Shear-induced Rigidity in Granular Materials

Statistical Mechanics with Friction

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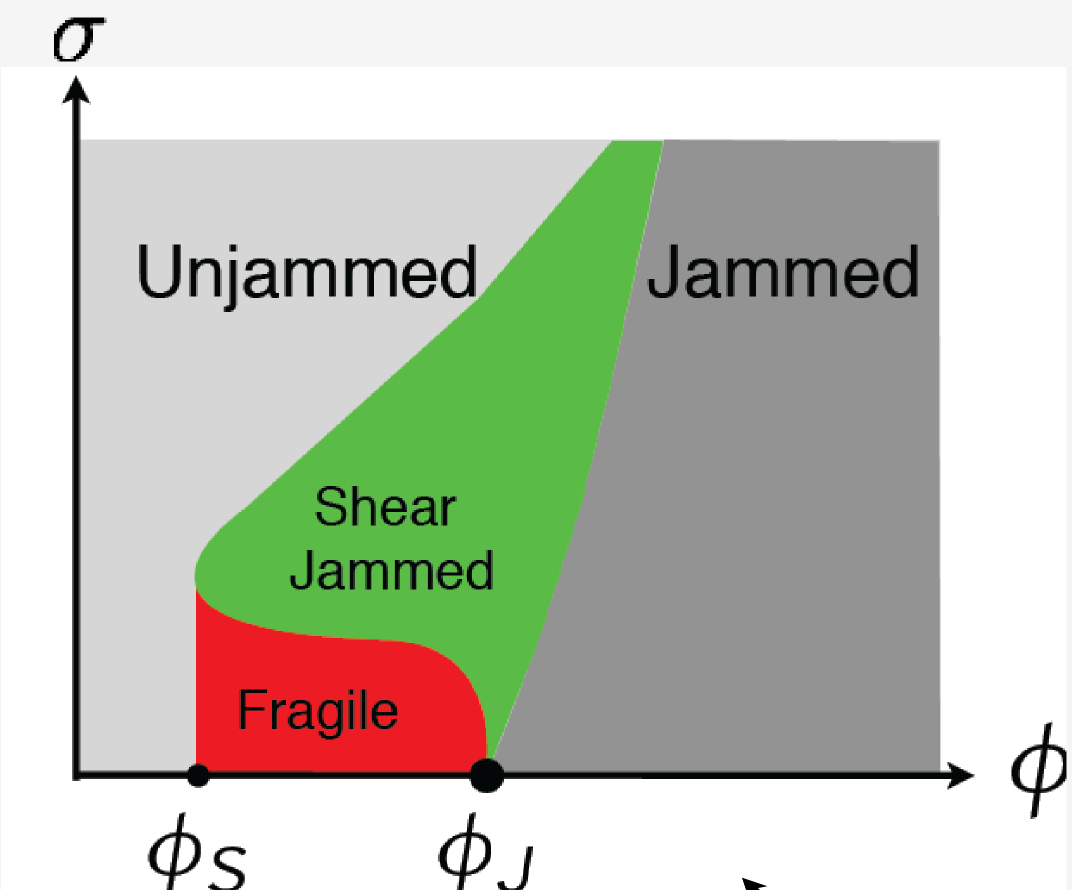
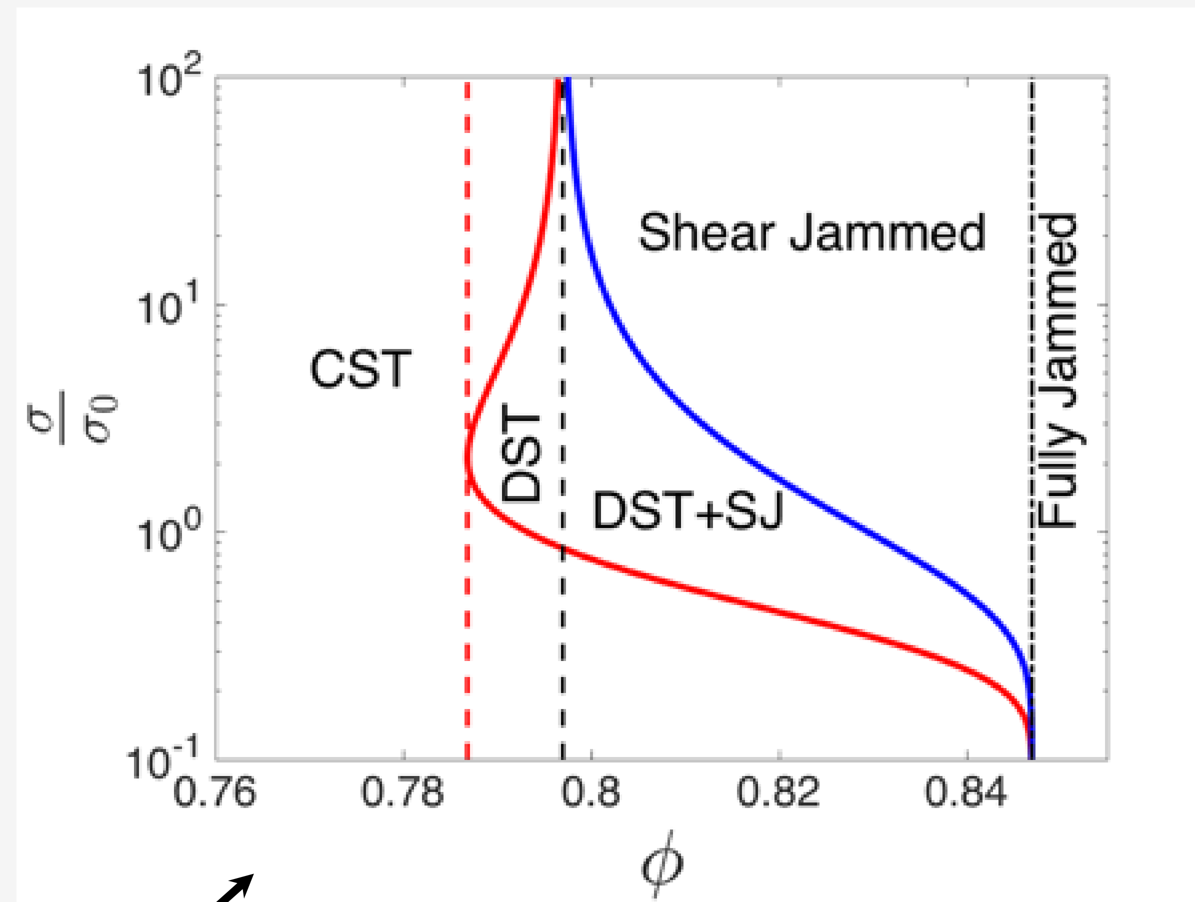




Dry Grains Suspensions

All states are "jammed" these are static states

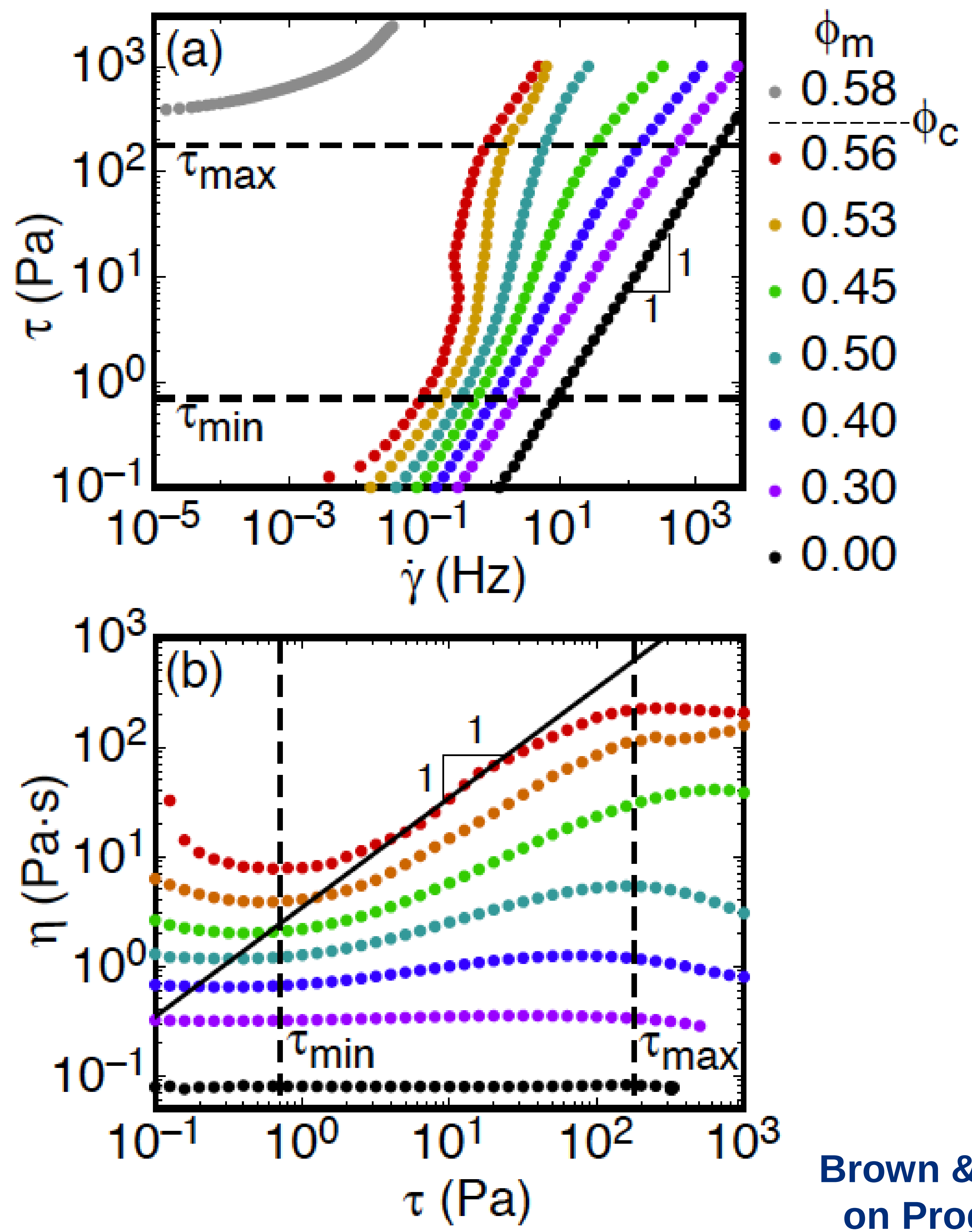
Flowing States (Non-equilibrium steady states)

ATHERMAL, FRICTIONAL MATERIALS

- Collection of macroscopic objects
- Purely repulsive, contact interactions.
- No thermal fluctuations to restore or create contacts
- Friction: Forces are independent degrees of freedom. No Hamiltonian, not energy minima
- States controlled by driving at the boundaries or body forces: shear, gravity
- Non-ergodic in the extreme sense: stays in one configuration unless driven

