Computation in Markets with Risk

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Build it and they will come

- Markets = equilibrium = complementarity ($\approx$ coupling)
- PATH solver for large scale mixed complementarity problems

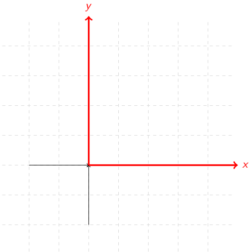
$$0 \leq F(x) \perp x \geq 0$$

- Nonsmooth Newton method, efficient linear algebra, available in modeling systems: GAMS, MPSGE, AMPL, AIMMS, Julia, Pyomo
- Used in models such as PIES, MERGE, VEMOD, MARKAL, TIMES, KAPSARC, ISEEM, MESSAGE, TEA, TIGER, Gemstone
- Models of Tobin, Nordhaus, Romer
- Frequently used in CGE analyses (GTAP data available), traffic, structural analysis
- Policy analyses such as Uruguay round, NAFTA, USMCA, Brexit

MIP formulations for Complementarity

Set $y_i = F_i(x)$, then (disjunction)

$$0 \le y_i, \quad y_i x_i = 0, \quad x_i \ge 0$$



If we know upper bounds on x_i and y_i we can introduce binary variable z_i and model as:

$$0 \le x_i \le M z_i, \quad 0 \le y_i \le M(1 - z_i)$$

or (without bounds)

$$(x_i, y_i) \in SOS1$$

(or use indicator variables to turn on "fixing" constraints).
Works if bounds are good and problem size is not too large. Issues with bounds on multipliers not being evident. c.f. Optimal topology problems.

Nonsmooth alternatives and approximations (NLPEC)

Alternative: generate generalized derivatives of nonsmooth reformulations

- PATH uses (PC^1) normal map
- Min-map $\min(x_i, y_i) = 0$
- Fischer-Burmeister $\Phi(x) = 0$

$$\phi(a, b) = 0 \iff 0 \leq a \perp b \geq 0$$

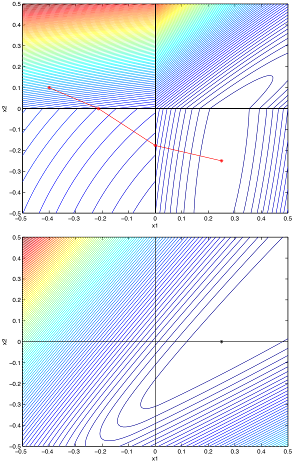
$$\Phi_i(x) \equiv \sqrt{x_i^2 + y_i^2} - x_i - y_i$$

- Smoothing (drive parameter μ to 0)

$$0 = \phi_\mu(x_i, y_i), \quad i = 1, 2, \ldots, n$$

$$\phi_\mu(a, b) := \sqrt{a^2 + b^2 + \mu} - a - b$$

- Relaxation $y_i x_i \leq \mu$
- Penalization $+\lambda \sum_{i=1}^{n} y_i x_i$



Extended Mathematical Programming (EMP)

- Optimization models improve understanding of underlying systems and facilitate operational/strategic improvements under resource constraints
- Problem format is old/traditional

$$\min_x f(x) \text{ s.t. } g(x) \leq 0, h(x) = 0$$

- Extended Mathematical Programs allow annotations of constraint functions to augment this format.
- Give several examples of this: complementarity problems, bilevel programming, multi-agent competitive models, risk

The PIES Model (Hogan) - Optimal Power Flow (OPF)

$$\min_x c(x) \qquad \text{cost}$$

$$\text{s.t. } Ax \geq q \qquad \text{balance}$$

$$Bx = b, x \geq 0 \qquad \text{technical constr}$$

- $q = d(\pi)$: issue is that π is the multiplier on the "balance" constraint
- Such multipliers (LMP's) are critical to operation of market
- Can try to solve the problem iteratively (shooting method):

$$\pi^{new} \in \text{multiplier}(OPF(d(\pi)))$$

Alternative: Form KKT of QP, exposing π to modeler

$$L(x, \mu, \lambda) = c(x) + \mu^T(d(\pi) - Ax) + \lambda^T(b - Bx)$$

$$\begin{aligned}
0 \leq & -\nabla_\mu L = Ax - d(\pi) & \perp \quad & \mu \geq 0 \\
0 = & -\nabla_\lambda L = Bx - b & \perp \quad & \lambda \\
0 \leq & \quad \nabla_x L = \nabla c(x) - A^T\mu - B^T\lambda & \perp \quad & x \geq 0
\end{aligned}$$

