A Joint Segmentation/Registration model and Deformation-informed PCA

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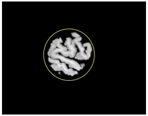
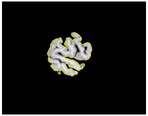
Outline

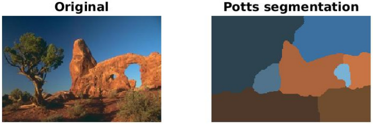
- Image **segmentation** : aims to partition a given image into relevant constituents or to delineate the contours inside the image for further analysis and understanding.



Initial contour segmentation



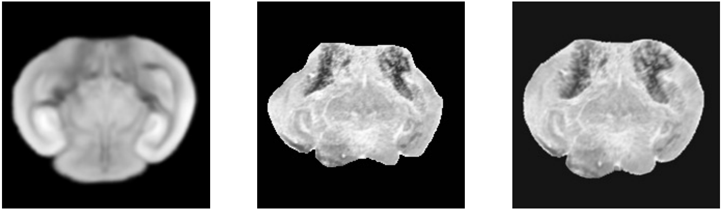
Obtained contour segmentation



Region segmentation.

- Challenges : definition of meaningful constituents is ambiguous and is subject to the applications and to the subjective human interpretation.
- Applications : object detection, scene parsing, organ reconstruction, tumor detection, etc.

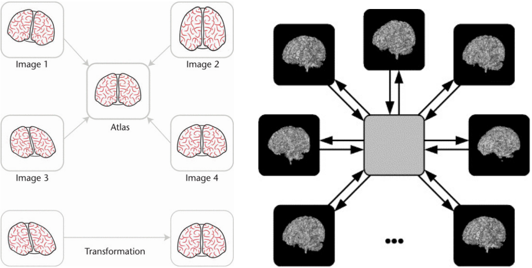
- Image **registration**: Given two images called **Template** (T) and **Reference** (R), registration consists in determining an optimal diffeomorphic transformation φ such that the deformed Template ($T \circ \varphi$) image is aligned with the Reference (R).



From left to right : Reference R; Template T (mouse atlas and gene expression data); deformed Template $T \circ \varphi$.

- Challenges : under-constrained problem $\Rightarrow$ ill-posedness, non-linearity, non-convexity, high dependency to the considered application.
- Applications : shape tracking, multi-modality fusion, computer-aided diagnosis and disease follow-up, atlas generation, etc.

- **Atlas generation**: construct a statistical representative image and an associated set of coordinated transformations from an ensemble of images.



Atlas generation schemes involving deforming and registering all images to the unknown atlas (Raj *et al.* [7], Joshi *et al.* [3]).

- Challenges : same as the registration ones with one more difficulty since the Reference to which the images should be mapped is unknown.

- Applications : characterization of the expected structure and variability of a population through a statistical analysis (PCA for instance), compare different populations (healthy/unhealthy for example), shape a-priori, etc.

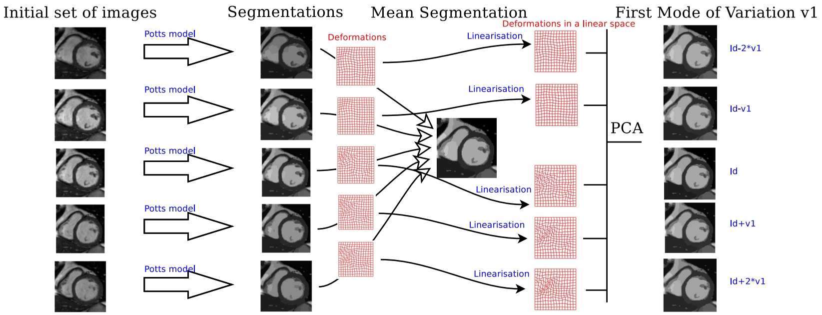


Figure: Overview of our framework

- **Main ideas for joint segmentation/registration models** :
  - As structure matching and intensity distribution comparison rule registration, combining both tasks into a single framework sounds relevant.
  - Registration is seen as prior information to guide segmentation and to overcome the difficulty of weak boundary definition.
  - Accurate segmented structures drive the registration process correctly based on geometrical and topological features.
- **Difficulty** : lies in the construction of such a relevant functional since the problem is underconstrained and involves nonlinearity and nonconvexity.

- **Proposed Methodology:**

  $\Rightarrow$ introduction of an original geometric dissimilarity measure based on segmentation principles and shape comparisons allowing for joint segmentation and registration:

  $\hookrightarrow$ Potts model(Potts [6], Storath *et al.* [10]) in order to segment each image of the dataset.
  $\hookrightarrow$ Non local shape descriptors inspired by the Potts model for segmentation to match regions.

