Hydraulic Modelling of Collection Networks
Study Group 2: Maths Foresees project report

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Abstract

This report describes the response to the 'hydraulic modelling of collection networks for civil engineering applications' challenge set by Sweco at the second Environmental Modelling in Industry Study Group. The solution was developed in workshops hosted by the Turing Gateway to Mathematics at the Newton Institute in Cambridge on 3-6 April 2017. The industry standard modelling practice of omitting the explicit representation of the collection network, road gullies for example, was examined by the team. This approach was found to significantly underestimate the exchange of water between 2D and 1D model domains. A solution was proposed in which the gullies are replaced by a line source/sink of carefully chosen intensity in order to represent the lumped behaviour of multiple road gullies. Two separate 2D surface models, based on the full shallow water equations and their diffusive wave approximation respectively, were used to test the solution. By comparing the results of simulations in which the gully sinks are explicitly modelled with those containing a line sink the performance of the parametrisation was found to be excellent. Future test cases are proposed, most importantly involving flows with strong coupling between the 2D surface flow and 1D sewer flow, which should be carried out before implementation of the parametrisation is recommended.
1 Introduction to the challenge

Integrated hydrodynamic models, that capture the behaviour of the coupled sewer network and surface flows, are essential engineering tools for developing urban flood management interventions. Examples of 2D surface flow models developed in the UK are Urban Inundation Model (UIM) (Chen et al., 2007; Leandro et al., 2009), JFLOW (Yu and Lane, 2006), LisFLOOD-FP (Neal et al., 2011), HiPIMS (Liang and Smith, 2015; Xia et al., 2017), CADDIES Weighted Cellular Automata 2D (WCA2D) Guidolin et al. (2016), and InfoWorks ICM (Integrated Catchment Modelling) (Innovyze, 2018). Similar models have been developed across the world (e.g. Fan et al., 2017), see Henonin et al. (2013) for a comprehensive review.

The equation set used to describe the surface flow is usually based on the fully nonlinear shallow water equations (SWE hereafter) (e.g. Hi-PIMS) or a high-friction approximation of the SWE known as the diffusive wave approximation (e.g. UIM and JFLOW). Numerical implementations vary widely and utilise various further approximations, for example the WCA2D model uses simple transition rules rather than the full SWE to reduce the computational time eightfold with minimal loss in accuracy, and the LisFLOOD-FP model includes a number of numerical schemes to simulate the propagation of flood waves along channels (sometimes using the ‘kinematic wave’ approximation in which the water slope term is also neglected) and across 2D floodplains.

The sub-surface or sewer network flows are generally modelled as a 1D network of pipe flows, with the flow in each sewer pipe network usually described by the Saint-Venant equations. Results are used to calculate the rainfall-runoff hydrographs and the flow conditions in the drainage network. Popular urban sewer network models include EPA SWMM (Storm Water Management Model) (Rossman, 2015), which is coupled to numerous surface flow models including InfoWorks ICM, MOUSE (Modelling of Urban Sewers) (Hernebring et al., 2002), and SIPSON (Simulation of Interaction between Pipe flow and Surface Overland flow in Networks) (Djordjević et al., 2005) which has been coupled to UIM. An in-depth recent review of surface and sub-surface models is provided by Teng et al. (2017). The sub-surface and surface models are coupled together by a semi-empirical treatment of the flow at inlets and outlets (see e.g. Chen et al., 2007).

