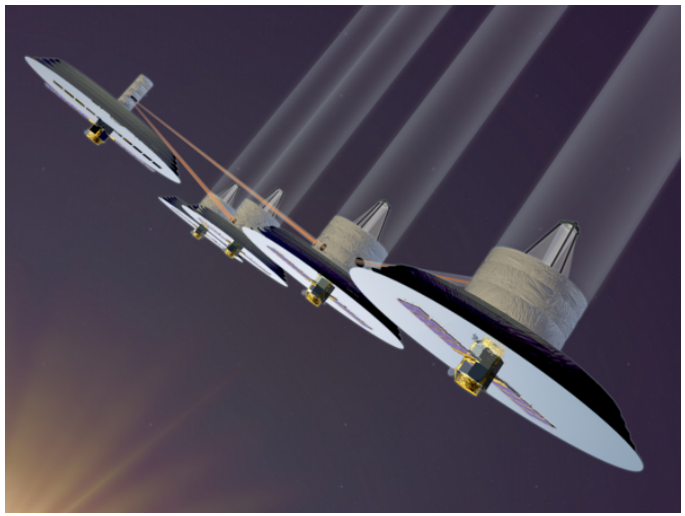
Optimal control

sports motion

space mission design

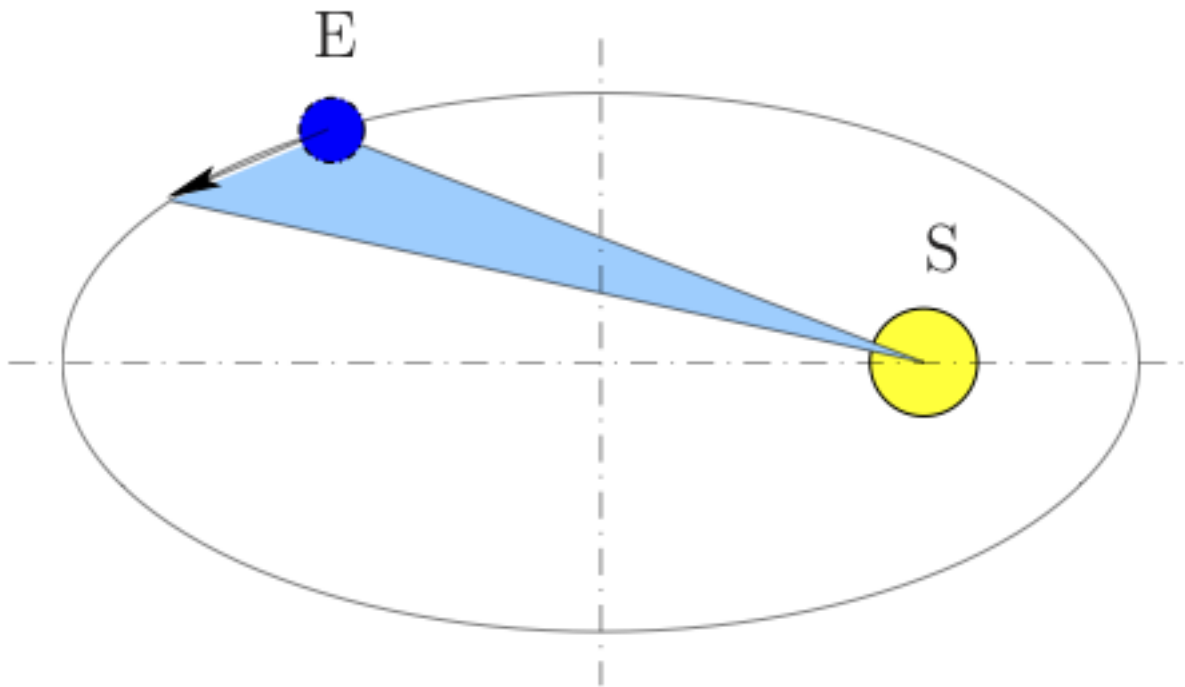


drive strategies

- need for numerical methods
- challenges: reliability, efficiency, robustness, ...
- our approach:
  account for geometric properties of the system

Structure preserving integration

- motivating example: Kepler problem (two-body-problem)



- goal: preserve geometric properties, e.g., energy, volume in phase space, momentum maps due to symmetries
- key ingredient: "symplectic integrator" for Hamilton / Lagrangian dynamics

Structure preserving integration

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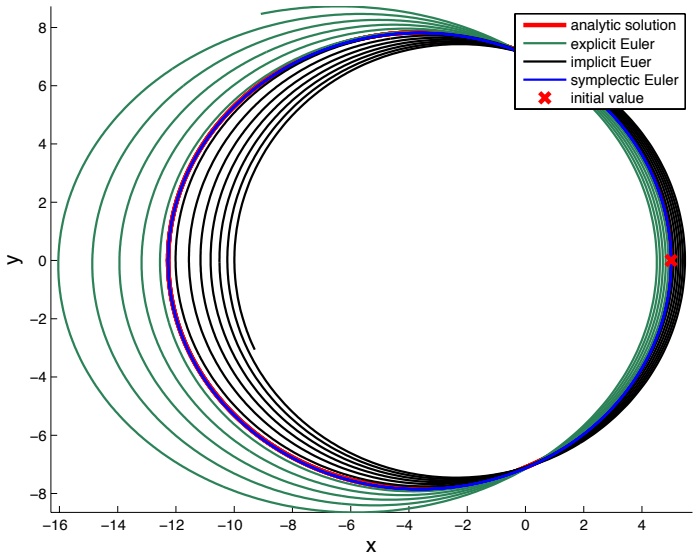
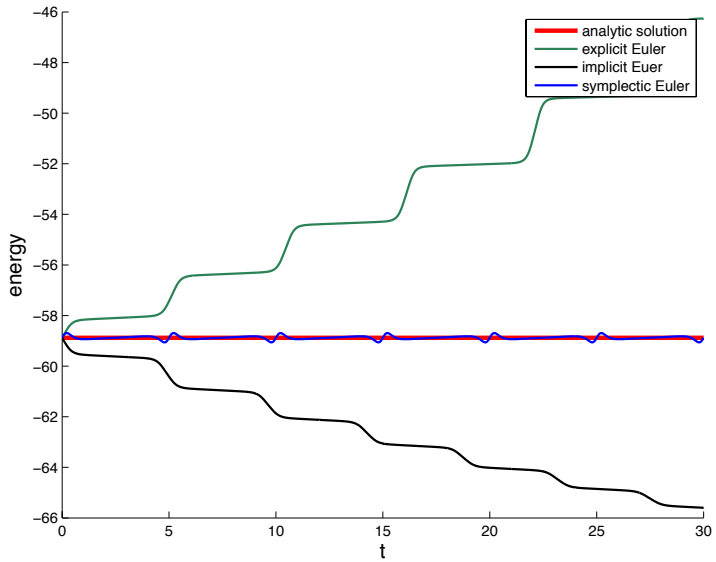
configuration space



energy over time



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Lagrangian dynamics: variational principle

- Lagrangian $L : TQ \to \mathbb{R}$
- stationary action

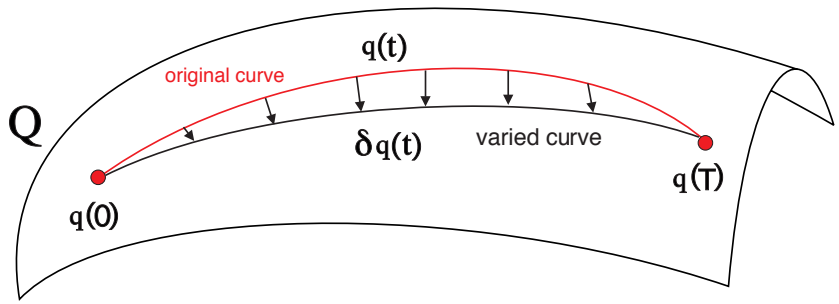
$$\delta \mathfrak{S} = \delta \int_0^T L(q, \dot{q}) \, \mathrm{d}t = 0$$

- Euler-Lagrange equations

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

- Lagrangian flow

$$F_L^t : TQ \to TQ$$



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- Lagrangian flow

$$F_L^t : TQ \to TQ$$

- discrete $L_d : Q \times Q \to \mathbb{R}$
- stationary discrete action

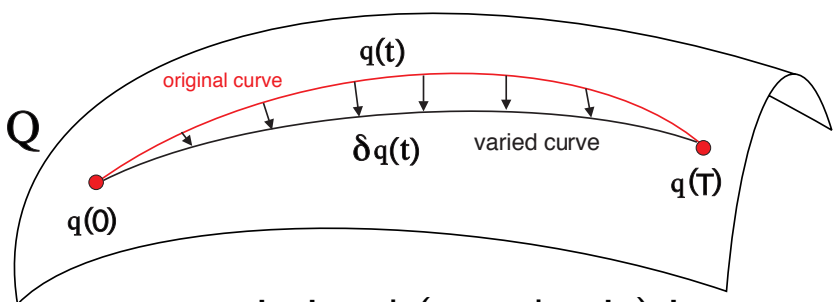
$$\delta\mathfrak{S}_d = \delta \sum_{k=0}^{N-1} L_d(q_k, q_{k+1}) = 0$$