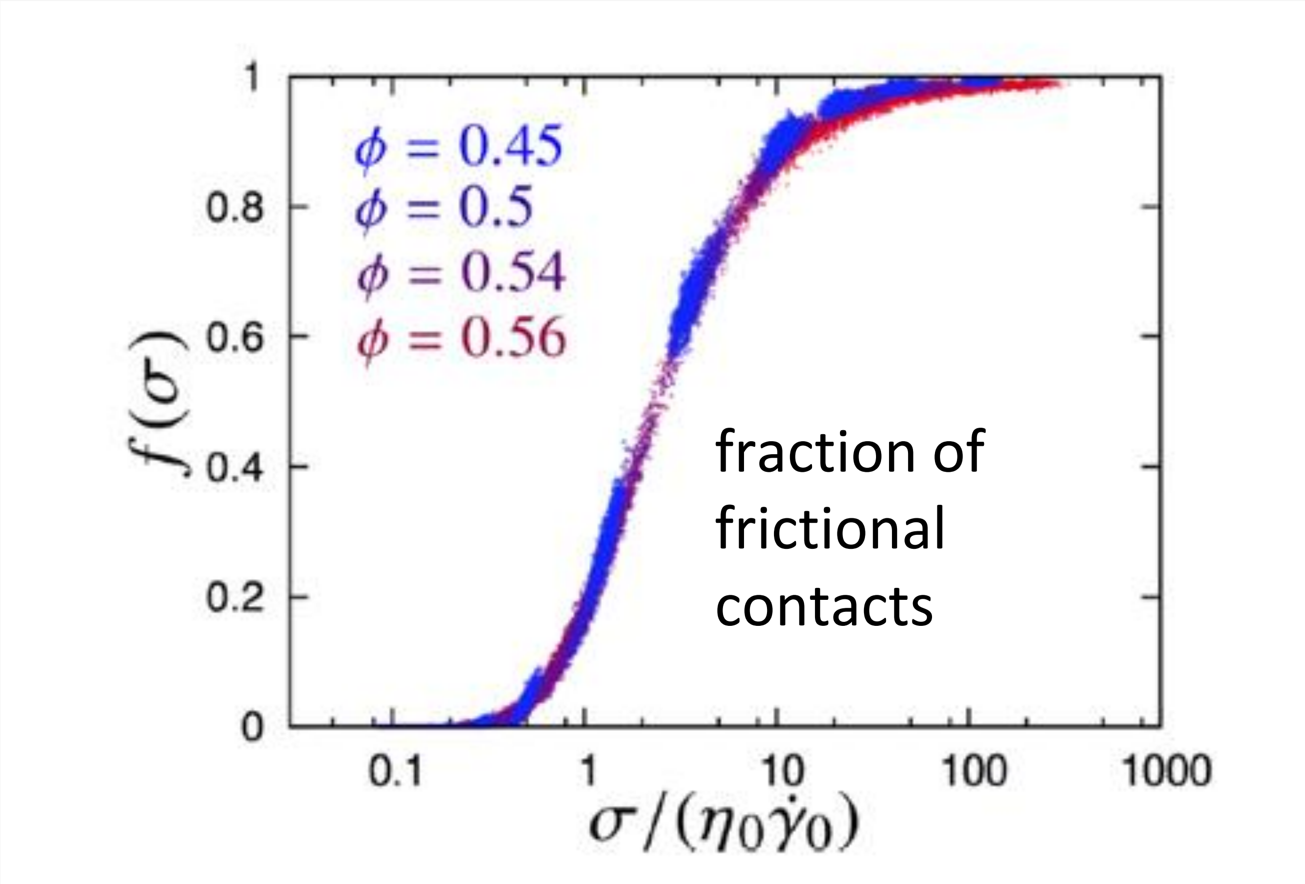
- No thermal or quantum fluctuations, yet states are characterized by broad distributions: definition of solid ? fluid ?
- Force Localization
- Large, avalanche-type, fluctuations characterize yielding
- Stress-driven transitions

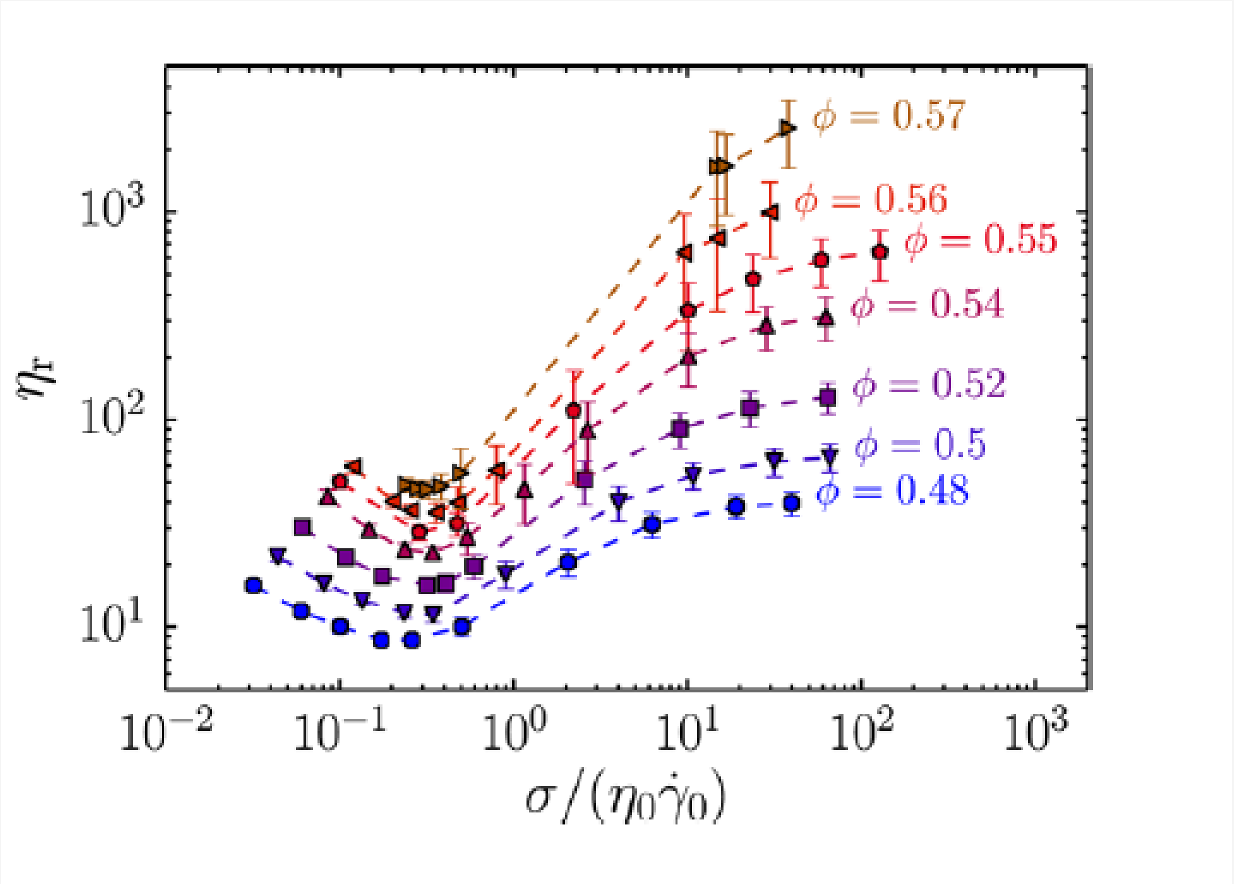
Discontinuous Shear Thickening (DST)



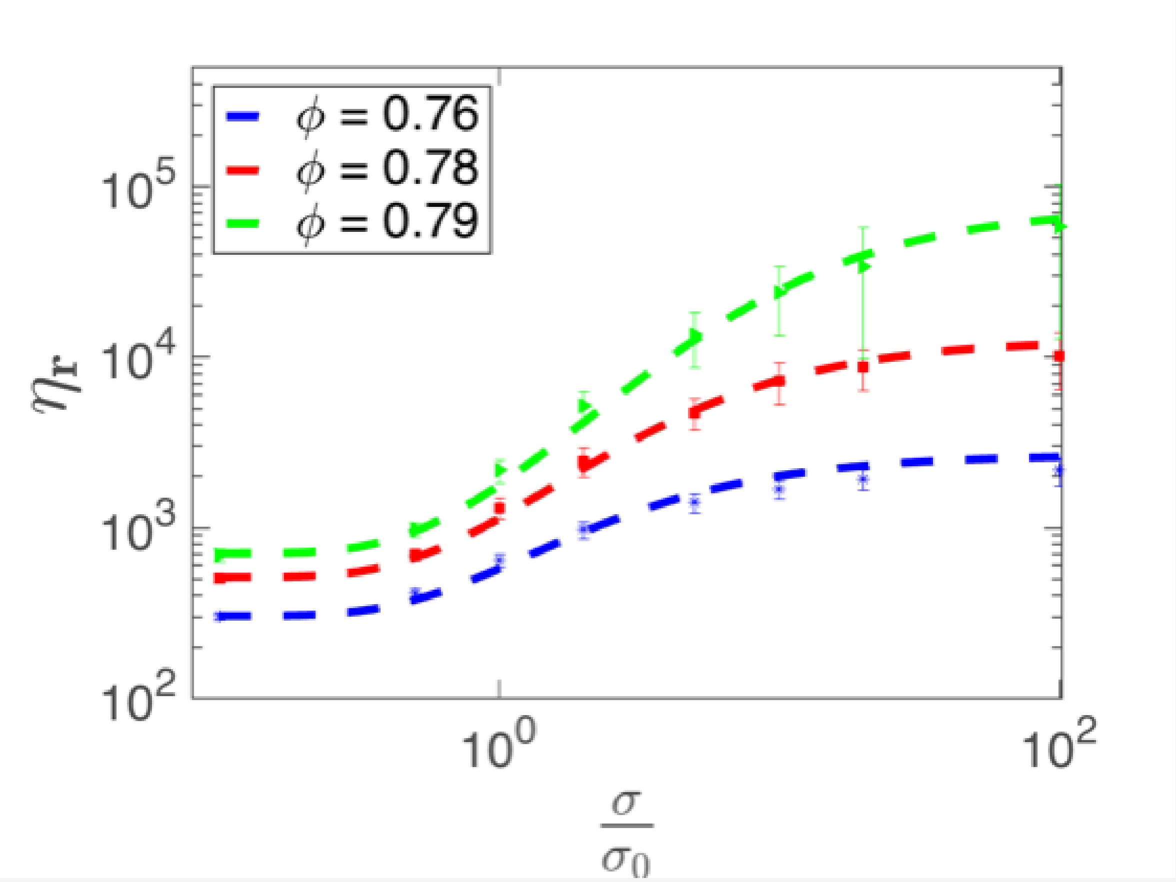
Brown & Jaeger: Reports on Progress in Physics (2013)

DST Phenomenology (Simulations)



