Modelling Financial Uncertainty for Regulated Insurance Companies:

The Case of Equity Risk

Dr M. Derevyanko
Senior Risk Quant, Aviva Plc

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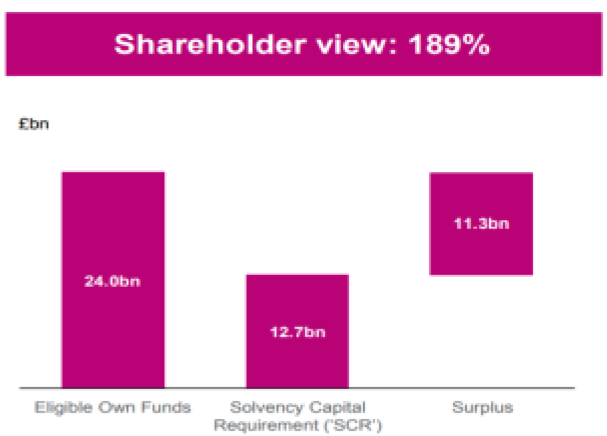
Regulatory environment for insurance companies

Solvency II regulation

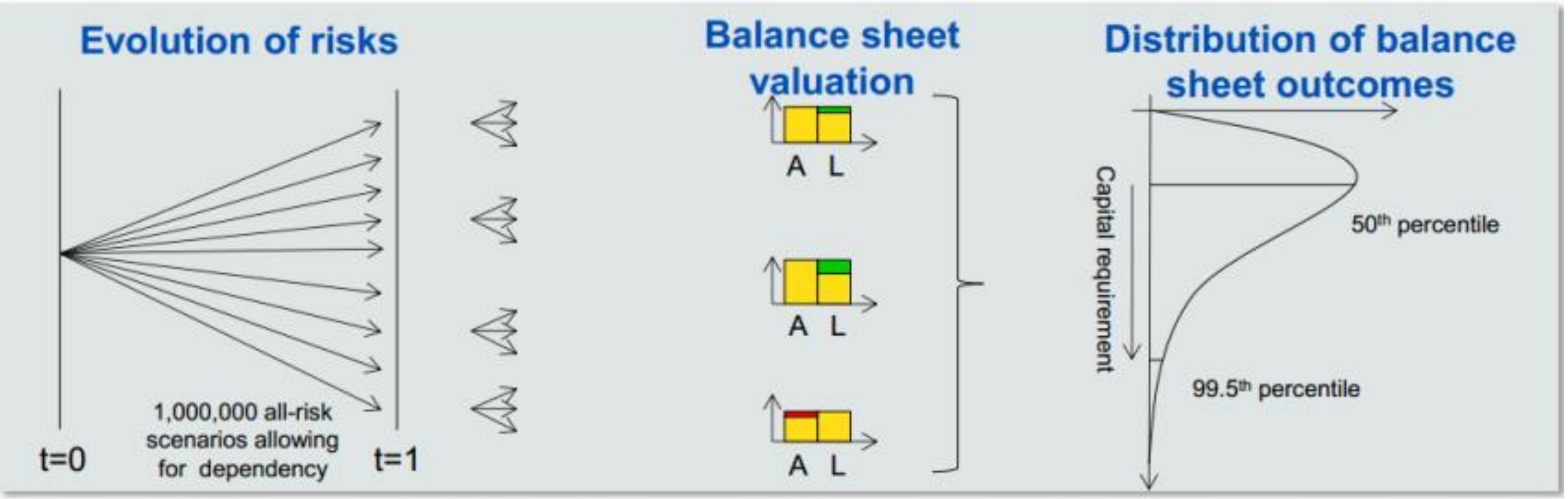
- It is a EU-wide set of rules (legislation) which defines how insurance companies are funded and governed;
- Its main objective is the adequate protection of policyholders and beneficiaries.
- It defines the Minimum and the Solvency Capital Requirements (SCR). The SCR level of capital is designed to absorb significant losses at a 99.5% confidence level over 1 year i.e. sufficient to cover liabilities following a 1-in-200 year event.
- It gives a choice of methods to calculate 1-in-200 capital requirements: the Standard Formula or Internal Model (or hybrid).
- Internal model is a risk management system to analyse the overall risk position, to quantify risks and to determine the economic capital.

