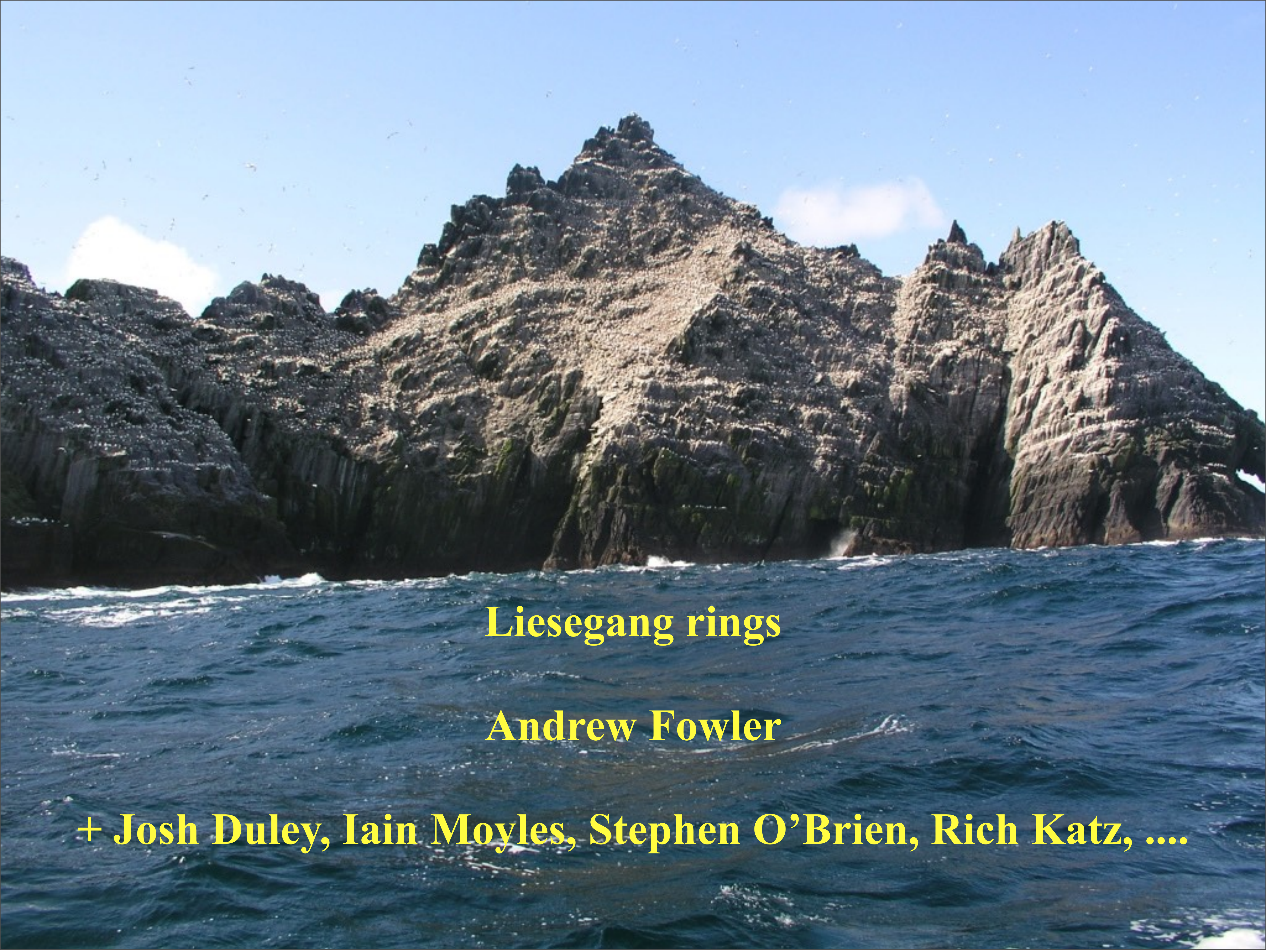
Liesegang rings
Andrew Fowler
+ Josh Duley, Iain Moyles, Stephen O'Brien, Rich Katz, ...

