Diffeomorphic Image Registration Models by New Constraints

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Outline

1. Introduction
2. Diffeomorphic Image Registration Models
3. New Constraint
4. Conclusion and Future Work

Image Registration and Its Variational Framework

Image Registration

Image registration is to find an optimal geometric transformation between corresponding image data. In practice, the concrete type of the geometric transformation as well as the notions of optimal and corresponding depend on the specific application.

Variational Framework

Given two images R, T, and a positive regularizing parameter $\alpha \in \mathbb{R}^+$, find a deformation $\mathbf{u}$, such that

$$\min_{\mathbf{u}} \mathcal{J}[\mathbf{u}] := \mathcal{D}[R, T(\mathbf{x} + \mathbf{u})] + \alpha \mathcal{S}[\mathbf{u}].$$

where $\mathbf{u}(\mathbf{x}) = \mathbf{y}(\mathbf{x}) - \mathbf{x}$.

Modersitzki J. Numerical methods for image registration[M]. Oxford University Press on Demand, 2004.

Diffusion Model

Diffusion Model

$$\min_{\mathbf{u}} \frac{1}{2}\int_\Omega (T(\mathbf{x}+\mathbf{u}) - R)^2 \mathrm{d}\mathbf{x} + \frac{\alpha}{2}\int_\Omega \sum_{l=1}^{2} |\nabla u_l|^2 \mathrm{d}\mathbf{x}$$

Euler-Lagrange Equation of Diffusion Model

$$(T(\mathbf{x}+\mathbf{u}) - R)\nabla_{\mathbf{u}} T(\mathbf{x}+\mathbf{u}) - \alpha\Delta\mathbf{u} = 0$$

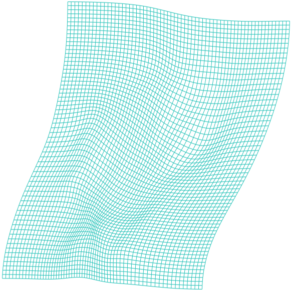
subject to $\langle \nabla u_l, \mathbf{n}\rangle = 0$ on $\partial\Omega$ for $l = 1, 2$.

Modersitzki J. Numerical methods for image registration[M]. Oxford University Press on Demand, 2004.

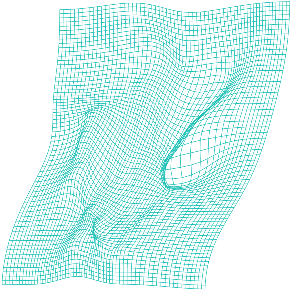
Chumchob N, Chen K. A robust multigrid approach for variational image registration models[J]. Journal of Computational and Applied Mathematics, 2011, 236(5): 653-674.

Diffeomorphic Mapping

Many relevant applications. e.g., in medical imaging, require that plausible transformations are diffeomorphic, i.e., smooth mappings with a smooth inverse.



(a) smooth mapping



(b) mapping with foldings

Diffeomorphic Mapping

How to control the mapping?

- control the Jacobian determinant of the transformation ($\det(J_{\mathbf{y}})$)

$$\min_{\mathbf{u}} \mathcal{J}[\mathbf{u}] := \mathcal{D}[R, T(\mathbf{x}+\mathbf{u})] + \alpha\mathcal{S}[\mathbf{u}] + \beta\mathcal{S}_1[\mathbf{y}]$$

- Large Deformation Diffeomorphic Metric Mapping (LDDMM)

$$\min_{\mathcal{T},v} \mathcal{D}(\mathcal{T}(\cdot,1),R) + \alpha\mathcal{S}(v)$$

$$\text{s.t. } \partial_t\mathcal{T}(\mathbf{x},t) + v(\mathbf{x},t)\cdot\nabla\mathcal{T}(\mathbf{x},t) = 0 \quad \text{and} \quad \mathcal{T}(\mathbf{x},0) = T,$$

where $v : \Omega\times[0,1]\to\mathbb{R}^2$ is the velocity and $\mathcal{T} : \Omega\times[0,1]\to\mathbb{R}$ is a series of images.

Burger M, Modersitzki J, Ruthotto L. A hyperelastic regularization energy for image registration[J]. SIAM Journal on Scientific Computing, 2013, 35(1): B132-B148.

Dupuis P, Grenander U, Miller M I. Variational problems on flows of diffeomorphisms for image matching[J]. Quarterly of applied mathematics, 1998: 587-600.

Control the Beltrami Coefficient

Quasi-conformal mapping, Beltrami equation and Beltrami coefficient

$f : \mathbb{C} \to \mathbb{C}$ is quasi-conformal provided that it satisfies the following Beltrami equation:

$$\frac{\partial f}{\partial \bar{z}} = \mu(f) \frac{\partial f}{\partial z}$$

for some complex-valued function μ satisfying $\|\mu\|_\infty < 1$.