InfoWorks ICM developed by Innovyze has been widely used by engineering consultancies, including Sweco, to quantify urban flood risk for a broad range of clients, including: local government, executive agencies, statutory corporations, public utility providers, contractors, consultants, and developers. InfoWorks ICM couples a 1D subsurface sewer network model and a 2D street-level surface water model. The two model domains are typically linked at manholes. The physical link between the sewer and the surface is referred to as the collection network, and consists of a large number of road drains (gullies), manholes, and the associated pipework that delivers water to the main sewers. In practice, the largest part of the collection network, gullies, are not represented in the InfoWorks ICM model due to the computational and resourcing overhead. Several modelling problems occur due the
absence of gullies in the model. These are described in the original challenge document as follows:

1. Surface water is collected by, and issues from, manholes rather than the collection network.
2. Collection and flooding occurs at the wrong locations.
3. Not enough water is collected. For example, manholes become inlet limited during short heavy rainfall events.
4. Storage and attenuation associated with the collection network is not explicitly accounted for.
5. Surface water pools indefinitely rather than charging the network during the recession of a storm.

The results from a typical Infoworks ICM simulation can be seen in Figure 1. Exchange between model domains occurs at the manholes highlighted with concentric circles, and surface water flow can be determined from the direction of arrows. Water can be seen to pool in topographic lows that do not have a manhole to drain them. In reality surface water should recede though the collection network at this stage of the flood.

The two main reasons that gullies are not included in Infoworks ICM are:

- The lack of available gully survey data, including: location, level, type, dimension etc. It would be prohibitively expensive for an engineering consultancy company to attempt to compile such data.

- Explicit modelling of gullies would also add considerably to the computational cost of the model. Figure 2 shows a plan of sewers (blue) and a gully network (green) under a main road. In InfoWorks ICM only the main sewer (central blue line) and manholes would be represented. Including the gully and secondary sewer network explicitly would increase the computational cost manyfold, as many more nodes would be needed in the 1D collection network, and also indirectly as shorter time-steps would be necessary to resolve the flow in the smaller gully channels.

The Challenge: The limitations of the industry standard modelling package described above have been identified by the industry sponsor over many years of experience in flood risk management using Infoworks ICM. This practical experience has led Sweco to set the following challenge, which can be summarised as:

- Based on the premise that explicit modelling of the gullies is not possible, find a method of parametrising the influence of gullies in the collection network without resolving them explicitly.
Figure 1: This figure shows a typical Infoworks ICM simulation where exchange between model domains occurs at the manholes highlighted with concentric circles, and surface water flow can be determined from the direction of arrows. Water can be seen to pool in topographic lows that do not have a manhole to drain them. In reality surface water should recede though the collection network at this stage of the flood.

Figure 2: A typical collection network (green lines) and trunk surface water sewer (blue lines).

In Section 2, the proposed solution to the challenge is described together with the method of testing this solution, in broad conceptual terms. In section 3 the two models
used to test our idea, together with their underlying physics and engineering, are described in detail. In section 4, the results of model experiments used to test the solution are described, while in section 5 some further suggestions are made, and in 6 conclusions are drawn.

2 The proposed solution and the idealised model testbed approach

Proposed solution: The proposed solution was to parametrise the gully network in the model by a network of line source/sinks, located along the edges of the roads. The idea was to design the line source/sink profiles so that, over a suitably averaged length of road, they have identical inflow/outflow properties as the set of gullies they are replacing. The key advantage of this solution was that it required very little knowledge of the details of the gullies (e.g., their exact locations). In fact, only the (approximate) spatial density and size of the gullies was needed.

Model testbed approach: In order to test the utility of the proposed solution, a simple model problem which captured the key elements of surface flow/sub-surface flow interaction was designed. The key performance indicator was whether or not a simulation with a suitably designed line-source/sink can accurately reproduce the model behaviour when the gully network was resolved explicitly.

