Rheology of dense (complex) suspensions

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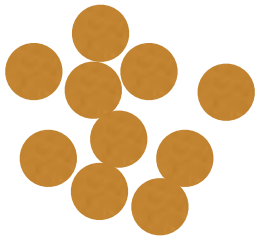
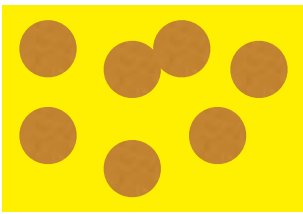
Schlumberger

Suspension of particles in fluids
in cosmetics, food, in petroleum industry,
in construction, in landslides….

dense
granular
medium
+
complex fluid



dense
complex
Suspensions



A very brief history of viscous suspensions

dilute suspension

dense suspension

jamming

0

ϕ_m

ϕ

time

1906
Einstein

Single particle

$$\eta = \eta_f(1 + \frac{5}{2}\phi + \alpha\phi^2..)$$

1970,72
Batchelor
Green

Pair interaction

1988 =>...
Brady, Bossis
Nott, Morris,..

hydrodynamic interactions

towards Constitutive equations

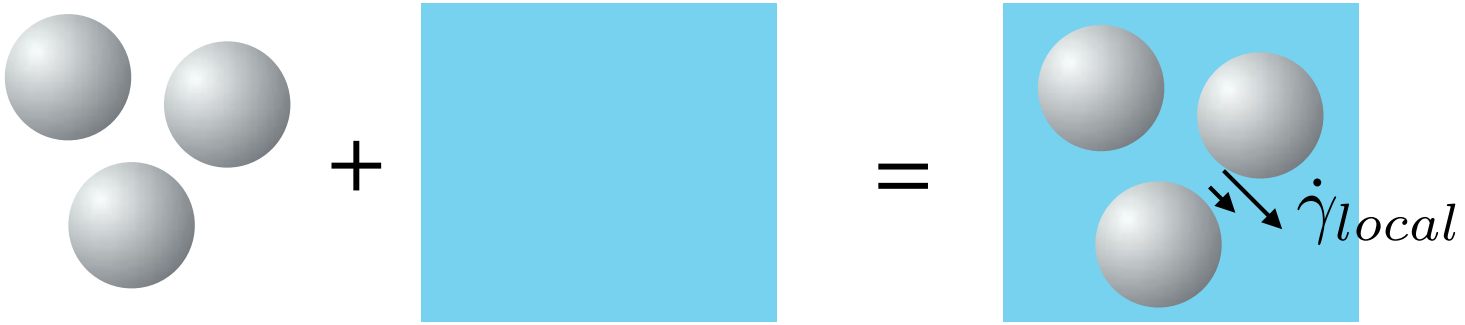
2007 =>...
Olson teiltel07, Tighe et al 09,
Heussinger, Andreotti et al 10,12
Wyart 12,16, Berthier12,...

steric/elastic interactions

understanding the divergence

A very naive view….

Adding particles => « Amplification » of the rheology



η_f $\eta_s(\phi_p) > \eta_f$

particles => amplification of the local shear rate

$$< \dot{\gamma}_{\text{local}} > = \mathcal{F}(\phi_p)\dot{\gamma}$$

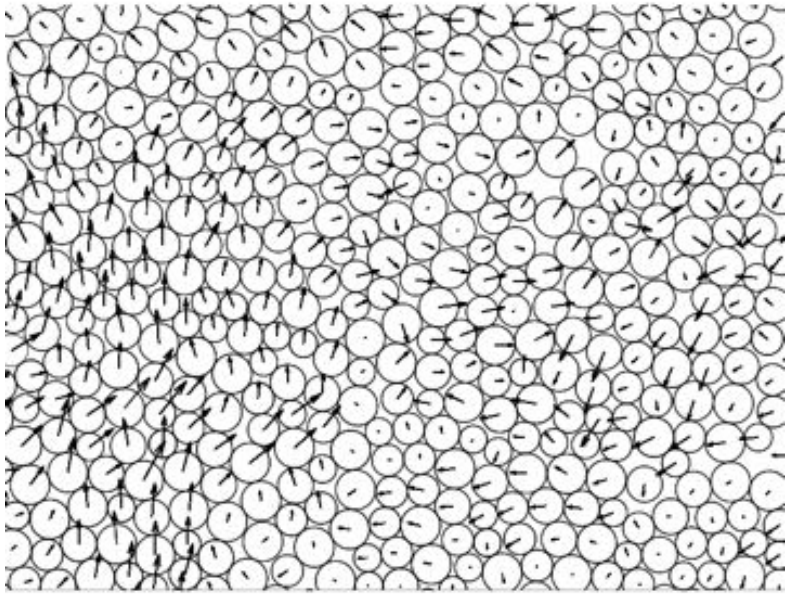
lever function

hydrodynamic origin in dilute regime
geometrical interlocking origin in dense regime

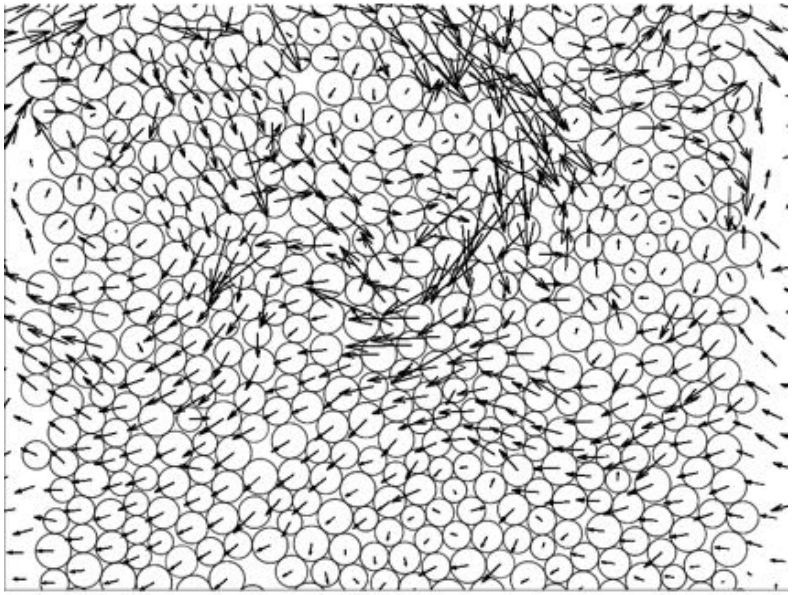
Chateau *et al.*, JOR 2008 Lerner et al 2012 De giuli et al PRE 2016

The lever function

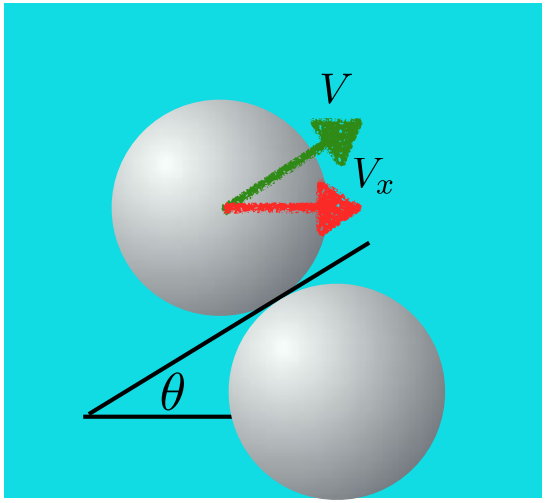
simulation from Degiuli et al PRE 2016

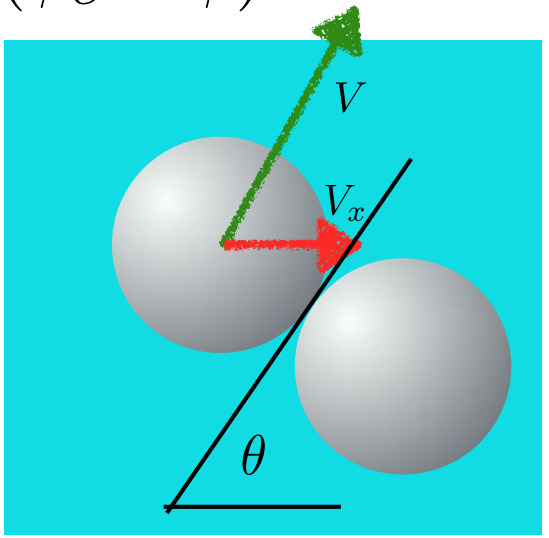


$(\phi_c - \phi) = 0.06$

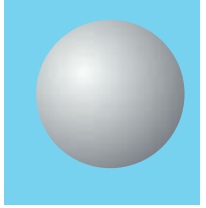


$(\phi_c - \phi) = 0.01$



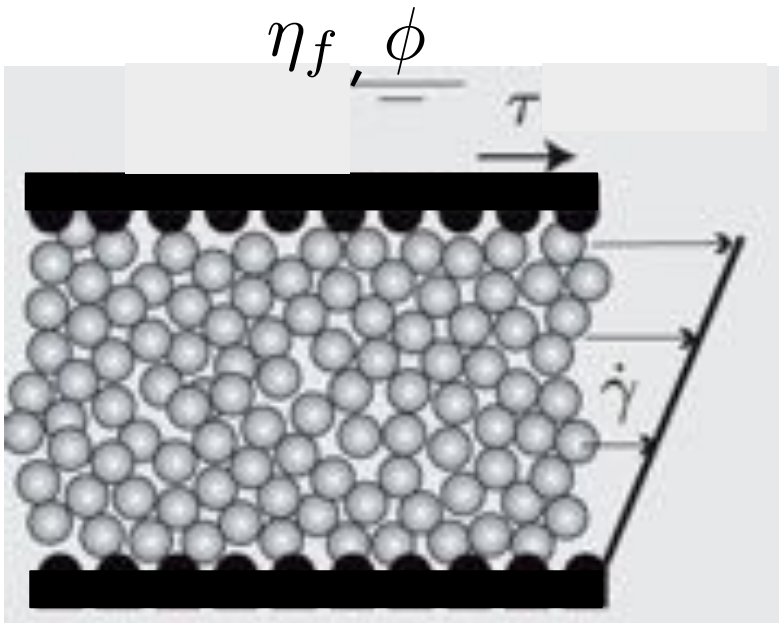


Rheology of spheres in a viscous liquid



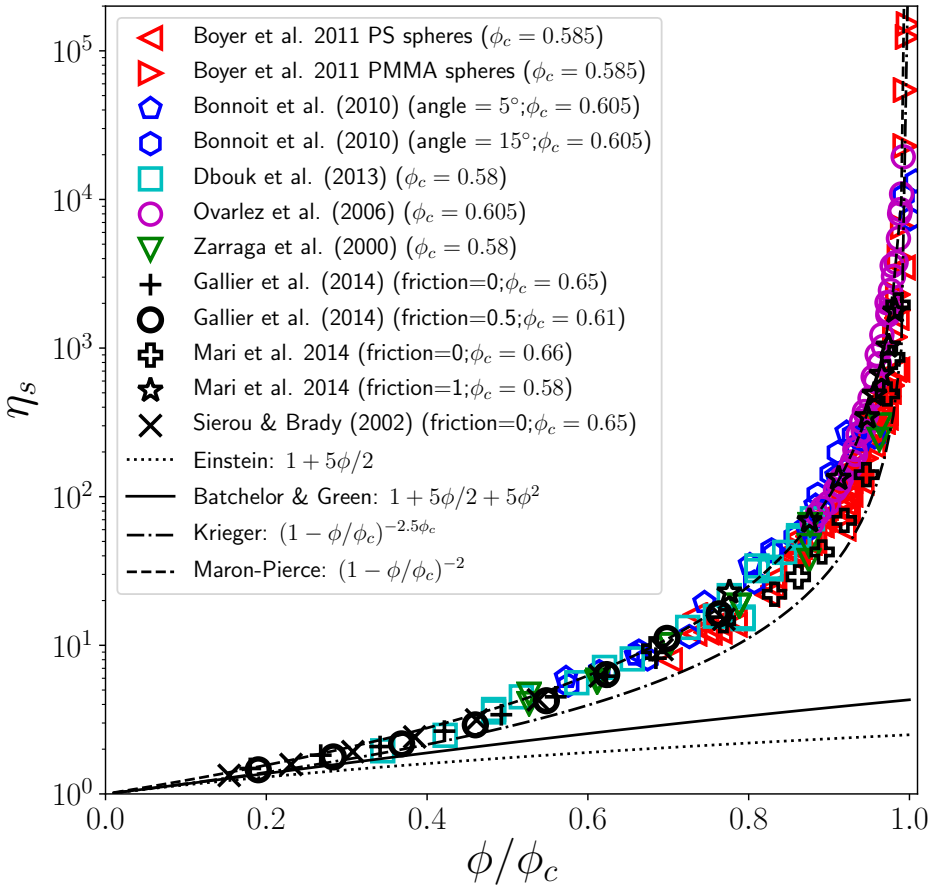
The shear viscosity

Shear stress $\tau = \eta_s(\phi)\eta_f\dot{\gamma}$

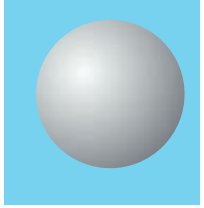


no brownian motion, rigid frictional particles, no inertia

=> stress is viscous



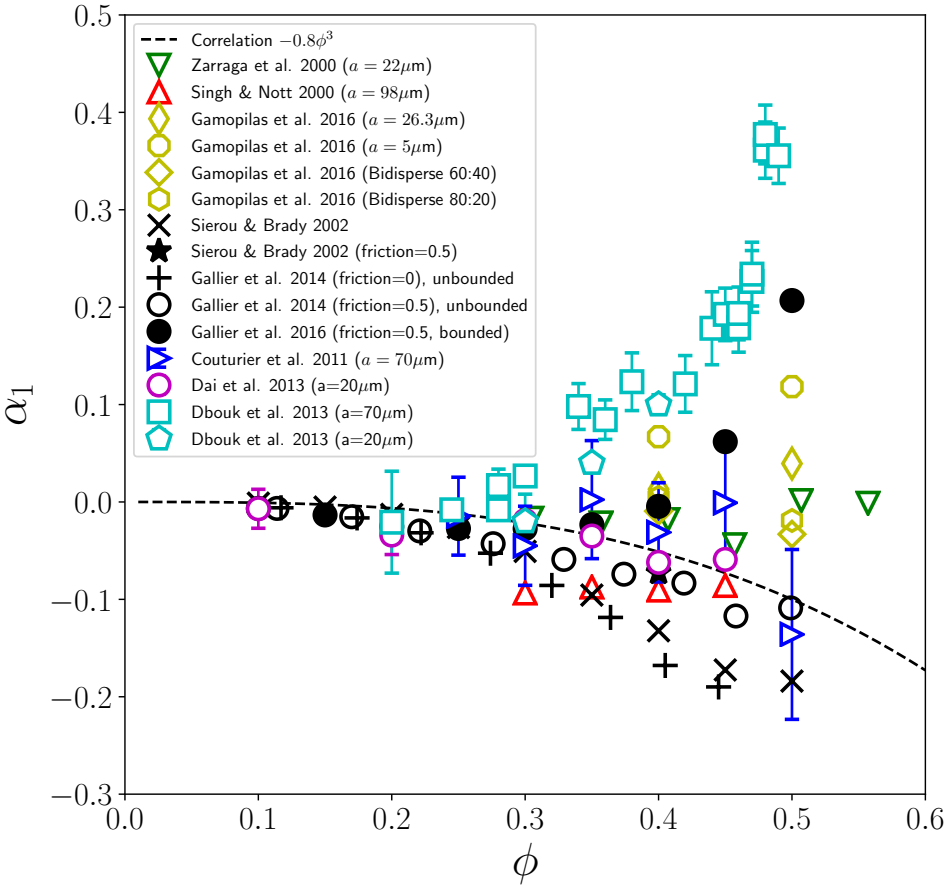
Rheology of spheres in a viscous liquid

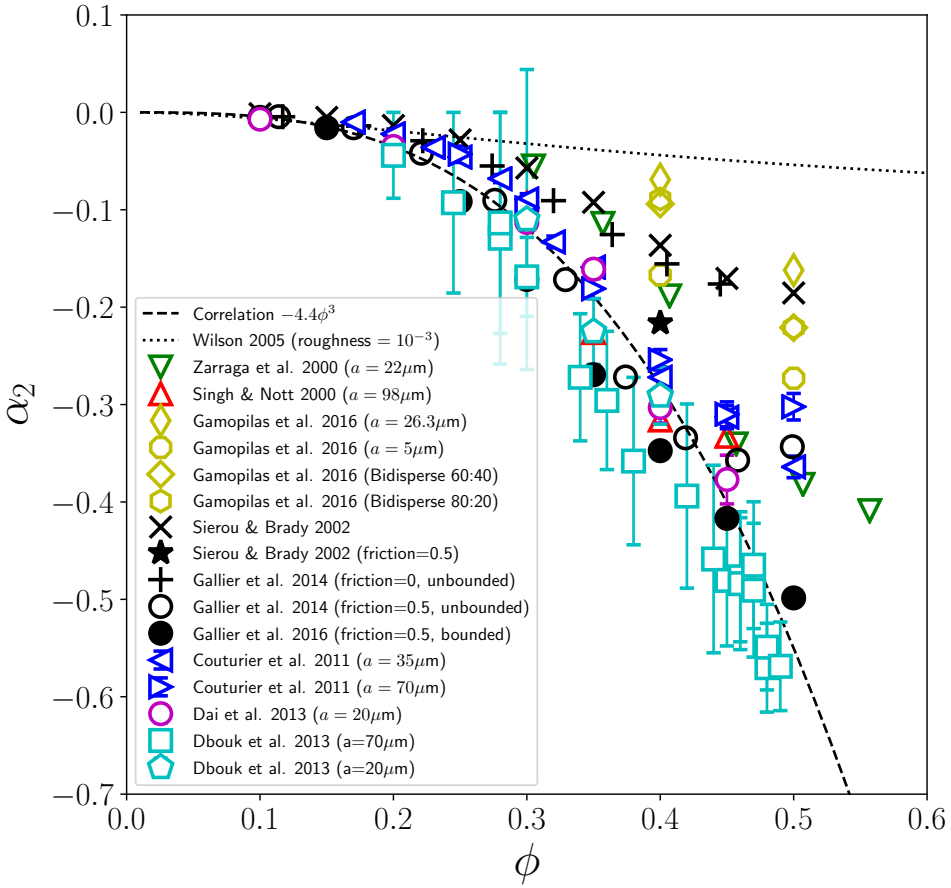


Normal stress differences

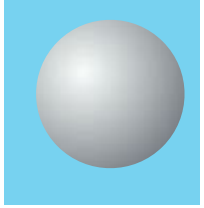
$$N_1 = \eta_f \eta_s(\phi)\alpha_1(\phi)\dot{\gamma}$$