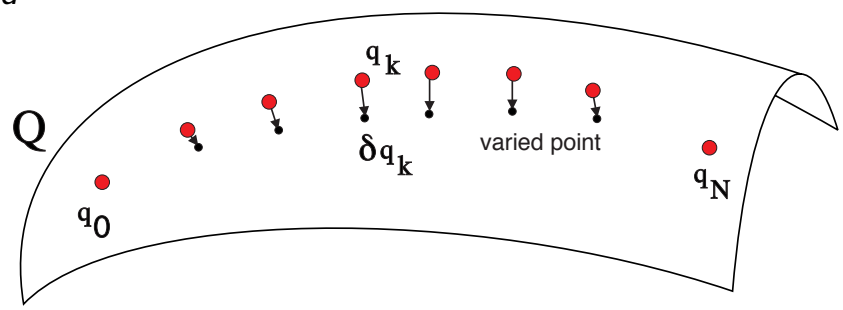
- discrete Euler-Lagrange eq.

$$D_1 L_d(q_k, q_{k+1}) + D_2 L_d(q_{k-1}, q_k) = 0$$

- discrete Lagrangian flow

$$F_{L_d}^h : Q \times Q \to Q \times Q$$



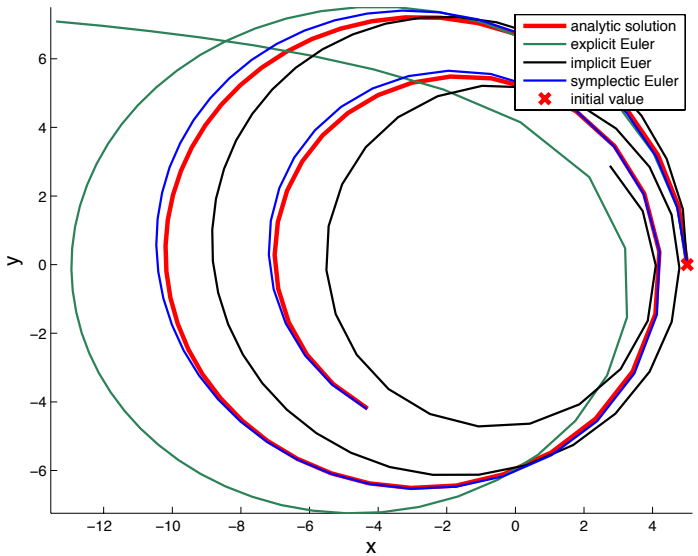


variational (symplectic) integrator [Suris 90], [Marsden, West 01]

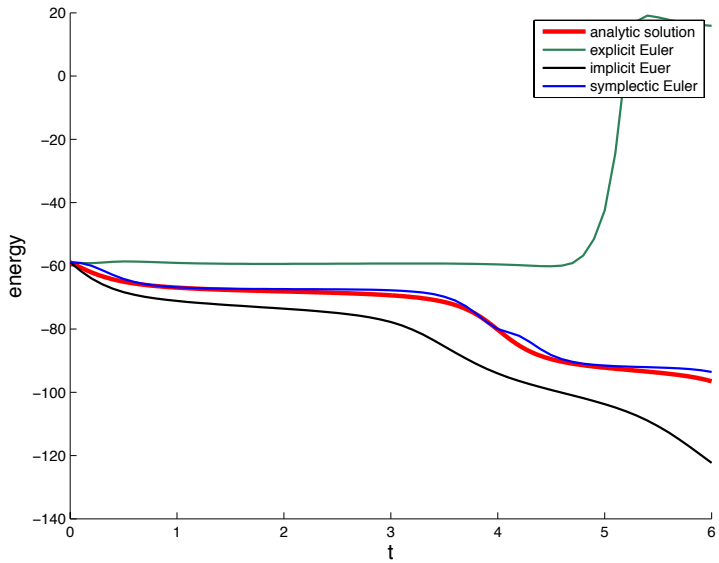
Variational integrators for dissipative systems

- motivation: Kepler problem with drag

configuration space



energy dissipation



- construction of "symplectic" integrators based on "modified" variational principles
- more accurate dissipation rates for long-time integrations

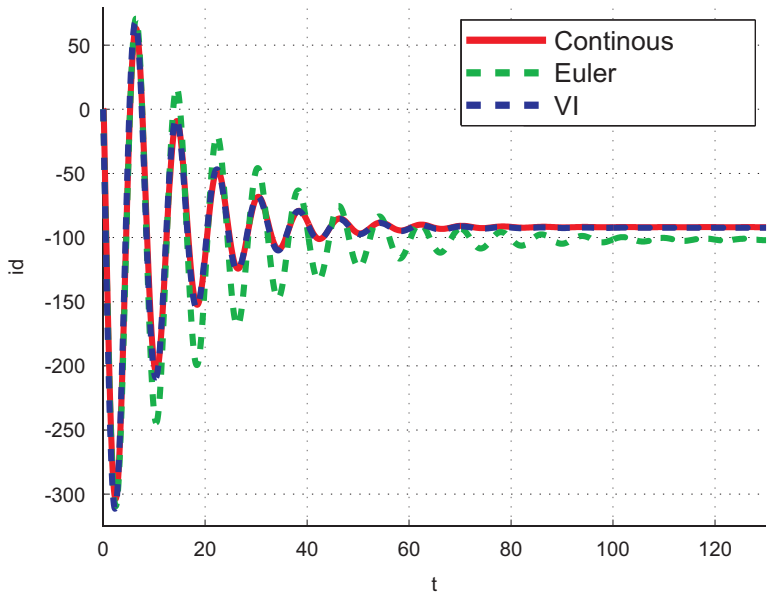
[Kane et al. 00], [McLachlan, Quispel 06], [Tsang et al. 15], [Modin, Söderlind 11], [Jiminez, O. 18], [Limebeer, O., Haddadfarshi 19]

Interior permanent-magnet synchronous motors

(VI for electric systems: : [O., Tao, Cheng et al. 2013])

- application in hybrid electric vehicles
- real-time implementation of simulation based motor model on hardware $\Rightarrow$ need of (fast) explicit integrators
- state-of-the-art: explicit Euler $\Rightarrow$ symplectic Euler?

voltage step response at $\omega = 785s^{-1}$



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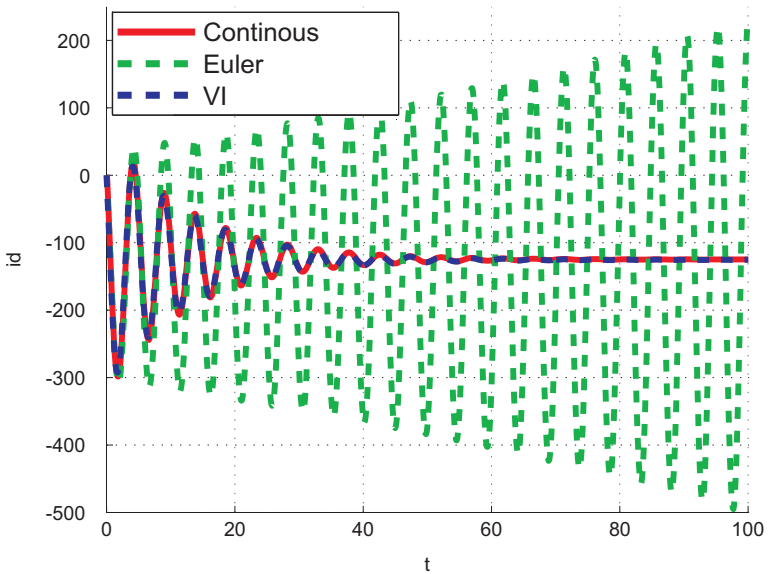

- high model accuracy in the transient and stationary state
- no instabilities at higher motor speeds [Specht, O. et al. 13]

Optimal control problem

$$\min_{x(\cdot),u(\cdot),(T)} J(x,u) = \int_0^T C(x(t),u(t))\,\mathrm{d}t + \Phi(x(T))$$

$$\begin{aligned}
&\text{s.t. system equation} && \dot{x}(t) &&= F(x(t),u(t)),\\
&\text{initial value} && x(0) &&= x_0,\\
&\text{path constraints} && 0 &&\leq h(x(t),u(t)),\\
&\text{final point constraint} && 0 &&= r(x(T))
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Discrete optimal control problem (DMOC) [O., Junge, Marsden 05]

$$\min_{q_d,u_d,(h)} J_d(q_d,u_d) = \sum_{k=0}^{N-1} C_d(q_k,q_{k+1},u_k) + \Phi_d(q_{N-1},q_N,u_{N-1})$$

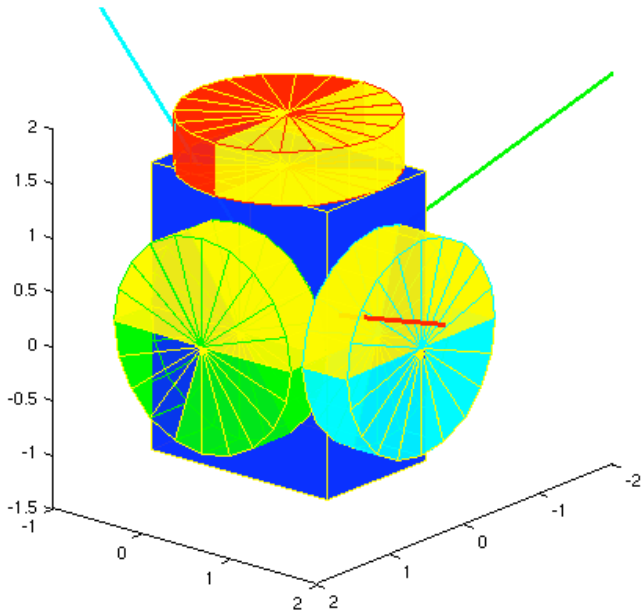
$$\text{s.t. DEL} \quad D_2L_d(q_{k-1},q_k) + D_1L_d(q_k,q_{k+1}) + f_{k-1}^+ + f_k^- = 0$$