Our ideal model testbed coupled a two-dimensional (2D) surface model based on the SWE to a one-dimensional (1D) sewer network model, based on the Saint-Venant equations. Mathematical details of this model have been provided in Section 3. This set-up captures the key fluid dynamics modelled by the InfoWorks ICM model. The coupling between the surface and sewer network models turned out to be too difficult to accomplish in the limited time period available to the study group (3 days). Hence, efforts were concentrated on one key aspect of the problem, namely the identification of a parametrisation for the gullies in the case where they act as ‘sinks’ only. Effectively the test bed makes the approximation that the sewers have infinite capacity. Subsequently the group produced two 2D surface models. The first was a fully nonlinear 2D SWE model based on a finite volume code developed at the University of Warwick (for details see Nwaigwe, 2016). The second was a 2D diffusive wave model based on finite difference methods, similar to the basic dynamical set-up UIM (Chen et al., 2007) or JFLOW (Yu and Lane, 2006). The second model was developed in parallel as it was easier to couple to a 1D drainage network for future applications and thus, test different flow and coupling scenarios.

A straight road configuration, with a constant gradient of 0.02, was selected as our model testbed, see Figure 3. The road was chosen to be 100 m long and features three manholes, separated equidistantly by 30 m and a total of 20 gullies (i.e. 10 on each side of the road) positioned 10 meters apart. Note that this ratio of gullies to manholes is roughly
consistent with Figure ???. The road orographic profile including raised pavements and a road surface with camber was modelled as seen in Figure 4.

![2D Surface of road and 1D Sewer Network](image)

Figure 3: Graphic representation of 2D surface model and below ground 1D sewer model.

![Orographic profile](image)

Figure 4: Orographic profile of the pavements and road surface configuration.

### 3 Model descriptions

This section presents the mathematical details of the two surface water models, used to produce the results to be discussed in Section 4 below. The full details for coupling of the models to the 1D sewer network model have also been provided.

#### 3.1 Description of the 1D sewer model and 2D surface models

InfoWorks ICM integrates 1D and 2D hydrodynamic models, both the above- and below-ground elements of catchments. The initial approach was to model the collection network using the SWE in 2D to represent surface water flow above the ground, and connect this to a 1D SWE representing the sewer network under the ground, see Figure 3.
While the results produced by the group were produced using uncoupled 2D surface water models, further work was undertaken on the 1D sewer network model and coupling with 2D surface model. Hence, the description of the 1D model has been included in this section, along with the 2D shallow water model.

### 3.1.1 1D sewer network modelling

The 1D sewer network is modelled using the Saint-Venant equations (1)-(2)

\[
\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0, \tag{1}
\]

\[
\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left( Q^2 A \right) + gA \left( \cos \theta \frac{\partial h_{1D}}{\partial x} - S_0 + \frac{Q |Q|}{K^2} \right) = 0. \tag{2}
\]

The quantities appearing in (1)-(2) are where
- \( Q \) - discharge (m³/s),
- \( A \) - cross-sectional area (m²),
- \( h_{1D} \) - free-surface height (m),
- \( g \) - acceleration due to gravity (m²/s),
- \( \theta \) - angle of ground with horizontal (degrees),
- \( S_0 \) - slope of ground,
- \( K \) - conveyance (m³/s).

The sewer channel pipe here is represented in 1D and hence its shape is parametrised in the Saint-Venant equations (1)-(2) using the relationship \( A(h_{1D}) \), where \( h_{1D} \) is the free-surface height within the sewer channel and \( A \) is the corresponding cross-sectional area of the sewer pipe.

A sewer network is a closed environment by design, i.e. not a free surface problem which the Eqs. (1)-(2) solve, posing a problem when the sewers become 'filled'. This difficulty is circumvented using the well-known Preissmann open-slot concept (Cunge and Wegner, 1964, following a suggestion of Preissmann) allowing for the change in the dynamics from a free-surface flow to a filled-pipe flow. The Preissmann open-slot is a clever engineering fix (see Figure 5), where the pipe cross-section is extended upwards in a thin slot, allowing the whole problem to be solved as a free surface problem using Eqs. (1)-(2).

When the water level \( h_{1D} \) enters the virtual slot, the physical interpretation is that the pipe is filled, and the pressure at the top of the pipe is given by the hydrostatic pressure associated with the water in the slot. The great advantage of the Preissmann open-slot is that the virtual water level \( h_{1D} \) also gives the height of the water in manholes and gullies, allowing discharge to be easily modelled, as described below.
Figure 5: Schematic illustration of the Preissman open-slot sewer cross-section.