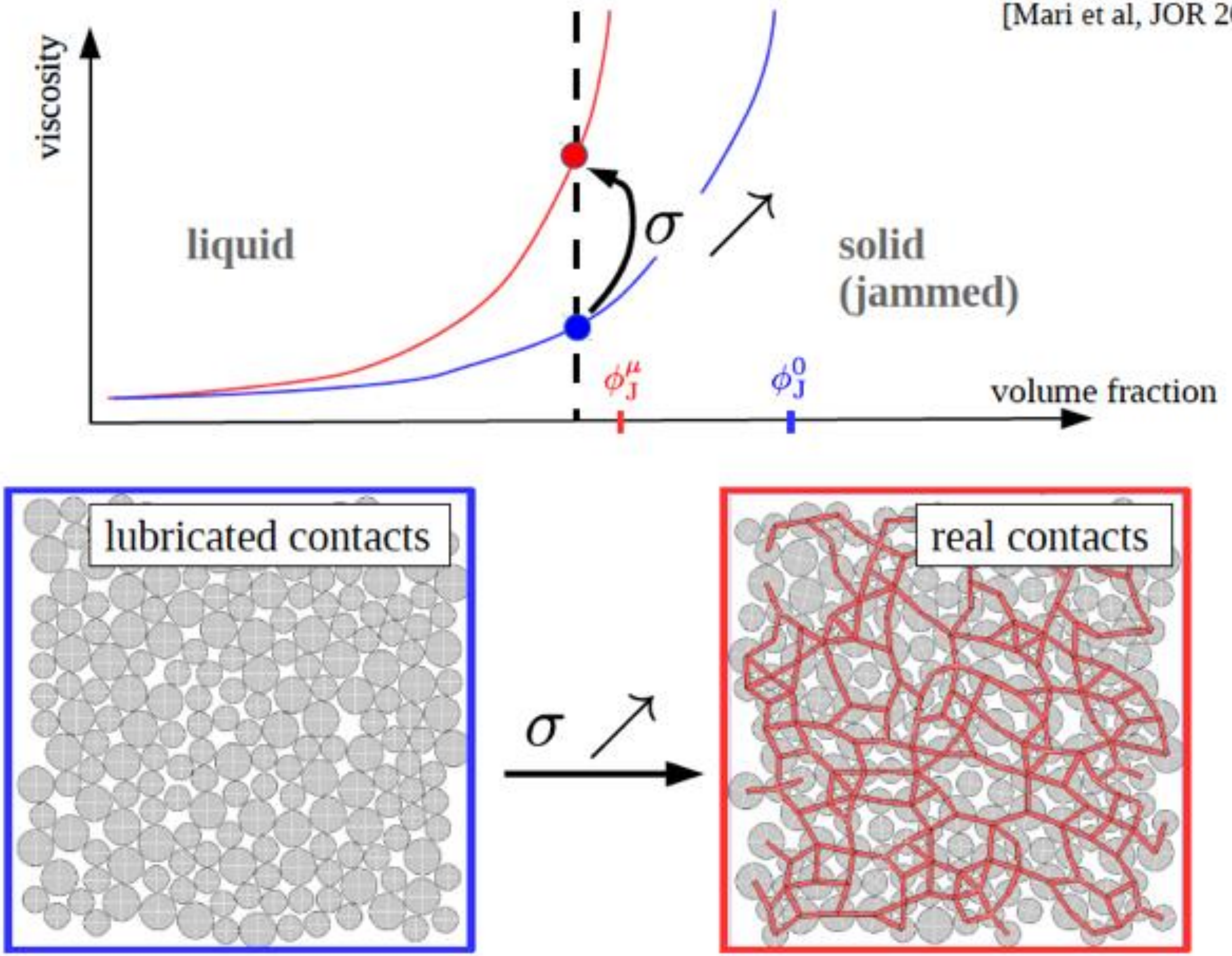


2D



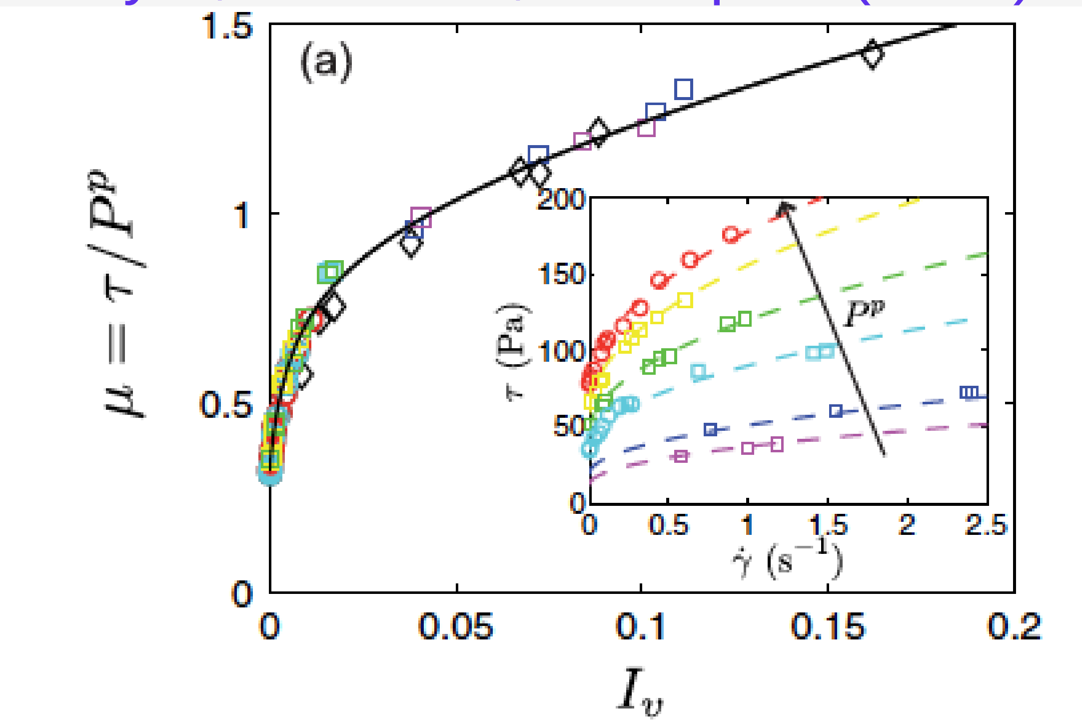
Mean-field theory based on contacts (DST)

[Fernandez et al, PRL 2013]
[Seto et al, PRL 2013]
[Heussinger, PRE 2013]
[Wyart and Cates PRL 2013]
[Mari et al, JOR 2014]



Viscosity: Suspension Rheology

Boyer, Guazzelli, Pouliquen (2011)



- A "universal" relationship between the macroscopic friction coefficient and the viscous number at constant pressure.
- When a suspension is sheared at constant volume, the shear and normal viscosities can be expressed in terms of the friction coefficient and the viscous number.

$$\sigma = \eta_s \dot\gamma$$

rate-independent but viscosity depends on packing fraction

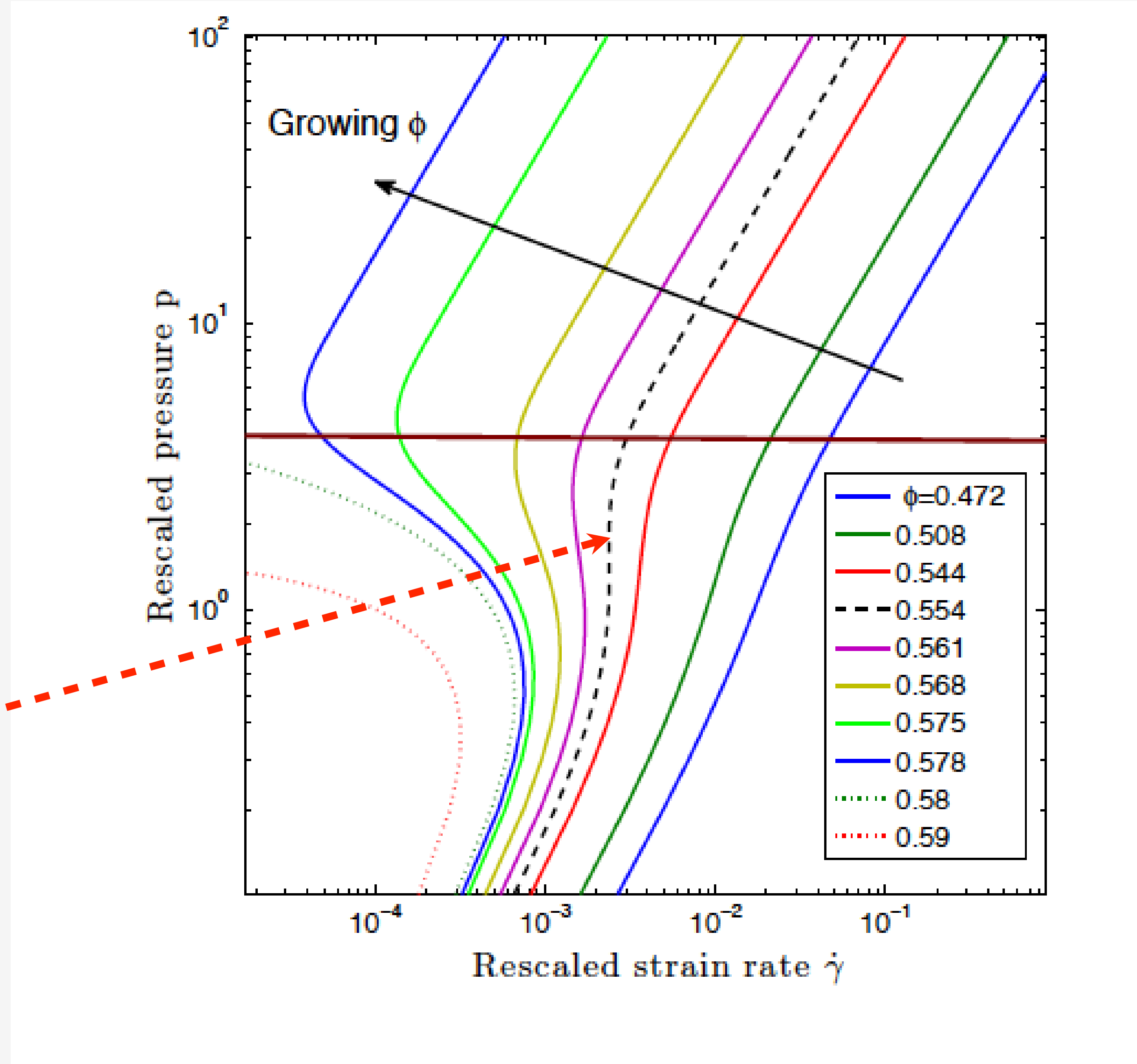
$$\phi_J(\sigma) = f(\sigma/\sigma^*)\phi_m + (1 - f(\sigma/\sigma^*)\phi_J$$

$$\eta_s(\phi) \propto \frac{1}{(\phi_J(\sigma)-\phi)^2}$$

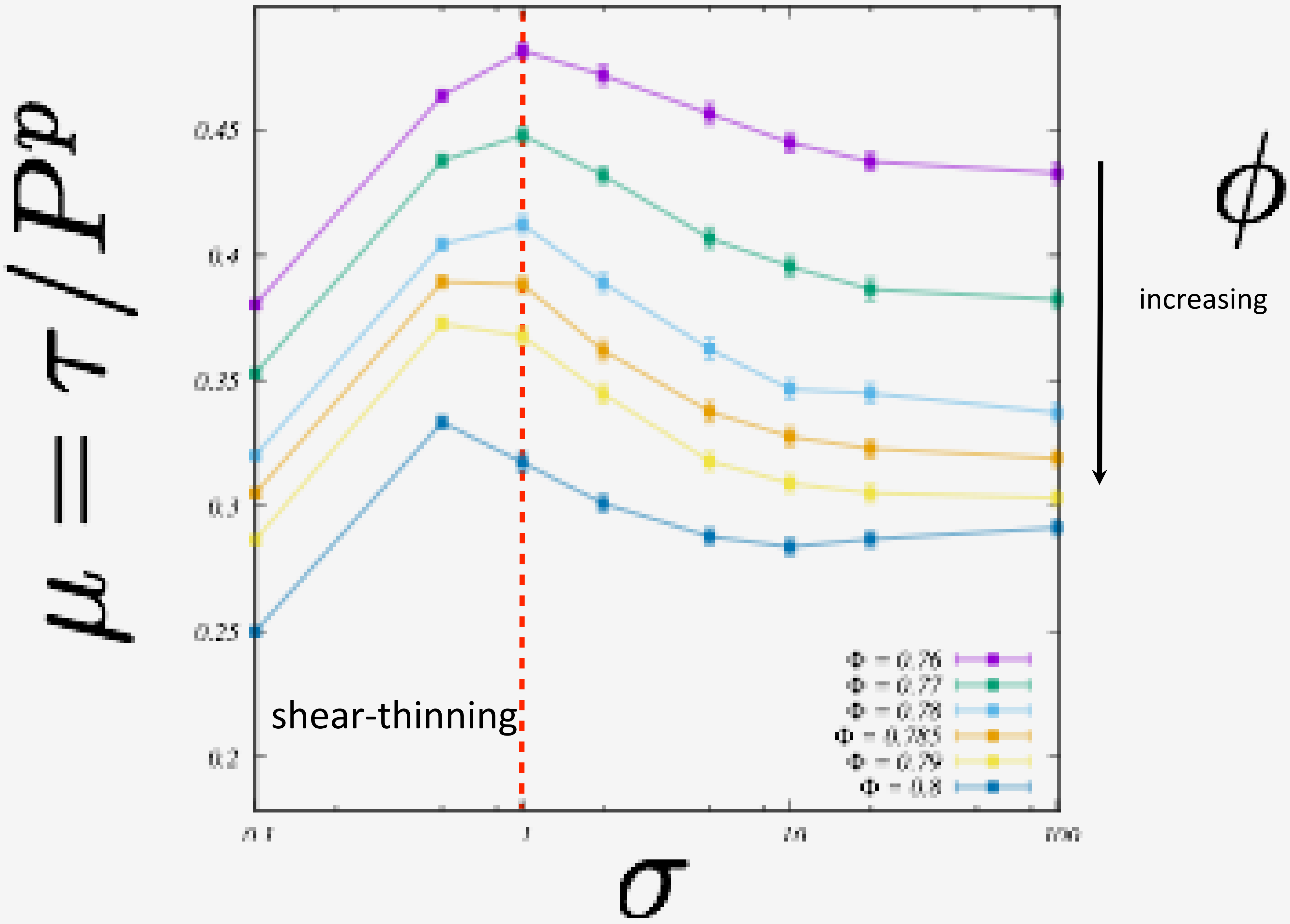
DST is regime in which the viscosity scales linearly with stress

$$\eta \propto \sigma$$

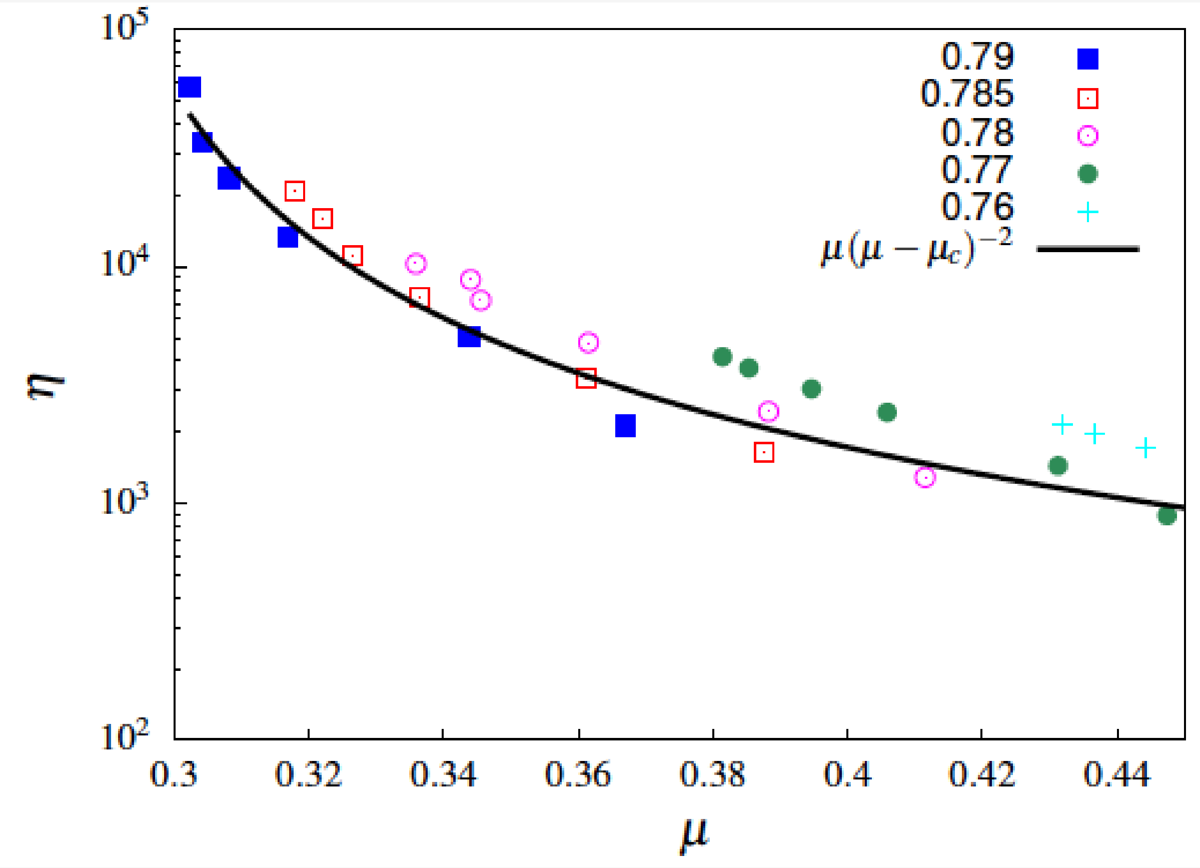
DST phase boundary: $\frac{d\dot\gamma}{d\sigma} = 0$



