Particle filters in high-dimensions

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Particle filtering

Let $\mathcal{X}$ be the state-space, and $\mathcal{Y}$ be the observation space of a state space model. We assume the dynamics are as follows:

$$X_t = f(X_{t-1}) + \zeta_t$$
$$Y_t = h(X_t) + \eta_t,$$

where ζ_t and η_t are independent random variables, and $X_0 \sim \mu_0$.

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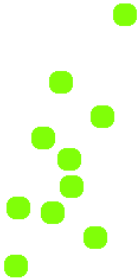
Particle filtering achieves this using a particle approximation to the true distribution.

Particle filtering - applications

- Climate modelling
- Oceanography
- Multi-target tracking

Particle filtering



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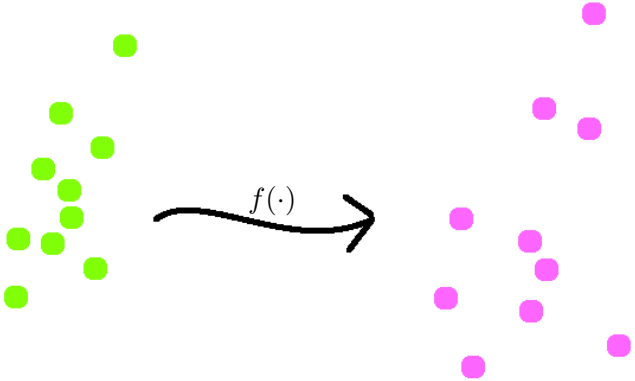
Particle filtering



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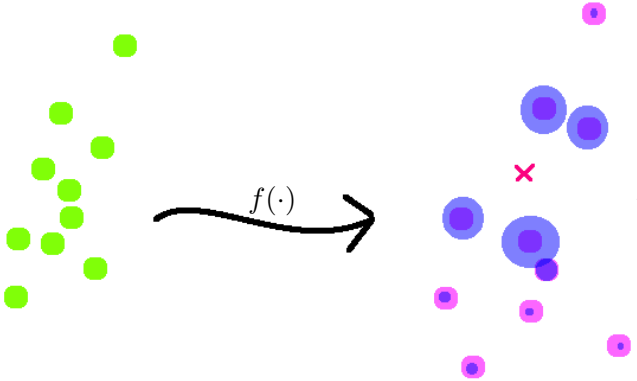
$\{X_{1,i}\}_{i=1}^N$

Particle filtering

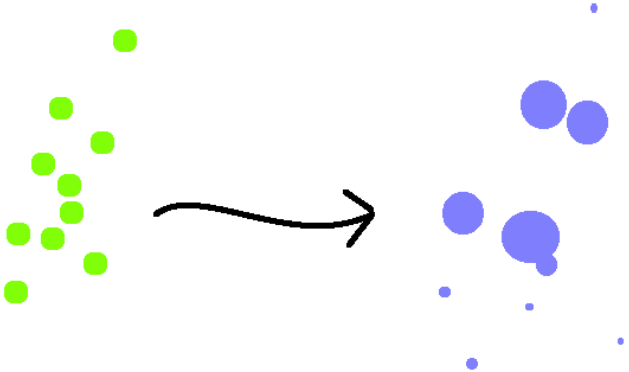


$w_i \propto L(X_{1,i}|y_1)$

$\{X_{0,i}\}_{i=1}^N \sim \mu_0$

$\{X_{1,i}, w_i\}_{i=1}^N$

Particle filtering



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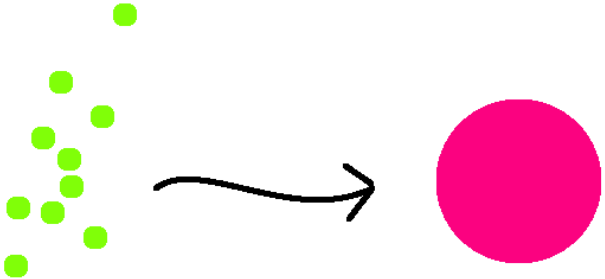
Particle filtering - in high dimensions



$\{X_{0,i}\}_{i=1}^N \sim \mu_0$

$\{X_{1,i}, w_i\}_{i=1}^N$

Particle filtering - the ideal case

$\{X_{0,i}\}_{i=1}^N \sim \mu_0$

$\{X_{1,i}, w_i = \frac{1}{N}\}_{i=1}^N$

Transport maps

Ideally, one wants to find a transport map $T_t : \mathcal{X} \to \mathcal{X}$ such that, if $\{\hat{X}_i\}_{i=1}^N$ is a sample from the prior, then $\{T_t(\hat{X}_i)\}_{i=1}^N$ is a sample from the posterior (the filtering distribution).

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This may be achieved by finding a flow f which solves the PDE $\partial_t \pi_t = -\nabla \cdot (\pi_t f)$, where π_t is a curve of measures with π_0 being the prior and π_1 being the posterior. [Heng et al. 2015]

Then a transport map is given by the final location of a particle X_i under the flow f.

Future research

To tackle the challenges which particle filters face in high-dimensional settings, promising research directions include:

- Improving on inaccurate (but fast) approximations obtained using Ensemble Kalman Filters.
- Exploit time-forgetting properties and space-forgetting properties to accelerate existing methods.
- ...

Thank you for listening!