3.1.2 2D surface water modelling and exchange terms

The 2D surface flow is modelled by the full shallow water equations,

\[
\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = q_{1D},
\]

\[
\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left( hu^2 + \frac{gh^2}{2} \right) + \frac{\partial (huv)}{\partial y} = -gh(S_{0,x} - S_{f,x}) + q_{1D}u_{1D},
\]

\[
\frac{\partial (hv)}{\partial t} + \frac{\partial}{\partial y}\left( hv^2 + \frac{gh^2}{2} \right) + \frac{\partial (huv)}{\partial x} = -gh(S_{0,y} - S_{f,y}) + q_{1D}v_{1D}
\]

where

- \( h \) - water depth,
- \( u, v \) - velocities in \( x \) and \( y \) directions,
- \( S_{0,x}, S_{0,y} \) - ground slope in \( x \) and \( y \) directions,
- \( S_{f,x}, S_{f,y} \) - friction slopes in \( x \) and \( y \) directions,
- \( q_{1D} \) - source discharge per unit area,
- \( u_{1D}, v_{1D} \) - velocity components of source discharge in \( x \) and \( y \) directions.

The modelling of the fluid exchange occurring at inlets and outlets, between the surface flow and sewer network, is described in detail in Chen et al. (2007). Chen et al. describe how, depending on the relative heights of the flow on the surface versus that in the sewer network, the discharge \( q_{1D} \) can be modelled at a sewer opening located at height \( h_m \), with
perimeter $w$ and area $A_m$. Here we take $h_m = 0$ (ground level), non-zero $h_m$ corresponds to the situation where the manhole outlet is raised slightly above road level. The discharge is given by the following formulae, in which $h_u = \max(h, h_{1D})$ and $h_d = \min(h, h_{1D}),$

$$q_{1D} = \begin{cases} 
-c_w w \sqrt{2g} (h - h_m)^{\frac{3}{2}}, & h > h_m > h_{1D}, \\
-sgn(h - h_{1D}) c_w w \sqrt{2g} (h - h_m)^{\frac{1}{2}} (h_u - h_d), & h, h_{1D} > h_m, h_u < A_m/w \\
-sgn(h - h_{1D}) c_w A_m \sqrt{2g} (h_u - h_d)^{\frac{1}{2}}, & h, h_{1D} > h_m, h_u > A_m/w 
\end{cases}$$

The three cases correspond to

1. the weir discharge formula for flow into an opening (here $c_w = \frac{2}{3}$);
2. the corresponding formula for a ‘partially submerged’ weir, which includes the possibility of flow out of the hole;
3. the orifice discharge formula, which applies to the situation where the flow is deep enough, or sewer pressure high enough, for the inflow/outflow to span the entire manhole.

### 3.2 Proof of concept in a simplified setting: the diffusion wave approximation

To test the proposed gully parametrisation schemes a so-called ‘diffusion wave’ model, which is in fact widely used in operational flood modelling (e.g. Yu and Lane, 2006; Chen et al., 2007), has also been implemented. The model is valid when the main balance in the shallow-water momentum equations (4)-(5) is between the pressure gradient due to the interface slope and the friction terms, i.e. inertia, can be neglected completely.

#### 3.2.1 Set of Equations and range of validity

The equations of the diffusion wave model are given by

$$h_t + \nabla \cdot (uh) = -\sum_{i=1}^{N} Q_i \delta(x - x_i) + r(x, t), \quad (7)$$

$$u = \frac{h^{\frac{2}{3}}}{n} \frac{1}{|\nabla (b + h)|^{\frac{1}{2}}} \hat{u}, \quad (8)$$

$$\hat{u} = -\frac{\nabla (b + h)}{|\nabla (b + h)|}. \quad (9)$$
where \( h \) and \( \mathbf{u} = (u, v) \) are the water depth and velocity as in (3-5).