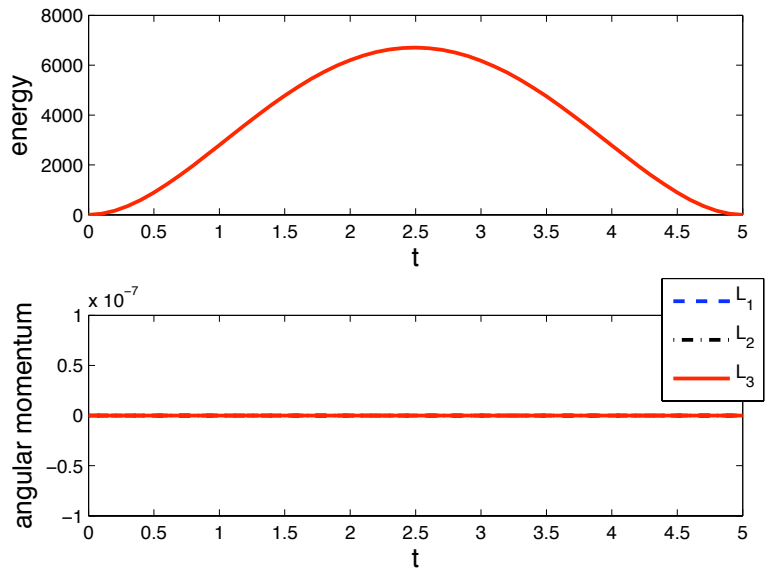
$$\begin{aligned}
&\text{initial condition} && s_d(q_0,q_1,u_0,q^0,\dot{q}^0) &&= 0\\
&\text{path constraints} && h_d(q_k,q_{k+1},u_k) &&\geq 0\\
&\text{final constraint} && r_d(q_{N-1},q_N,u_{N-1},q^T,\dot{q}^T) &&= 0
\end{aligned}$$

Structure preservation: satellite reorientation

[Leyendecker, O., Marsden, Ortiz 08]

- ⊕ symplectic-momentum consistent
  - system is invariant under rotations
  - exact preservation of total angular momentum





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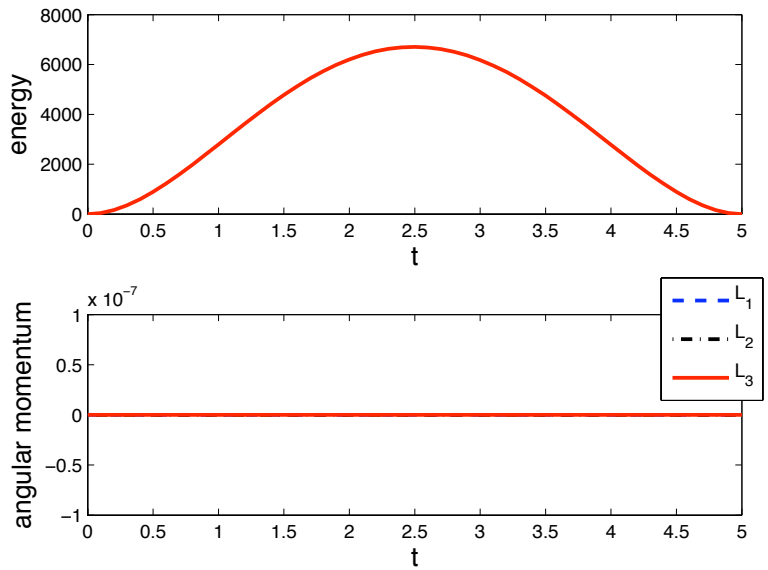

The adjoint system

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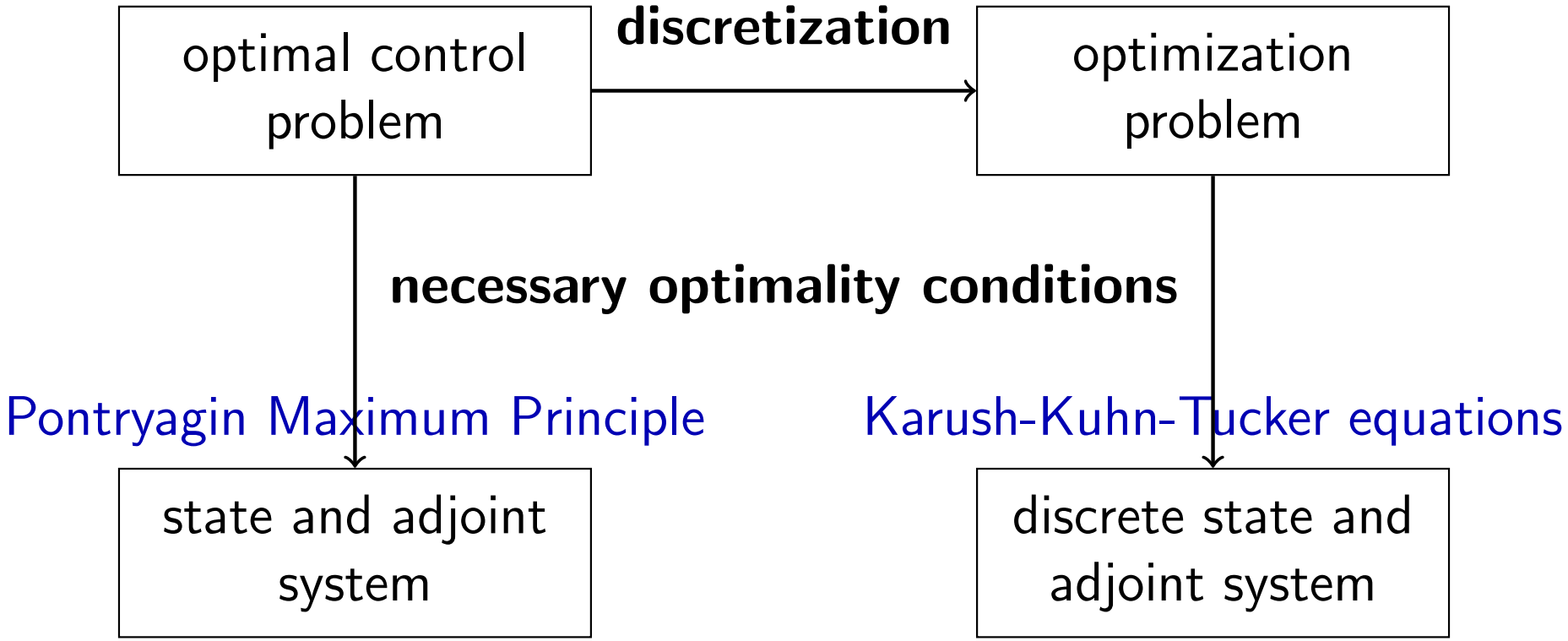

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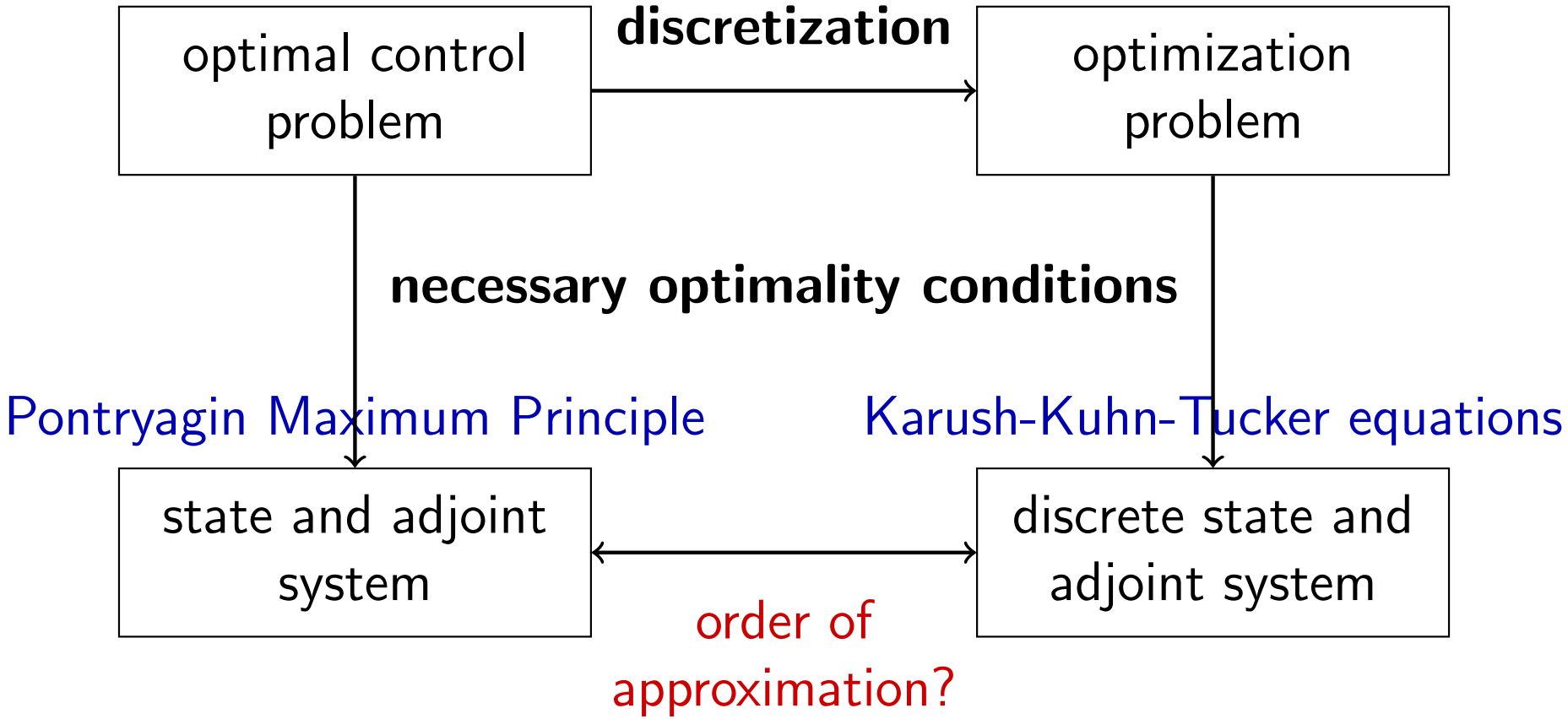