Frictional Contacts & Stress Anisotropy (DST)



DST: theory based on stress anisotropy



Characteristic decrease in anisotropy

Boyer, Guazzelli, Pouliquen (2011)

$$\eta = \frac{\mu(I)}{I} \longrightarrow \eta = \frac{\mu}{I(\mu)}$$

$$\eta \propto \frac{\mu}{(\mu-\mu_c)^2}$$ BUT $\mu(\sigma, \phi)$

The DST transition is identified by:

$$\frac{d\dot{\gamma}}{d\sigma} = 0$$

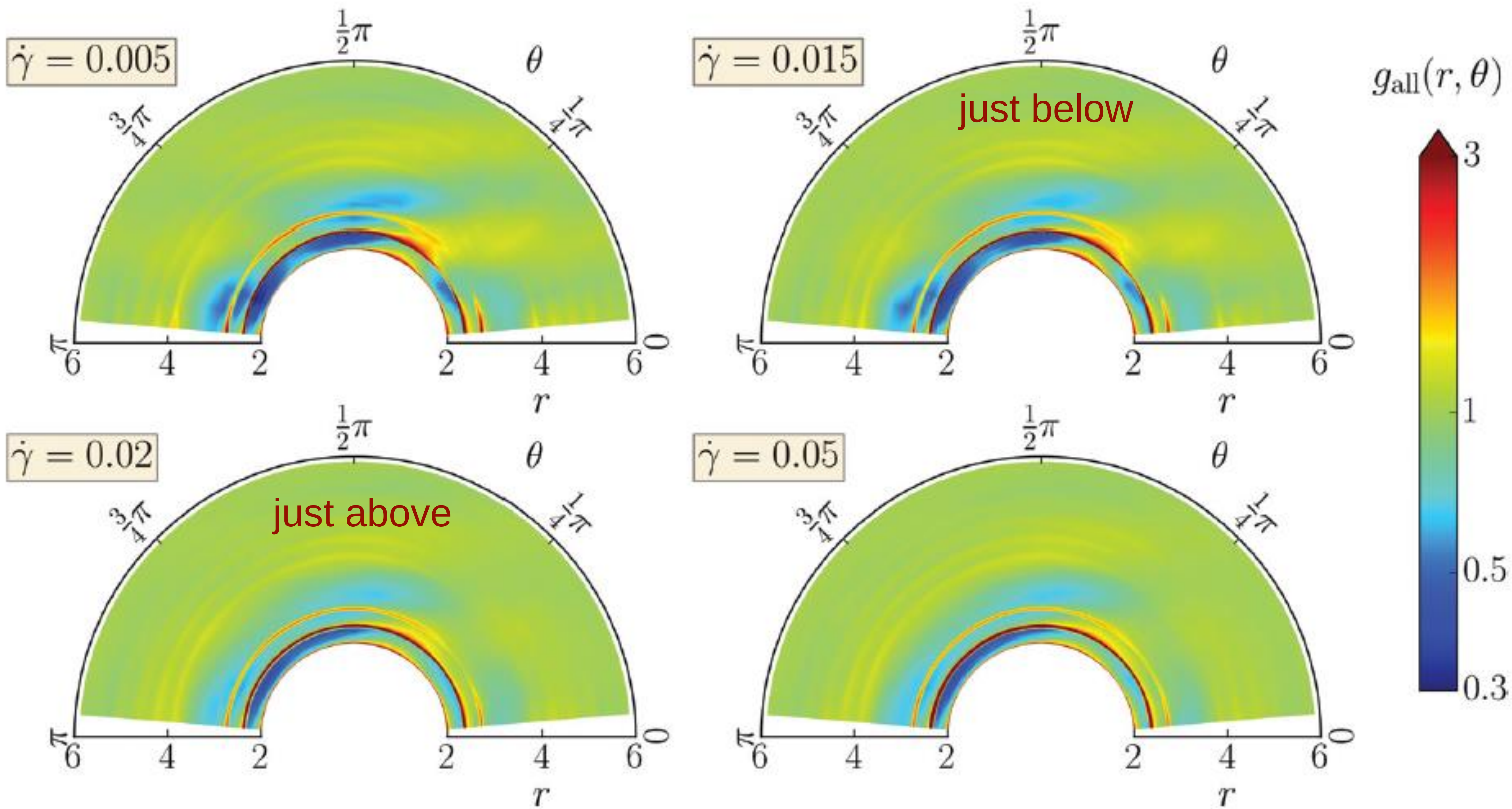
Using the "new" constitutive relation, we can write this condition as

$$\frac{\sigma}{\mu}\frac{d\mu}{d\sigma} = \frac{\mu-\mu_c}{\mu+\mu_c}$$

A theory for the macroscopic friction coefficient $\mu(\sigma, \phi)$

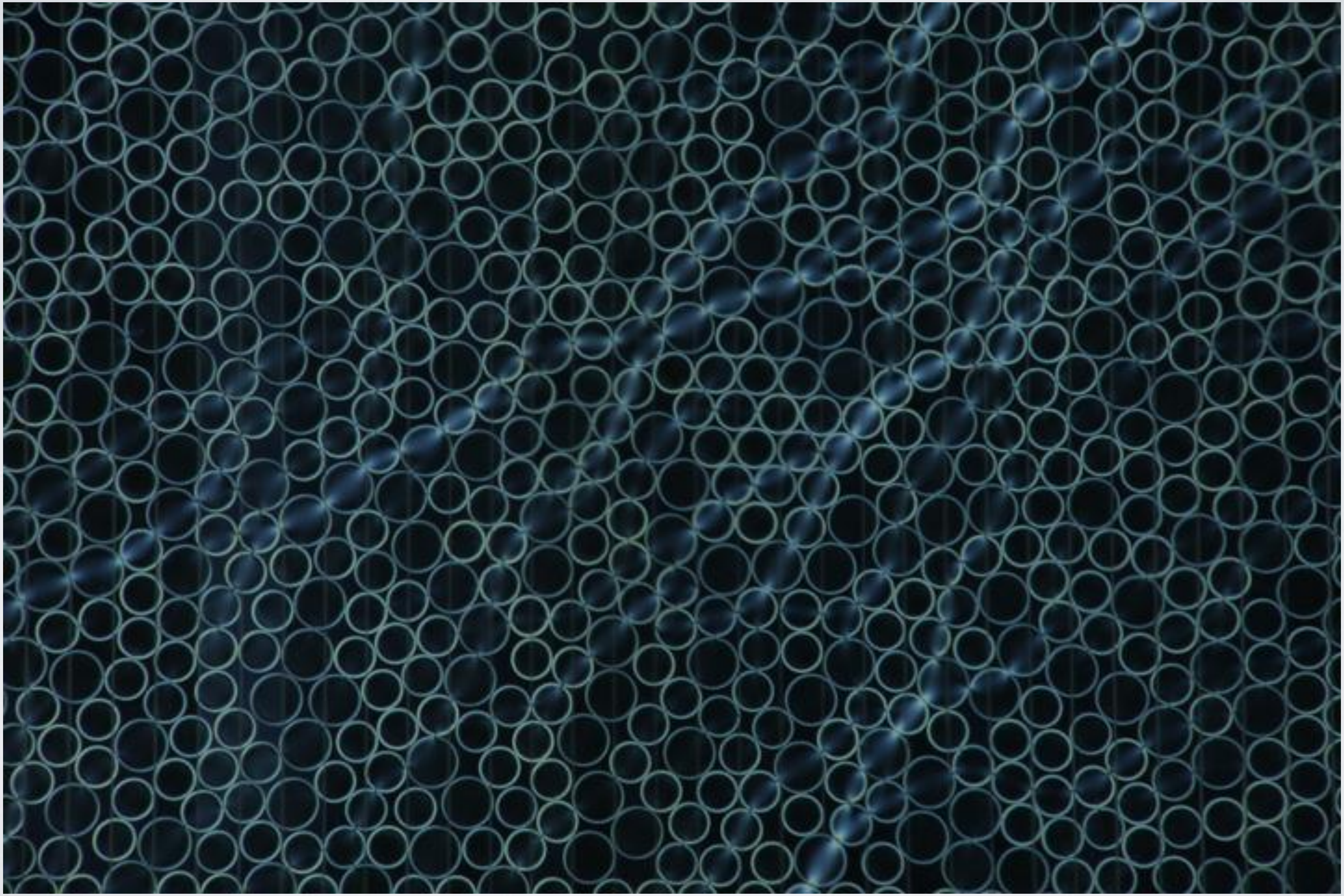
from microscopic correlations
(arXiv: 1804.03155, to appear in PRL)

DST Microstructure: Pair Correlation Functions

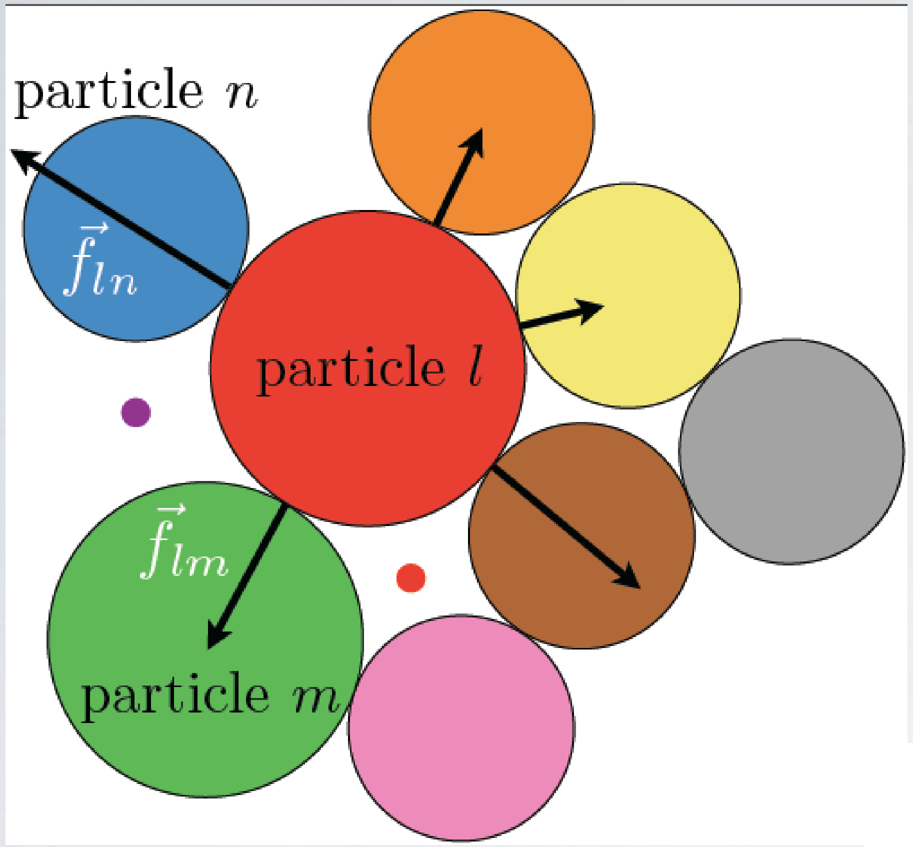


Mari, Seto, Morris, Denn (JOR, 2014)