To obtain eqns. (7-9) it is necessary to make the following choices in eqns. (3-5)

\[
q_{1D} = -\sum_{i=1}^{N} Q_i \delta(x - x_i) + r(x, t) \tag{10}
\]

\[
(S_{0,x}, S_{0,y}) = \nabla b \tag{11}
\]

\[
(S_{f,x}, S_{f,y}) = -\frac{n^2 |\mathbf{u}| \mathbf{u}}{h^{4/3}} \tag{12}
\]

These choices can be justified as follows:

- The first term on the right hand side in eqn. (10) models an array of ‘small’ gullies and manholes, centred on \( x_i \), each having total discharge \( Q_i \). The second term \( r(x, t) \) is a smooth source per unit area (e.g. rain).

- It is natural to express the bed slope as the gradient of the bed height field \( b(x) \).

- This choice for the friction slope corresponds to Manning friction, which is a standard model for turbulent flow over a rough surface (variants of which apply to both channel and 2D flow). The constant \( n \) in Eq. (12) is known as the Gaukler-Manning coefficient, and its value has been found experimentally for a wide range of surfaces. It has dimensions: \( \text{Time} \times \text{Length}^{-1/3} \) (\( \text{sm}^{-1/3} \)).

Neglecting inertial terms in eqns. (4-5) now gives the balance

\[-\frac{n^2 |\mathbf{u}| \mathbf{u}}{h^{4/3}} = \nabla (b + h) \tag{13}\]

from which (8) is obtained.

The balance represented by eqn. (13) can be justified more formally by first non-dimensionalising eqns. (3)-(5), using a typical depth \( H \) as the units for \( h \), \( \sqrt{gH} \) for the velocity units, a typical length \( L \) over which the interface varies as the distance unit, and \( L/\sqrt{gH} \) as the timescale. The result is that the balance eqn. (13) is justified provided that

\[
\frac{L_f}{L} \ll 1, \quad \text{where} \quad L_f = \frac{H^{4/3}}{n^2 g} \tag{14}
\]

is the length scale above which frictional effects dominate in the flow. Taking \( n = 0.017 \) \( \text{sm}^{-1/3} \) as a typical Gaukler-Manning coefficient for asphalt, \( H = 0.3 \) m for a typical urban flood depth, and \( g = 9.81 \) \( \text{ms}^{-2} \) gives \( L_f \approx 70 \) m.
3.2.2 Numerical Implementation

Eqs. (7)-(9) constitute a nonlinear diffusion equation with various sources and sinks. Such an equation is straightforward to implement numerically using a finite volume method with a regular grid (equivalently, in this case, a finite difference method on a staggered grid). The following simple spatial and temporal discretisation is used,

\[ h_{m,n}^{(p+1)} = h_{m,n}^{(p)} - \Delta t \left( \frac{F_{m+1,n}^{(p)} - F_{m-1,n}^{(p)}}{\Delta x} + \frac{G_{m,n+1}^{(p)} - G_{m,n-1}^{(p)}}{\Delta y} \right) + \Delta t \frac{S_{m,n}}{\Delta x \Delta y}. \]  

(15)

Here \( h_{m,n}^{(p)} \) denotes the average layer depth in a rectangular cell with dimensions \( \Delta x \times \Delta y \) centred on \( (x_m, y_n) \) at time \( t_p \). The time step \( \Delta t \) satisfies \( t_{p+1} = t_p + \Delta t \). The term \( S_{m,n}^{(p)} \) is the source term integrated over the cell. Fluxes \((F, G) = \mathbf{u} h\) are evaluated at the cell edges, in which the subscript \( m+1, n \) corresponds to the boundary between the cell centred on \( (x_{m+1}, y_n) \) and that centred on \( (x_m, y_n) \). The discretisation used is

\[ F_{m+1,n} = -\frac{1}{n} \text{sgn} \left( \frac{s_{m+1,n} - s_{m,n}}{\Delta x} \right) \left| \frac{s_{m+1,n} - s_{m,n}}{\Delta x} \right|^{1/2} \left( \frac{h_{m+1,n} + h_{m,n}}{2} \right)^{5/3} \]  