$$N_2 = \eta_f \eta_s(\phi)\alpha_2(\phi)\dot{\gamma}$$





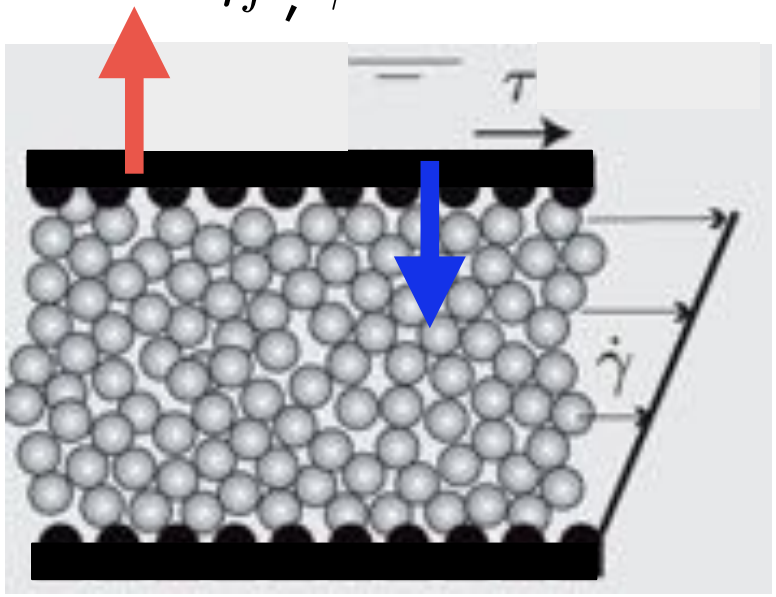
Rheology of spheres in a viscous liquid



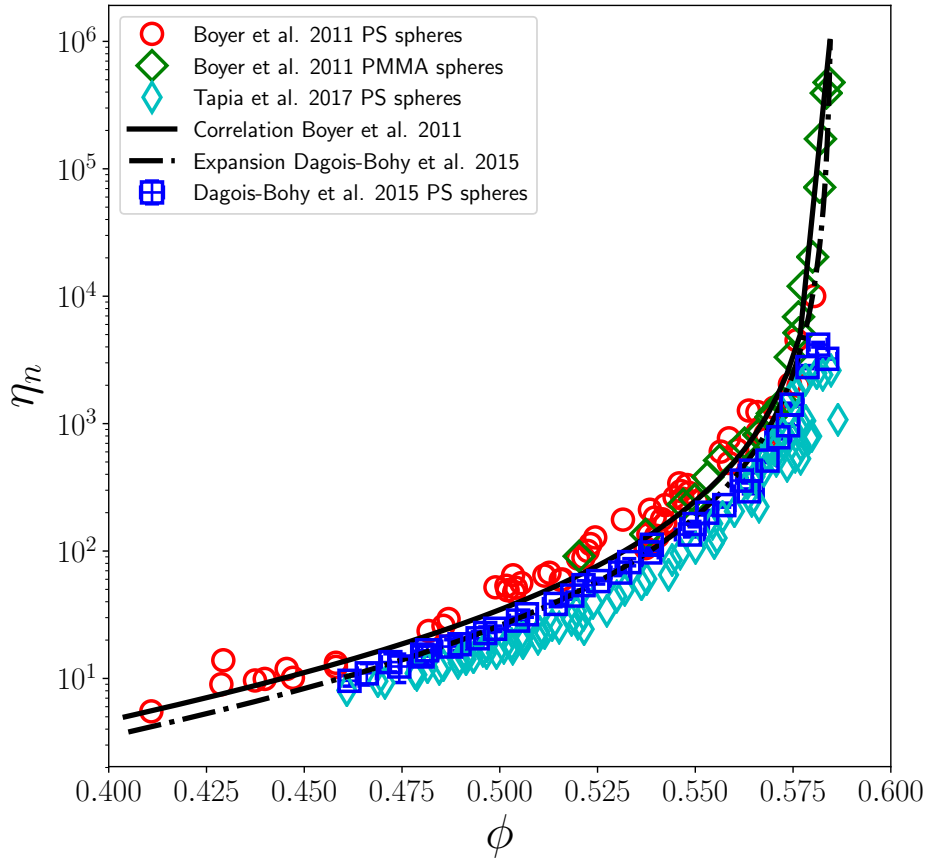
The normal viscosity

particle pressure $P_p = \eta_n(\phi)\eta_f\dot{\gamma}$

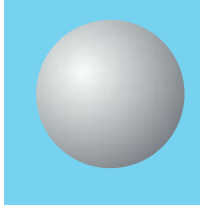
η_f , ϕ



Particles push on the wall
the wall pulls on water



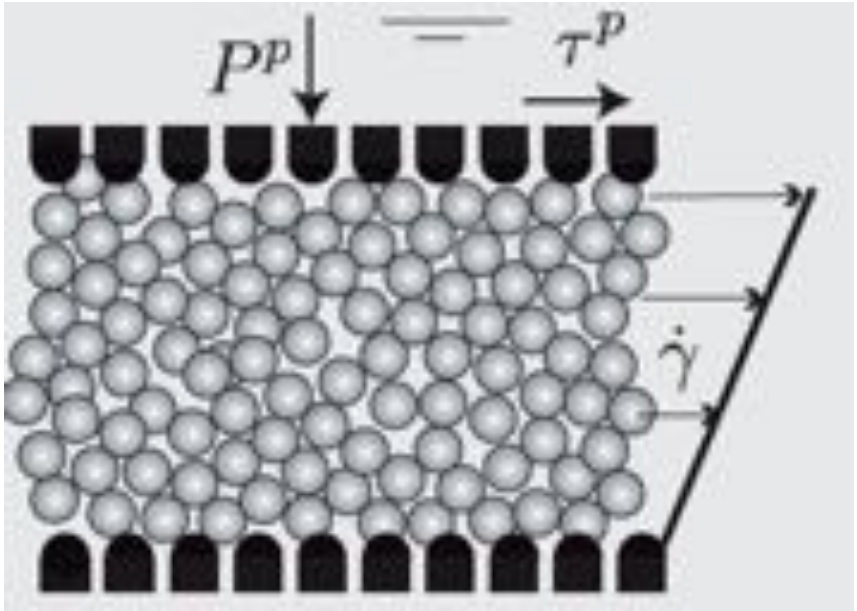
Rheology of spheres in a viscous liquid



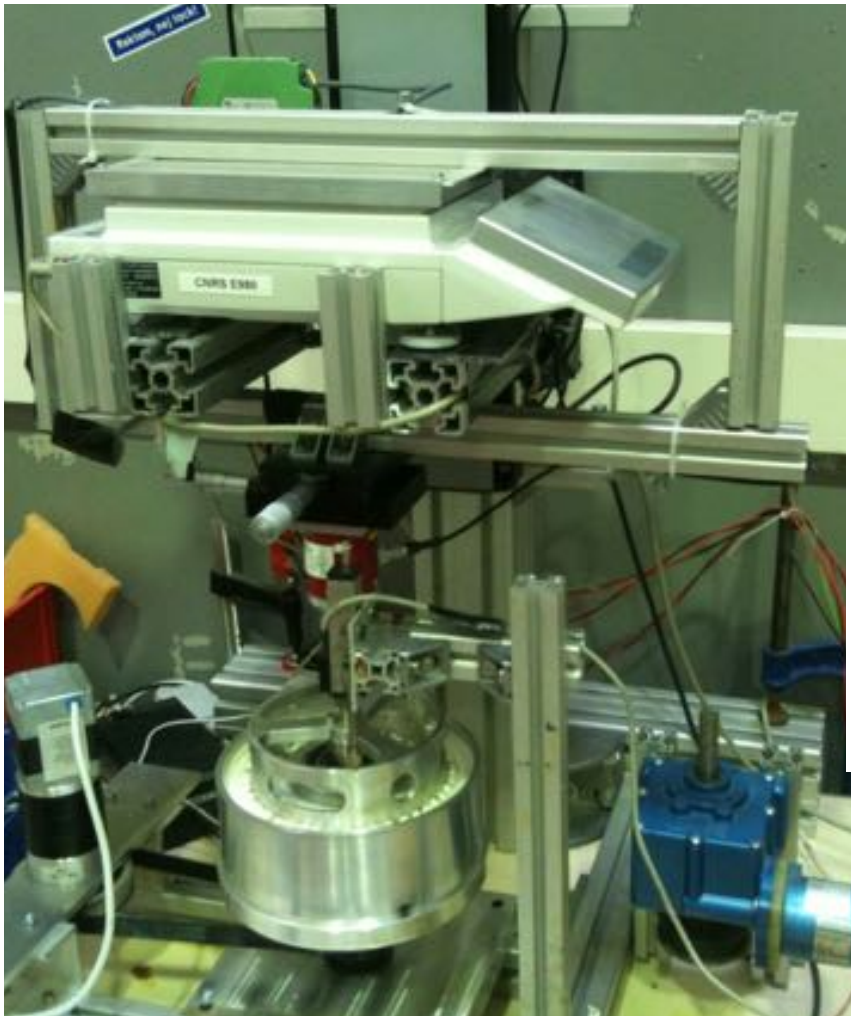
Measuring the normal viscosity

particle pressure $P_p = \eta_n(\phi)\eta_f\dot{\gamma}$

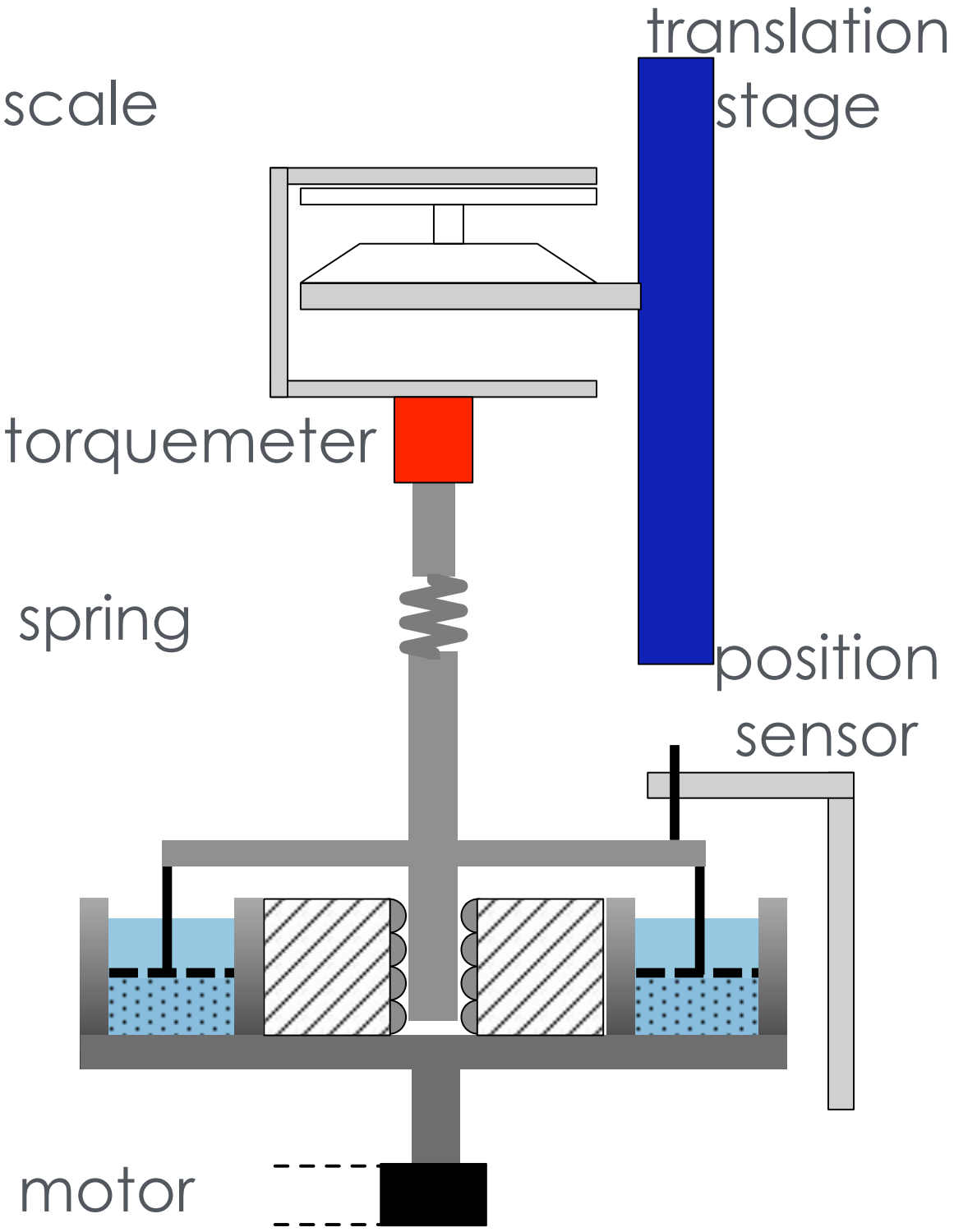
η_f , ϕ



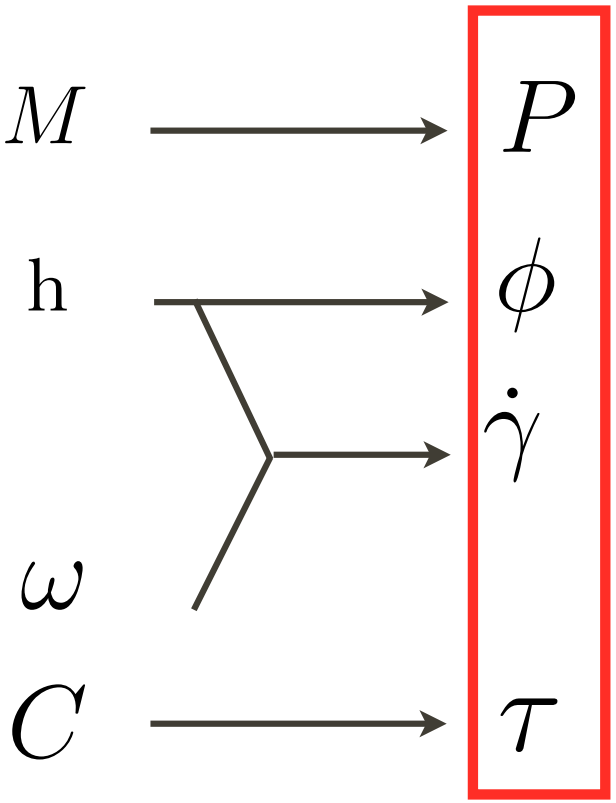
The pressure imposed rheometer

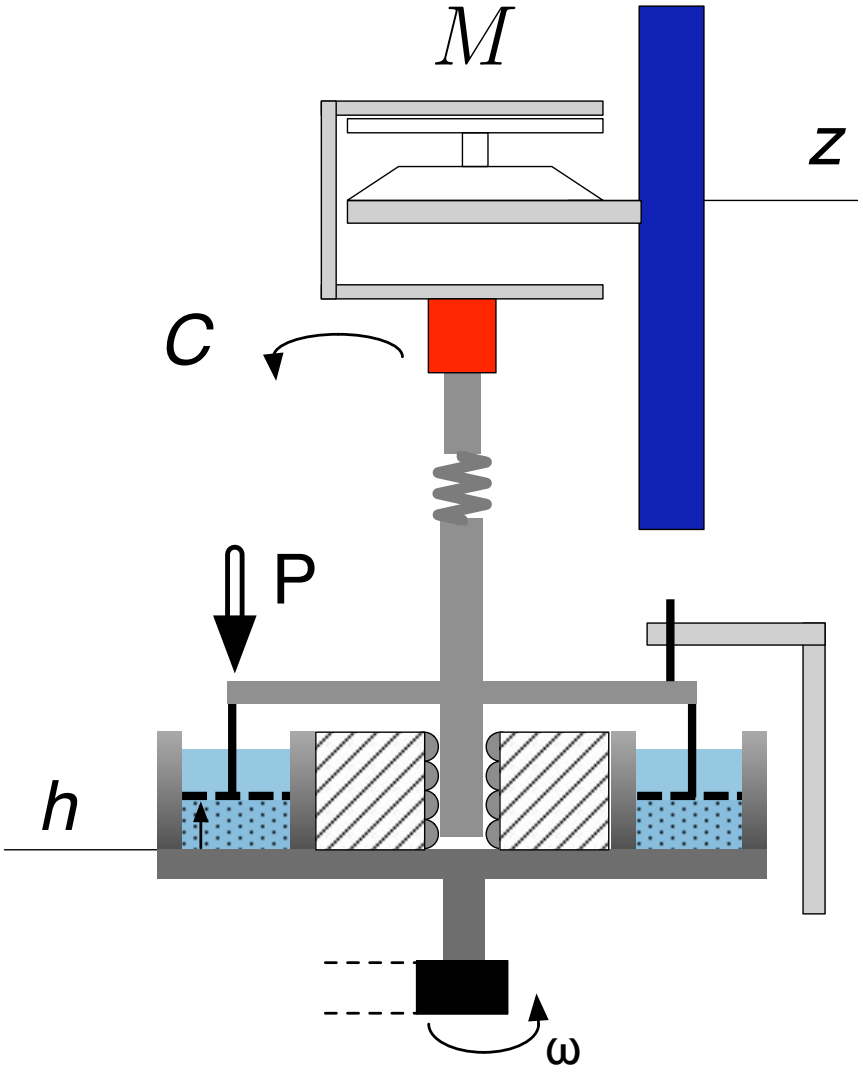


top and bottom plate
are rough

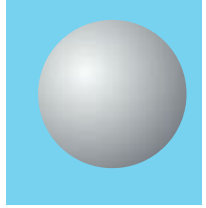


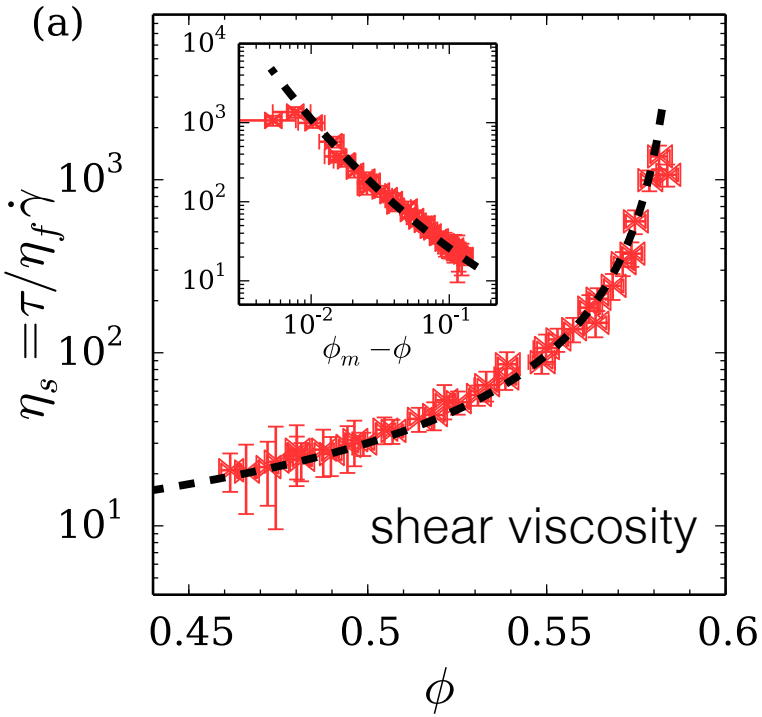
Measurements :

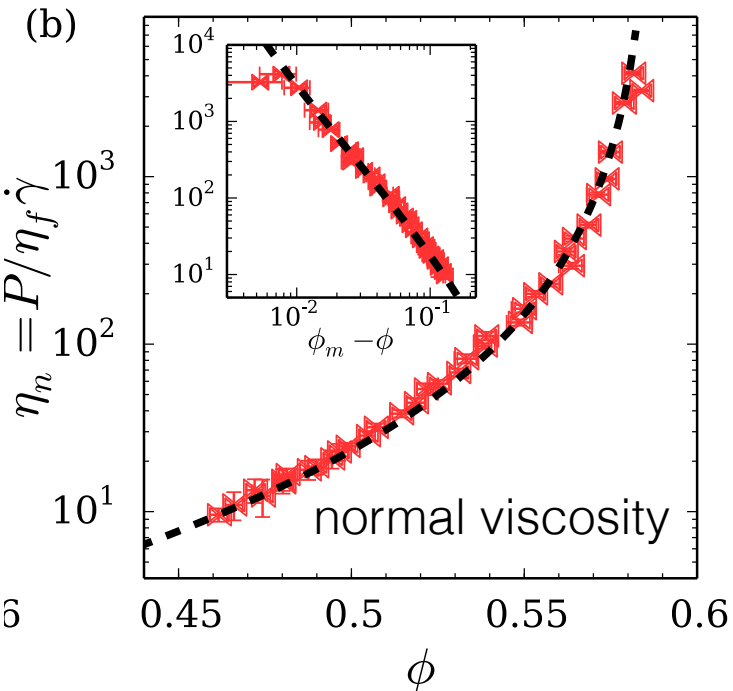