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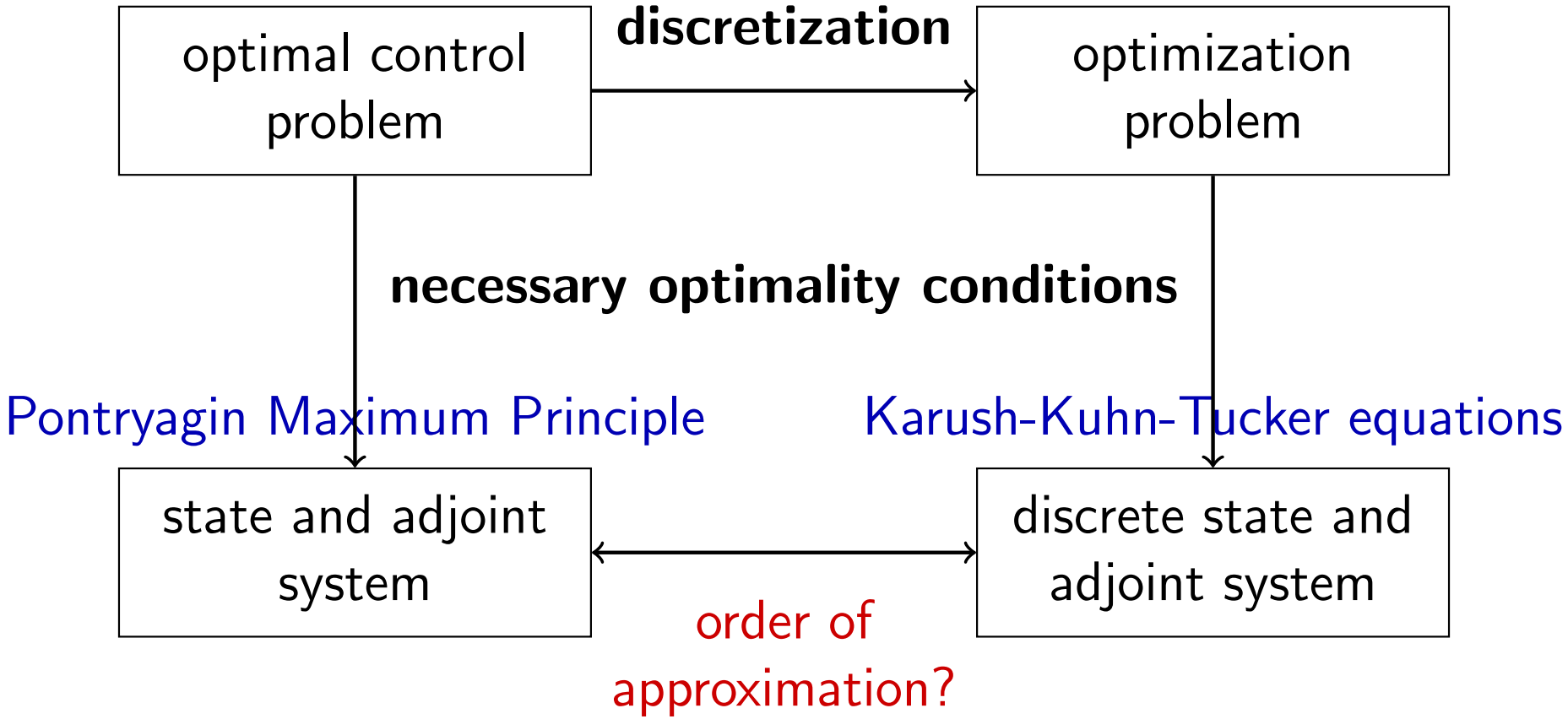

- RK methods: different schemes for state and adjoint system $\Rightarrow$ order loss [Hager 2000], [Bonnans at al. 2006]

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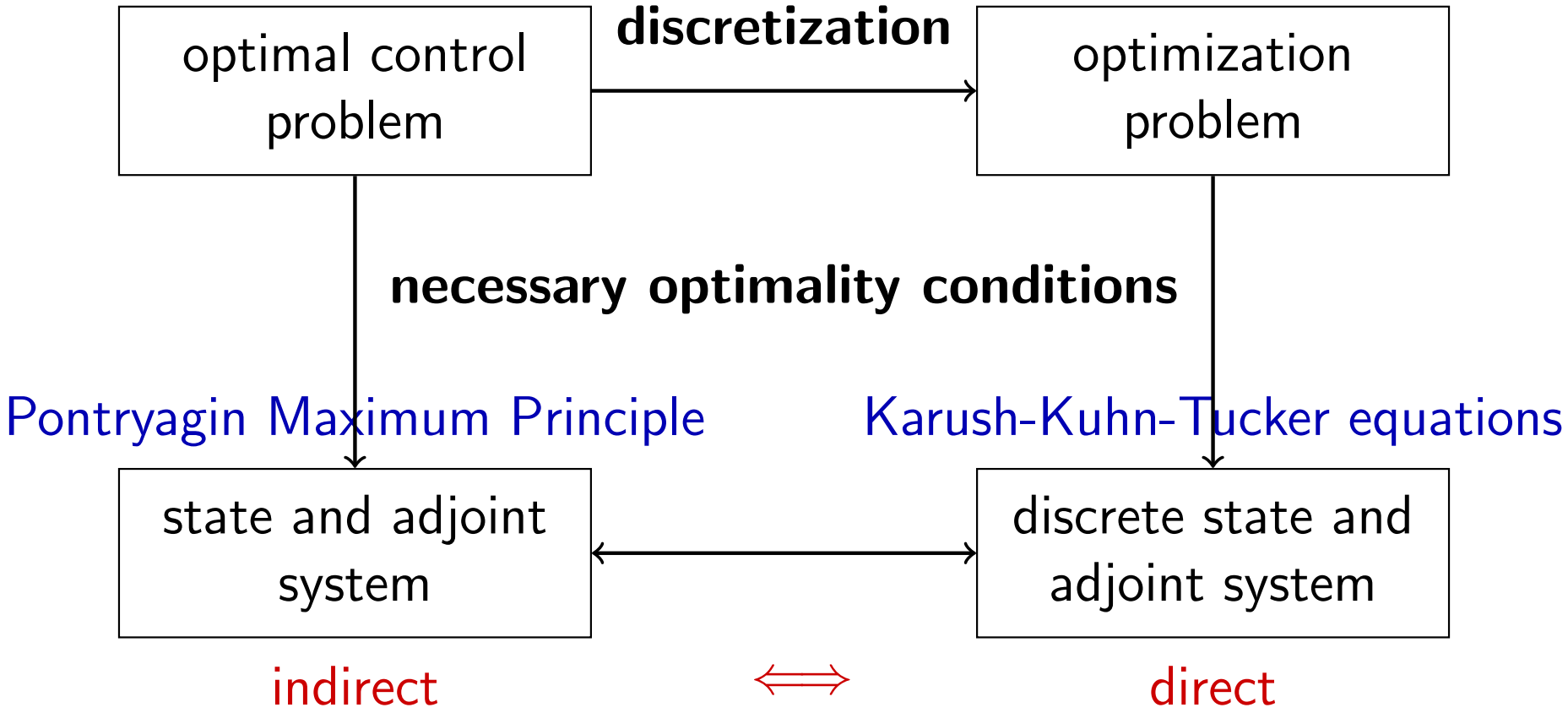
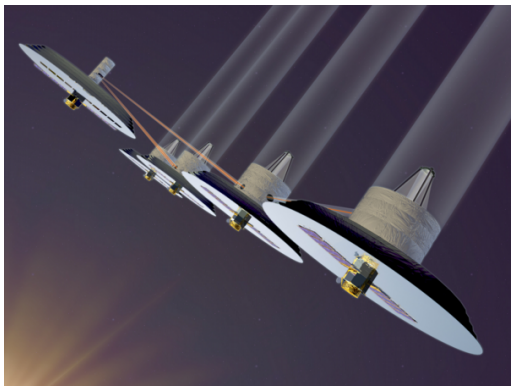