(16)

\[ F_{m-1,n} = -\frac{1}{n} \text{sgn} \left( \frac{s_{m,n} - s_{m-1,n}}{\Delta x} \right) \left| \frac{s_{m,n} - s_{m-1,n}}{\Delta x} \right|^{1/2} \left( \frac{h_{m,n} + h_{m-1,n}}{2} \right)^{5/3} \]  

(17)

where \( s_{m,n} = b_{m,n} + h_{m,n} \), with analogous expressions for \( G \). The source terms \( S_{m,n} = Q_i \) in cells containing a manhole or gully, which are located at points \( x_i \), and are zero otherwise (in all of our integrations the ‘rain’ term \( r = 0 \)).

3.2.3 Initial and boundary conditions

The domain of integration is taken to be an \( x \)-periodic channel (for computational ease) with dimensions \( L_x \times L_y \), with impermeable sidewalls at \( y = 0, L_y \). The bottom topography \( b(x) \) is chosen to model a sloping road which has pavements to each side and a realistic camber. Analytically the function \( b \) is given by

\[
b(x, y) = -\alpha x + F(y), \quad F(y) = \begin{cases} H_p & 0 < y < \frac{1}{6}L_y \\ \frac{6H_p}{L_y^2}(y - \frac{1}{6}L_y)(\frac{5}{6}L_y - y) & \frac{1}{6}L_y < y < \frac{5}{6}L_y \\ H_p & \frac{5}{6}L_y < y < L_y \end{cases}
\]  

(18)

Here, \( H_p \) is the pavement height, and \( H_c \) is the maximum camber height.

The initial condition for our main integrations is

\[ h(x, y, 0) = H \exp \left( -(x - x_0)^2 / 2L_w^2 \right), \]  

(19)
representing a sudden influx of flood water of maximum depth $H$, e.g. from a burst pipe, spread over a horizontal scale $L_w$. During the course of the integrations this water, as should be expected, flows down the slope. Note that the initial conditions have a characteristic length scale which is somewhat less than the frictional length scale (taking $L = L_w$ gives $L_f/L \approx 14$), and as a result inertial effects should be important in the early stages of such a flow. In other words, the diffusion wave equation will not be quantitatively accurate at the beginning of the simulation. That said, the equation can still be expected to be qualitatively reasonable, and by the end of the simulations, when $L_f$ is effectively reduced due to the flood water depth decreasing, and $L$ increased by the flood spreading, it will be quantitatively accurate.