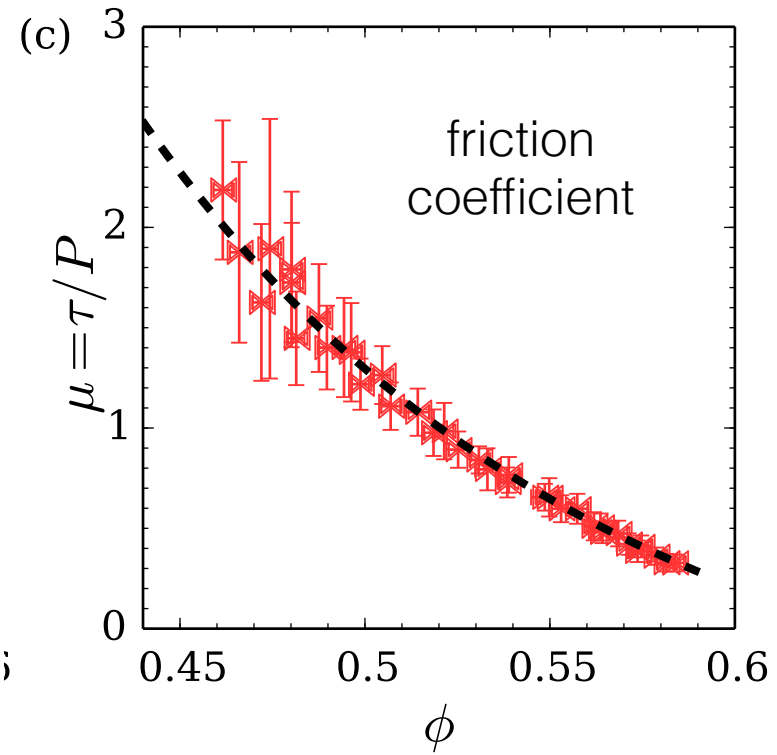


Rheology of spheres in a viscous liquid









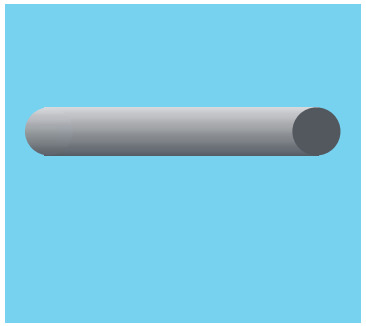
$$\tau = \eta_s(\phi)\eta_f\dot{\gamma}$$
$$P = \eta_n(\phi)\eta_f\dot{\gamma}$$

Simon Dagois-Bohy

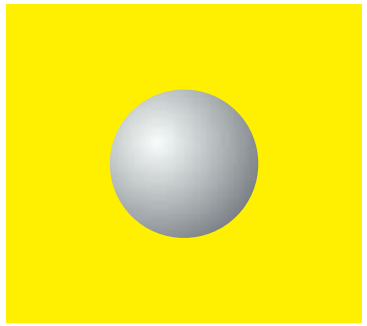
$$\eta_s(\phi) \propto (\phi_c - \phi)^{-2}$$
$$\eta_n(\phi) \propto (\phi_c - \phi)^{-2}$$

towards more complex systems?

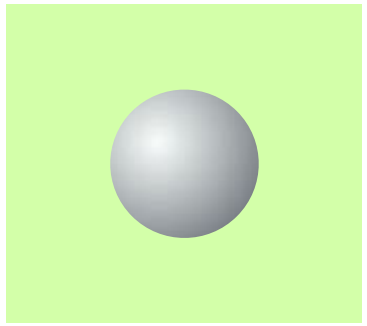
suspensions of fibres



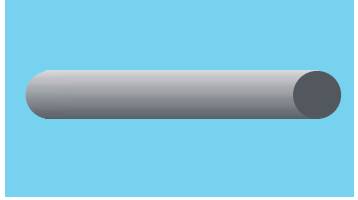
suspensions in a yield stress fluid



suspensions in cornstarch



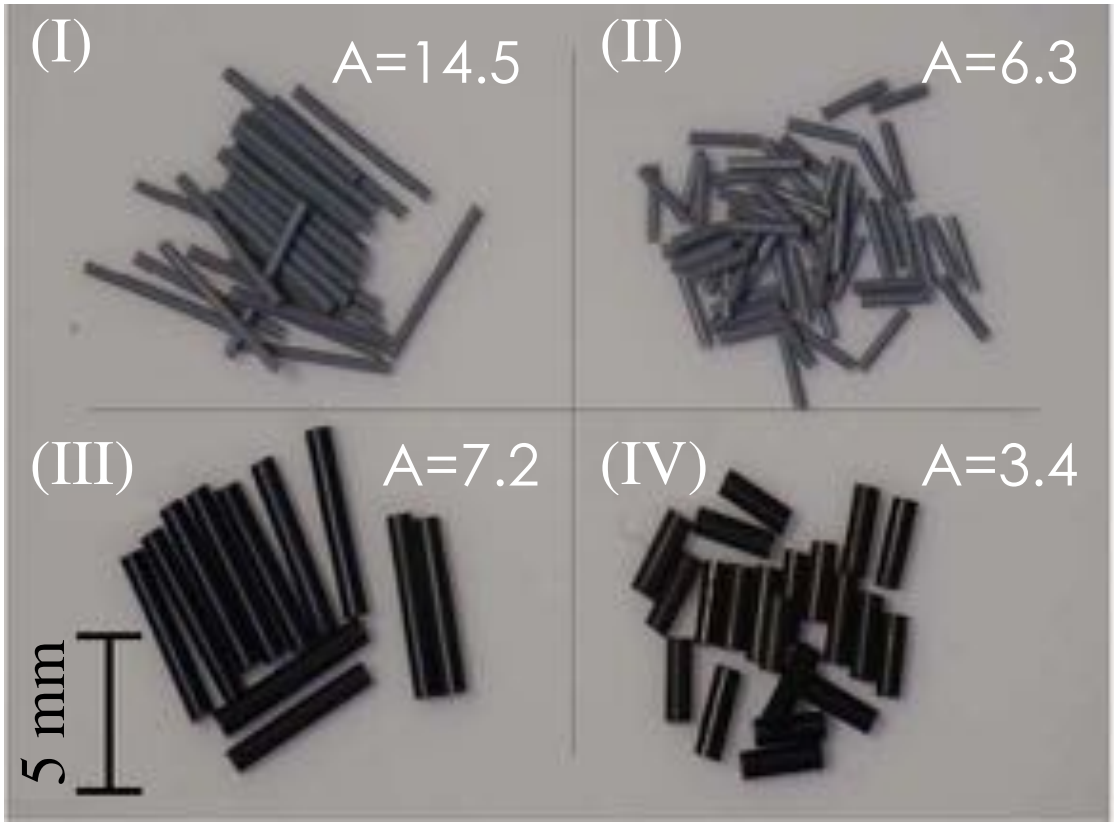
Dense suspension of fibres



A new parameter: aspect ratio A η, ϕ, A

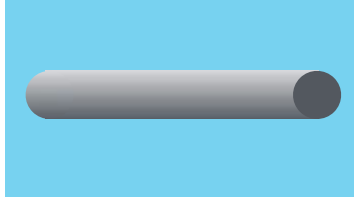
(b)

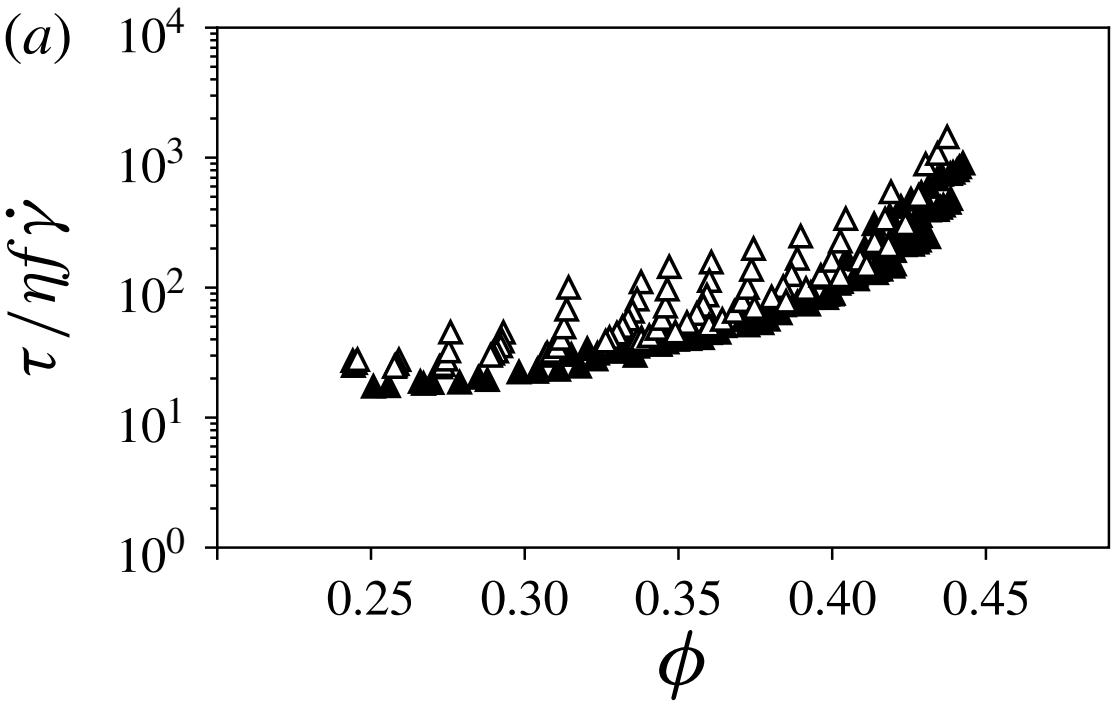
	A	L (mm)	d (mm)
(I)	14.5 ± 0.8	5.8 ± 0.1	0.40 ± 0.01
(II)	6.3 ± 0.4	2.5 ± 0.1	0.40 ± 0.01
(III)	7.2 ± 0.4	5.8 ± 0.2	0.81 ± 0.02
(IV)	3.4 ± 0.3	2.8 ± 0.1	0.81 ± 0.03