Analysis of the Lorenz Equations for Large r

By A. C. Fowler

....

can be thought of as a multiple scale approximation. A more elegant method of obtaining higher-order terms is that due to Kuzmak [8], extended by Luke [11], and by Kogelman and Keller [7] and Kevorkian and Cole [6]. In this method the fast time t^* is chosen to satisfy $dt^*/dt = \phi(\tau)$, where ϕ is chosen to keep the period P of the fast oscillation constant: in our case we would have $\phi \propto \sqrt{B}/K$.

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7. S. KOGELMAN and J. B. KELLER, Asymptotic theory of nonlinear wave propagation, *SIAM J. Appl. Math*. 24:352–361 (1973).

SIAM J. Appl. Math.
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OSCILLATIONS IN A MATURATION MODEL OF BLOOD CELL PRODUCTION*

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[6] A. S. Fokas, J. B. Keller, and B. D. Clarkson, *Mathematical model of granulocytopoiesis and chronic myelogenous leukemia*, Cancer Res., 51 (1991), pp. 2084–2091.

Three things ...

Recurrent precipitation and Liesegang rings

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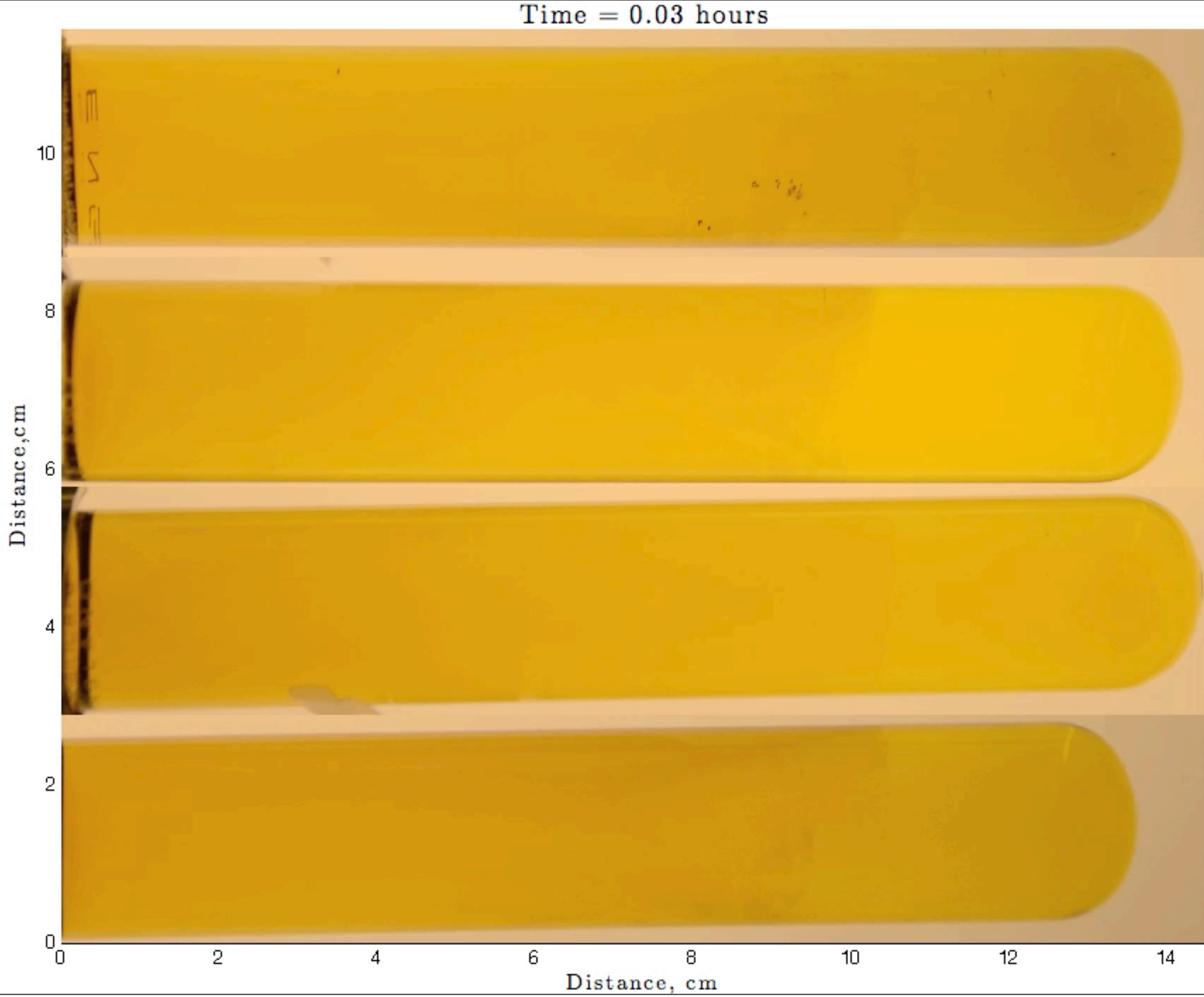
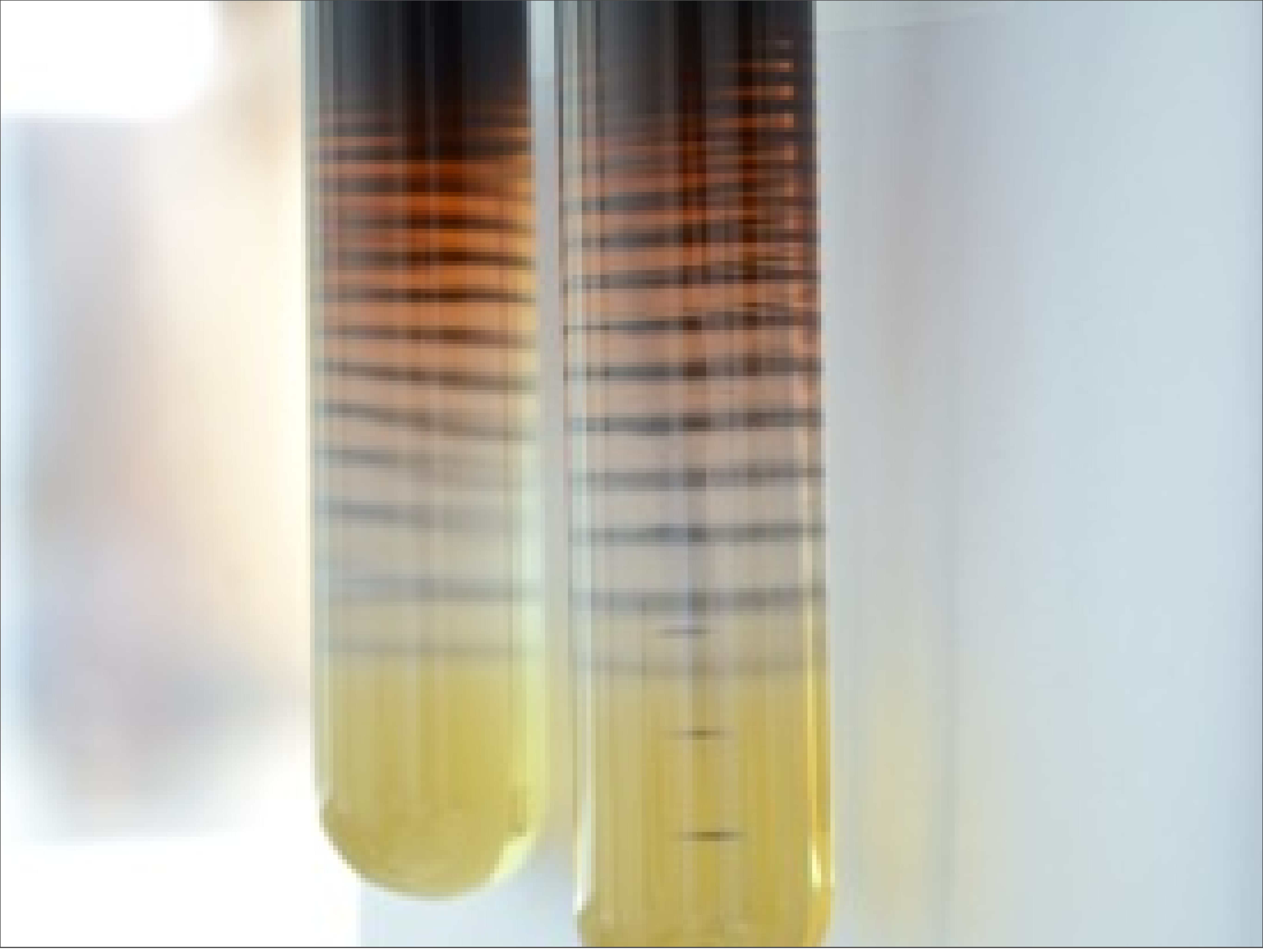
A mathematical formulation is presented of Wilhelm Ostwald's supersaturation theory of Liesegang ring formation. The theory involves diffusion of two reactants toward one another, their chemical reaction to form a product, the reverse reaction, and diffusion of the product. When the product concentration reaches a certain supersaturation value, it begins to precipitate. It is shown that this theory can lead to recurrent precipitation, resulting in rings or bands of precipitate. Conditions under which this occurs are determined. In addition, the locations and times of formation of the bands are calculated and shown to agree with experimental results.