The physical and numerical parameters shared by the integrations are listed in Table 1. In the integrations with point sinks (all integrations except 'just road'), the manholes and gully sinks are implemented in a similar fashion. The sink strength $Q_i$ is given by the weir discharge formula, i.e. the first term in (6) with $h_m$ set to zero, that is, it is assumed that the capacity of the sewer system is large compared to the size of our flood. Manholes, when present, are placed every $L_m = 33$ m in the middle of the road and have perimeter $w = 2$ m. Gullies, when present, are placed every $L_y = 10$ m on each side of the road and have perimeter $w = 0.4$ m. The line sink is implemented by replacing the gullies with a large number of 'mini-gullies' each with perimeter $w_l = (L_g/\Delta x)w$ in every grid-cell intersecting the road edge. As a result, the influence of the individual gullies are effectively 'smeared' over a line parallel to the kerb. The operational advantage of the line sink is that it can be implemented without any knowledge of the actual gully locations, only requiring the pre-existing knowledge of the locations of the road edges.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$n$</td>
<td>0.017 sm$^{-1/3}$</td>
<td>Gaukler-Manning coefficient (asphalt)</td>
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<tr>
<td>$L_x$</td>
<td>100 m</td>
<td>Channel length</td>
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<tr>
<td>$L_y$</td>
<td>6 m</td>
<td>Channel width</td>
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<tr>
<td>$\alpha$</td>
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<td>Road slope</td>
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<tr>
<td>$H_p$</td>
<td>0.2 m</td>
<td>Pavement height</td>
</tr>
<tr>
<td>$H_c$</td>
<td>0.05 m</td>
<td>Max. camber height</td>
</tr>
<tr>
<td>$H$</td>
<td>0.3 m</td>
<td>Max. depth at $t = 0$</td>
</tr>
<tr>
<td>$L_w$</td>
<td>5 m</td>
<td>Flood extent at $t = 0$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>0.05 s</td>
<td>Time-step</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>0.5 m</td>
<td>Grid-spacing ($x$)</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>0.5 m</td>
<td>Grid-spacing ($y$)</td>
</tr>
<tr>
<td>$T_{\text{max}}$</td>
<td>30 s</td>
<td>Integration length</td>
</tr>
</tbody>
</table>

Table 1: Table listing physical and numerical parameter values (all integrations).
4 Results

To investigate the parametrisation of the drains the following two scenarios have been compared:

- Drains are resolved directly in the grid, denoted manholes+gullies. This is a base case in which the collection network was redesigned to include gullies. Recall that, for an operational model, this scenario is prohibitively laborious and expensive to implement.

- Drains are parametrised by specifying a simple line source term (e.g. averaged over a few drains), denoted manholes+line sinks.

In order to explore further the influence of the different representations of sinks, further runs are performed, including one with sinks, denoted just road, and one with only manholes as sinks on the road, denoted as with manholes.

4.1 Results using the fully nonlinear shallow water equations (SWE)

Using the finite volume model (see Nwaigwe, 2016), simulations were performed solving the fully nonlinear SWE, for the following cases: 3 manholes (3MH), 10 gullies (10G) and 100 gullies (100G), the latter being effectively a line sink in accordance with our parametrisation. In the 100 gullies simulation the perimeter of each gully is reduced by a factor of 10, so that the same total sink strength, compared to the 10 gullies simulation is maintained. Both the 10G and 100G simulations include the three manholes. The three sink configurations (3MH, 10G, 100G) were performed for two different initial flooding conditions. In the first, the whole 2D domain was assumed to be inundated while in the second the road’s surface configuration has been modelled to redirect the water to the gullies for the sake of an improved realistic approach. The results of our simulations is summarised in Table 2. It is

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>3MH</th>
<th>10G</th>
<th>100G</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D domain completely inundated</td>
<td>14.95</td>
<td>15.38</td>
<td>15.34</td>
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<tr>
<td>Percentage lost</td>
<td>77.3%</td>
<td>79.4%</td>
<td>79.3%</td>
</tr>
<tr>
<td>Re-think initial conditions: Road camber redirecting the flow to the gullies</td>
<td>3.1%</td>
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Table 2: Result summary of the SWE simulations for three drainage configurations (10 gullies, 100 gullies and 3 manhole) for 2 different initial conditions

clear that in the first case, in which the whole 2D domain has been completely inundated,
the water collected from a few manholes or a line sink is almost equal. For the second configuration, in which the water flow on the road is redirected to the gullies, we can confirm that the manholes are not as effective as the gullies in the drainage process. Note that the simulation animations can be viewed following this link Results presentation on Thursday, April 6th 2017 (10:32 - 11:34).

4.2 Results using the diffusion wave equation

Figure 6: Results from the diffusion wave equation simulations. Dotted curve: Width-averaged water depth (m) at $t = 0$ (all simulations). Solid curves: Width-averaged water depths at $t = 30$ s. Black: road only simulation. Red: Road with manholes. Blue: road with manholes and gullies. Green: Road with manholes and line sink. Red arrows show manhole locations, and blue arrows gully locations - see text for details.