- EMP: Take original QP model, and add single annotation:
- empinfo: dualvar π balance
- Fixed point: replaces $\mu \equiv \pi$
- LCP/MCP is then solvable using PATH

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$$\begin{aligned} 0 &\leq Ax - d(\pi) & \perp \quad & \pi \geq 0 \\ 0 &= Bx - b & \perp \quad & \lambda \\ 0 &\leq \nabla c(x) - A^T\pi - B^T\lambda & \perp \quad & x \geq 0 \end{aligned}$$

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Supply function equilibria

$$\begin{array}{lll} \text{OPF}(\alpha): & \min_y & \text{energy dispatch cost } (y,\alpha) \\ & \text{s.t.} & \text{conservation of power flow at nodes} \\ & & \text{Kirchoff's voltage law, and simple bound constraints} \end{array}$$

α are (given) price bids, parametric optimization

Supply function equilibria

OPF(α):	$\min_y$	energy dispatch cost (y, α)
	s.t.	conservation of power flow at nodes
		Kirchoff's voltage law, and simple bound constraints

α are (given) price bids, parametric optimization

Leader($\bar{\alpha}_{-i}$):	$\max_{\alpha_i, y, \lambda}$	firm i's profit (α_i, y, λ)
	s.t.	$0 \leq \alpha_i \leq \hat{\alpha}_i$
		y solves OPF$(\alpha_i, \bar{\alpha}_{-i})$

Note that objective involves multiplier from OPF problem

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		y, λ solves KKT(OPF$(\alpha_i, \bar{\alpha}_{-i})$)

This is an example of an MPCC since KKT form complementarity constraints

Hierarchical models

- Bilevel programs:

$$\begin{array}{ll} \min\limits_{x^*,y^*} & f(x^*,y^*) \\ \text{s.t.} & g(x^*,y^*) \le 0, \\ & y^* \text{ solves } \min\limits_{y} v(x^*,y) \text{ s.t. } h(x^*,y) \le 0 \end{array}$$

- model bilev /deff,defg,defv,defh/;
  empinfo: bilevel f x deff defg min v y defv defh
- EMP tool automatically creates the MPCC

$$\begin{array}{lll} \min\limits_{x^*,y^*,\lambda} & f(x^*,y^*) & \\ \text{s.t.} & g(x^*,y^*) \le 0, & \\ & 0 \le \nabla v(x^*,y^*) + \lambda^T \nabla h(x^*,y^*) & \perp y^* \ge 0 \\ & 0 \le -h(x^*,y^*) & \perp \lambda \ge 0 \end{array}$$

Multi-player EPEC and security constraints

- $(\bar{\alpha}_1, \bar{\alpha}_2, \ldots, \bar{\alpha}_m)$ is an equilibrium if

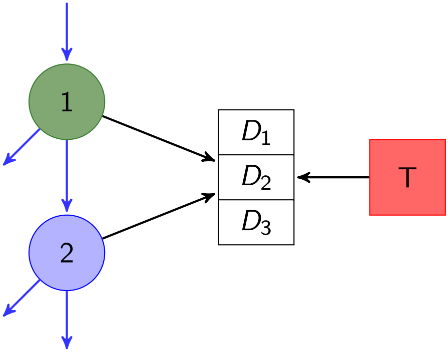
$$\bar{\alpha}_i \text{ solves Leader}(\bar{\alpha}_{-i}), \quad \forall i$$

- (Nonlinear) Nash Equilibrium where each player solves an MPCC
- MPCC is hard (lacks a constraint qualification)
- Nash Equilibrium is PPAD-complete (Chen et al, Papadimitriou et al)
- In practice, also require contingency (scenario) constraints imposed in the OPF problem

- Solution via "diagonalization"
- Model detail, data, forecast and aggregation level critical

Cascading hydro-thermal system: XMGD

- Two hydros on same river: '1' is above '2': spill or release with generation
- Thermal generator 'T' and consumer (risk neutral)



- Competing firms (collections of consumers, or generators in energy market)
- Each firm minimizes objective independently
- Look at joint ownership issues (firms represented colors: X, M, G)
- Label consumer as 'D' (but can be partitioned into 'D_1','D_2','D_3')