Aviva Plc

- Multi-national life, general and health insurance business. The largest in the UK with 32m customers.
- Regulated by the PRA and under Solvency II regime (Internal Model).
- It is a complex business with c. £475 billion under management.
- Key risks include (in order of significance):
  - Market and credit (financial) risks.
  - Specific insurance risks.
  - Operational risk.
- Solvency capital ratio is 189% (YE16).



Internal Model: key elements



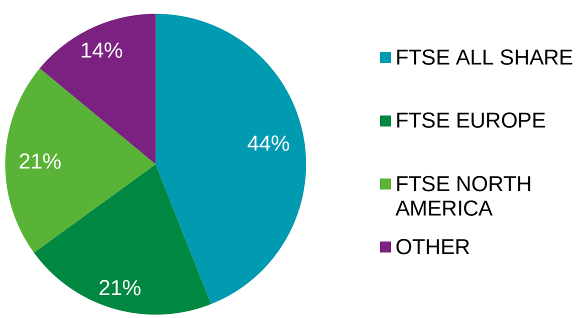
Risk Horizon	Balance Sheet Valuation	Calibration methodology	Risks to include	Quantification Methodology	Dependency and Aggregation
One year	Economic i.e. assets and liabilities are valued by using market consistent valuation framework	Value-at-Risk (99.5%)	• Market risks • Credit risks • Insurance risks • Operational risk	Monte Carlo simulation	Gaussian Copula with various adjustments

Challenges in modelling Equity Risk (and other risks)

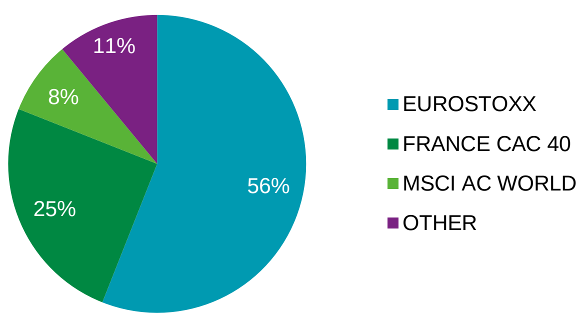
Definition: Equity risk is the risk of an adverse financial impact because of equity market movements.

Data length and frequency	We need to model annual returns (1 year VaR) but the amount of annual non-overlapping data is very limited. Modelling on 20-30 data points is meaningless.
Overlapping data	Traditional approach to risk calibration is to use overlapping data. However: • Window choice and frequency are (subjective) expert judgements. • High degree of autocorrelation (>95%) => hypothesis tests are biased. • Invalidates MLE fitting assumptions. • Goodness of Fit tests cannot be directly applied.
Dynamic SCR calculations	We need to be able to estimate equity risks (VaR) instantaneously if the portfolio composition changes through time => instant capital impacts can support investment decisions
Equity (financial) returns stylized facts	• Leptokurtosis i.e. fat tailed return distribution. • Volatility clustering i.e. tendency for volatility to appear in bunches • Leverage effect i.e. the tendency for volatility to rise more following a large price fall.
Dependency structure	Funds portfolios consist of a number of benchmark indices. Therefore, the dependency structure (i.e. correlation) between the indices should be explicitly modelled.

UK Fund



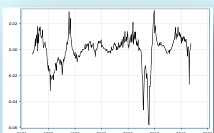
EU Fund

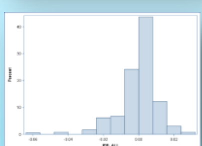


Market Risk calibration process: overview

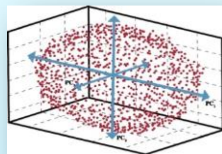
Monthly Data

Monthly Data

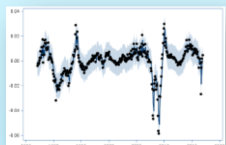




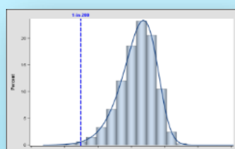
Calibration Methodology



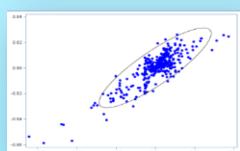
Principal Component Analysis (PCA) is a ***dimensionality reduction*** techique which is used to model highly correlated systems such as interest rate term structures.



Time series techiques aim to model the historic behaviour of individual risk factors by accounting for the series autoregressive properties and different error term distrbutions .