Time = 0.03 hours
10
8
6
4
2
0
Distance,cm
0
2
4
6
8
10
12
14
Distance, cm



$$2\mathrm{AgNO_3} + \mathrm{K_2Cr_2O_7} \rightarrow \mathrm{Ag_2Cr_2O_7} + 2\mathrm{KNO_3}$$

$$2\mathrm{A} + \mathrm{B} \ \underset{k_-}{\overset{k_+}{\rightleftharpoons}} \ \mathrm{C} \xrightarrow{p} \mathrm{D}$$

$$p = \begin{cases} q[c - c_s]_+ & \text{if} \quad c \geq c_n \ > c_s \text{ or } d > 0, \\ 0 & \text{if} \quad c < c_n \text{ and } d = 0, \end{cases}$$

$$
\begin{aligned}
a_t &= D_A a_{xx} - 2r, \\
b_t &= D_B b_{xx} - r, \\
c_t &= D_C c_{xx} + r - p, \\
d_t &= p,
\end{aligned}
$$

$$r = k_+ a^2 b - k_- c$$

$$
\begin{aligned}
a_t &= a_{xx} - 2\varepsilon r, \\
b_t &= \delta b_{xx} - r, \\
\nu c_t &= \nu\delta c_{xx} - p + r, \\
d_t &= p,
\end{aligned}
$$

$$
p = \begin{cases} [c-\alpha]_+ & \text{if} \quad c \geq 1 \text{ or } d > 0, \\ 0 & \text{if} \quad c < 1 \text{ and } d = 0, \end{cases}
$$

$$
r = \Lambda(\lambda a^2 b - c).
$$

total dichromate

$$B = b + \nu c$$

$$\begin{aligned} B_t &= \delta B_{xx} - p, \\ d_t &= p, \end{aligned}$$

$$c = AB, \quad A = \frac{\lambda a^2}{1 + \lambda \nu a^2},$$

$$p = \begin{cases} [AB - \alpha]_+ & \text{if} \quad AB \geq 1 \text{ or } d > 0, \\ 0 & \text{if} \quad AB < 1 \text{ and } d = 0. \end{cases}$$

$$A(\theta) = \frac{\lambda \operatorname{erfc}^2\theta}{1 + \lambda\nu \operatorname{erfc}^2\theta}, \quad \theta = \frac{x}{2\sqrt{t}}$$

Suppose that $A(\theta)$ is slowly varying in space, and that s is slowly varying in time;

$$\sqrt{A(\Theta)} - \frac{1}{\sqrt{A(\Theta)}} = (1-\alpha)\sqrt{\pi t}\tanh\left(\sqrt{\frac{A(\Theta)}{\delta}}s\right), \qquad \Theta = \frac{s}{2\sqrt{t}}.$$