- **Proposed Methodology:** *(continuation)*

$\Rightarrow$ introduction of a deformation model in a nonlinear elasticity framework.

$\hookrightarrow$ Shapes to be matched are viewed as isotropic, homogeneous, hyperelastic materials and more precisely as Ogden materials (see Ciarlet's book [1]).
$\hookrightarrow$ Addition of two original constraints to ensure the deformations are bi-Lipschitz homeomorphisms.
$\hookrightarrow$ Hyperelasticity is a suitable framework when dealing with large and nonlinear deformations.
$\hookrightarrow$ *Rubber, filled elastomers, biological tissues* are often modelled within the hyperelastic framework.

- **Proposed Methodology:** *(continuation)*

  Observation (Rumpf *et al.* [8]) : the arithmetic mean x of observations $(x_i)_{i=1}^{M}$ can be interpreted as the minimizer of the total elastic deformation energy in a system where the average x is connected to each x_i by an elastic spring under the Hooke's law.

  $\Longrightarrow$ introduction of a mean segmentation given by the particular deformed configuration that minimizes the total nonlinear energy required to deform each segmentation so that it is aligned to this mean configuration.

Prior related works suggest jointly treating segmentation and registration. Among others:

- Droske *et al.* ([2]): *combine the general Mumford and Shah functional and registration via nonlinear elasticity principles*;
- Ozeré, Gout and Le Guyader ([5]): *combine a weighted total variation to align the edges, and the modified stored energy function of a Saint Venant-Kirchhoff material.*

Prior works on joint segmentation/registration/shape averaging :

- Rumpf and Wirth ([8]): *combine the Ambrosio-Tortorelli phase field approximation of the Mumford and Shah functional, generation of a mean shape, and the stored energy function of an Ogden material.*

- $\Omega \subset \mathbb{R}^2$: open bounded and connected subset of $\mathbb{R}^2$ with boundary $\partial\Omega$ of class $\mathcal{C}^1$.
- $T_i : \bar{\Omega} \to \mathbb{R}$ the i-th Template image, for $i = 1, \cdots, M$ where M is the total number of images.
- For **theoretical purposes:**
  - T_i are assumed to be compactly supported on Ω.
  - T_i are assumed to be Lipschitz continuous.
- $\varphi_i : \bar{\Omega} \to \mathbb{R}^2$: deformation (*or transformation*) from the Template T_i to the unknown mean.
- $\theta_{T_i} : \bar{\Omega} \to \mathbb{R}$ corresponds to the segmentation of the Template T_i, and $\theta_R : \bar{\Omega} \to \mathbb{R}$ represents the mean segmented atlas (unknown of our problem).
- u_i: associated displacement s.t. $\varphi_i = \text{Id} + u_i$.

- **Construction of the nonlinear-elasticity-based regularizer**:

  - $\Rightarrow$ proposed regularizer on each φ_i based on the coupling of the stored energy function W_O of an Ogden material and on a term controlling that the Jacobian and the inverse Jacobian remain small.
    $\Rightarrow$ the deformation map does not exhibit contractions or expansions that are too large and is a bi-Lipschitz homeomorphism.
  - To sum up, the regularization can be written as

$$E_{reg}(\varphi_i) = \int_\Omega W(\nabla \varphi_i)\, dx,$$

with

$$W(F) = W_O(F) + \mathbf{1}_{\{\|.\|_{L^\infty(\Omega, M_2(\mathbb{R}))} \leq \alpha\}}(F) + \mathbf{1}_{\{\|.\|_{L^\infty(\Omega, M_2(\mathbb{R}))} \leq \beta\}}(F^{-1}).$$

- **Construction of the Template segmentation**:

Potts model

D $u^* = \underset{u \in \mathbb{R}^s}{\operatorname{argmin}} E_{seg}(u) = \|\nabla u\|_0 + \|u - f\|_2^2$, with f the observed image.

C $u^* = \underset{u \in \left\{ \begin{array}{c} u = \sum_{l=1}^{N} c^l \theta_l, \\ \theta_l \in BV(\Omega, \{0,1\}), \\ \sum_{l=1}^{N} \theta_l = 1, \text{ a.e.} \end{array} \right\}}{\operatorname{argmin}} E_{seg}(u) = \sum_{l=1}^{N} TV(\theta_l) + \int_\Omega \sum_{l=1}^{N} \theta_l (c^l - f)^2 \, dx.$

Interpretation

Approximation in the L^2 sense of the image f by N regions whose characteristic functions are respectively θ_l with a constant intensity c^l for each $l = 1, \cdots, N$ minimizing the length of the overall edges.