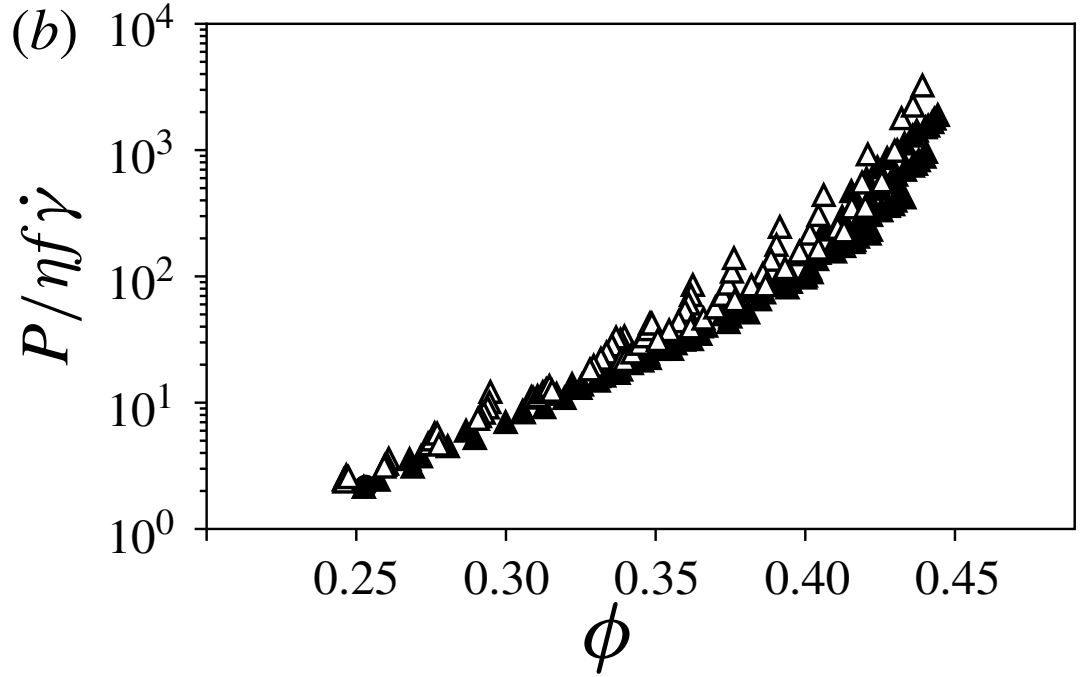


Franco Tapia (Aix-Marseille univ)

Dense suspension of fibres

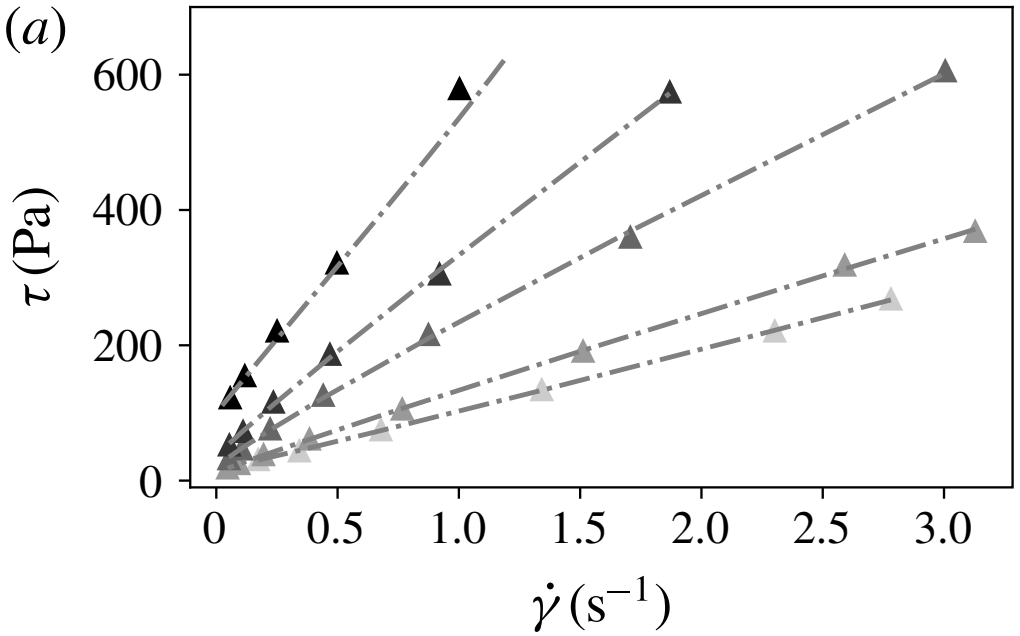


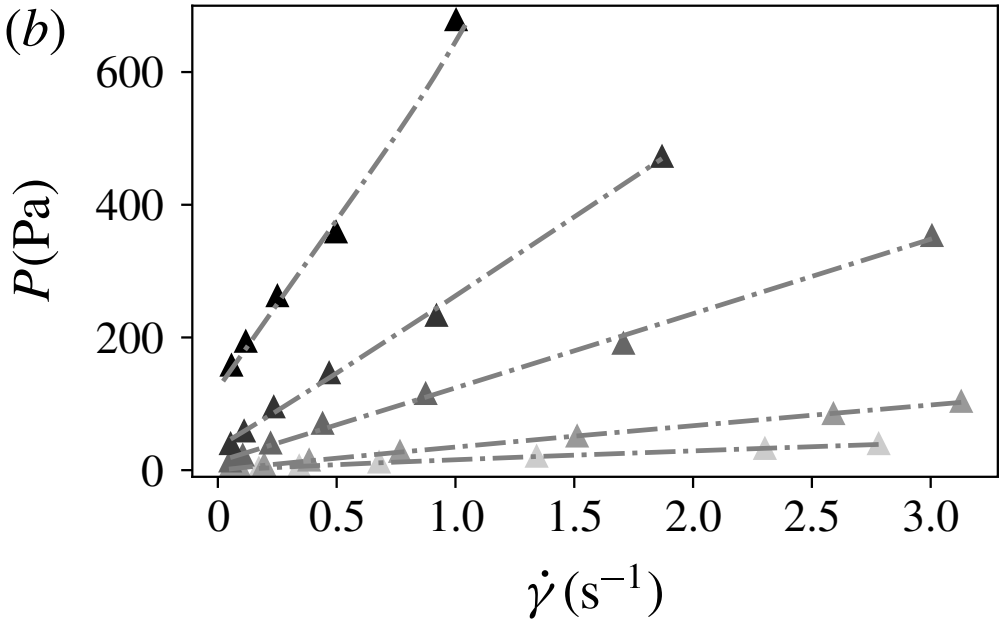




No viscous scaling…?
an apparent shear thinning?

Dense suspension of fibres

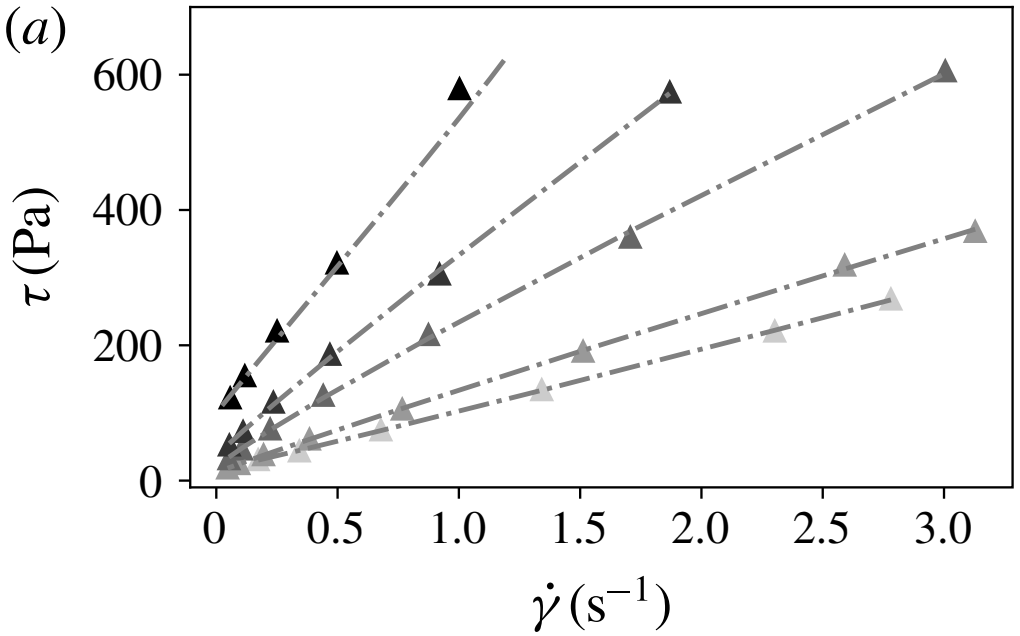


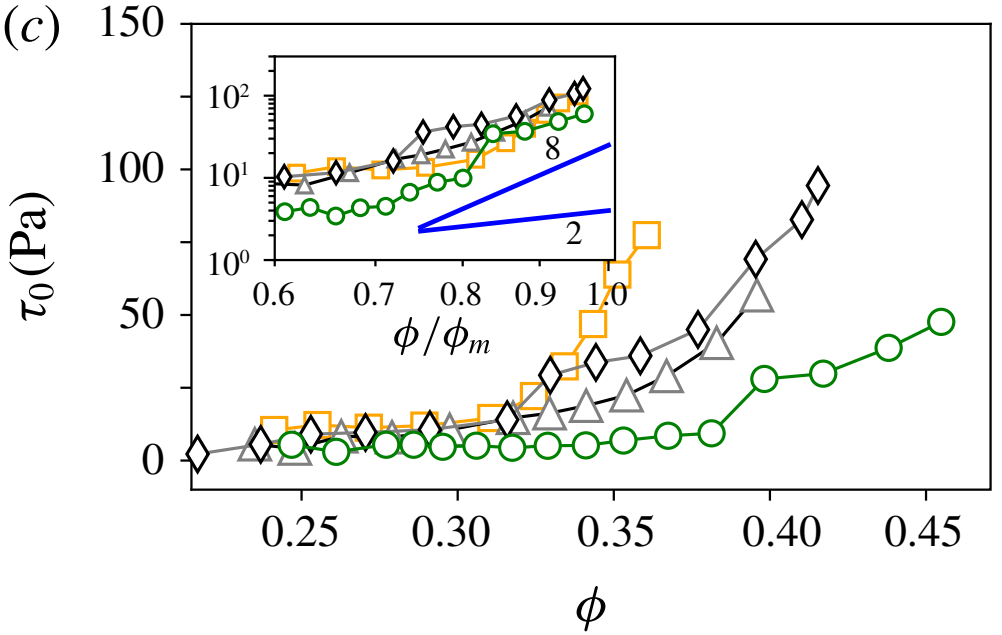


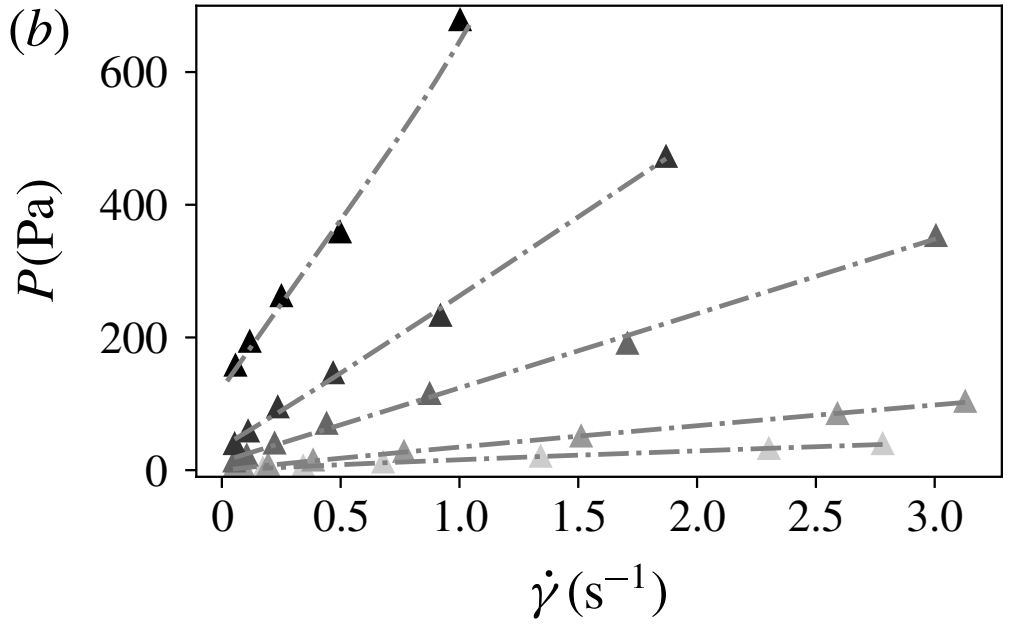
a yield stress!

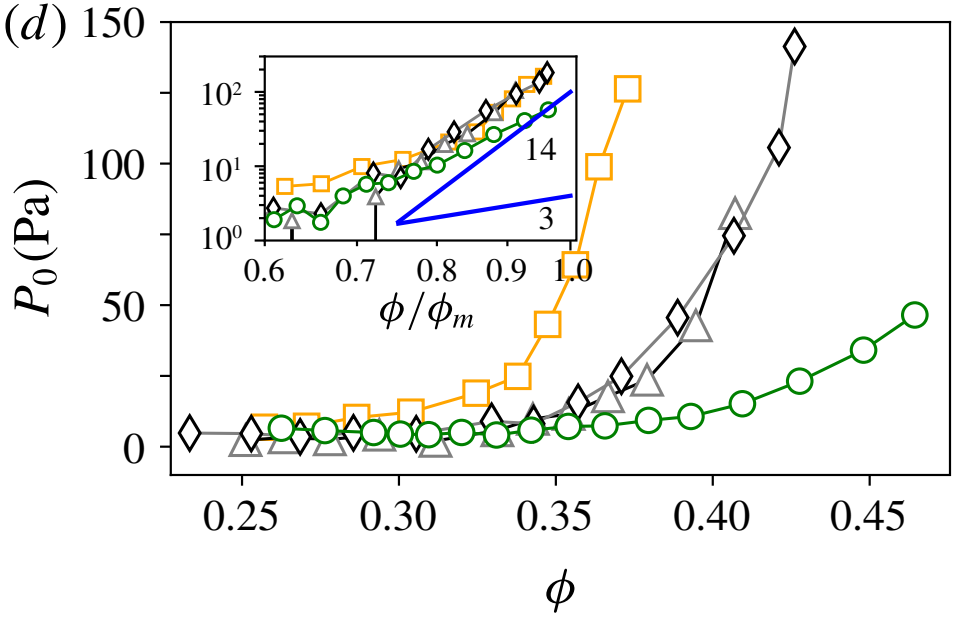
Dense suspension of fibres

a yield stress!