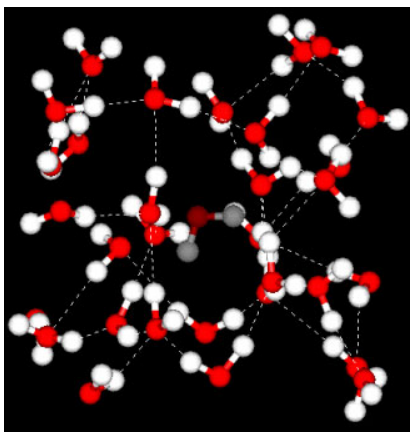
- RK methods: different schemes for state and adjoint system $\Rightarrow$ order loss [Hager 2000], [Bonnans at al. 2006]
- Variational integrator: state and adjoint scheme coincide $\Rightarrow$ same order [O. 2008], [Campos, O., Trélat 2015]

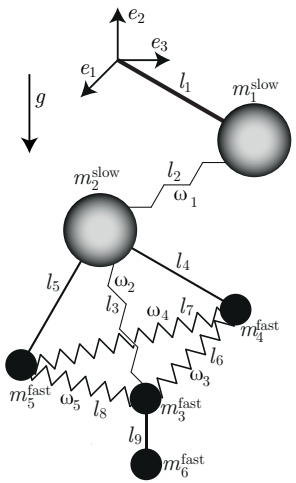
Extensions and applications

optimal control in biomechanics
(British Cycling Performance Support Team)

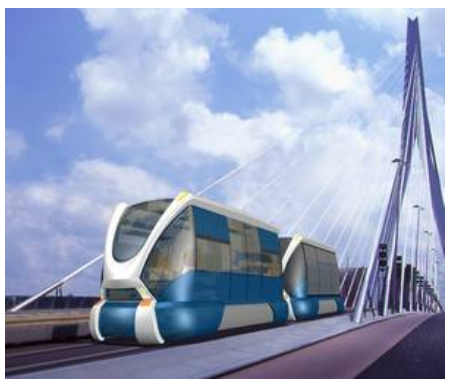


space mission design:
hierachical approaches for networks





optimal control of multirate systems



drive technology

The Technology-Network:
Intelligent Technical Systems OstWestfalenLippe . Germany
it's owl

Exploit geometric and dynamical structures

So far

- preserve system structure
- more accurate algorithms in particular regarding invariances and qualitative system properties
- drawbacks: only local solutions, not real-time capability

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Goal

- make optimal solutions "better" (local methods)
- make algorithms faster

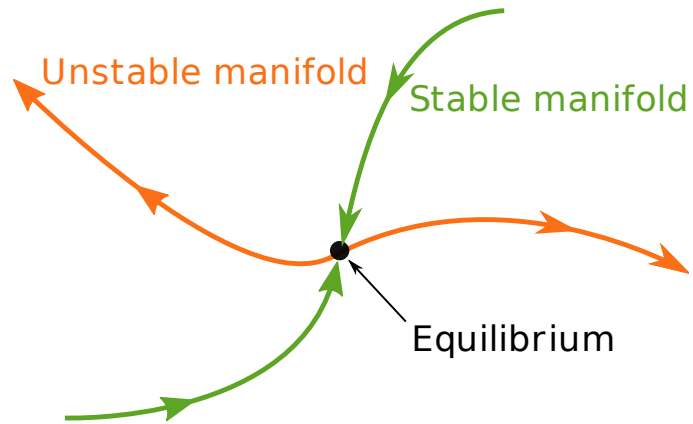
Exploit system's structure to

- find better initial guesses for local optimization
- exploit symmetries to search in space of all solutions more efficiently

Exploit structure in optimal control

[Moore, O., Marsden 12]

- use of dynamical system structures to provide better initial guesses for local optimization
- Example: "Shoot the Moon" [Koon et al. 00]
  - aim: fuel efficient transfer from Earth to Moon orbit
  - strategy: exploit natural dynamics of the system



- (un-)stable manifold for a fixed point x_0

$$W^s_{loc}(x_0) = \{x \in U | \phi^t(x) \to x_0 \text{ for } t \to \infty \text{ and } \phi^t(x) \in U \ \forall t \geq 0\}$$

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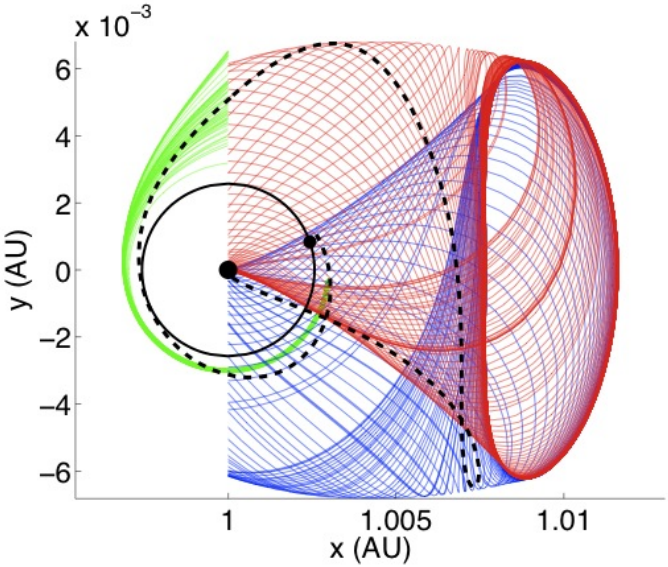

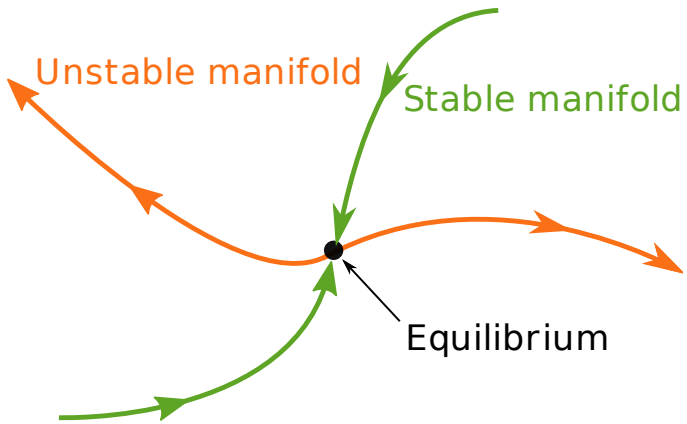

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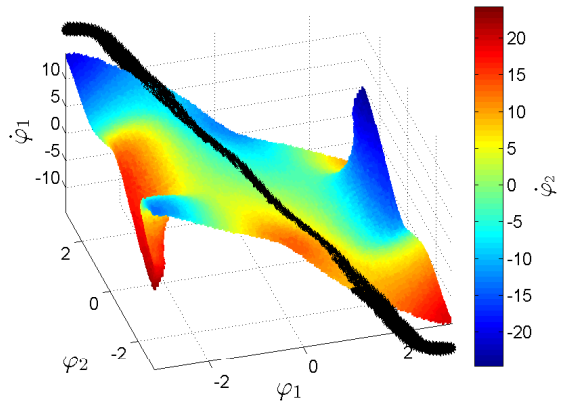
Exploit structure: optimal up-swing of a pendulum

double pendulum on the cart

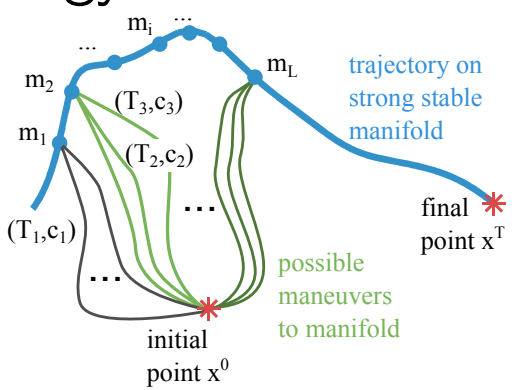
Control Engineering and Mechatronics, UPB

(strong) stable manifold

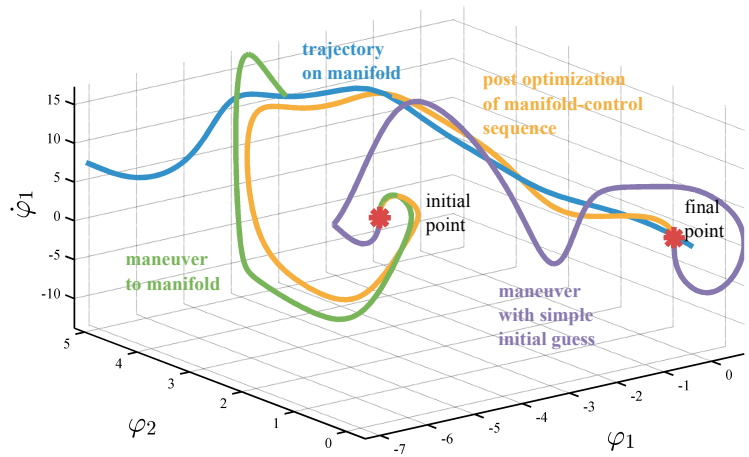


solution strategy

(T,c)	maneuver with time T and cost c
$m_1,\ldots,m_L$	final points of subproblems



comparison of solutions



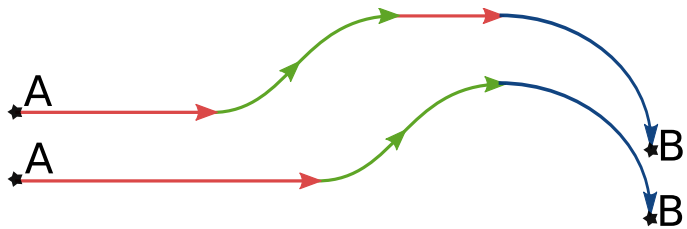
[Flasskamp, Timmermann, O., Trächtler 2014]
improvement by factor 3.7

Exploit structure: motion planning

[Flasskamp, O., Kobilarov 2012]

- use of symmetries in the dynamical system to search in space of all solutions more efficiently
- solution strategy: motion primitives

1. compute short and simple pieces of trajectories ...
   $\rightarrow$ motion primitives [Frazzoli, Daleh, Feron 2005]
2. ... which can be combined to sequences in various ways.