Figure 6 shows results from the four main numerical experiments described in section 3.2.3: just road, manholes only, manholes+gullies, and manholes+line sinks.
In all four simulations the flood water runs down the slope as expected, travelling around 60 m over the 30 s integration. Comparing results from just road (black curve), manholes only (red), and manholes+gullies (blue) cases it is evident that the net effect of discharge through the manholes and gullies after $t = 30$ s is firstly to reduce the flood volume, and secondly to reduce the distance the flood travels down the road. A natural goal for any parameterisation of sinks is to quantitatively capture the extent of volume reduction and front speed reduction. Also shown in Figure 6 is the result from the manholes+line sinks simulation (green). It shows that parameterising the gullies by a line sink is remarkably accurate, with the only noticeable difference between the manholes+gullies and manholes+line sinks simulations being due to small deviations in $h$ at the gully locations in the former.

In summary, as for the SWE experiments, the diffusion wave experiments show that the line sink parametrisation of the gullies is very accurate under realistic parameter settings, at least for this flow set-up.

5 Further suggestions

Two separate 2D surface water models have been implemented in order to develop and test a line sink parametrisation to represent the sinks in the road from the gullies. Due to the time constraint we were not able to couple either of these models to the 1D sewer network and to further test and develop the parametrisation scheme. An important issue that has to be addressed is related to adding the drained surface water using the line sinks into the 1D sewer network model. Our suggestion is to add the aggregate sink volume around each manhole directly into the manhole connection, since adding a source term at each grid-point into the sewer network model would be computationally very expensive for any realistic model and further it could potentially make the whole system unstable. The next step would be to test the developed parametrisation scheme in a coupled surface water model and sewer network system.

6 Conclusions and outlook

To summarise, we have implemented two 2D surface water models; full shallow water equation model and a simplified diffusion wave equation model. Neither of these models were coupled to a 1D underground sewer network model due to the time constraints. However, a line sink representation was developed and implemented for both SWE and DWE models, which were run on an idealised road setting with pavements, road cumber, manholes, and gullies (or their representation).

We found that, for the given initial and boundary conditions, both models removed surface water through line-sinks (e.g. gully sink term averaged over a few drains) of the
same order as in the case of directly resolving gullies, thus, we found that parametrising gullies by a line sink produces very accurate results in comparison to directly resolving each gully in the model. Parametrisation did, however, produce a smoother solution, see Figure 6, since the sink term was distributed between all the neighbouring model cells. Finally, including only manholes as sinks in the surface water model clearly does not remove enough water from the road, which is an issue the InfoWorks ICM model is facing while a simple line sink parametrisation of gullies could solve this problem in this much more realistic model.

Naturally, the above conclusions come with many caveats. Before any attempt is made to implement the line sink method in an operational model, we would recommend further tests in idealised settings. In particular, the behaviour of the parametrisation in flows which capture the full range of coupling outcomes between the 2D surface flow and the 1D sewer network must be tested. Before operational implementation is considered, it is also necessary to give some thought to how to best obtain key parameters for the line sinks, such as average gully inlet perimeters, and gully spacings. Notwithstanding these caveats, we are confident that, properly implemented, our chosen method could improve the performance of any urban flood model in which gully representation is absent.

Acknowledgments

We acknowledge Albert Chen, who kindly and patiently answered our questions on the theory and practice of flood modelling, during the workshop. At the time of writing, a video presentation on the progress at the end of the workshop is available for viewing at, and further details on other aspects of the workshop are available on the Turing Gateway to Mathematics. The original problem specification supplied by Sweco is available on the same web page. We would like to thank the Turing Gateway to Mathematics and the Isaac Newton Institute for hosting the event and Maths Foresees team for organizing this stimulating workshop. The support of the EPSRC Living with Environmental Change - Maths Foresees Network is gratefully acknowledged.

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