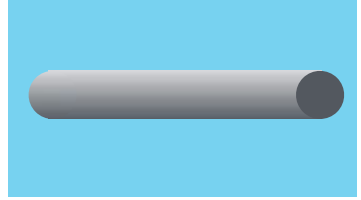


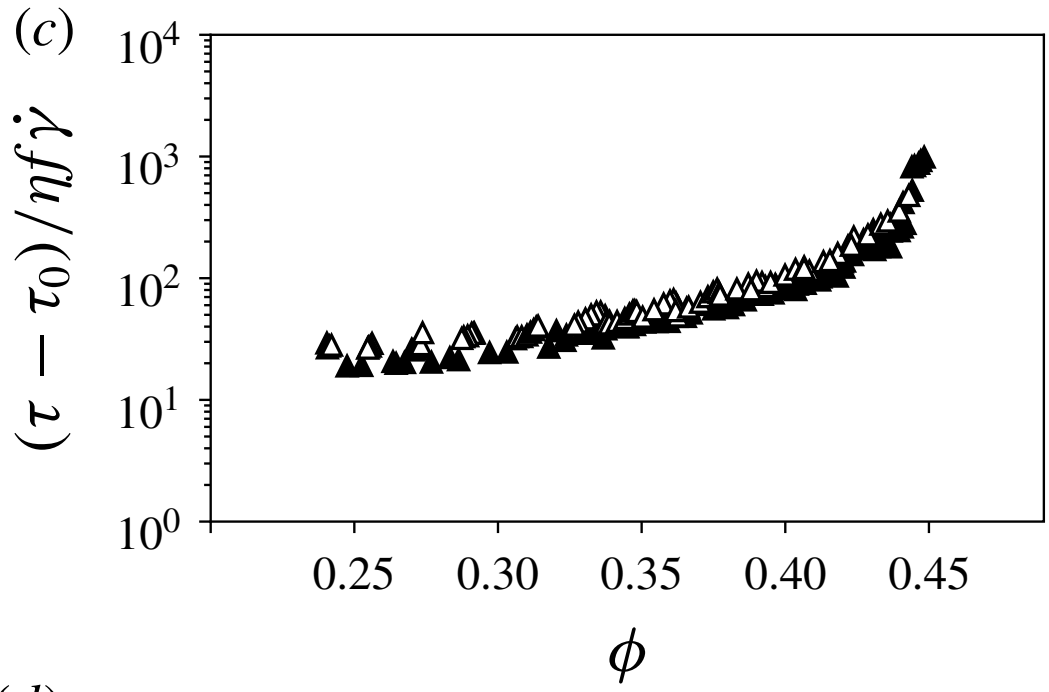


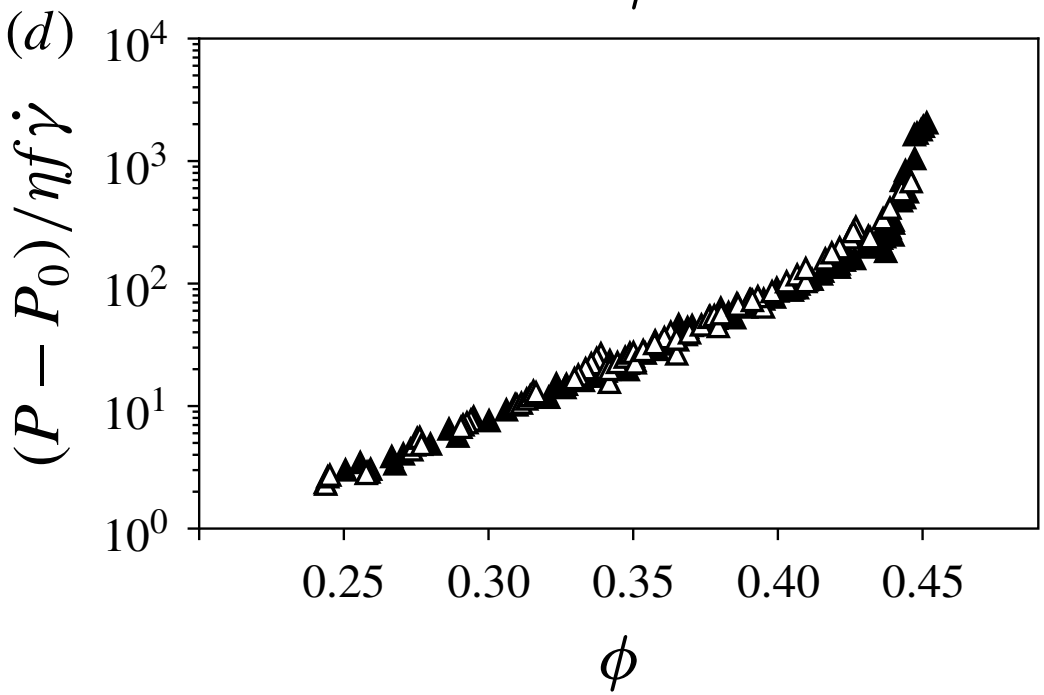




Dense suspension of fibres



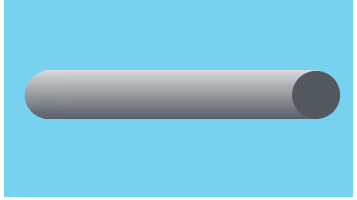


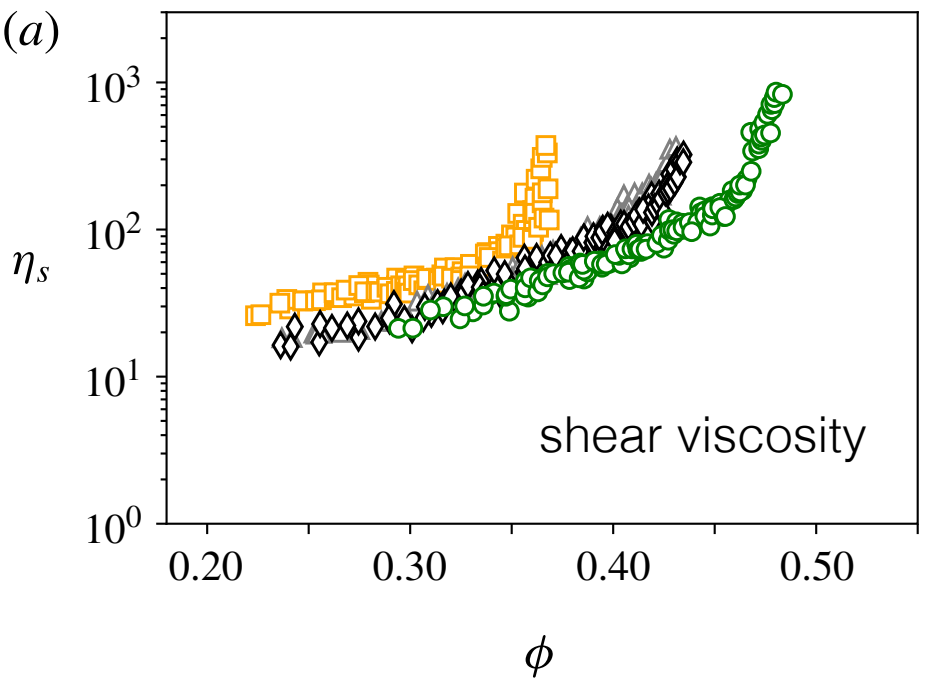


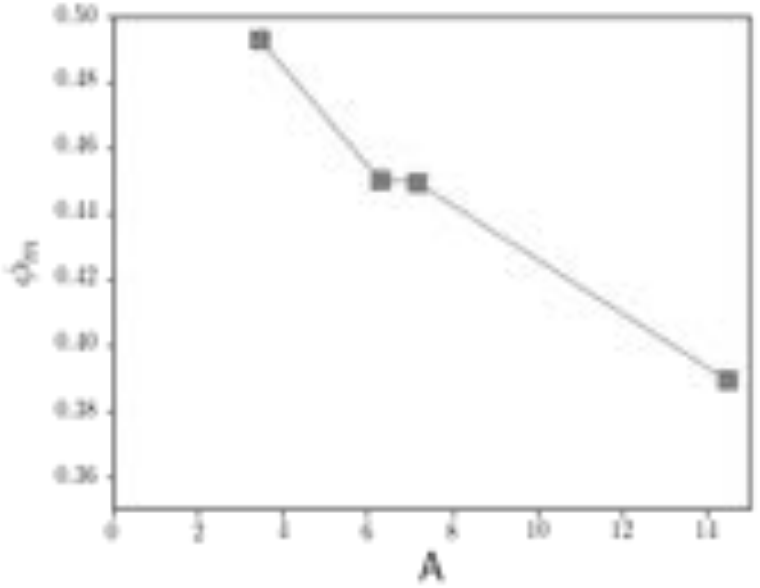
origin of the yield stress?

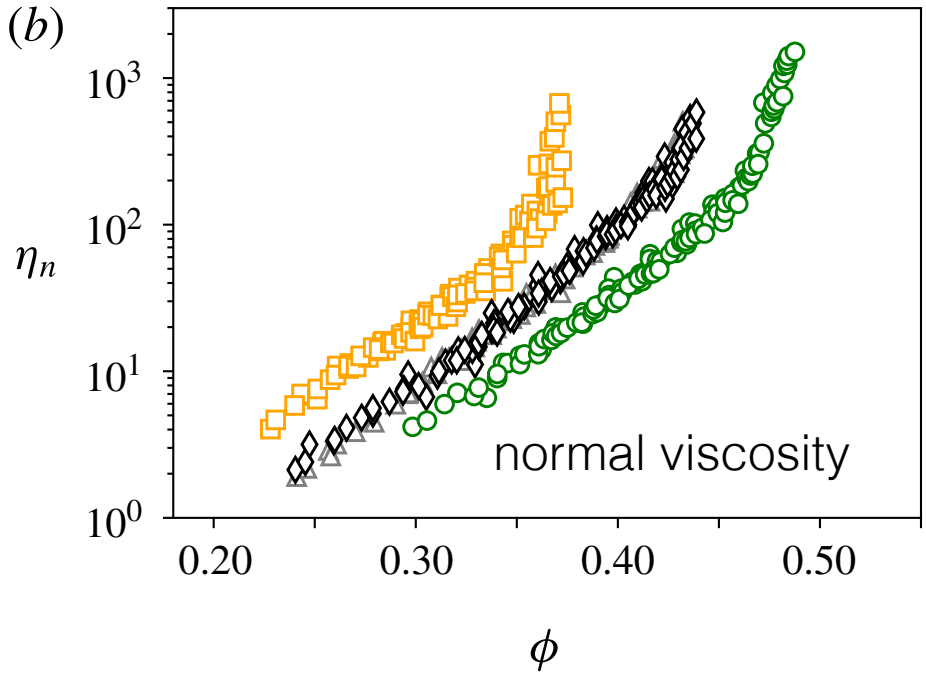
-other interactions?
-elasticity?
-finite size effect+softness?

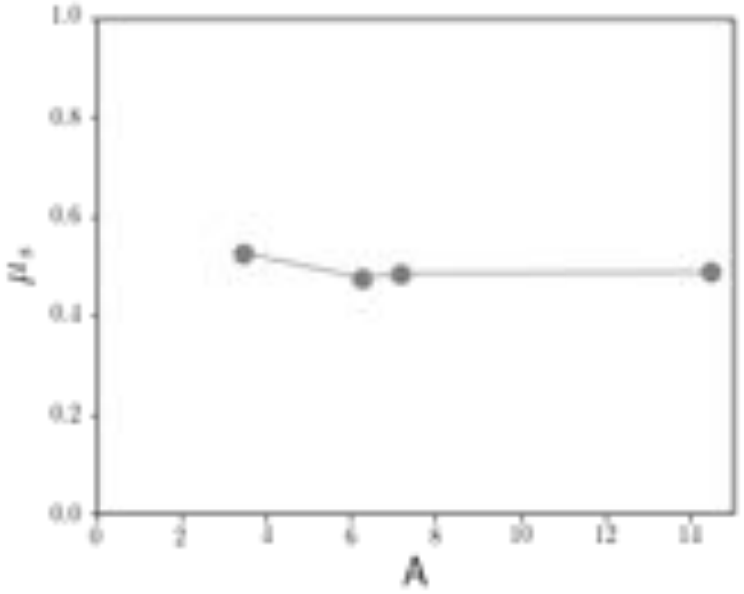
Dense suspension of fibres



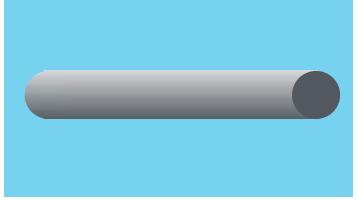


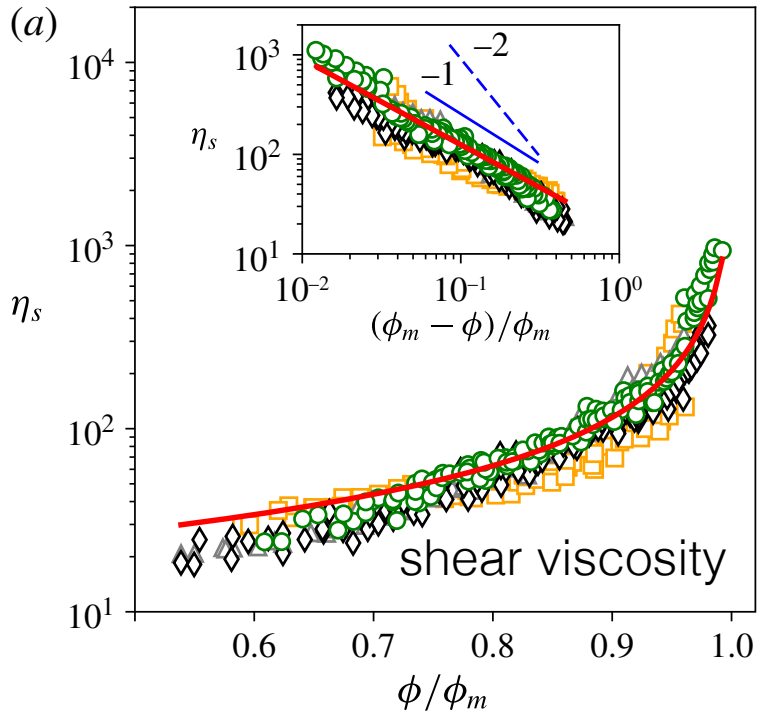


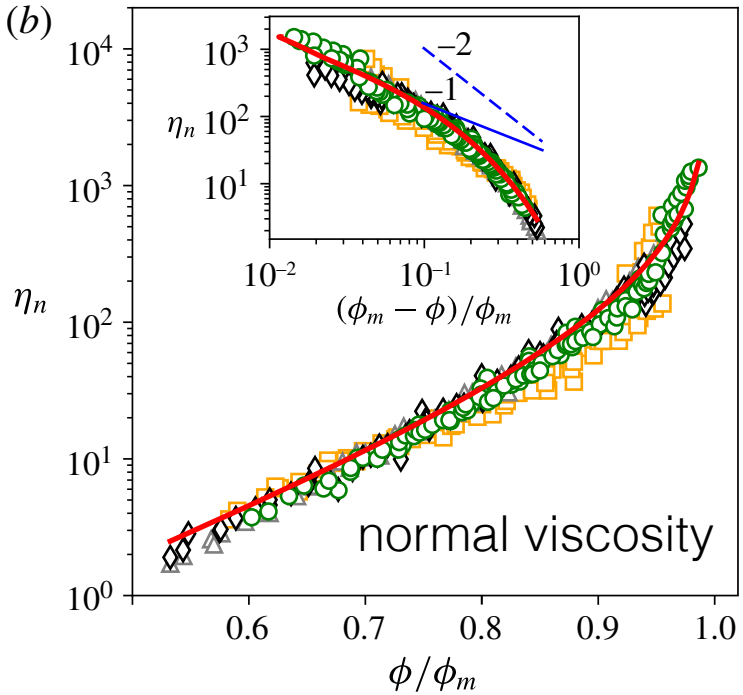


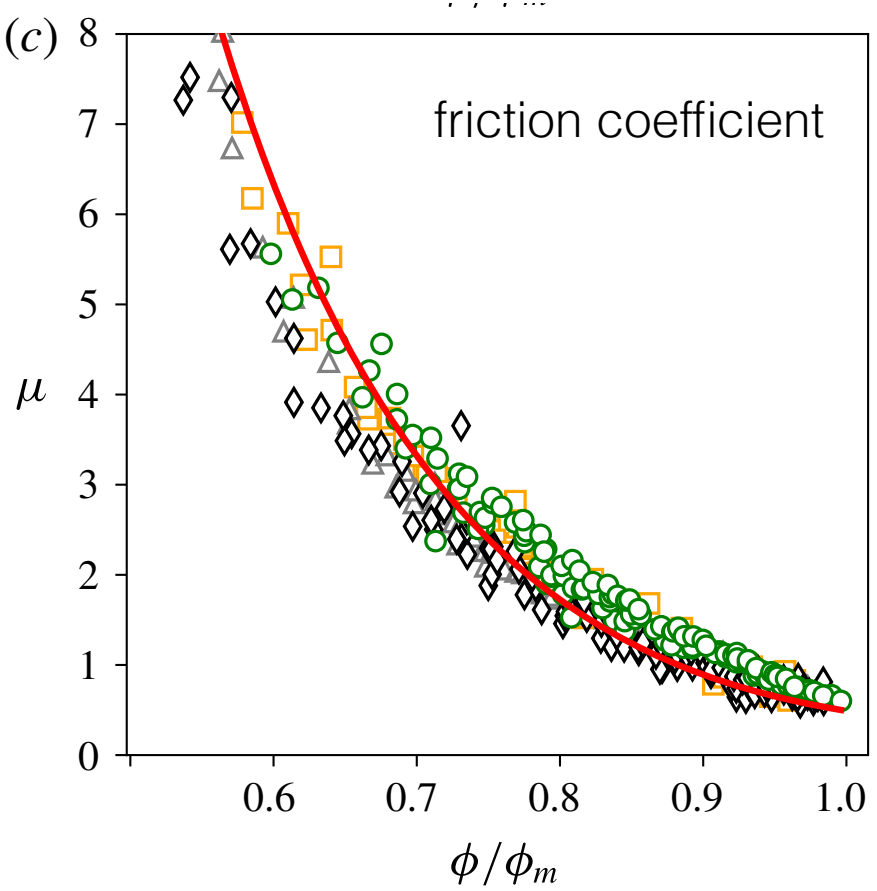


Dense suspension of fibres





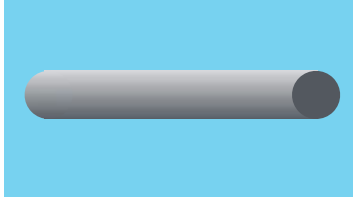




influence of A
=> rescaling by $\phi_m(A)$

$$\eta_{s,n}(\phi/\phi_m) \propto (1-\phi/\phi_m)^{-1}$$

Dense suspension of fibres



- a yield stress is observed (origin?)

- Aspect ratio seems to influence the rheology only through the maximum volume fraction

- the divergence is different from spheres (1 instead of 2?)