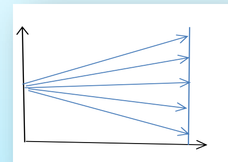


Statistical ***distribution fitting*** is an approach to fit a parametric probability distribution which best explains the underlying data properties.



Dependency modelling utilises copula theory to model the correlation structure between underlying risk factors.

Simulation and Aggregation



Stochastic simulation is used to generate a large number of scenarios (states of the world) from a given distribution/time series model and underlying dependency

Temporal Aggregation

$$\text{Annual} = \sum \mathit{Monthly}$$

Time series (temporal) aggregation is a technique to obtain annual non-parametric risk distribution by summing up simulated high-frequency (e.g.

Goodness of Fit testing and Benchmarking

Final Annual Parametric or Non-parametric Risk distribution

Input to Internal Model

Univariate modelling: GARCH-type models

To capture stylized facts we use discrete volatility models of GARCH-type. Specifically we estimate:

- AR(1) model with constant volatility.
- Generalised ARCH model (GARCH) with possible AR(1) process .

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{j=1}^{p} \gamma_j h_{t-j} \qquad p \geq 0, q > 0, \omega > 0, \alpha_i \geq 0, \gamma_j \geq 0$$

- Quadratic GARCH model (QGARCH) with possible AR(1) process. In the QGARCH model the lagged errors centres are shifted from zero to some constant values:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i (\varepsilon_{t-i} - \psi_i)^2 + \sum_{j=1}^{p} \gamma_j h_{t-j}$$

- Glosten, Jaganathan and Runkle (1993) (GJR-GARCH) model with possible AR(1) process. In the GJR model there is an extra slope coefficient for each lagged squared error:

$$h_t = \omega + \sum_{i=1}^{q} (\alpha_i - 1_{\varepsilon_{t-i}<0} \psi_i) \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \gamma_j h_{t-j}$$

- $y_t = \sqrt{h_t} \varepsilon_t$

Model Specifications	Distribution of residuals
AR(1)	$N(0,1)$, $t(\upsilon,0,1)$, $t(\upsilon,0,1,\lambda)$
AR(1)-GARCH(1,1)	$N(0,1)$, $t(\upsilon,0,1)$, $t(\upsilon,0,1,\lambda)$
AR(1)-QGARCH(1,1)	$N(0,1)$, $t(\upsilon,0,1)$, $t(\upsilon,0,1,\lambda)$
AR(1)-GJR-GARCH(1,1)	$N(0,1)$, $t(\upsilon,0,1)$, $t(\upsilon,0,1,\lambda)$

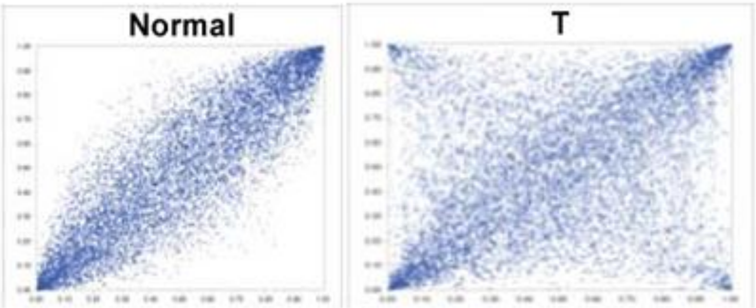
Dependency structure: Copulas

Modelling dependency structure is a difficult task. The common approaches applied in the practice can be briefly summarised as follows:

• The normal linear VaR model, in which it is assumed that the distribution of risk factor returns is multivariate normal. The biggest disadvantage of the approach is the normality assumption underpinning the return distributions.

• The historical simulation model, which makes minimal assumptions about the risk factor return distribution. The model is widely used in banks but it has a limited value for estimating 1-year VaR.

• The copula approach with Monte Carlo simulation. The approach is chosen due to its flexibility. Specifically, a copula is a function that links univariate marginal distributions to their multivariate distribution. It is a powerful tool as it does not require any assumptions on marginal distribution functions and it allows decomposing any N-dimensional joint distribution function into N marginals and a copula.



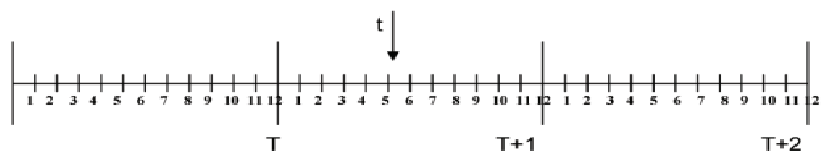
Temporal aggregation

Rationale:

- Economic/financial time series are indexed by time. The choice of the frequency clearly influences the estimation results. It is logical to assume that estimated models for different frequencies should be related.
  - The annual data can be considered as a function of the monthly data due to the aggregation process.
  - the annual model is also a function of the monthly model.