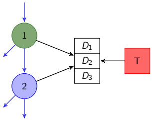
Average inflow 0.6

- T_{ab} encodes the water network, water prices are multipliers on:

$$x_a(n_-) + \sum_b T_{ab} u_b(n) + \omega_a(n) \geq x_a(n)$$

- Allows interaction with other water uses (irrigation, tourism, conservation)
- Ownership of both hydros is not beneficial with competitive pricing of water

XMGD
TotRA = 87351
SysRA = 92763
SysRN = 93109



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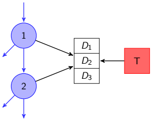

Average inflow 0.6 vs. low inflow 0.1

- Ownership of both hydros is not beneficial with competitive pricing of water

XMGD
TotRA = 87351
SysRA = 92763
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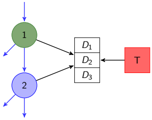

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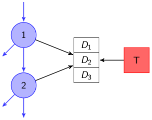
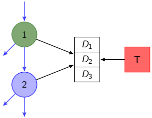
SysRA = 92763
SysRN = 93109

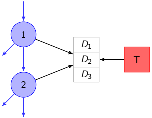


- Not true: risk averse and low inflows shows advantage to co-ownership of hydros

XMGD
TotRA = 62382
SysRA = 65269
SysRN = 65375

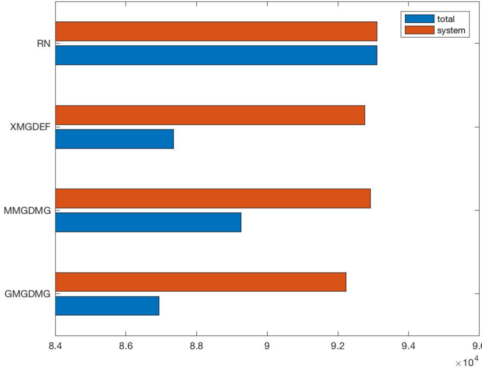


MMGD
TotRA = 62552
SysRA = 65371
SysRN = 65375



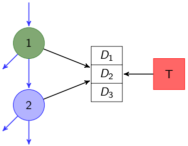
Vertical integration/asset swaps

- SysRN and TotRN in risk neutral case, followed by SysRA and TotRA for three cases depicted on left

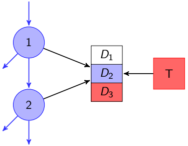


- Vertical integration and risk matters!

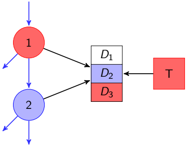
Base: $XMGD_1D_2D_3$



Vertical integration: $MMGDMG$



VI & Asset Swap: $GMGDMG$



Top-down, bottom-up equilibrium (simple Nash case)

$$\forall i : \min_{x_i \in X_i} f_i(x_i, x_{-i}, \pi)$$

(detailed optimizations) coupled with the market definition:

$$0 \le H(x, \pi) \perp \pi \ge 0$$

- Optimization problems might be large LP or QP models of particular sectors
- Diagonalization frequently fails
- Complication: Optimizations are multi-stage risk-averse stochastic programs

$$\forall i : \min_{x_i \in X_i} f_i(x_i^1, x_{-i}^1, \pi^1) + \rho(g_i(x, \pi, \omega))$$

- empinfo: OVF cvarup ρ z θ p
- EMP/PATH has difficulty with these problems

Proximal Algorithm (in SELKIE)

$$\forall i : \min_{x_i \in X_i} f_i(x_i, x_{-i}, \pi) + \frac{1}{2}(x_i - \bar{x}_i)^T \Lambda (x_i - \bar{x}_i)$$

$$0 \leq H(x, \pi) + \Lambda^{-1}(\pi - \bar{\pi}) \perp \pi \geq 0$$

- Choice of Λ is critical for efficiency
- Best choice for Λ motivated by local models of $\pi(x)$ and $x(\pi)$ (Rutherford)
- Individual optimization problems become strongly convex quadratic programs
- Stabilized (trust region) diagonalization scheme
- Amenable for parallel computation
- Alternative: Dantzig-Wolfe decomposition

Conclusions

- Markets naturally modeled via complementarity
- Solvers exist for medium to large scale problems
- Frameworks (EMP) exist to streamline model transformations
- empinfo: dualvar, bilevel, equilibrium, vi, OVF
- Very large scale models (many agents with many instruments acting strategically) with risk are hard
- Decomposition/diagonalization methods (SELKIE) are effective when sensitivity information is exploited
- New algorithms enable solution of more detailed, authentic problems and address underlying policy questions
- Evaluation via simulation computations and out-of-sample testing