- **Construction of the dissimilarity measure**:

Distance measure criterion

D $E_{dist}(\theta_R, (\theta_{T_i}, \varphi_i)_{i=1,\cdots,M}) = \frac{1}{M}\sum_{i=1}^{M} \|\nabla(\theta_R - \theta_{T_i} \circ \varphi_i)\|_0,$

C $E_{dist}(\theta_R, (\theta_{T_i}, \varphi_i)_{i=1,\cdots,M}) = \frac{1}{M}\sum_{i=1}^{M}\sum_{l=1}^{N} TV(\theta_{R,l} - \theta_{T_i,l} \circ \varphi_i),$

with $\theta_R = \sum_{l=1}^{N} c_R^l \theta_{R,l}$, $\theta_{T_i} = \sum_{l=1}^{N} c_{T_i}^l \theta_{T_i,l}$.

Interpretation

It aims at minimizing the length of the contours defined by the difference between the deformed segmentation $\theta_{T_i} \circ \varphi_i$ of the Template T_i and the mean segmentation θ_R. $\Longrightarrow$ aligns the edges of each homogeneous region.

Functional minimization problem

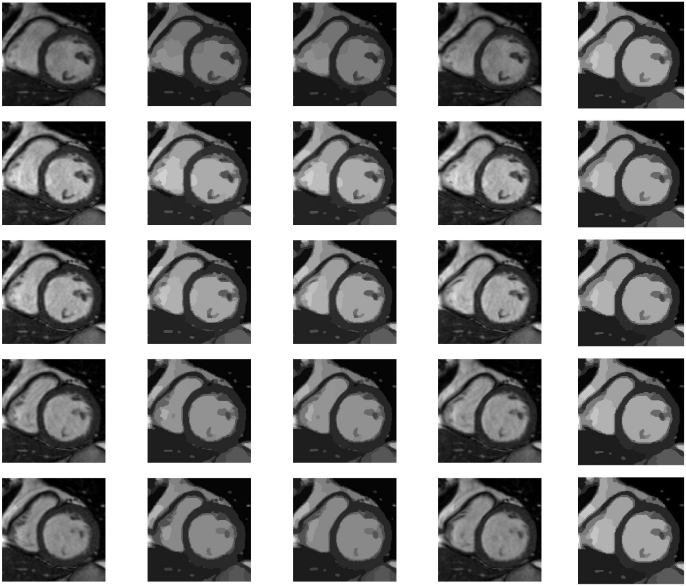
$$\inf \Big\{ I(\theta_R, (\theta_{T_i}, \varphi_i)_{i=1,\cdots,M}) = E_{dist}(\theta_R, (\theta_{T_i}, \varphi_i)_{i=1,\cdots,M}) + \frac{1}{M}\sum_{i=1}^{M} (E_{reg}(\varphi_i) + E_{seg}(\theta_{T_i})) + E_{seg}(\theta_R) \Big\}. \quad \text{(P)}$$

Numerical difficulties: nonlinearity in $\nabla\varphi_i$, the presence of $(\nabla\varphi_i)^{-1}$ and the composition $\theta_{T_i} \circ \varphi_i$ in the distance measure criterion.

- Inspired by the work of Negrón Marrero [4], we introduce auxiliary variables V_i simulating $\nabla\varphi_i$, W_i simulating $(\nabla\varphi_i)^{-1}$, and $\tilde{\theta}_{T_i}$ simulating $\theta_R - \theta_{T_i} \circ \varphi_i$ for each $i = 1, \cdots, M$, and use an L^p-penalization method to ensure their closeness to the initial variable.

- We use an alternating optimization scheme in which we solve the subproblem with respect to each unknown alternatively.

#	T_i	θ_{T_i}	$\theta_{T_i} \circ \varphi_i$	$T_i \circ \varphi_i$	θ_R
1					
2					
3					
4					
5					



Principal Component Analysis

Principal Component Analysis (PCA) : statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables into a set of values of linearly uncorrelated variables called principal components. This transformation is defined in such a way that the first principal component has the largest possible variance, and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components. The resulting vectors (each being a linear combination of the variables) are an uncorrelated orthogonal basis set.

Requirement: the variables need to live in a linear space.