Stress Space



2D Systems: Force Tilings



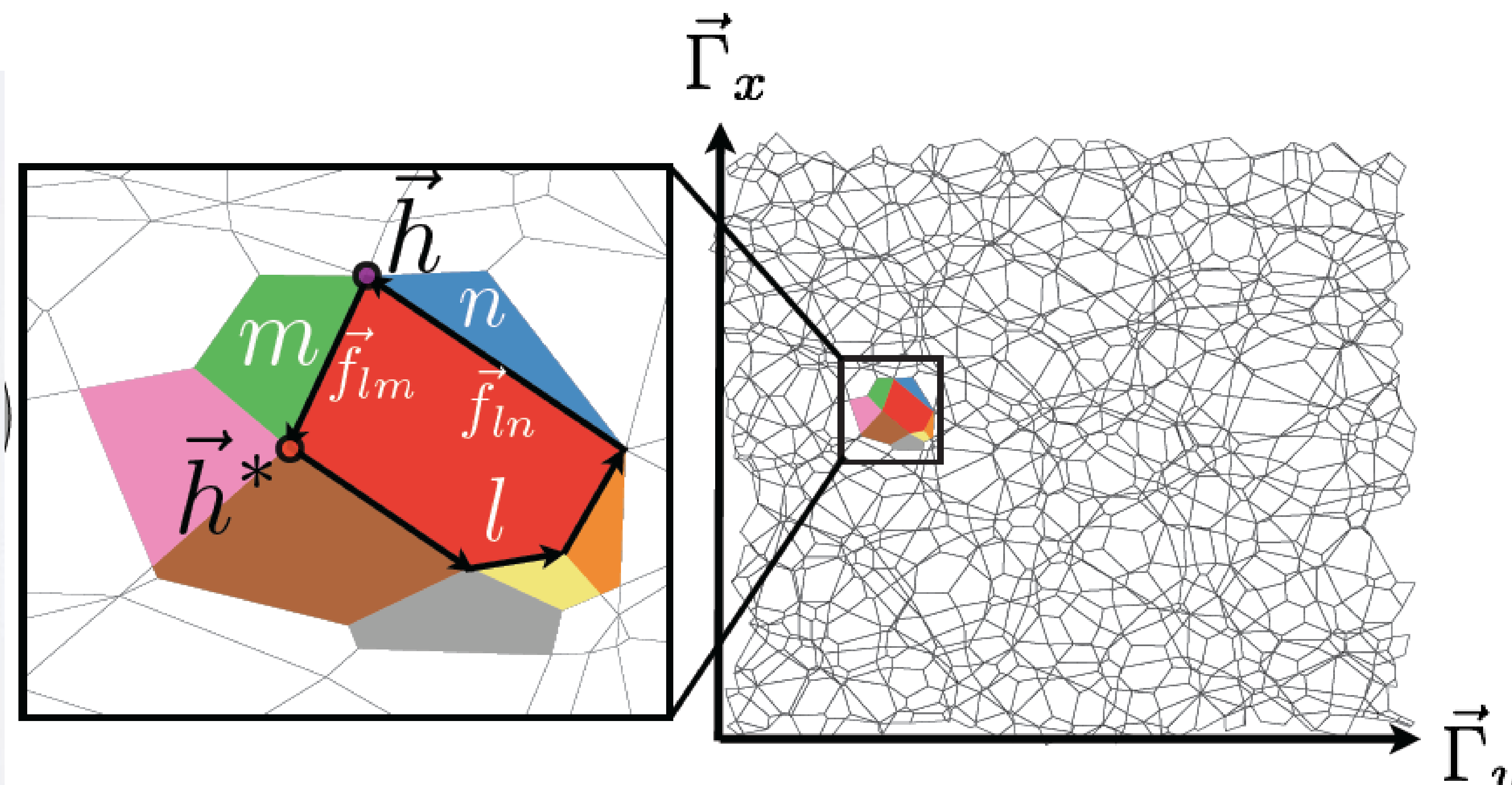
Forces at contacts can have normal and tangential components. Impose force balance on every grain, and use Newton's third law

Applied to dry grains and theory of shear jamming

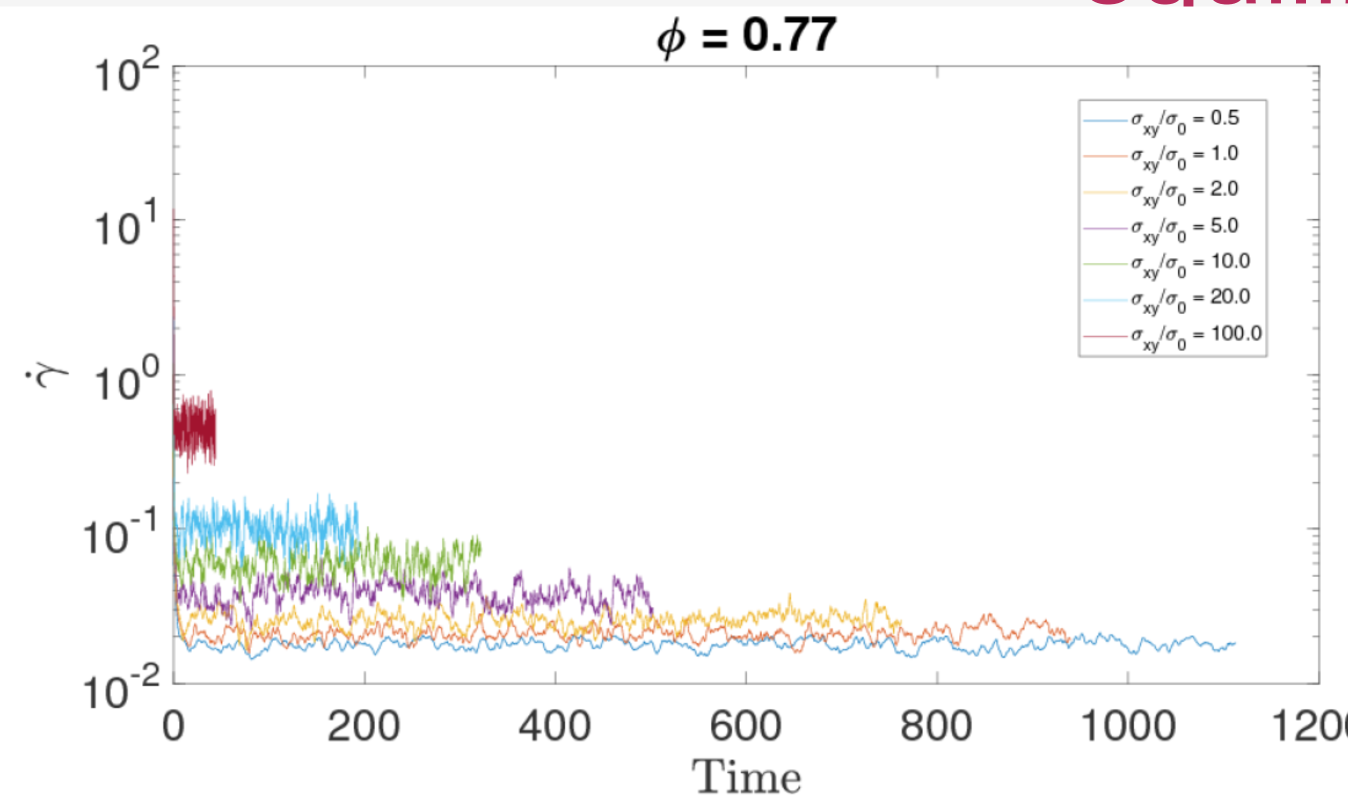
Force Moment Tensor

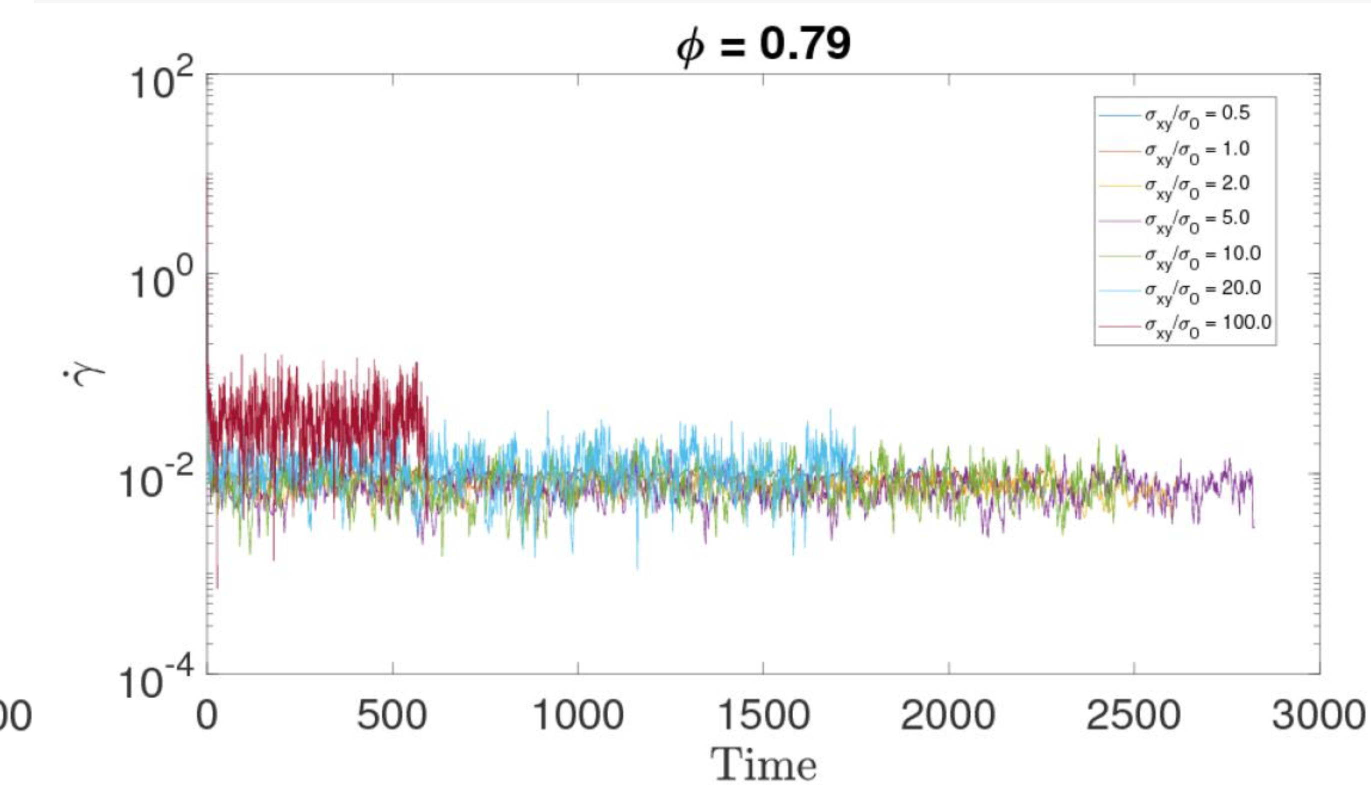
$$\hat{\Sigma} = \sum_{ij} \vec{r}_{ij} \otimes \vec{f}_{ij}$$

$$\hat{\Sigma} = \begin{pmatrix} L_y\Gamma_{yx} & L_y\Gamma_{yy} \\ -L_x\Gamma_{xx} & -L_x\Gamma_{xy} \end{pmatrix}$$



Non-equilibrium steady states sampled in DST simulations: Instantaneously in mechanical equilibrium