3. search in this library for the optimal sequence.

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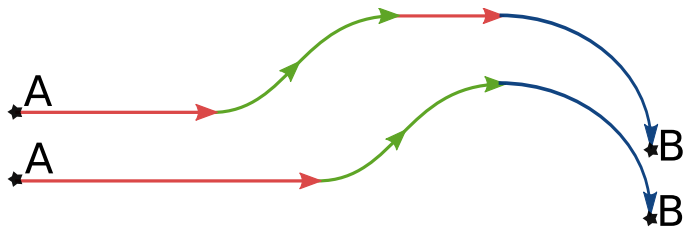

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candidates for motion primitives

- force free motion (invariant manifolds)
- connecting optimal control maneuvers
- in addition: exploit symmetries

Exploit structure: Lie group symmetry

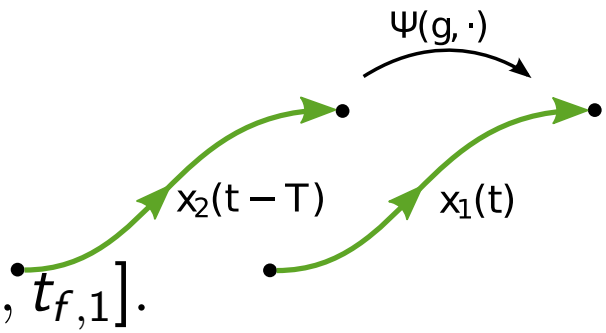
Consider Lie group action $\Psi : \mathcal{G} \times Q \to Q$ with $L \circ \Psi_g^{TQ} = L$.

Two trajectories $\pi_1 : [t_{i,1}, t_{f,1}] \mapsto (x_1(t), u_1(t))$ and $\pi_2 : t \in [t_{i,2}, t_{f,2}] \mapsto (x_2(t), u_2(t))$ are equivalent, if

(i) $t_{f,1} - t_{i,1} = t_{f,2} - t_{i,2}$ and

(ii) there exists $g \in \mathcal{G}, T \in \mathbb{R}$ s. t.

$$\begin{aligned} x_1(t) &= \Psi(g, x_2(t-T)) \\ u_1(t) &= u_2(t-T) \end{aligned} \qquad \forall t \in [t_{i,1}, t_{f,1}].$$



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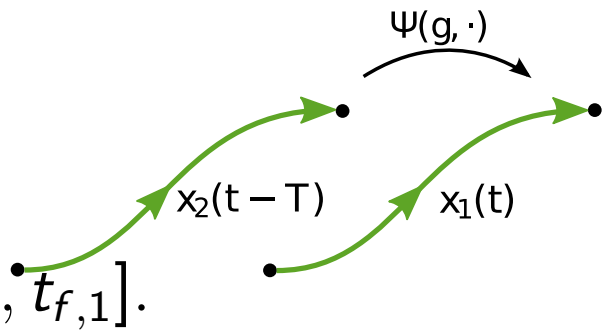

$\Rightarrow$ abstraction to an equivalence class: motion primitive

[Frazzoli, Daleh, Feron 2005]

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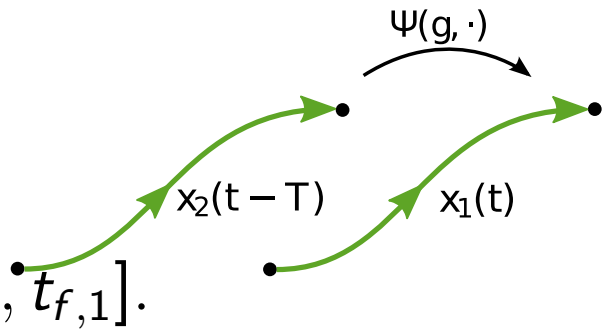
(ii) there exists $g \in \mathcal{G}, T \in \mathbb{R}$ s. t.

$$\begin{aligned} x_1(t) &= \Psi(g, x_2(t-T)) \\ u_1(t) &= u_2(t-T) \qquad \forall t \in [t_{i,1}, t_{f,1}]. \end{aligned}$$



$\Rightarrow$ abstraction to an equivalence class: motion primitive

[Frazzoli, Daleh, Feron 2005]

Trajectories $(x(t), u(t))$ which can be expressed as

$$\begin{aligned} x(t) &= \Psi(\exp(\xi\, t), x_0) \\ u(t) &= u_0 = const. \quad \forall t \in [0, T] \end{aligned}$$

with $\xi \in \mathfrak{g}$ (Lie algebra) and $\exp : \mathfrak{g} \to \mathcal{G}, \xi \mapsto \exp(\xi) \in \mathcal{G}$ are called trim primitives (trims)

Exploit structure: trims, manifolds, and maneuver

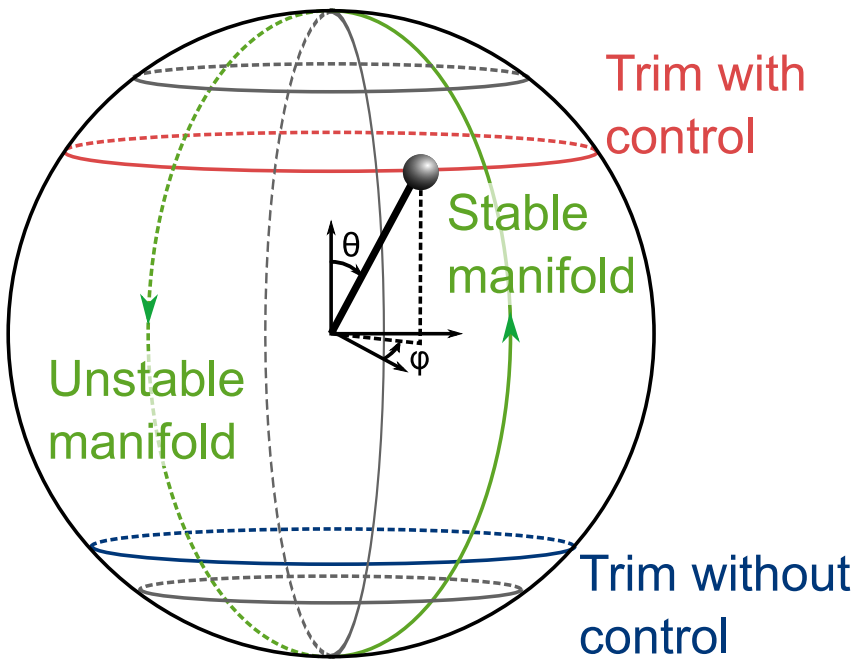
- example: 3D pendulum
- horizontal-vertical decomposition

$$\text{hor}_{(\varphi,\theta)} = \{(q,\dot{q}) \in TQ \,|\, J = 0\} = \{(\varphi,\theta,\dot{\varphi},\dot{\theta}) \,|\, \dot{\varphi} = 0\}$$

(vertical motion)

$$\text{ver}_{(\varphi,\theta)} = \{\xi_Q(q) \,|\, \xi \in \mathfrak{g}\} = \{(\varphi,\theta,\xi,0) \,|\, \xi \in \mathbb{R}\}$$

(horizontal motion)



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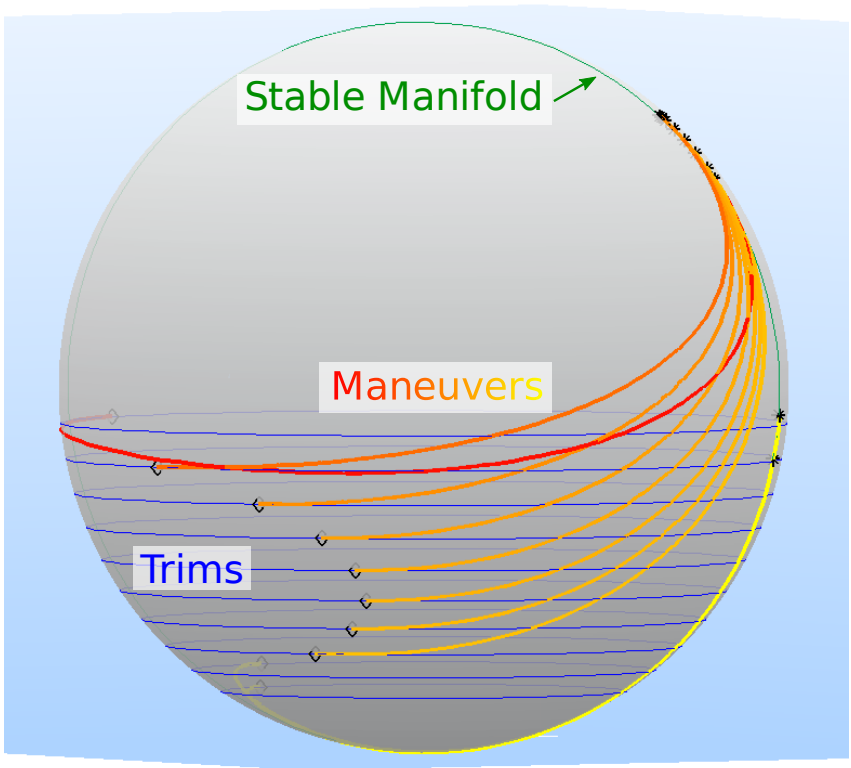
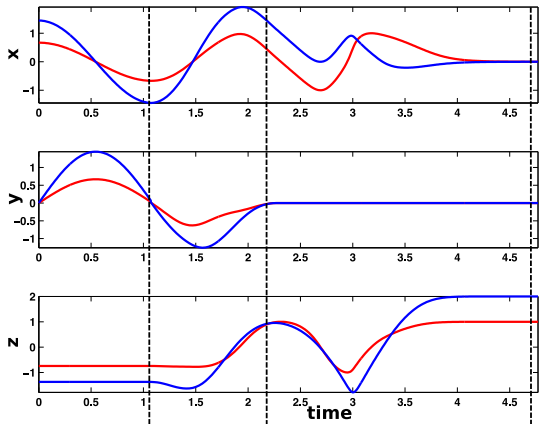