- Objective : statistical analysis of the dataset through a Principal Component Analysis on the deformations to retrieve the main modes of variations inside the population.

- Difficulty : our deformation maps do not live in a linear space.

- Solution : get a good representation of our deformations in a linear space equipped with a scalar product.

Cauchy stress tensor

$$\sigma = \frac{\partial W}{\partial F}(\varphi)\mathrm{Cof}\nabla\varphi$$

Interpretation

In an equilibrium position, we have :

$$\forall y, \vec{n}, t(y, \vec{n}) = \sigma(y)\vec{n},$$
$$f(y) = -\mathrm{div}\sigma,$$

$t(y, \vec{n})$ is the pressure applied to the material at the boundary point y in the normal direction $\vec{n}$, $f(y)$ is the inner volumetric force applied at the point y inside the material.

The tensor thus relates the forces applied to the material and the corresponding deformation.

- **Observation** (Rumpf *et al.* [9]): The classical covariance tensor can be identified with the covariance tensor of the displacements obtained by adding a small fraction of the i-th spring force under the Hooke's law.
  $\Longrightarrow \min_{v_i} \int_\Omega W(x, \mathrm{Id} + \delta v_i) + \delta^2 \int_\Omega \mathrm{div}\sigma_i : v_i \, dx.$
  σ_i is the Cauchy stress tensor corresponding to the deformation φ_i.
- We apply a Taylor development to W and get back to the linearized elasticity equation, with $\epsilon(v_i) = \frac{\nabla v_i + \nabla v_i^T}{2}$:

$$\min_{v_i \in H^1(\Omega, \mathbb{R}^2)} \int_\Omega \mu Tr(\epsilon(v_i)^2) + \frac{\lambda}{2} Tr(\epsilon(v_i))^2 + \delta^2 \int_\Omega \mathrm{div}\sigma_i : v_i \, dx,$$

  whose solution is in the linear $H^1(\Omega, \mathbb{R}^2)$ space.
- PCA performed on the obtained displacements v_i.

- Drawback : loss of the initial nonlinear nature of the deformation. $\Longrightarrow$ PCA performed on the Cauchy stress tensors directly after noticing they belong to the linear space $L^2(\Omega, M_2(\mathbb{R}))$ and get $(\sigma_{pca,i})$.
- Resolution of the following problem to get back to the deformations by assuming the correspondence between the forces and the displacements is one-to-one which is true at least locally :

$$\min_{v_i} \sum_{k=1}^{M} \int_\Omega W(\nabla((\mathrm{Id} + \delta v_i) \circ \varphi_k))\, dx + \delta^2 \int_\Omega \mathrm{div}\sigma_{pca,i} : v_i\, dx$$
$$+ \mathbf{1}_{\{\|.\|_{L^\infty(\Omega, M_2(\mathbb{R}))} \leq \alpha\}}(\nabla v_i)$$

- Change of vision and consider this problem as an interpolation/approximation one. Find the best approximation of our deformation fields belonging to $H^3(\Omega, \mathbb{R}^2)$ such that the linear deformation tensor $\frac{\nabla v + \nabla v^T}{2}$ is equal to the complete initial deformation tensor $\frac{\nabla u_i + \nabla u_i^T + \nabla u_i^T \nabla u_i}{2}$, with u_i the displacement associated to the deformation φ_i obtained previously.
- Drawback : this interpolation problem is too constrained and might create some memory storage issues.
- We propose to relax it as an approximation one by solving :

$$\min_{v \in H^3(\Omega, \mathbb{R}^2)} |v|^2_{H^3(\Omega, \mathbb{R}^2)} + \|u_i - v\|_2^2 + \frac{\gamma}{2} \|(\nabla v + \nabla v^T) - (\nabla u_i + \nabla u_i^T + \nabla u_i^T \nabla u_i)\|_2^2$$

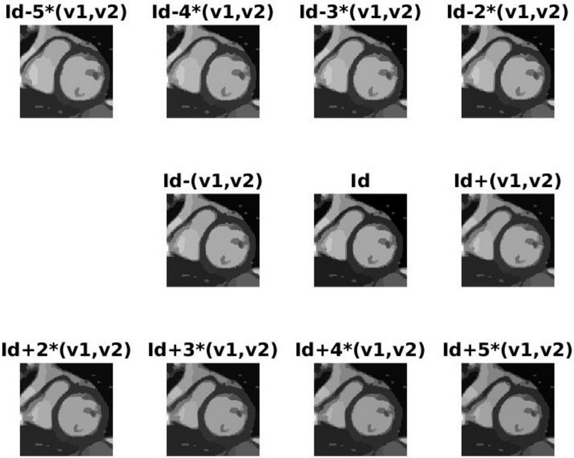


Figure: First mode of variation obtained with the first method.