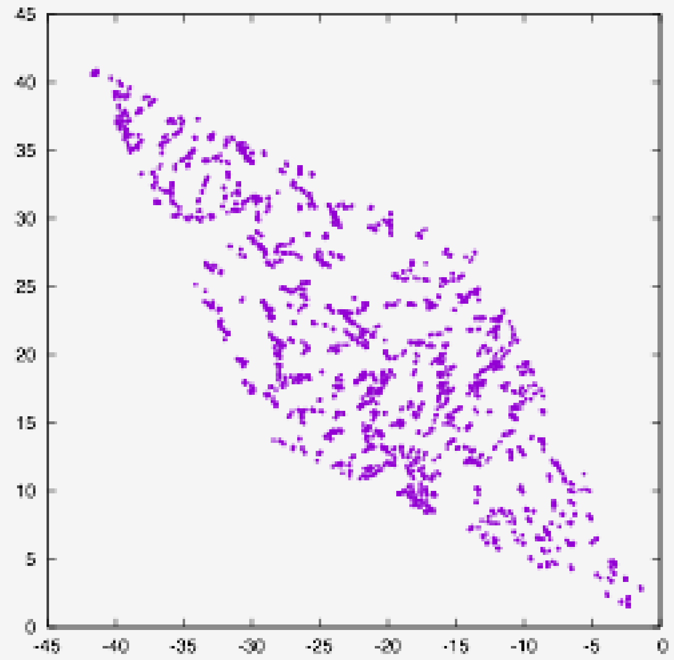




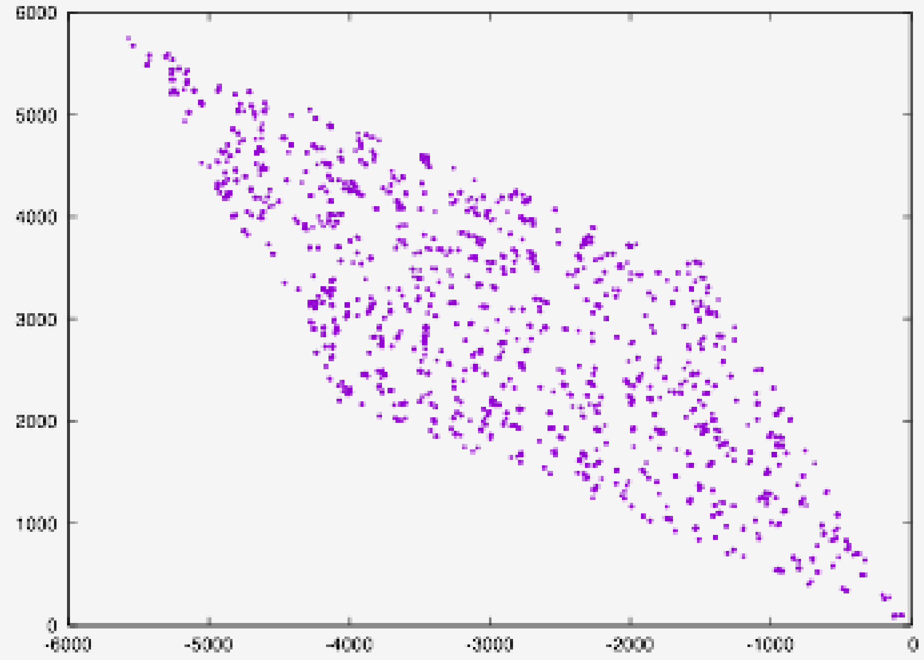
- Unlike dry granular materials, these are flowing states: particles have velocities
- “Contact forces”: lubricated and solid-on-solid frictional
- “Body forces”: Stokes drag
- Generalized force tilings using graph Laplacian of contact network
- (Kabir Ramola & BC, Journal of Statistical Physics, 2017)

Point Patterns: Vertices of Force tilings

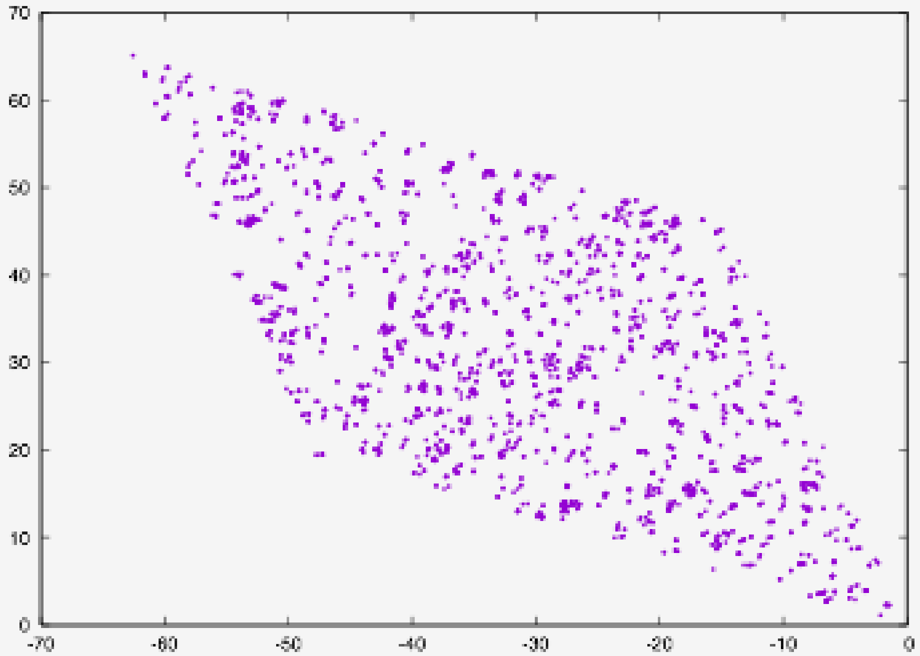
(0.76, 1)



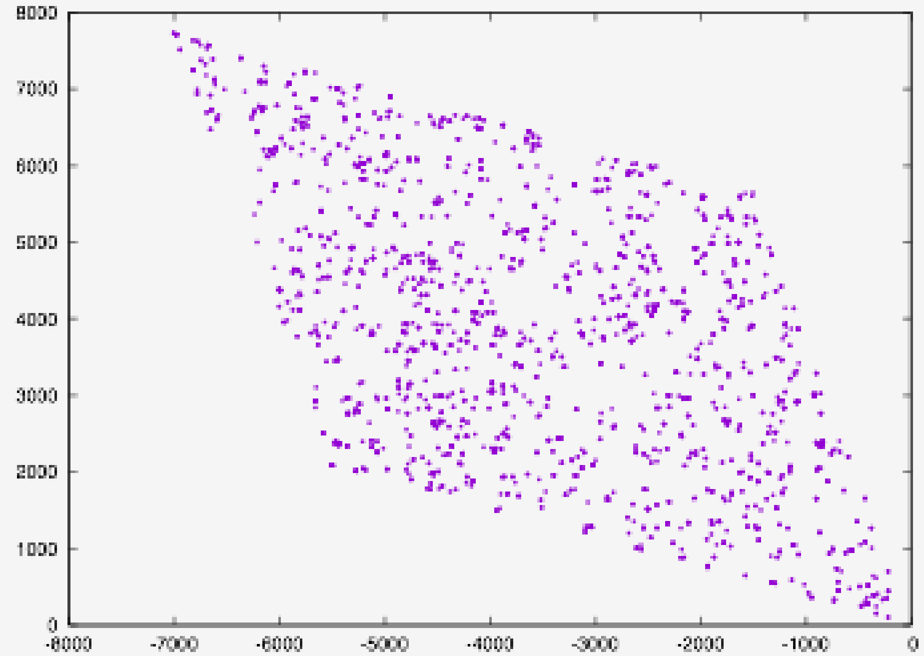
(0.76, 100)



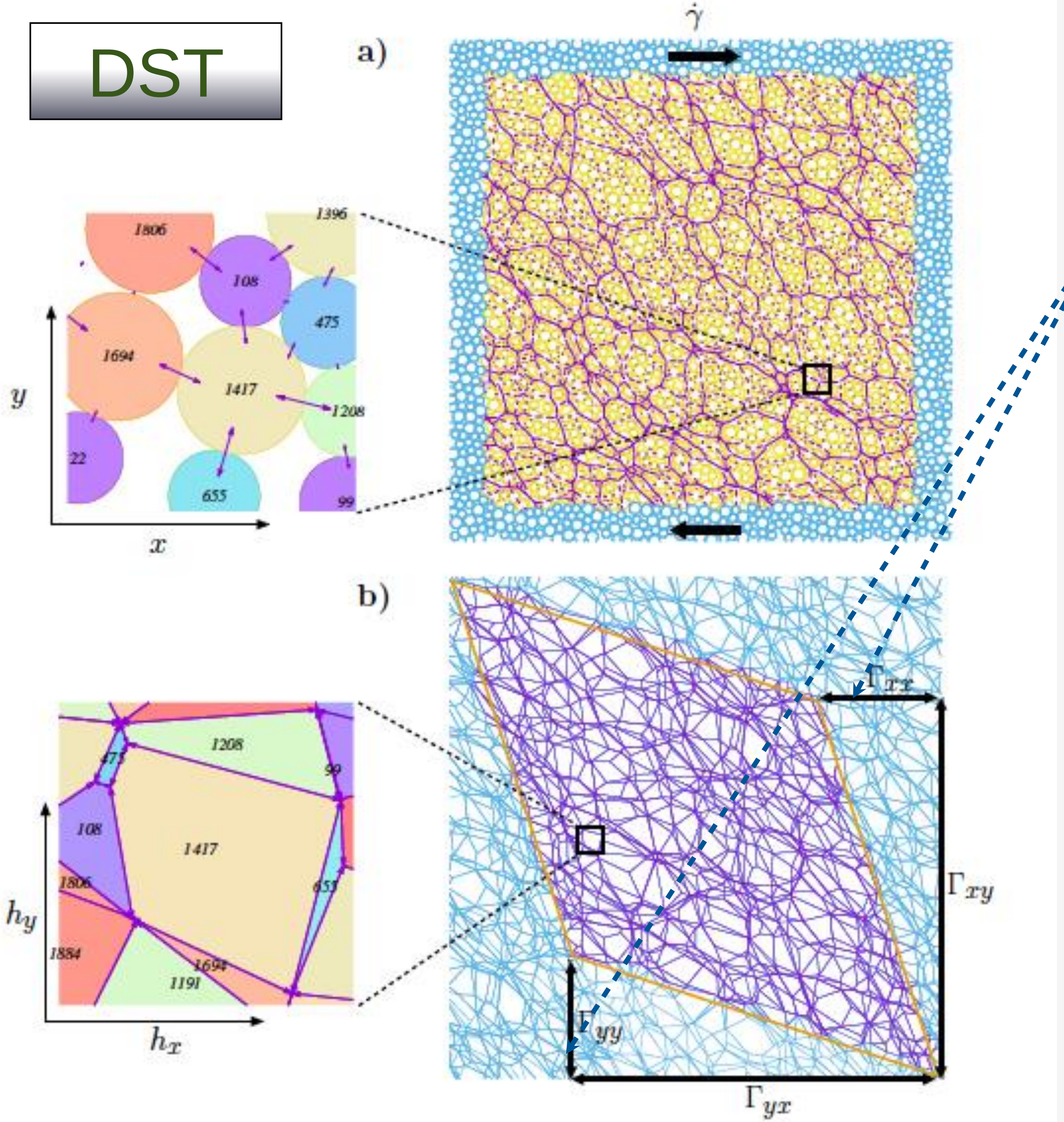
(0.8, 1)



(0.8, 100)



The set of points is represented by "height vectors" : $\{\vec{h}_i\}$



Shape can Fluctuate

Fixed in stress-controlled simulations

$$\mu = \frac{\tau}{P} = \frac{\sqrt{(N_1)^2 + 4\sigma^2}}{2P}$$

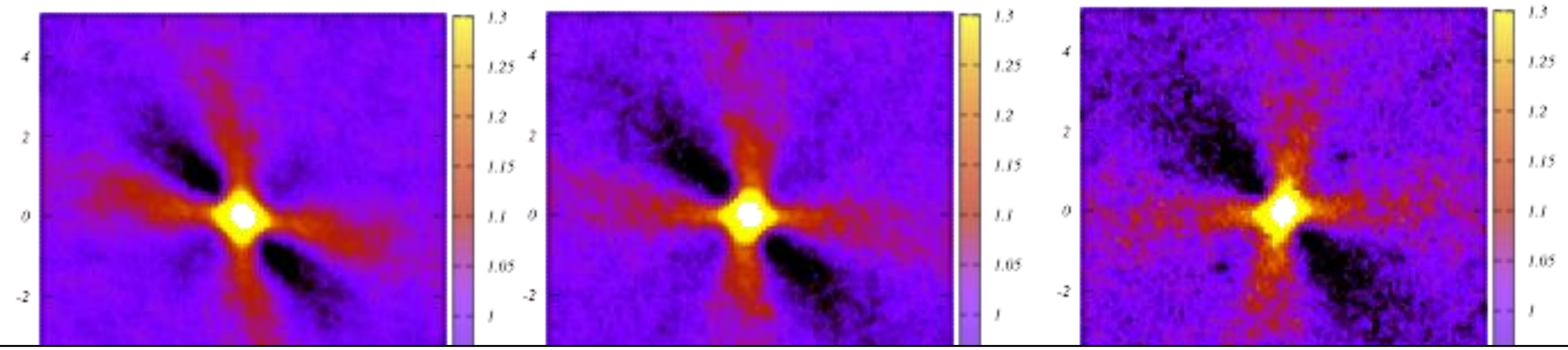
$$P = \frac{\Gamma_{yx} - \Gamma_{xy}}{2}$$