Exploit structure: motion planning

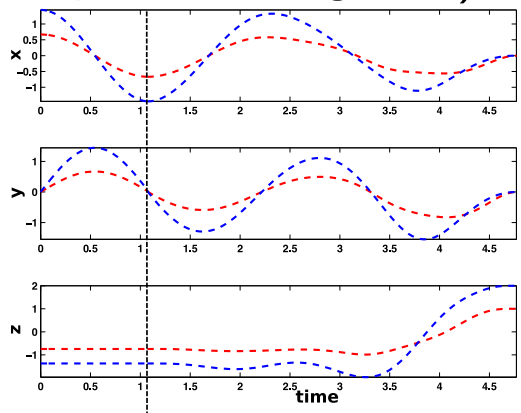
sequence of primitives:



trim | *man-euver* | *strongly stable manifold of upper fixed point* | *maneuver to fixed point*

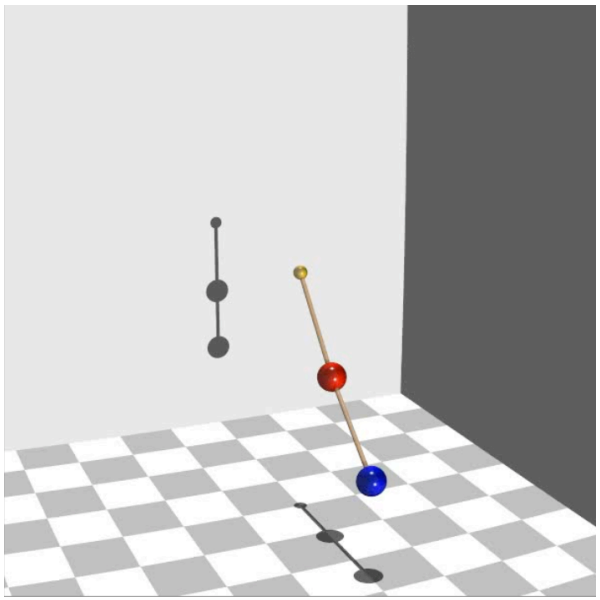
$\Rightarrow$ costs $= 288.81$

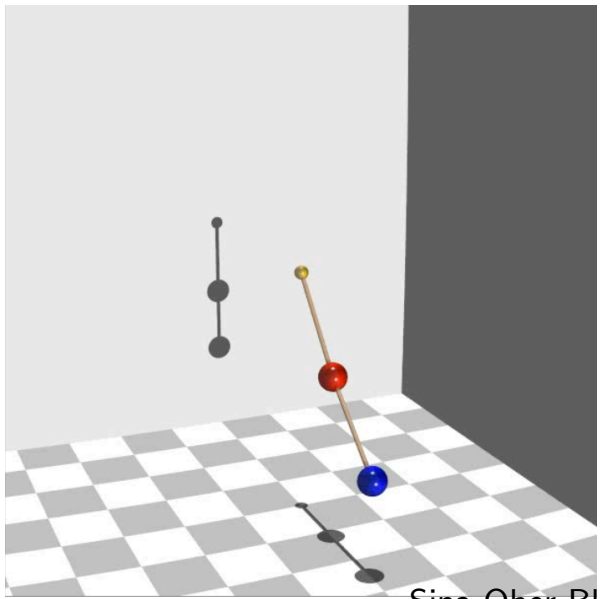
compared to a direct maneuver (with simple initial guess):



trim | *maneuver to fixed point*

$\Rightarrow$ costs $= 377.36$





Application: cruise control for an electric vehicle

- given route with velocity restrictions
- minimal arrival time and maximal state of charge
- consider decision preference
- real-time capable

with M. Dellnitz, S. Peitz, K. Schäfer and Hella KGaA Heuck & Co.

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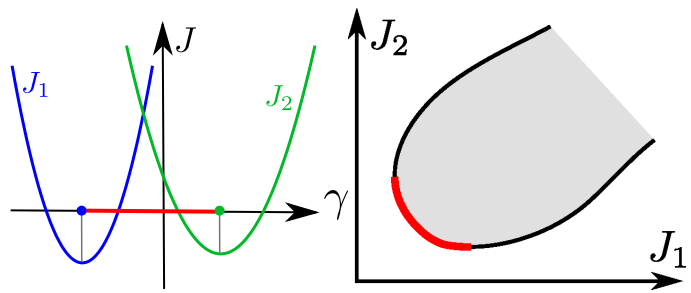
Problem and methods:

- Multiobjective optimal control problem



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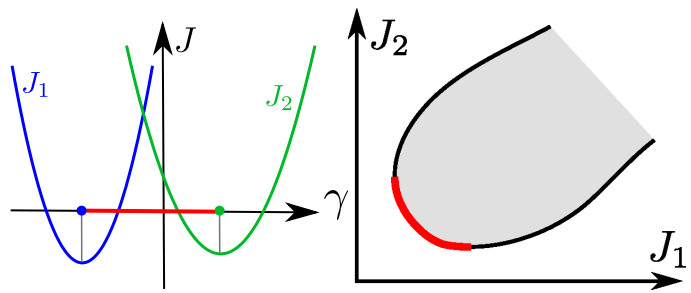
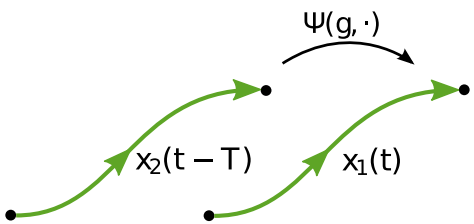
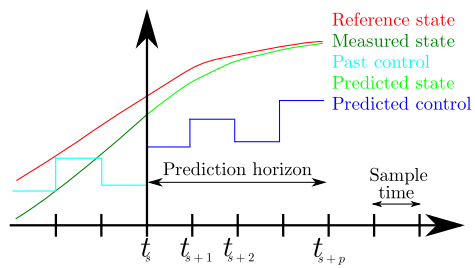


- Motion Primitives



- Model predictive control

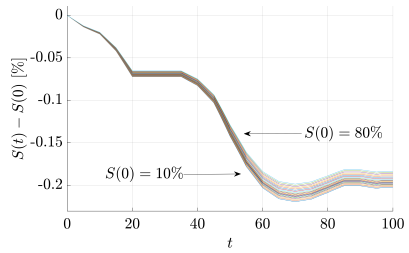


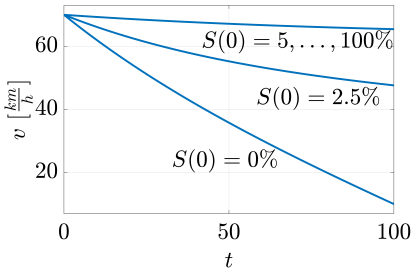
Application: cruise control for an electric vehicle

(i) Offline phase

1. identify symmetries

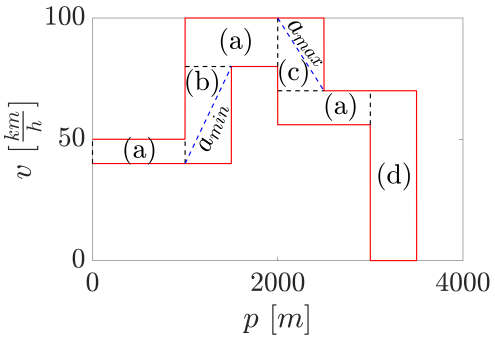
   (translational invariance w.r.t position and SOC)





2. compute Motion Primitives (Pareto optimal solutions) for different scenarios

   (identified by velocity constraints: constant / accelerate / decelerate / stop sign)



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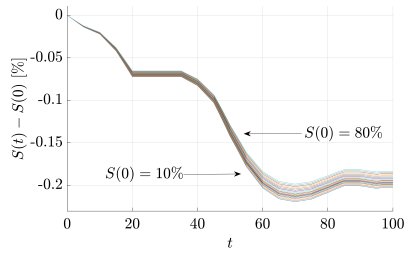