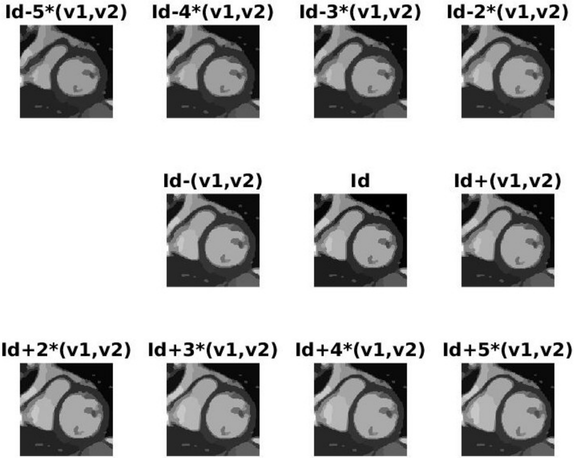


Figure: First mode of variation obtained with the second approach.

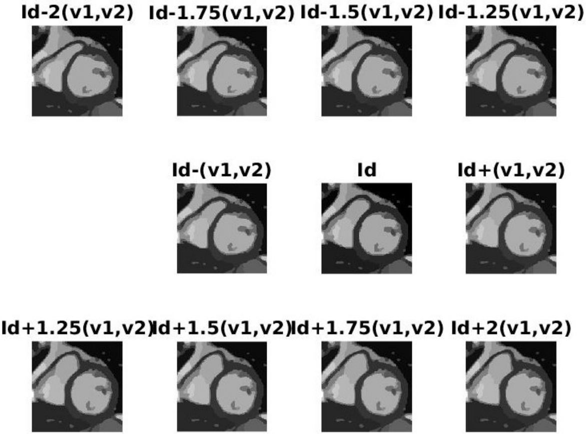
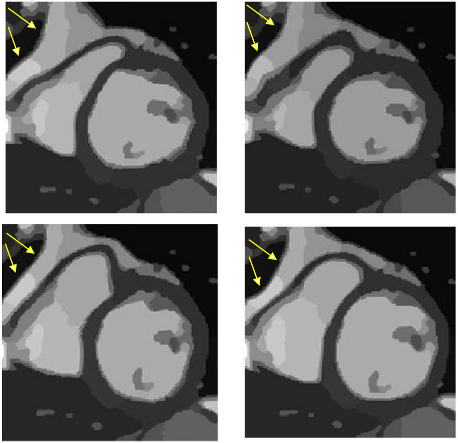


Figure: First mode of variation obtained with the third model.

Third Method : Id-2(v1,v2) Second Method : Id-5(v1,v2)



Third Method : Id+2(v1,v2) Second Method : Id+5(v1,v2)

Figure: Comparison of methods 2 and 3.

Summary of the developed model:

- Joint Registration/Segmentation/Atlas generation model based on the Potts model for segmentation and the nonlinear elasticity principles.
- Three different methods to approximate the obtained deformations in a linear space to perform PCA and retrieve the main modes of variations inside the studied population.
- Preliminary numerical simulations on cardiac MRIs.

[1] P. Ciarlet, *Elasticité Tridimensionnelle*, Masson, 1985.

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[4] P. V. N. Marrero, *A numerical method for detecting singular minimizers of multidimensional problems in nonlinear elasticity*, Numerische Mathematik, 58 (1990), pp. 135–144.

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[6] R. B. Potts, *Some generalized order-disorder transformations*, Mathematical Proceedings of the Cambridge Philosophical Society, 48 (1952), p. 106–109.

[7] M. Raj, M. Mirzargar, J. S. Preston, R. M. Kirby, and R. T. Whitaker, *Evaluating shape alignment via ensemble visualization*, IEEE Computer Graphics and Applications, 36 (2016), pp. 60–71.

[8] M. Rumpf and B. Wirth, *A nonlinear elastic shape averaging approach*, SIAM Journal on Imaging Sciences, 2 (2009), pp. 800–833.

[9] ———, *An elasticity-based covariance analysis of shapes*, International Journal of Computer Vision, 92 (2011), pp. 281–295.

[10] M. Storath and A. Weinmann, *Fast partitioning of vector-valued images*, SIAM Journal on Imaging Sciences, 7 (2014), pp. 1826–1852.

Thank you for your attention.