QCHR (Lam and Lui 2014)

$$\min_{\mathbf{y}} \int_\Omega |\nabla \mu(\mathbf{y})|^2 + \alpha \int_\Omega |\mu(\mathbf{y})|^p + \beta \int_\Omega (T(\mathbf{y}) - R)^2$$

subject to

- $\|\mu(\mathbf{y})\|_\infty < 1$ (bijectivity)
- $\mathbf{y}(p_i) = q_i$ for $1 \le i \le m$ (Landmark constraints)

Lam K C, Lui L M. Landmark-and intensity-based registration with large deformations via quasi-conformal maps[J]. SIAM Journal on Imaging Sciences, 2014, 7(4): 2364-2392.

New Constraint and Model

New Regularizer

$$|\mu(\mathbf{y})|^2 = \frac{(\partial_{x_1} y_1 - \partial_{x_2} y_2)^2 + (\partial_{x_1} y_2 + \partial_{x_2} y_1)^2}{(\partial_{x_1} y_1 + \partial_{x_2} y_2)^2 + (\partial_{x_2} y_1 - \partial_{x_2} y_1)^2} = \frac{|\nabla \mathbf{y}|^2 - 2\det(J_{\mathbf{y}})}{|\nabla \mathbf{y}|^2 + 2\det(J_{\mathbf{y}})}.$$

New Model

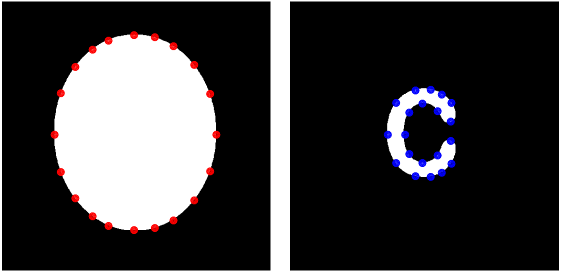
$$\min_{\mathbf{u}} \frac{1}{2}\int_\Omega (T(\mathbf{x}+\mathbf{u}) - R)^2 \mathrm{d}\mathbf{x} + \frac{\alpha}{2}\int_\Omega \sum_{l=1}^{2} |\nabla u_l|^2 \mathrm{d}\mathbf{x} + \beta \int_\Omega \phi(|\mu|^2)\mathrm{d}\mathbf{x}$$

Advantages of New Model

1. Control the Jacobian determinant of the transformation indirectly
2. Reduce unknowns and simplify the algorithm compared with QCHR

Zhang D, Chen K. A Novel Diffeomorphic Model for Image Registration and Its Algorithm[J]. Journal of Mathematical Imaging and Vision, 2018: 1-23.

Numerical Result: Example 1



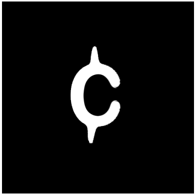
(c) Template T (d) Reference R

Numerical Result: Example 1

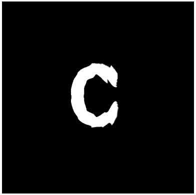
(e) our model

(f) Hyperelastic model

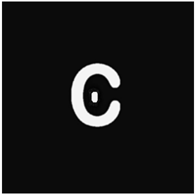
(g) LDDMM



(h) DDemons

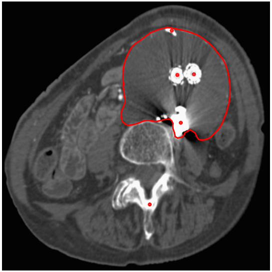


(i) QCHR

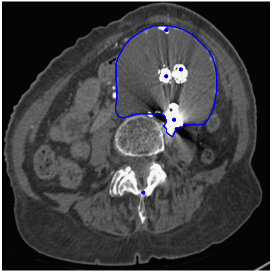


(j) Diffusion model

Numerical Result: Example 2



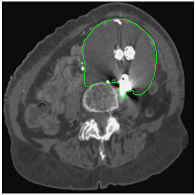
(k) Template T



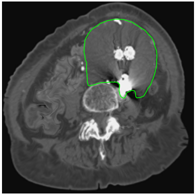
(l) Reference R

The pair of anonymized images are from the Royal Liverpool University Hospital.

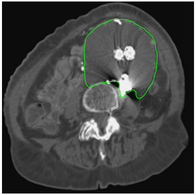
Numerical Result: Example 2



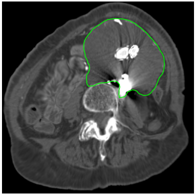
(m) Our Model



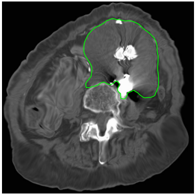
(n) Hyperelastic model



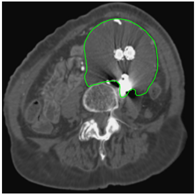
(o) LDDMM



(p) DDemons



(q) QCHR



(r) Diffusion model

Conclusion and Future Work

Conclusion:

1. Propose a new constraint based on quasi-conformal theory
2. New model based this new constraint can produce accurate and diffeomorphic registrations.

Future Work:

1. Develop the fast solver
2. Consider how to extend our idea to 3D registration
3. Try to apply our idea to multi-modality registration

Thank You