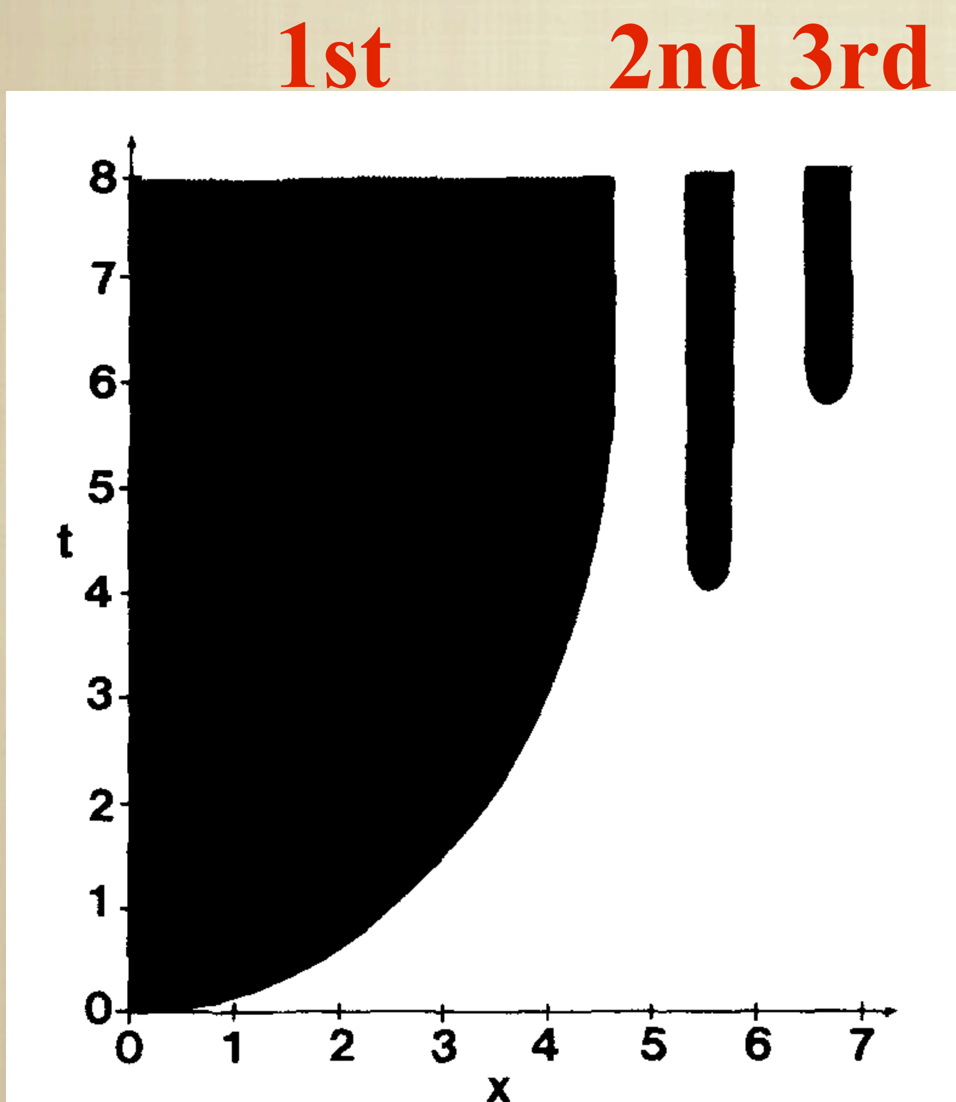


FIG. 1. The (x, t) plane, showing the first three precipitation bands shaded. The boundary of the first precipitation band $x=R(t)$ has been computed for the parameter values $k=10$, $\nu=2$, $\beta=1$, $c^*=0.2$, $c^s=0.05$, as described in Sec. VI. The second and third zones begin at $(x_2, t_2)=(5.53, 4.02)$ and $(x_3, t_3)=(6.64, 5.79)$, respectively. These points have been computed from the asymptotic solution Eq. (7.6) with $x_1=R^*=4.61$, $t_1=(R^*)^2/4\rho^2=2.79$, $\mu=1.20$, and $\rho=1.38$. The values of ρ and μ are the solutions of Eqs. (7.7) and (7.8) with the same parameter values as above. The boundaries of these two bands are schematic, and were not computed.