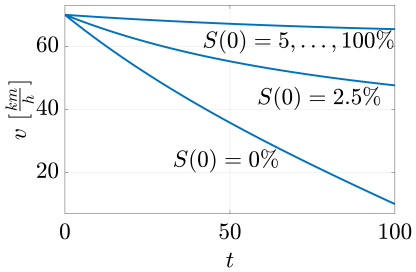

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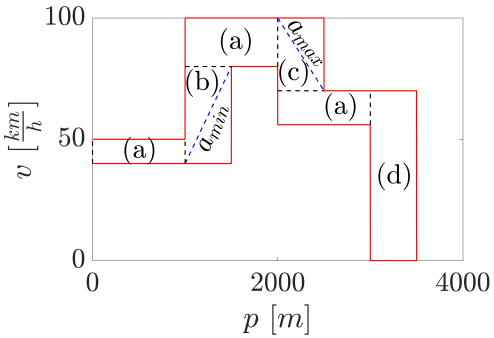
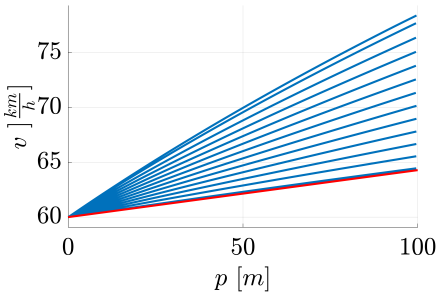
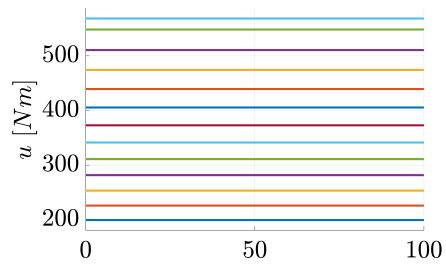


(ii) Online phase

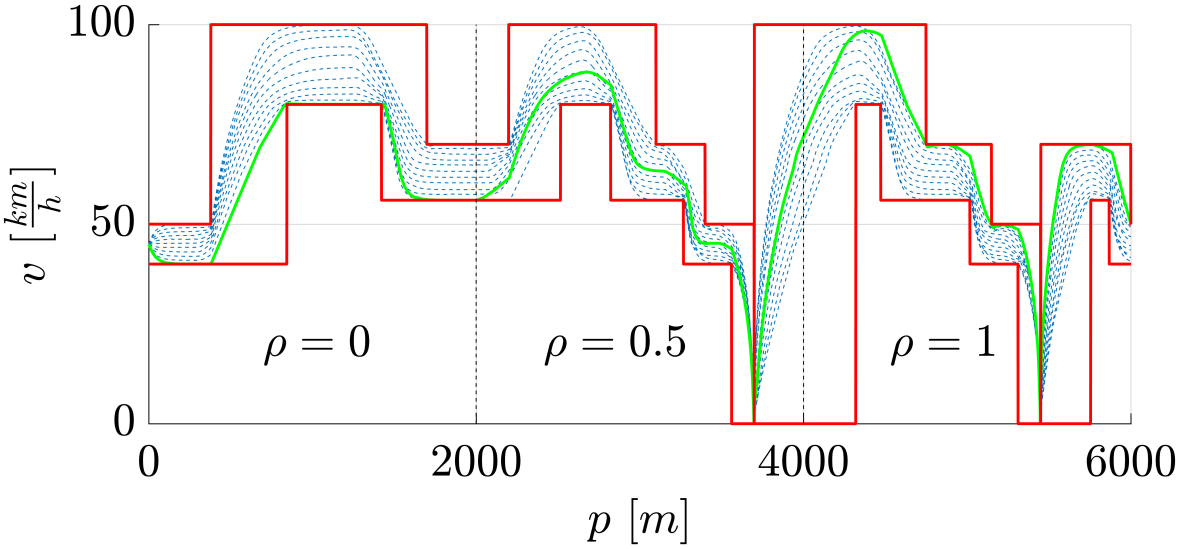
1. dependent on current velocity and constraints choose Pareto set
2. choose optimal Pareto point based on decision maker's preference $\rho \in [0, 1]$
3. apply solution in MPC framework; go back to 1.





Application: cruise control for an electric vehicle

MPC solutions according to different preferences



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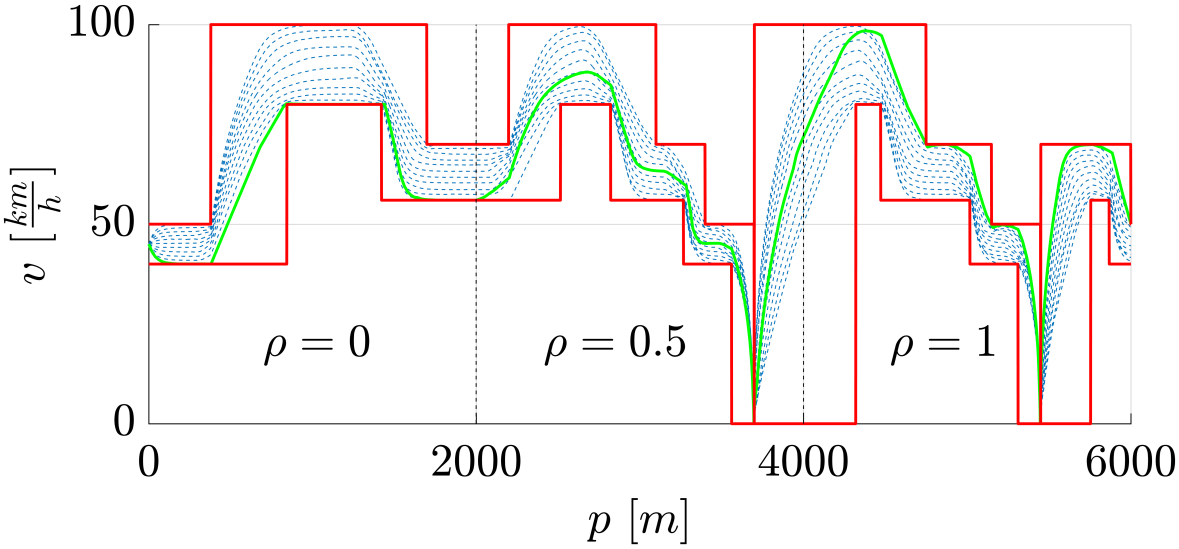
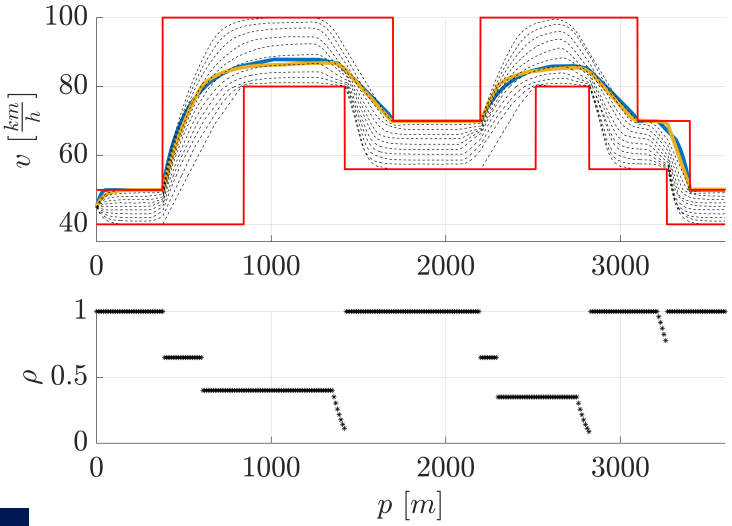
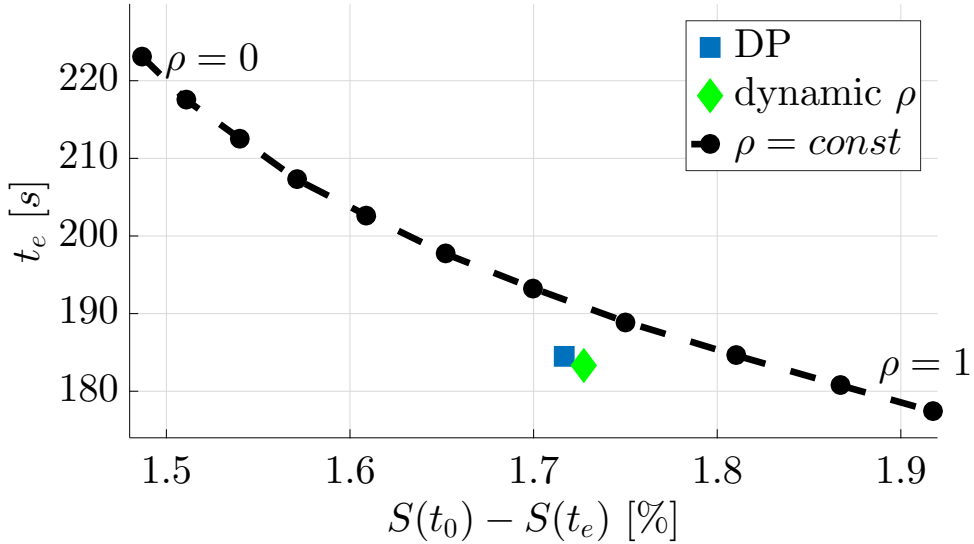


Solution with dynamic weighting ρ



Pareto front for entire track



Conclusion and future directions

Symplectic integration in optimal control: preserve structure

- symplectic-momentum preserving discretization
- good long-time behavior
- optimization and discretization commute: preserved accuracy in optimal control

Geometric concepts in optimal control: exploit structure

- invariant manifolds provide good inital guesses
- symmetry reduces computational complexity
- adapted for multiple objectives within MPC
- real-time capable in industrial applications: electric vehicle

Recent works

- motion primitives concepts with uncertain dynamics