$$N_1 = \Gamma_{yx} + \Gamma_{xy}$$

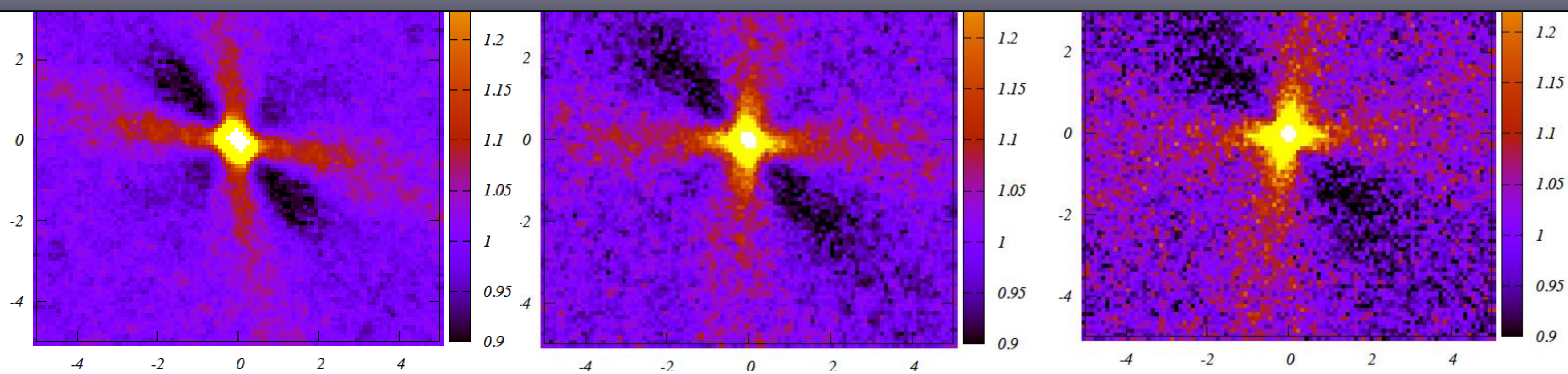
For simplicity, let's set normal stress difference to zero, then area of the box, A, is the single shape parameter.

$$\boxed{A = \sigma^2\left(\frac{1}{\mu^2} - 1\right)}$$

DST: Pair Correlation Functions



Can these changes in microscopic correlations in force-space lead to changes in $\mu(\sigma, \phi)$?



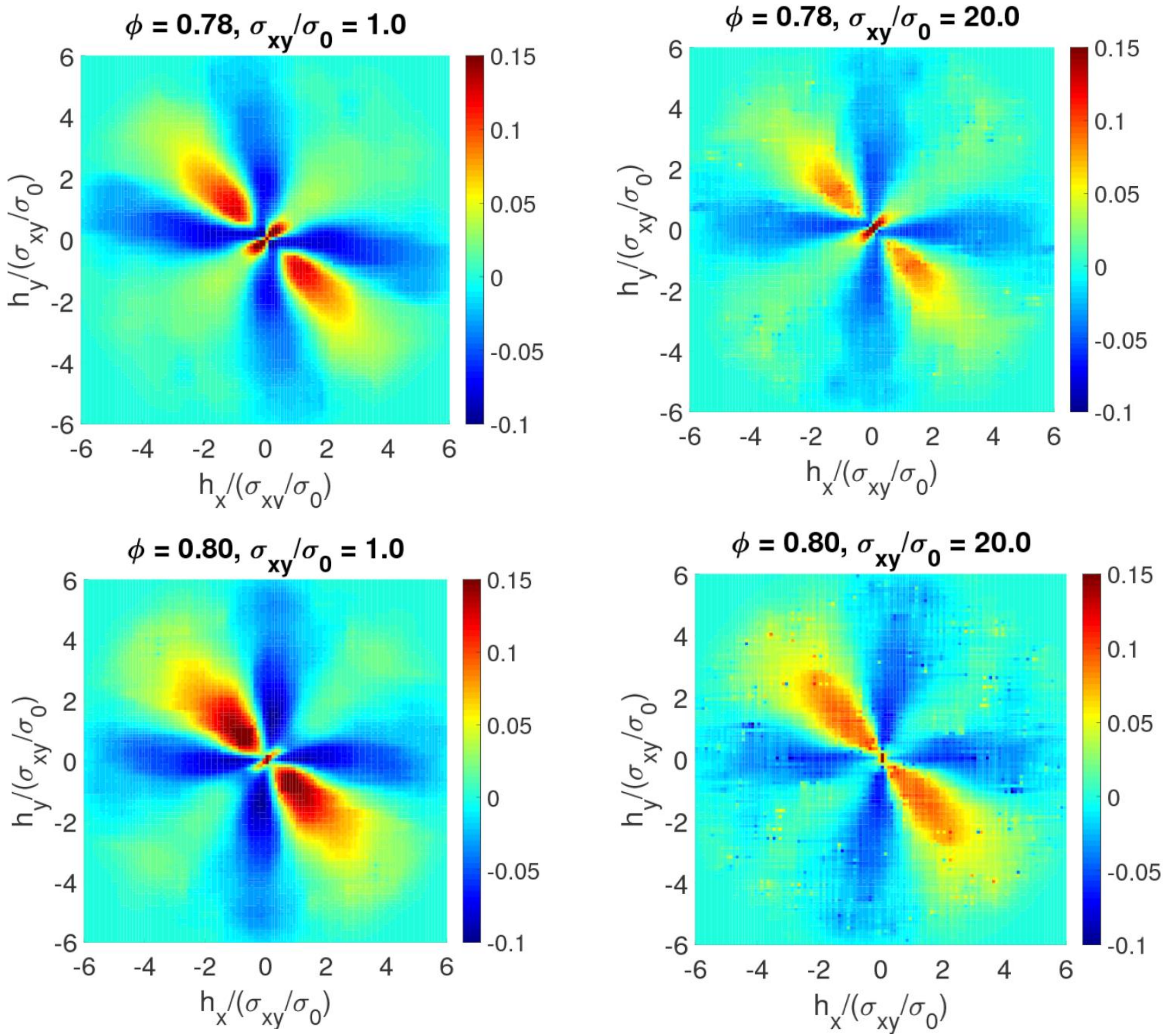
(0.76, 10) (0.78, 10) (0.8, 10)

blue-black regions: statistically larger forces, primarily along the compressive direction

yellow-red regions: statistically smaller forces, rotates with increasing packing fraction

with increasing stress: contrast decreases

DST: Pair Potentials

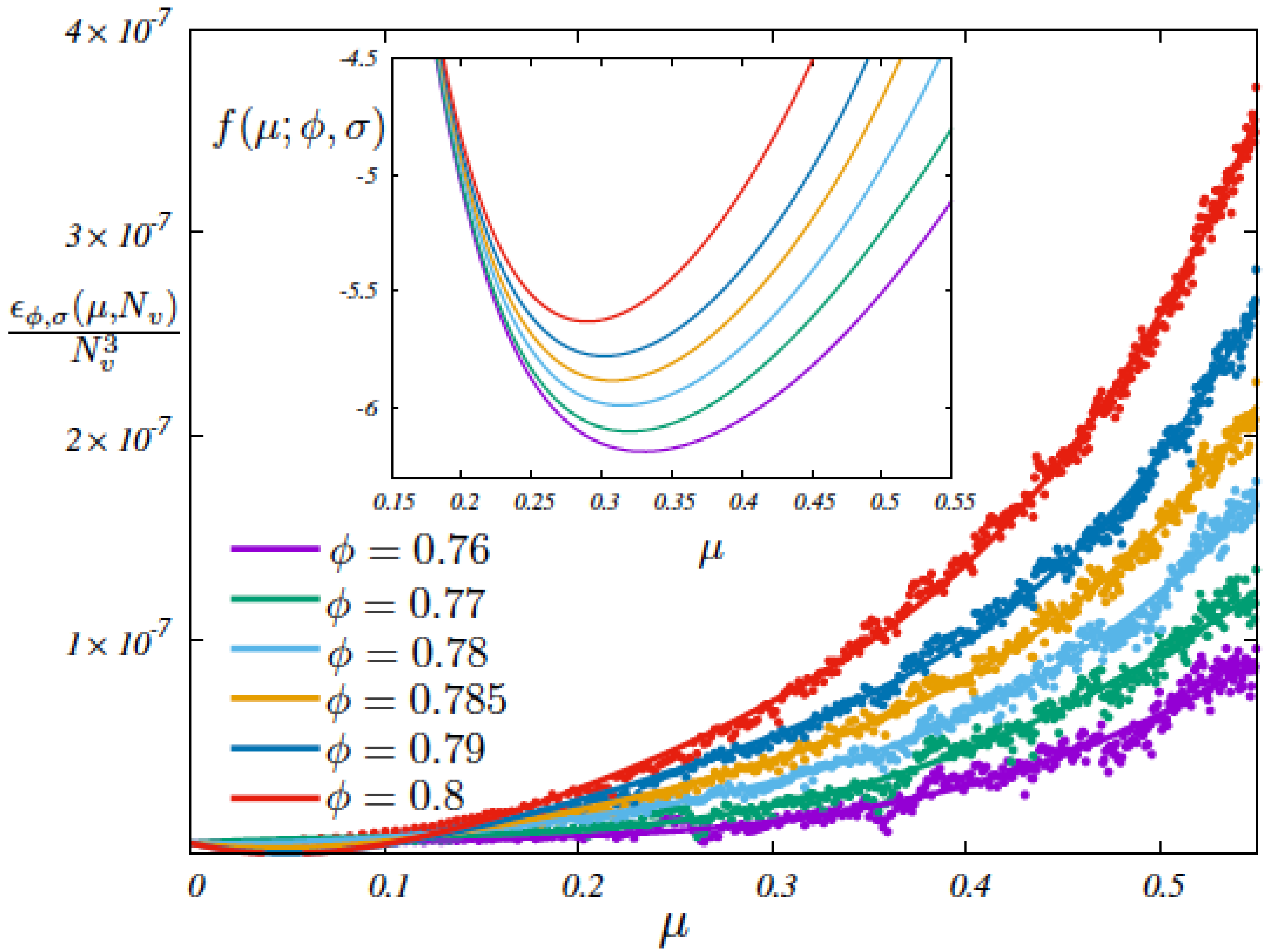


ST: Statistical Mechanics based on Edwards ensembl

No assumption about equiprobability: measure obtained from pair potential

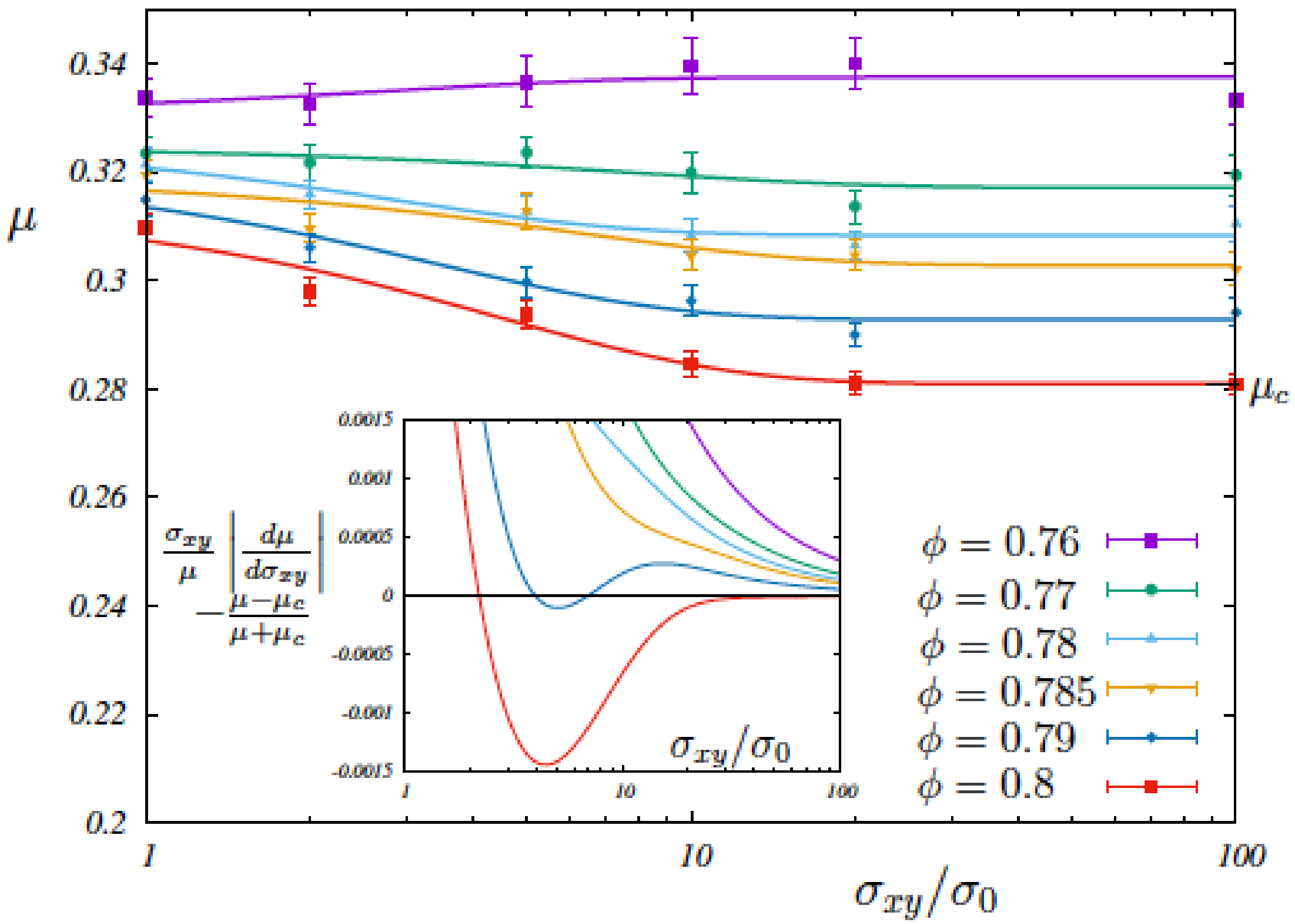
$$Z_{\phi,\sigma} = \frac{1}{N_v!}\int_0^\infty dA \exp\left(-N_v f_p^* A\right) \times \underbrace{\int_A \prod_{i=1}^{N_v} d\vec{h}_i \exp\left(-\sum_{i,j} V_{\phi,\sigma}(\vec{h}_i - \vec{h}_j)\right)}_{A^{N_v}\exp(-\epsilon_{\phi,\sigma}(A,N_v))}$$