Model:

- In general, the aggregate variable can be defined as $y_t^* = \sum_{j=0}^{k-1} y_{t-j} = (1 + L + \ldots + L^{k-1}) y_t$
- The aggregated series do not overlap, e.g. one year does not overlap with the next.



$$y_T^* = \sum_{j=0}^{11} y_{t-j} \qquad y_{T+1}^* = \sum_{j=0}^{11} y_{t-j}$$

- *Example*: a disaggregate AR(1) model has the following form:

$$(1 - \phi L) y_t = \varepsilon_t \,,$$

where $\varepsilon_t \sim (0, \delta^2)$. Assuming the aggregation frequency $k = 12$ (i.e. aggregating from monthly to annual data frequency) the aggregated series follow ARMA(1,1) process

$$(1 - \beta B) y_T^* = (1 + \eta B) \varepsilon_T^*$$

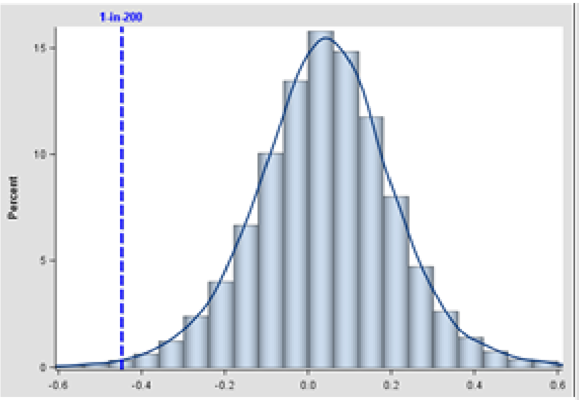
where $\beta = \phi^{12}$.

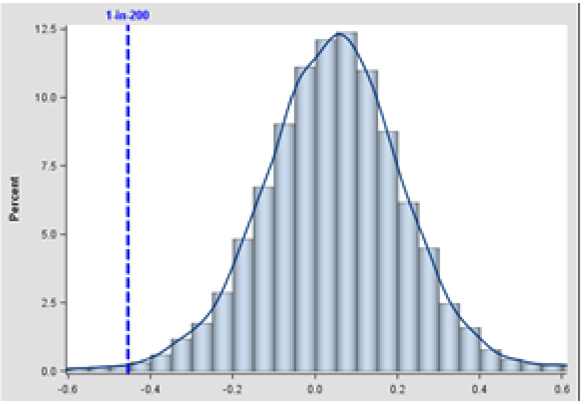
Calibration steps

In general, the methodology for calibrating conventional equity stresses consists of:

- ***Univariate data analysis of indices***: normality tests, non-linear dependency tests, unit root tests and so forth.
- We model univariate (i.e. marginal) equity returns by using an appropriate ***time series models*** of (asymmetric) GARCH-type.
- We utilise the ***copula*** theory to model and simulate the dependency structure between benchmark indices.
- For each index the simulated residuals together with the corresponding GARCH parametric model are used to calculate simulated monthly log returns. We apply the ***temporal aggregation*** technique to obtain annual non-overlapping equity returns from simulated monthly returns.
- We calculate final stresses based on the portfolio composition.

1-in-X	UK fund	EU Fund
1-in-4	-6.37%	-6.42%
1-in-10	-16.84%	-16.86%
1-in-20	-23.77%	-23.65%
1-in-50	-32.52%	-32.73%
1-in-100	-38.13%	-38.55%
1-in-200	**-44.85%**	**-45.50%**





Conclusion

- The main advantage of the framework is its flexibility i.e. modelling marginal distributions and dependency structure separately
- It utilises a number of theoretical approaches to model uncertainty i.e. financial models, copula theory and time series analysis.
- Closed form solution is available for a number of model specifications.
- The framework has been successfully applied to modelling of other risks including credit.
- It has been approved by the PRA as a part of the Internal Model.

Limitations:

- Model dependent approach i.e. model risk.
- Normality assumption for closed form solutions.
- Expert Judgement overlay is required.

Thank you