Franco tapia

Elisabeth Guazzelli

Jason Butler

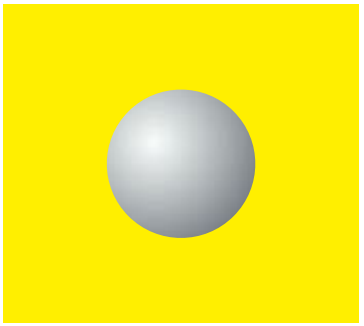
Saif shaikh

Tapia et al JFM, 827, R5 (2017)

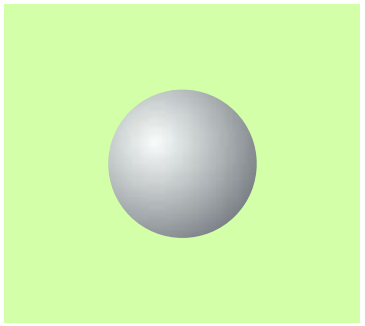
towards more complex systems?

suspensions of fibres

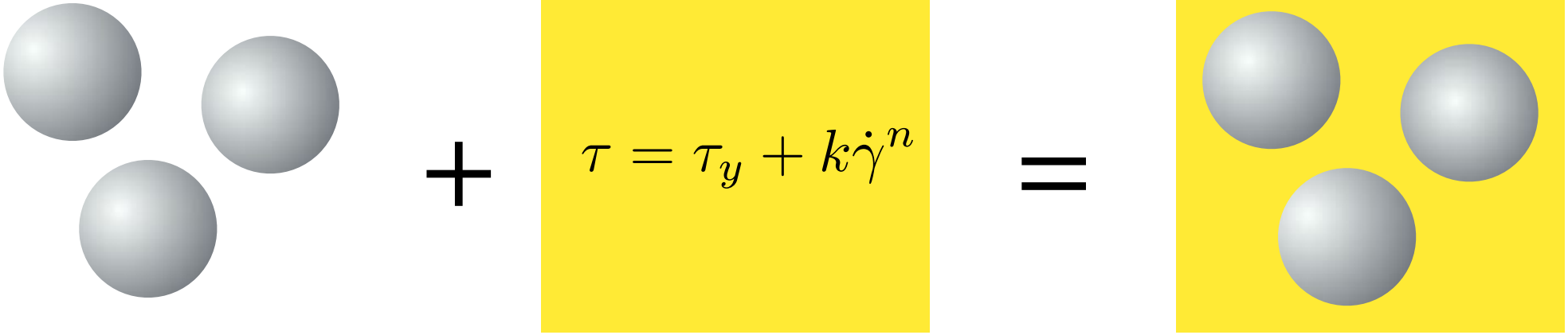
suspensions in a yield stress fluid



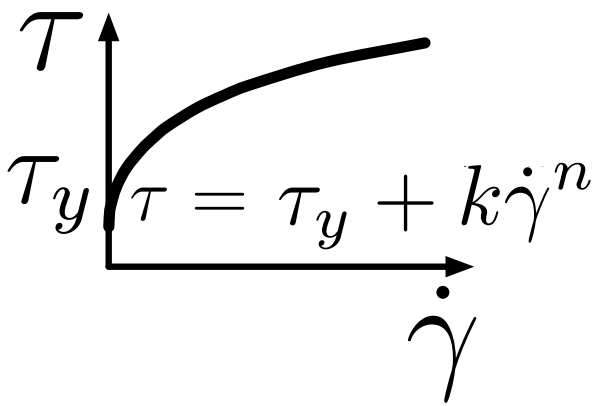
suspensions in cornstarch



Suspension in a yield stress fluid



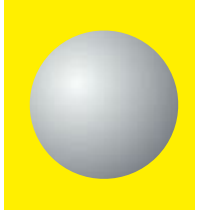
new parameters: τ_y, k



$$\frac{\tau}{k\dot{\gamma}^n} = \tau\left(\frac{\tau_y}{k\dot{\gamma}^n}, \phi\right),$$

$$\frac{P}{k\dot{\gamma}^n} = P\left(\frac{\tau_y}{k\dot{\gamma}^n}, \phi\right).$$

Suspension in a yield stress fluid



in the dense regime:
rheology controlled by geometrical constraints....

Same rheology as in newtonian fluids:

$$\tau = \eta_{ap}\eta_s(\phi)\dot{\gamma}$$

$$P = \eta_{ap}\eta_n(\phi)\dot{\gamma}$$

Same as in Newtonian fluid

with an apparent fluid viscosity:

$$\eta_{ap} = \frac{\tau_y + k\dot{\gamma}_{loc}^n}{\dot{\gamma}_{loc}}$$

$$\dot{\gamma}_{loc} = \mathcal{F}(\phi)\dot{\gamma}$$

$$\tau = \frac{\eta_s(\phi)}{\mathcal{F}(\phi)}\left(\tau_y + k\dot{\gamma}^n\mathcal{F}(\phi)^n\right)$$

$$P = \frac{\eta_n(\phi)}{\mathcal{F}(\phi)}\left(\tau_y + k\dot{\gamma}^n\mathcal{F}(\phi)^n\right)$$

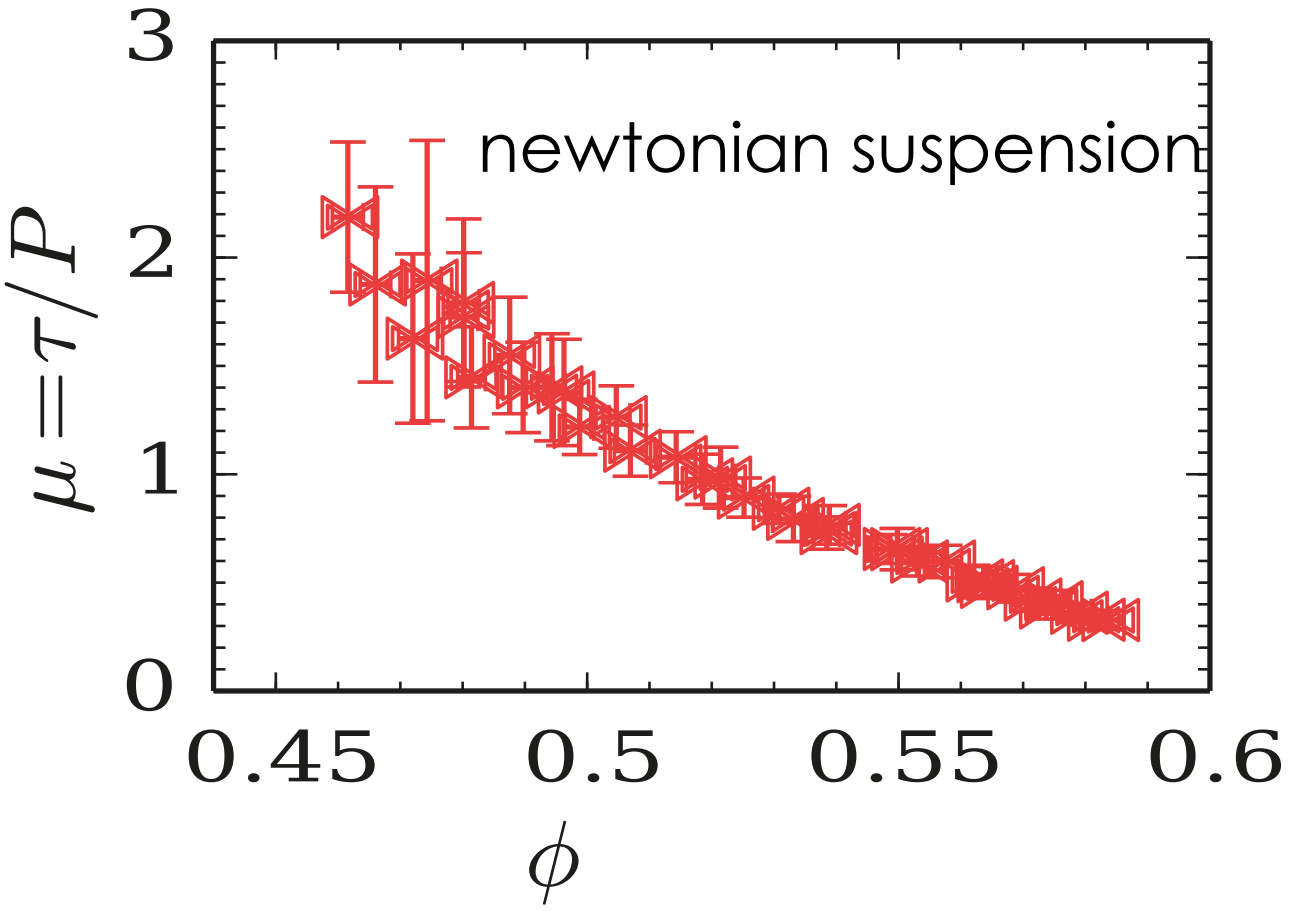
Dagois-Bohy et al JFM 2015

Suspension in a yield stress fluid

test of: ' The same viscosities as in the newtonian expression : '

$$\tau = \eta_{ap}\eta_s(\phi)\dot{\gamma} \qquad \frac{\tau}{P} = \frac{\eta_s}{\eta_n} = \mu(\phi)$$

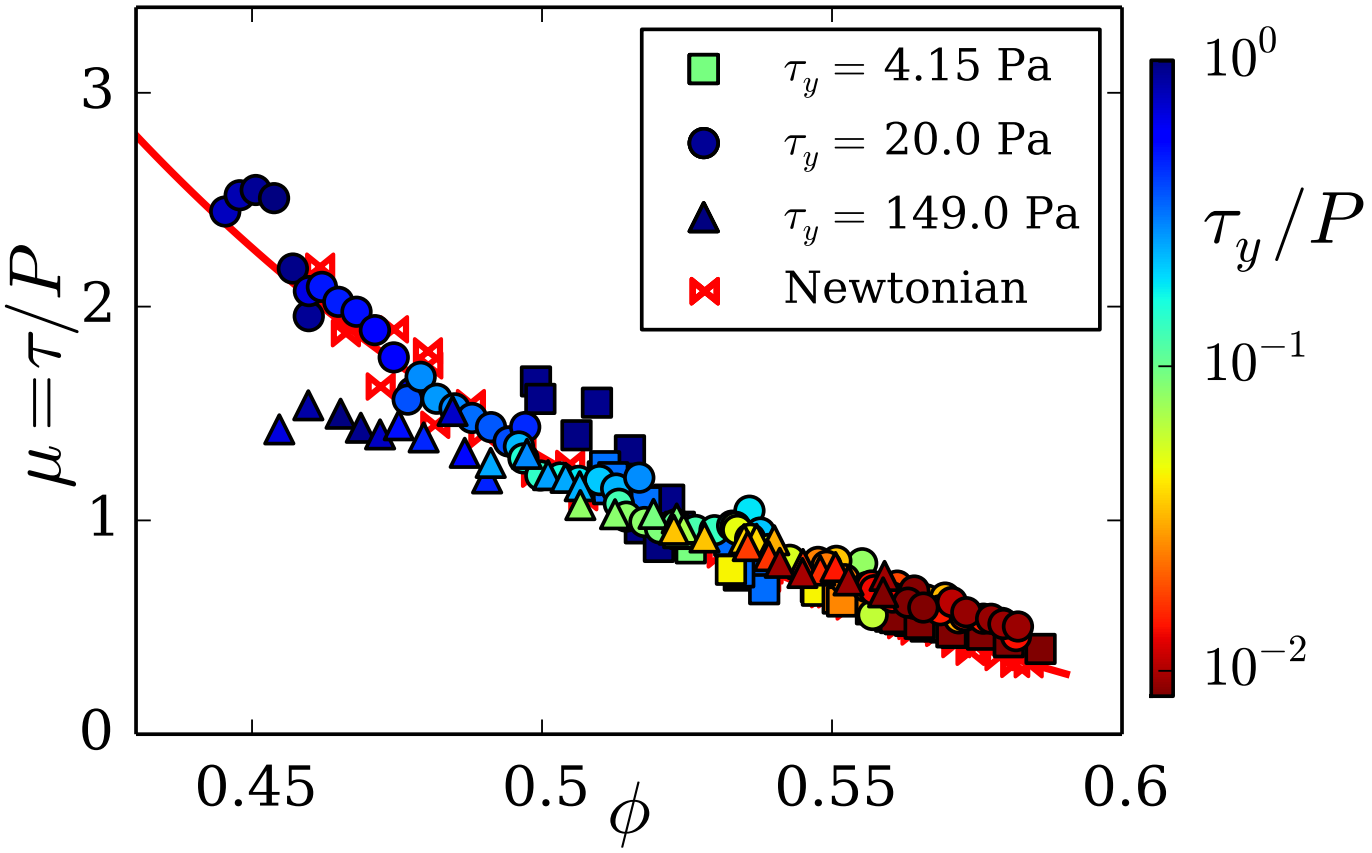
$$P = \eta_{ap}\eta_n(\phi)\dot{\gamma}$$



Suspension in a yield stress fluid

test of: ' The same viscosities as in the newtonian expression : '

$$\tau = \eta_{ap}\eta_s(\phi)\dot{\gamma} \qquad P = \eta_{ap}\eta_n(\phi)\dot{\gamma} \qquad \frac{\tau}{P} = \frac{\eta_s}{\eta_n} = \mu(\phi)$$

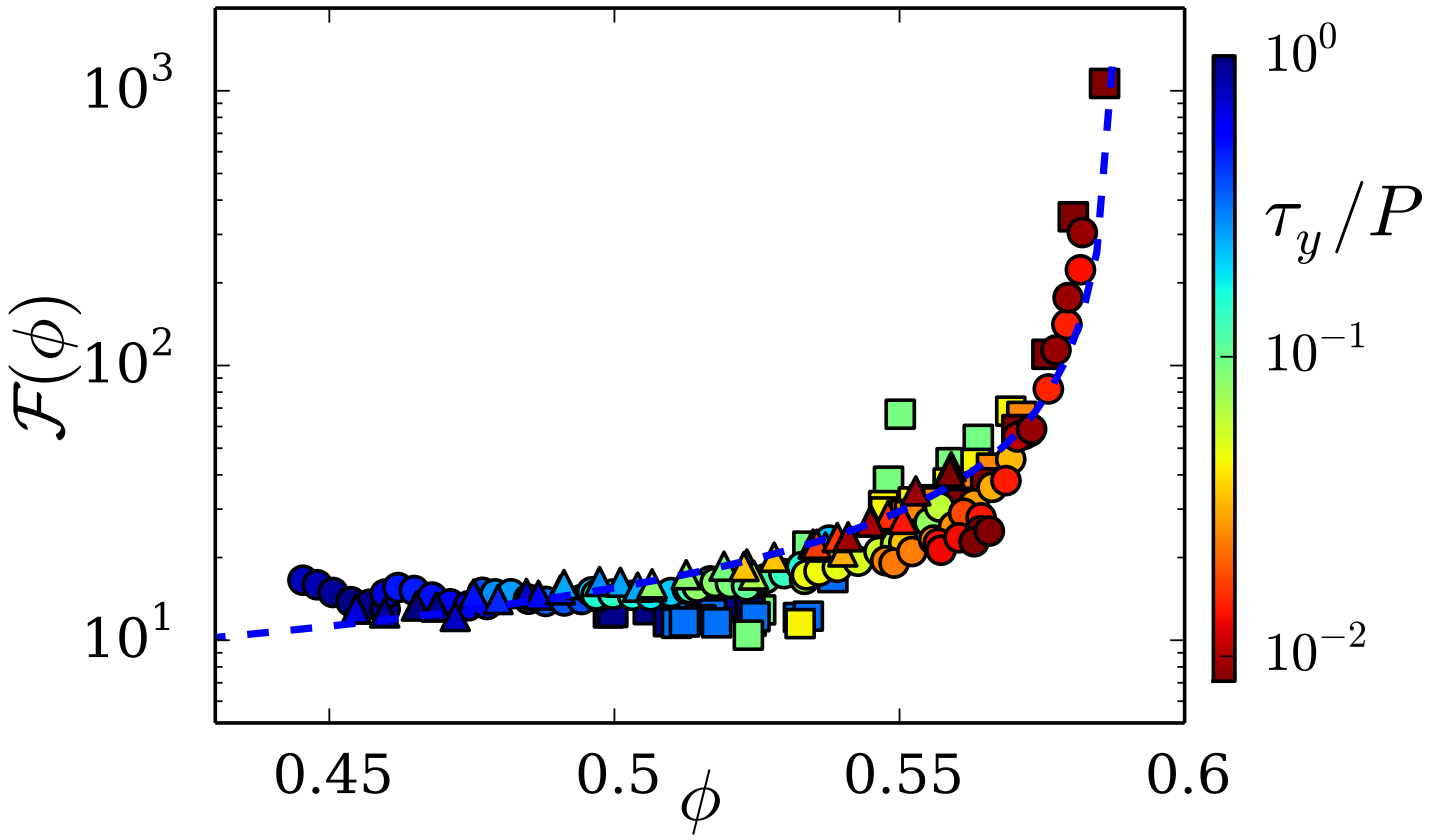


500µm polystyrene beads in Dodecane & water emulsion :

Suspension in a yield stress fluid

test of: 'the local shear rate function' :

$$\tau = \frac{\eta_s(\phi)}{\mathcal{F}(\phi)} \left(\tau_y + k\dot{\gamma}^n \mathcal{F}(\phi)^n\right)$$



Suspension in a yield stress fluid

$$\tau = \frac{\eta_s(\phi)}{\mathcal{F}(\phi)}\left(\tau_y + k\dot{\gamma}^n \mathcal{F}(\phi)^n\right)$$

$$P = \frac{\eta_n(\phi)}{\mathcal{F}(\phi)}\left(\tau_y + k\dot{\gamma}^n \mathcal{F}(\phi)^n\right)$$

in the dense regime, $\mu(\phi)$ and local shear rate are characteristic of the grains, not of the fluid (lever effect)..

suggests that for very dense suspensions, geometrical constraints is the key, hydrodynamics does not matter...