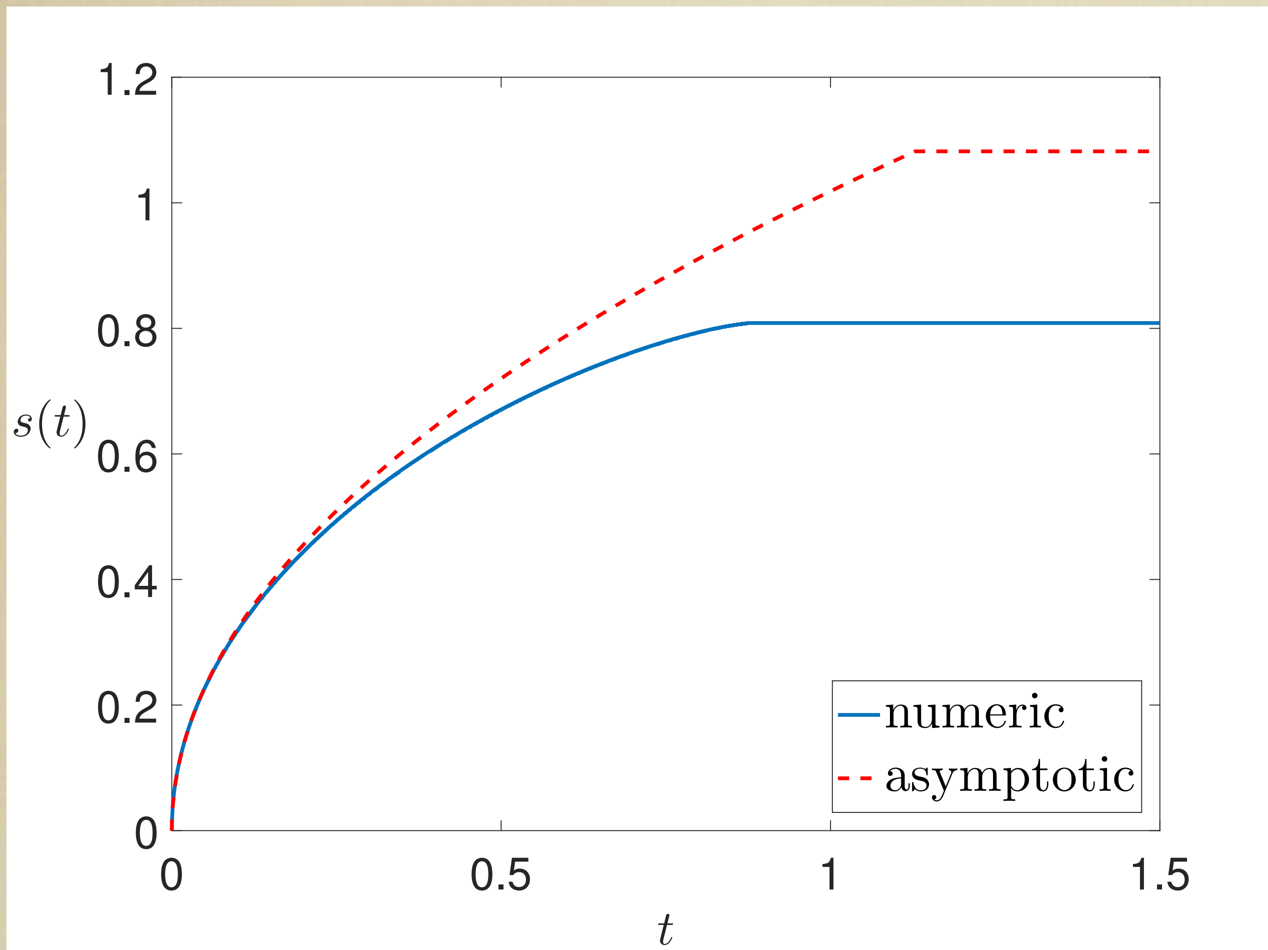
$$
\begin{aligned}
B_t &= \delta B_{xx} - p, \\
d_t &= p,
\end{aligned}
$$