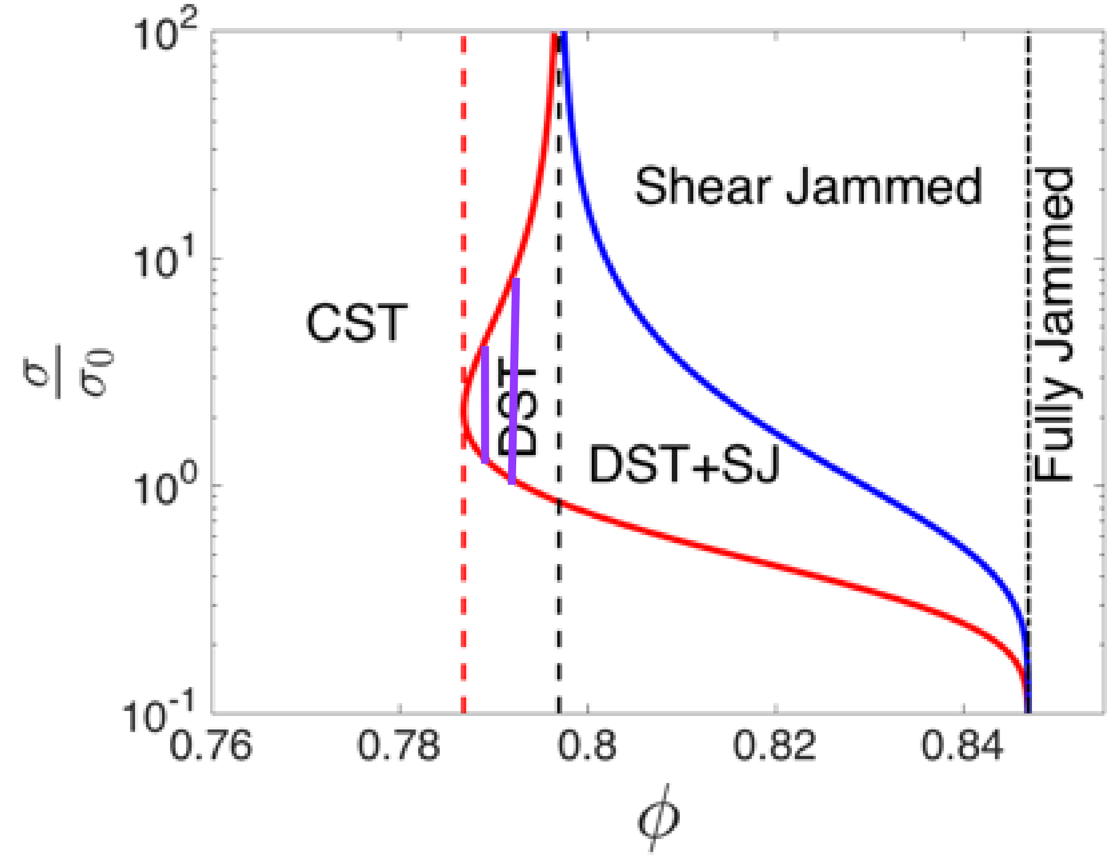
$$= \int_0^\infty dA \exp(-\mathcal{F}_{A;\phi,\sigma}).$$



$$\boxed{A = \sigma^2\left(\frac{1}{\mu^2} - 1\right)}$$

Results for stress anisotropy: Based on a theory of probability distributions from an effective pair potential in force space





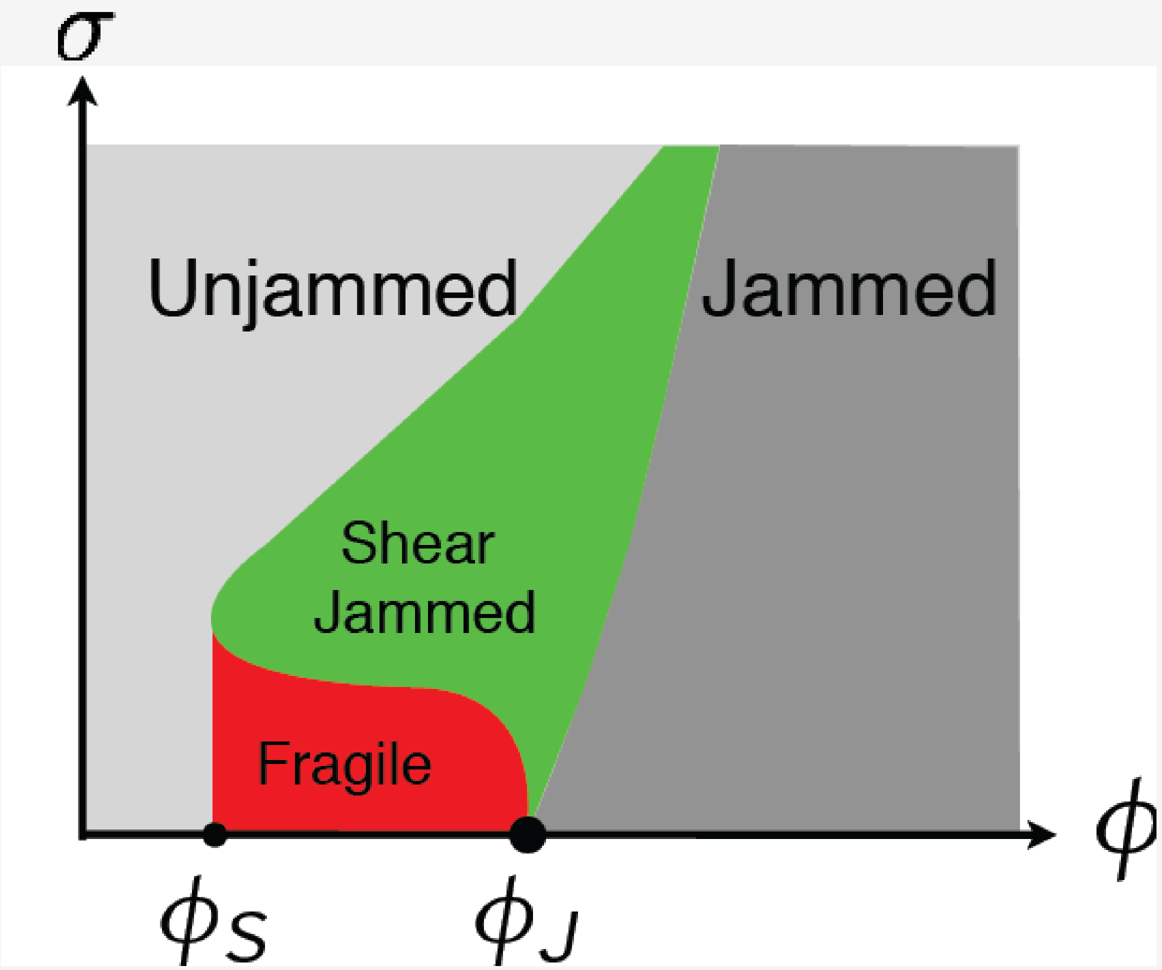
Get DST ~ 0.785-0.79 packing fraction: at 0.785 (0.5-2) & at 0.79 (1.5 - 6)

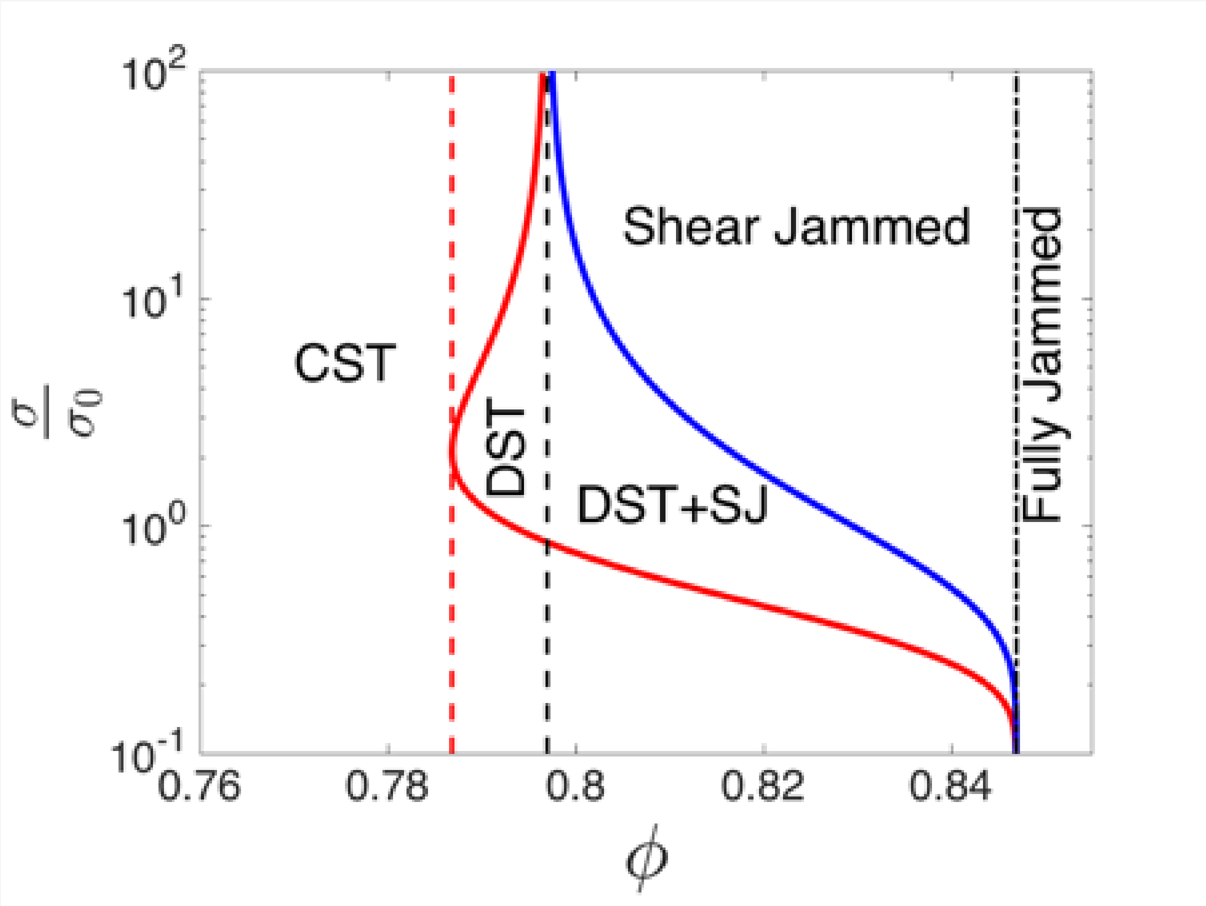
More importantly: we show that there is a definitive change in “microstructure” between the low and high viscosity states

in force space

Shear Jamming and Shear Thickening:

In transitions driven by friction





Force space reveals changes in correlations

Stress Ensemble for dynamically evolving steady states

Microscopic Theory for the Pair Correlations ?