Simon Dagois-Bohy

Sarah Hormozi

Elisabeth Guazzelli

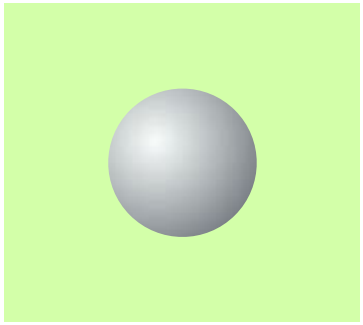
Dagois-Bohy et al JFM, 776, R2 (2015)

towards more complex systems?

suspensions of fibres

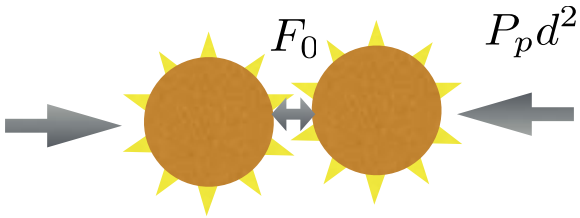
suspensions in a yield stress fluid

suspensions in cornstarch



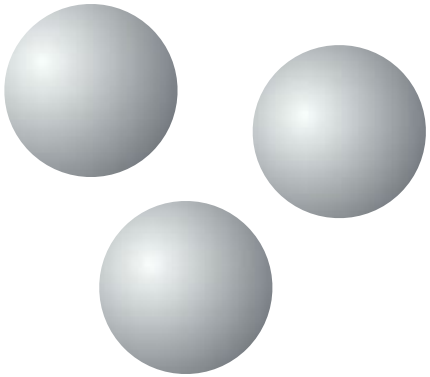
Suspension in cornstarch

Seto et al PRL 2013, Wyart and Cates 2014,…



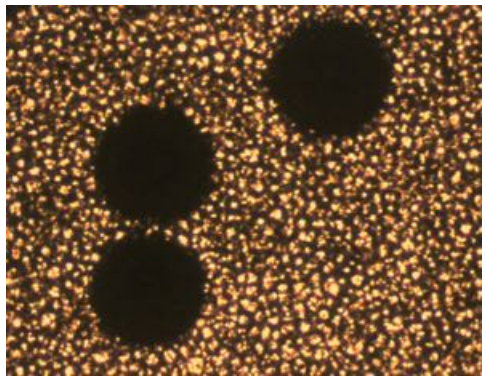
DS $P_p \propto F_0/d^2$

Suspension in cornstarch



+

= ???



Cornstarch: $d_{cs} = 13\mu m \quad \rho = 1.68 g/cm^3$

sylver coated PMMA Particles:

$106\mu m < d_p < 125\mu m \quad \rho = 1.34 g/cm^3$

PMMA Particles:

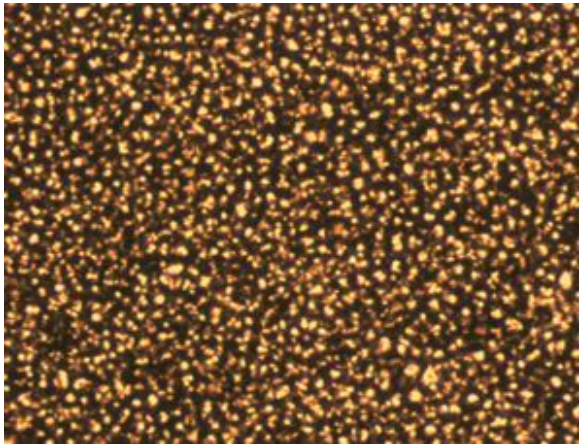
$45\mu m < d_p < 355\mu m \quad \rho = 1.20 g/cm^3$

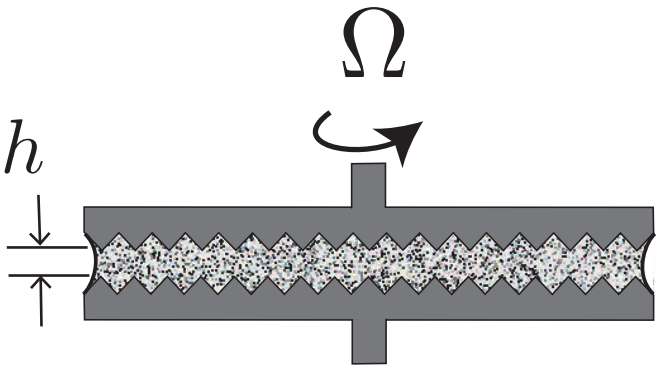
Fluid:

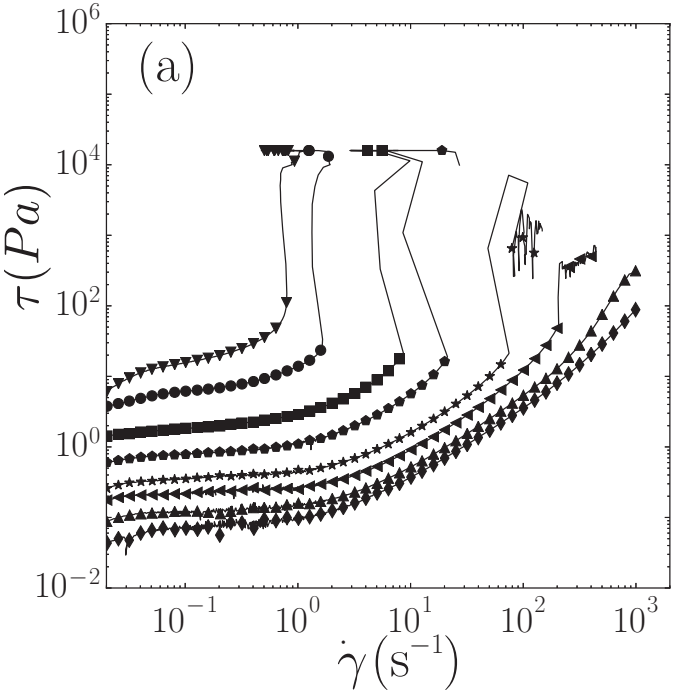
water+cesium chloride $\rho = 1.68 g/cm^3$

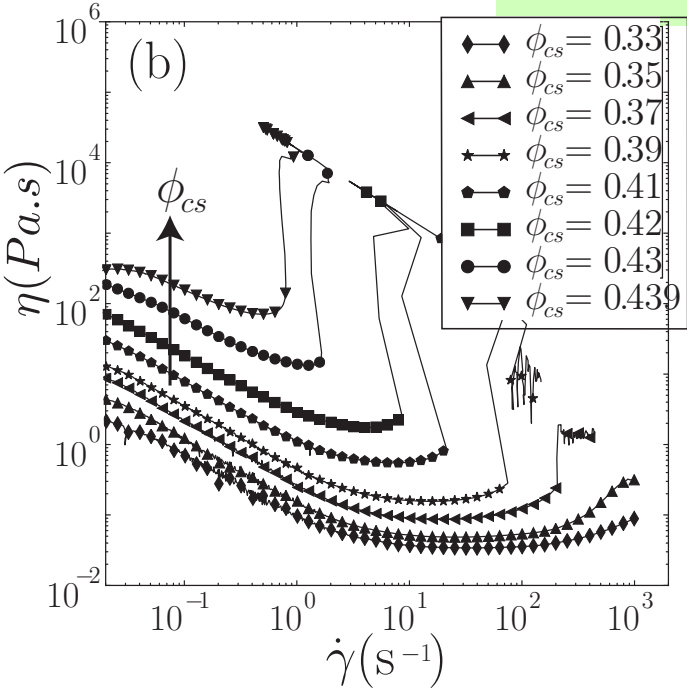
Suspension in cornstarch

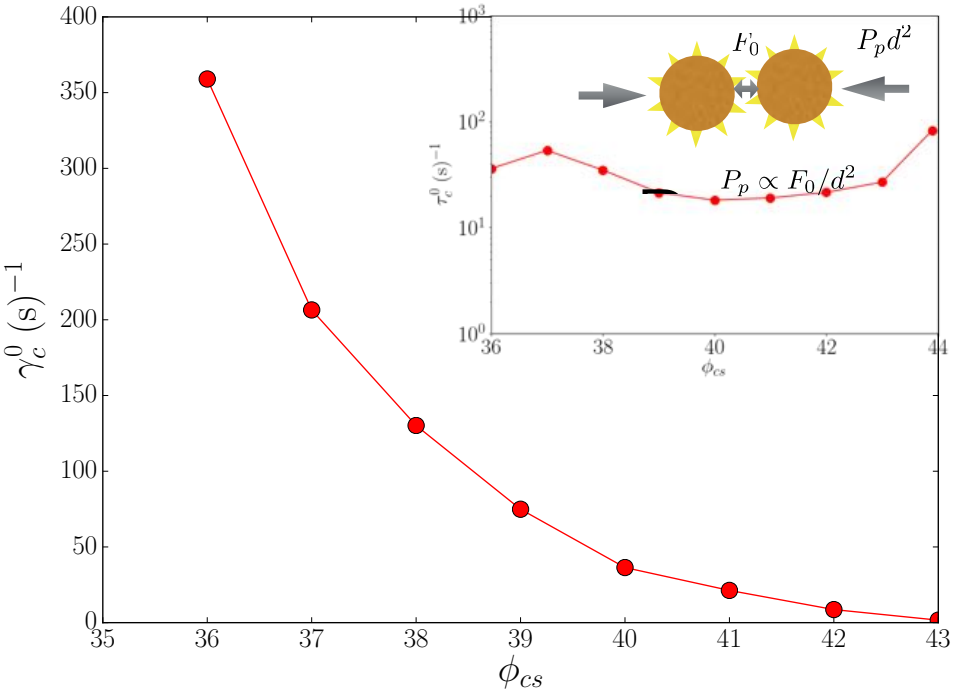
Rheology of pure Cornstarch:





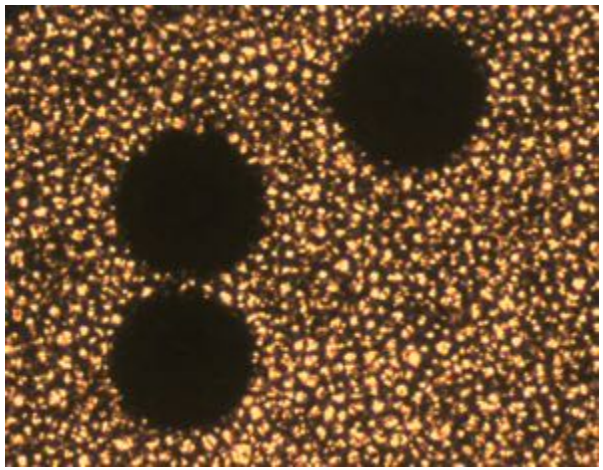


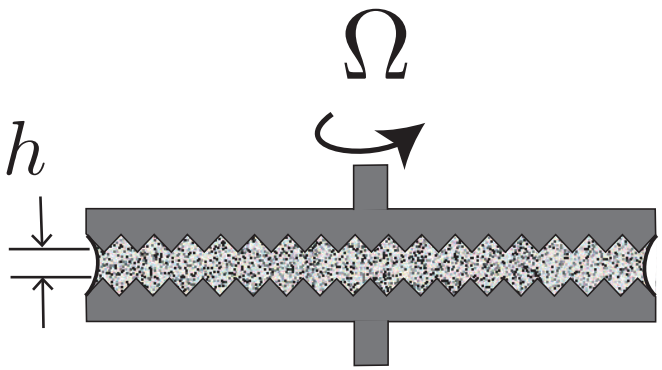


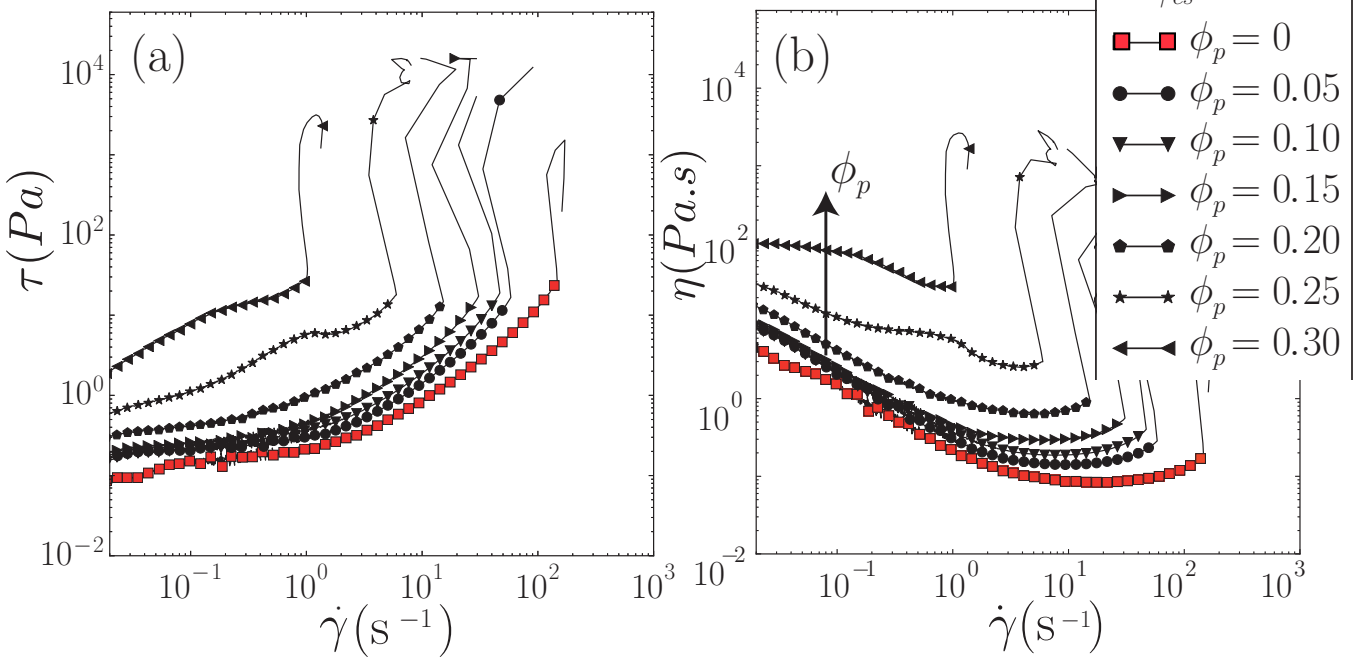


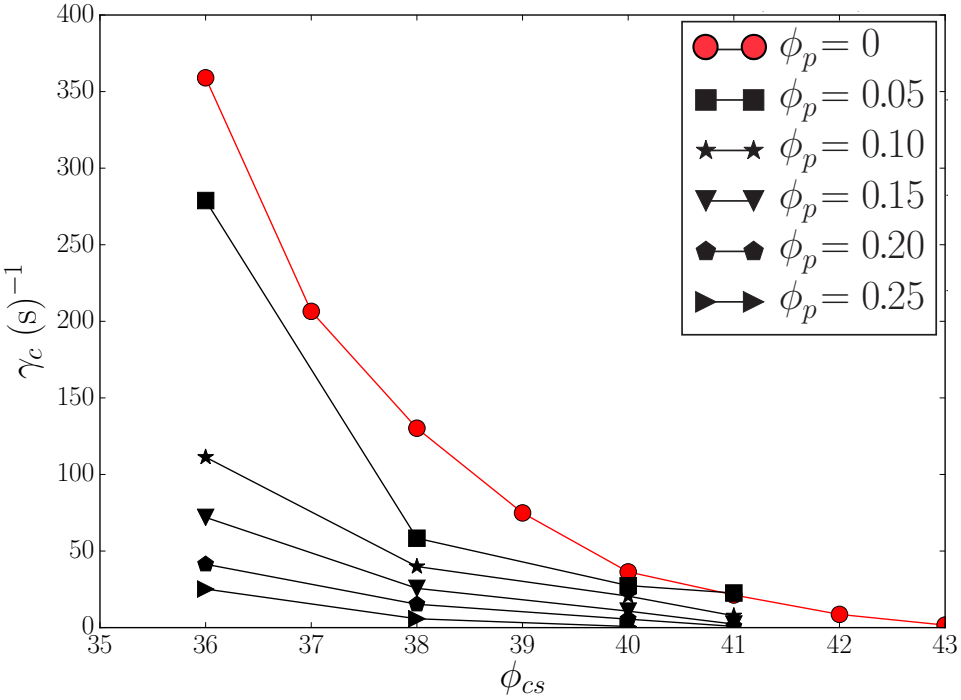
Suspension in cornstarch

Rheology of Cornstarch+ particles



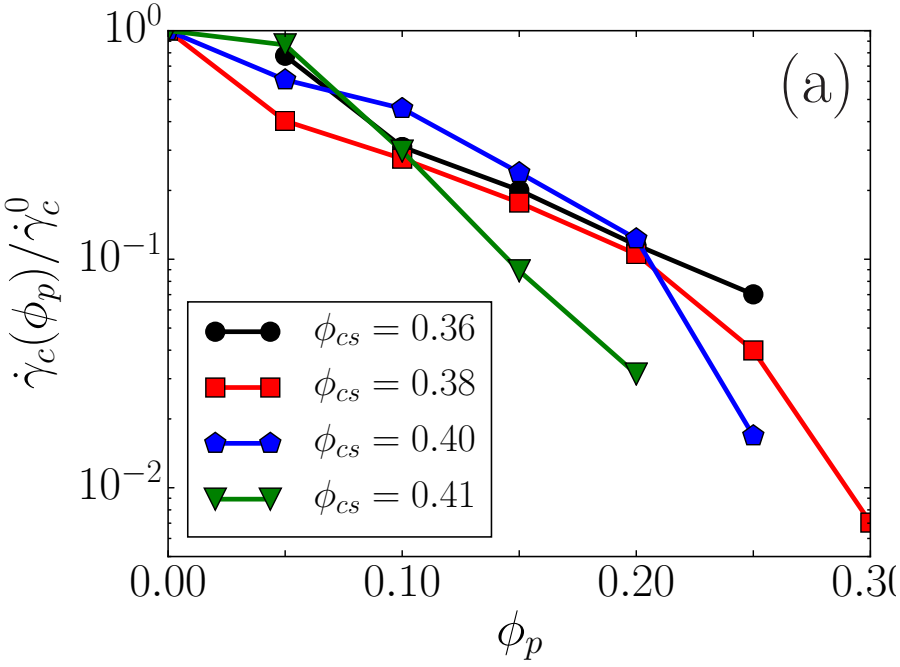


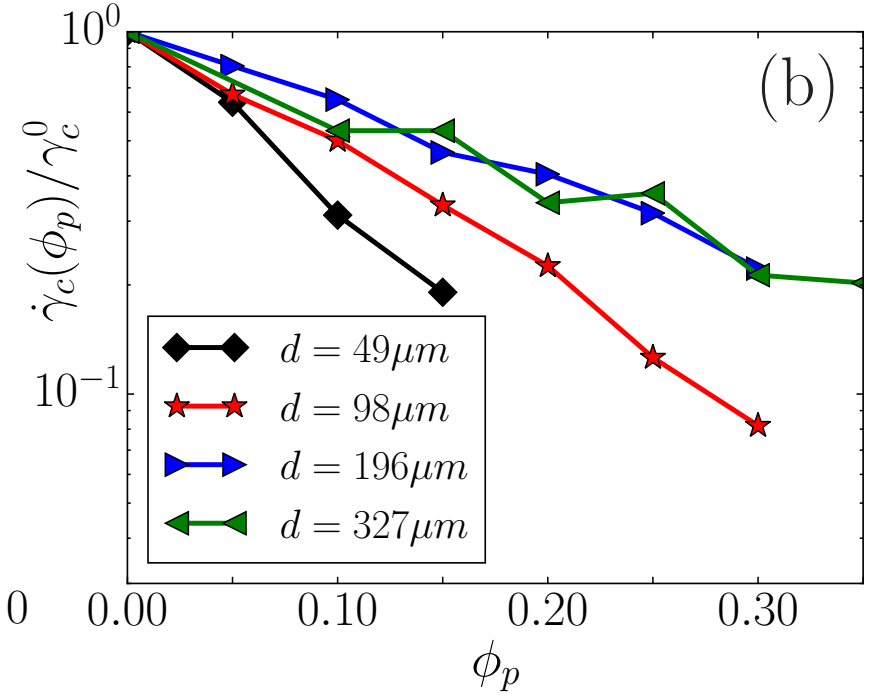




Suspension in cornstarch

shift of the critical shear rate



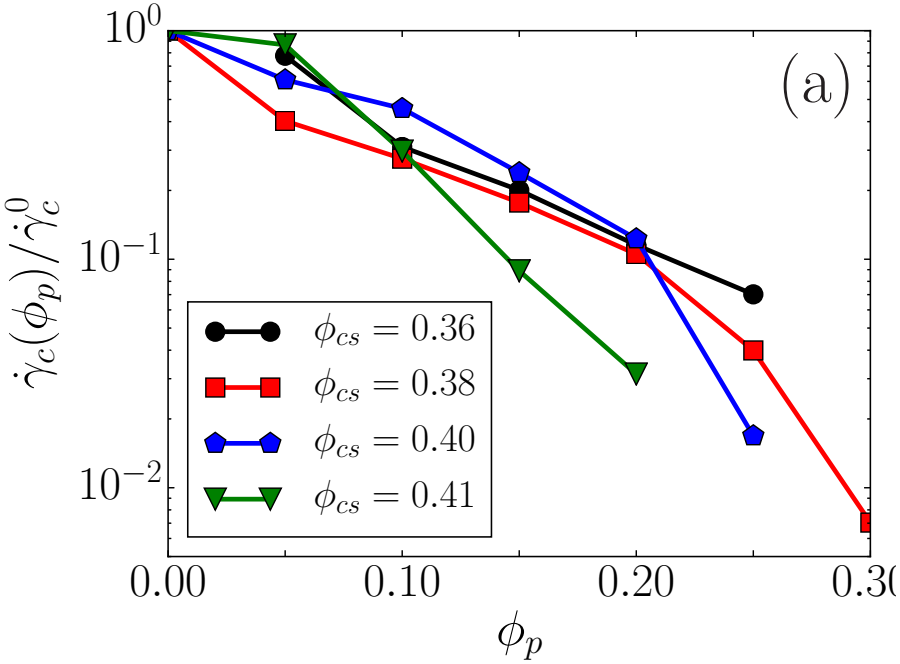


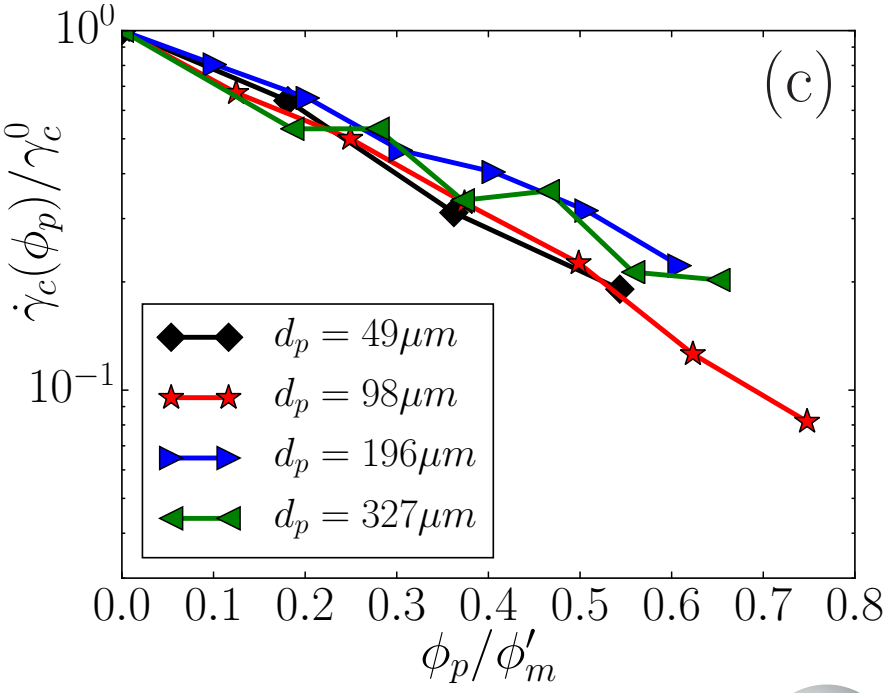
Independent of the cornstarch concentration

depends on the size ratio

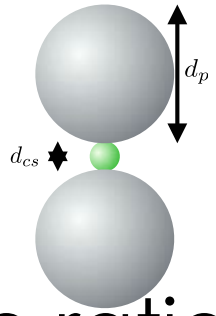
Suspension in cornstarch

shift of the critical shear rate





$$\phi_m^{'} = \phi_m d_p^3/(d_p + d_{cs})^3$$

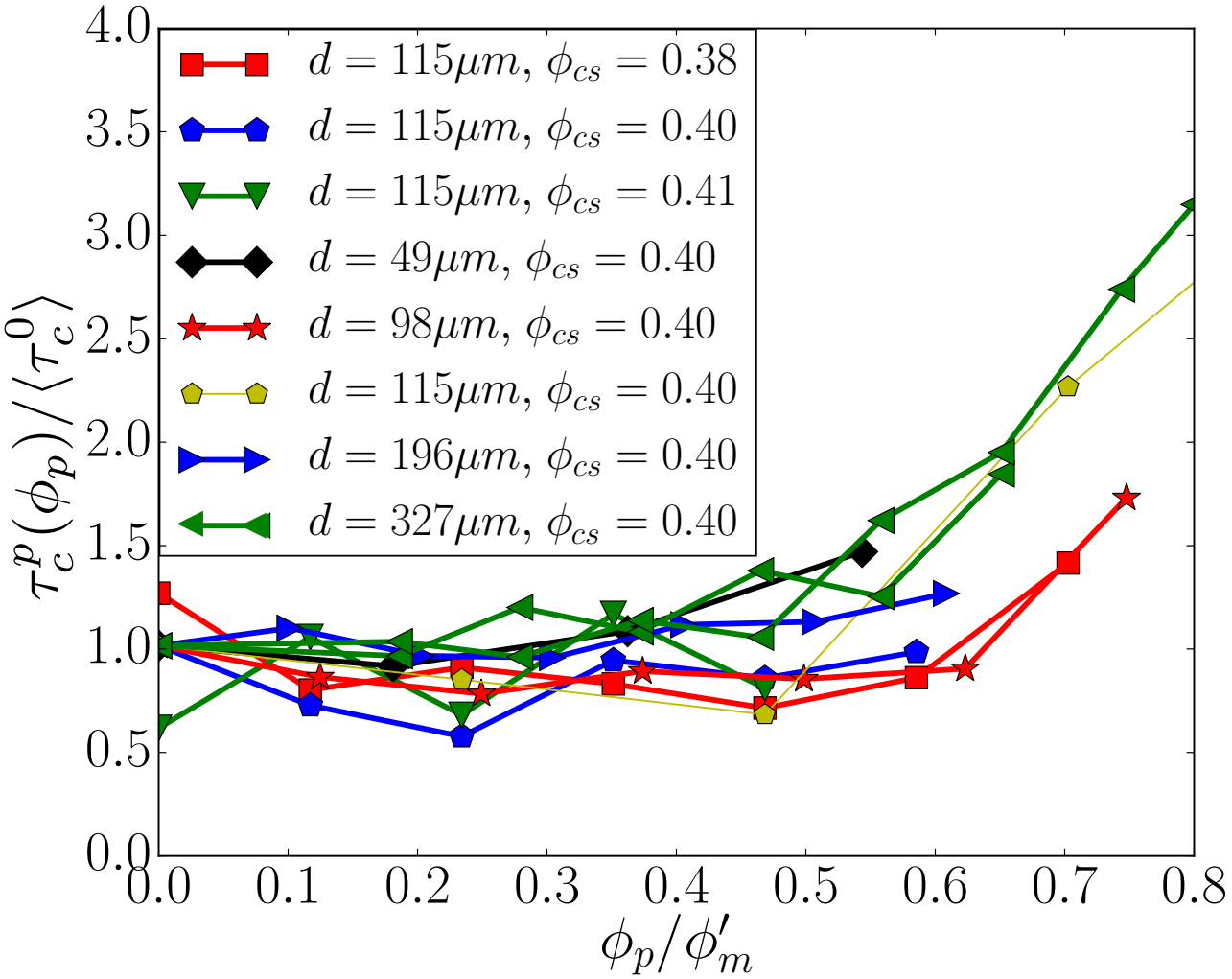


Independant of the cornstarch concentration

depends on the size ratio (a simple rescaling)

Suspension in cornstarch

influence on the critical shear stress

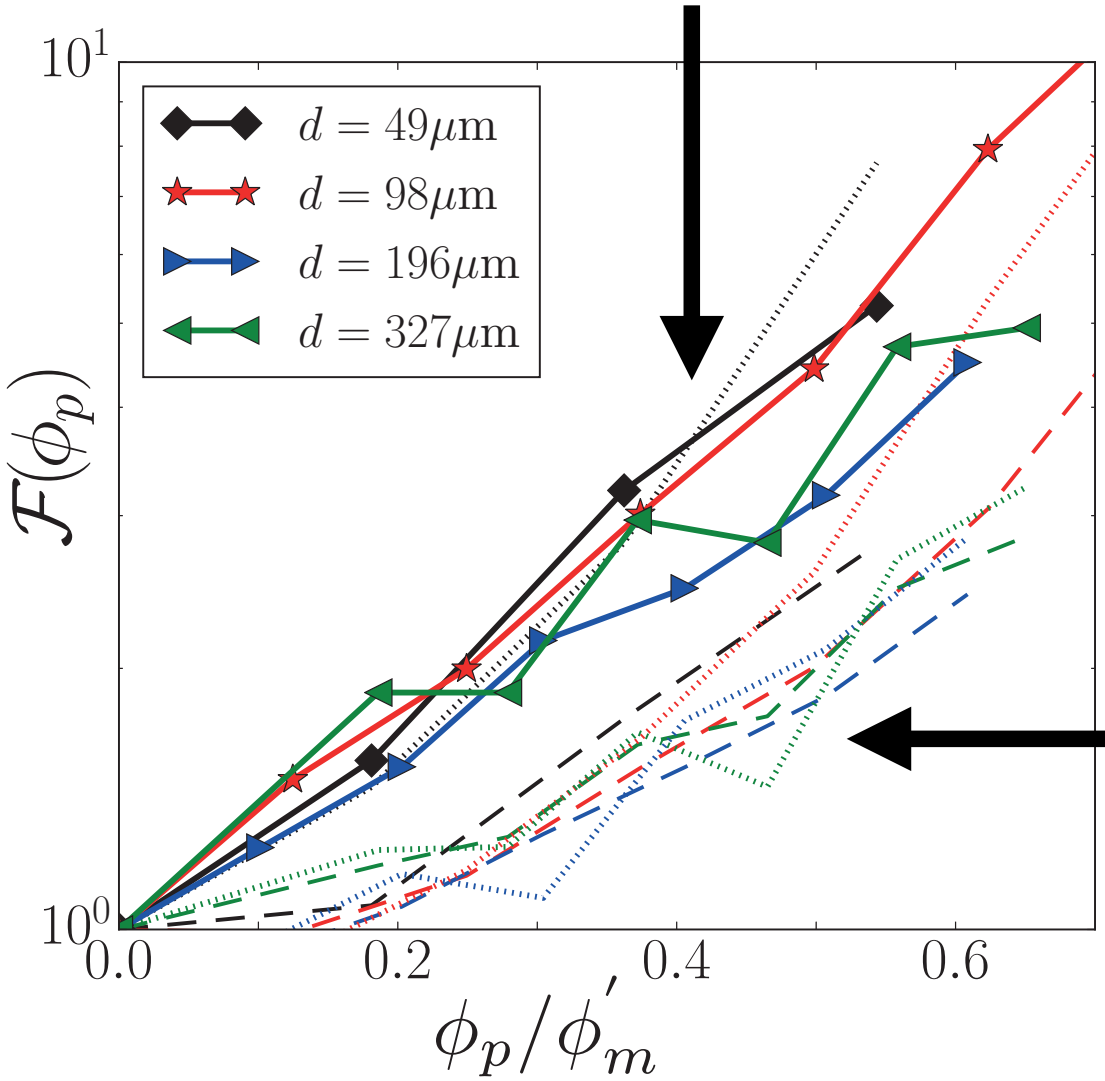


critical shear rate increases at high volume fraction..
(when granular stresses become important..?)

Suspension in cornstarch

lever function $< \dot{\gamma}_{\mathrm{local}} >= \mathcal{F}(\phi_p)\dot{\gamma}$

estimate from DST $\mathcal{F}(\phi) = \dot{\gamma}_c(0)/\dot{\gamma}_c(\phi)$



estimate from the viscosity prior to DST

$$\mathcal{F}(\phi) = \sqrt{\frac{\eta(\phi)}{\eta(0)(1-\phi)}}$$

Suspension in cornstarch

1) Adding Non Brownian particles in a shear thickening fluid shift the critical shear rate, but not so much the critical stress… (equivalent to increase the cornstarch concentration…)

2) influence of the particle is more dramatic on the DST

than on the bulk rheology…

Yasaman Madraki

Sarah Hormozi

Guillaume Ovarlez

Elisabeth Guazzelli

Madraki et al
Phys. Rev. Fluids **2**, 033301 (2017)

Concluding remarks :

even without chemistry,
suspensions of hard particles are rich….

adding particles amplifies the rheology…

A better understanding of those simple systems
might provide insights for more complex ones…

thanks..

Simon Dagois-Bohy (Univ. Lyon)

Franco tapia (Aix-Marseille univ)

Yasaman Madraki (Ohio Univ.)

Elisabeth Guazzelli (Aix-Marseille Univ)

Saif Shaikh (Univ. Florida)

Jason Butler (Univ. Florida)

Sarah Hormozi (Ohio Univ.)

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