$$
c = AB, \quad A = \frac{\lambda a^2}{1 + \lambda \nu a^2},
$$

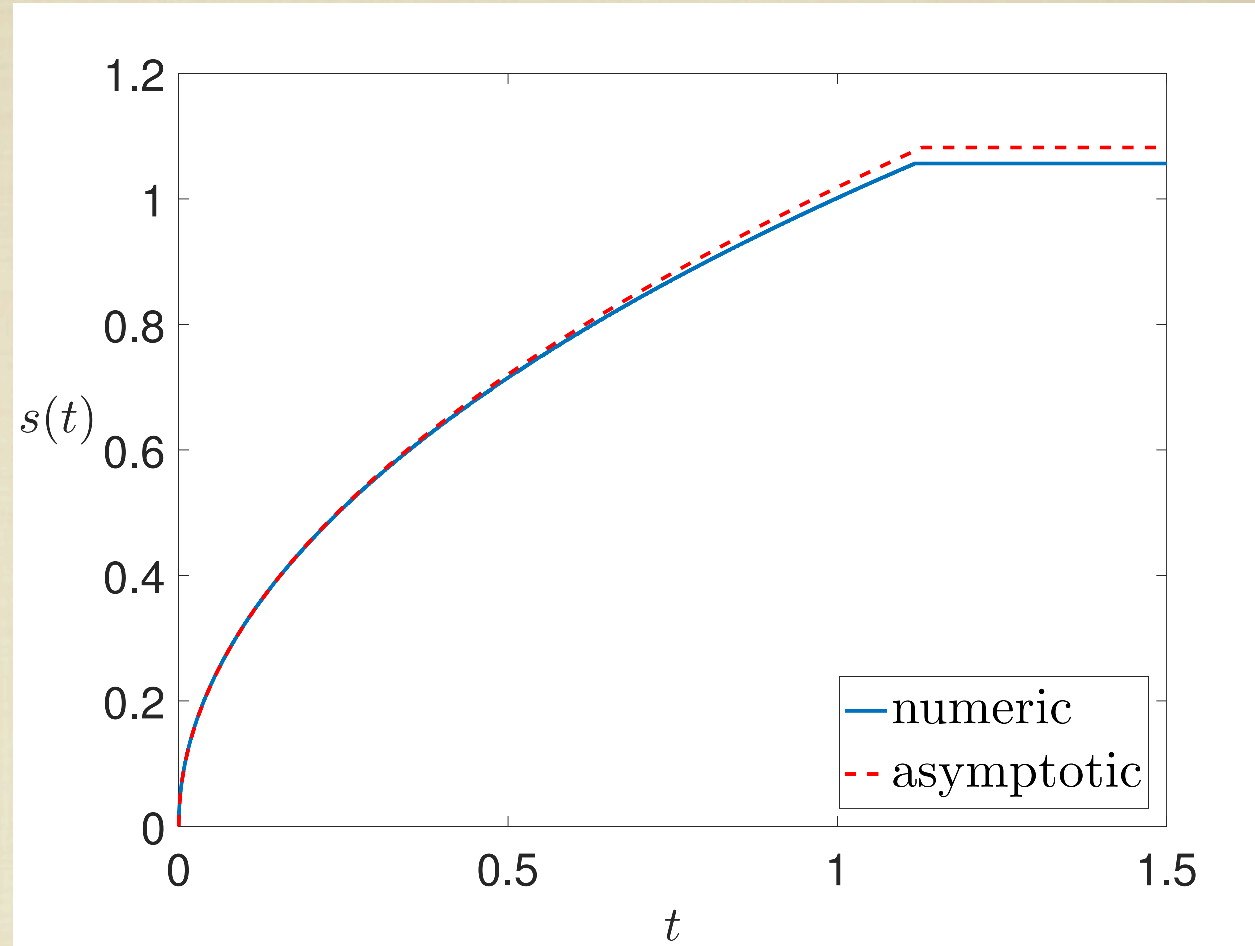
$$
p = \begin{cases} [AB - \alpha]_+ & \text{if} \quad AB \geq 1 \text{ or } d > 0, \\ 0 & \text{if} \quad AB < 1 \text{ and } d = 0. \end{cases}
$$

$\delta \ll 1$ ....

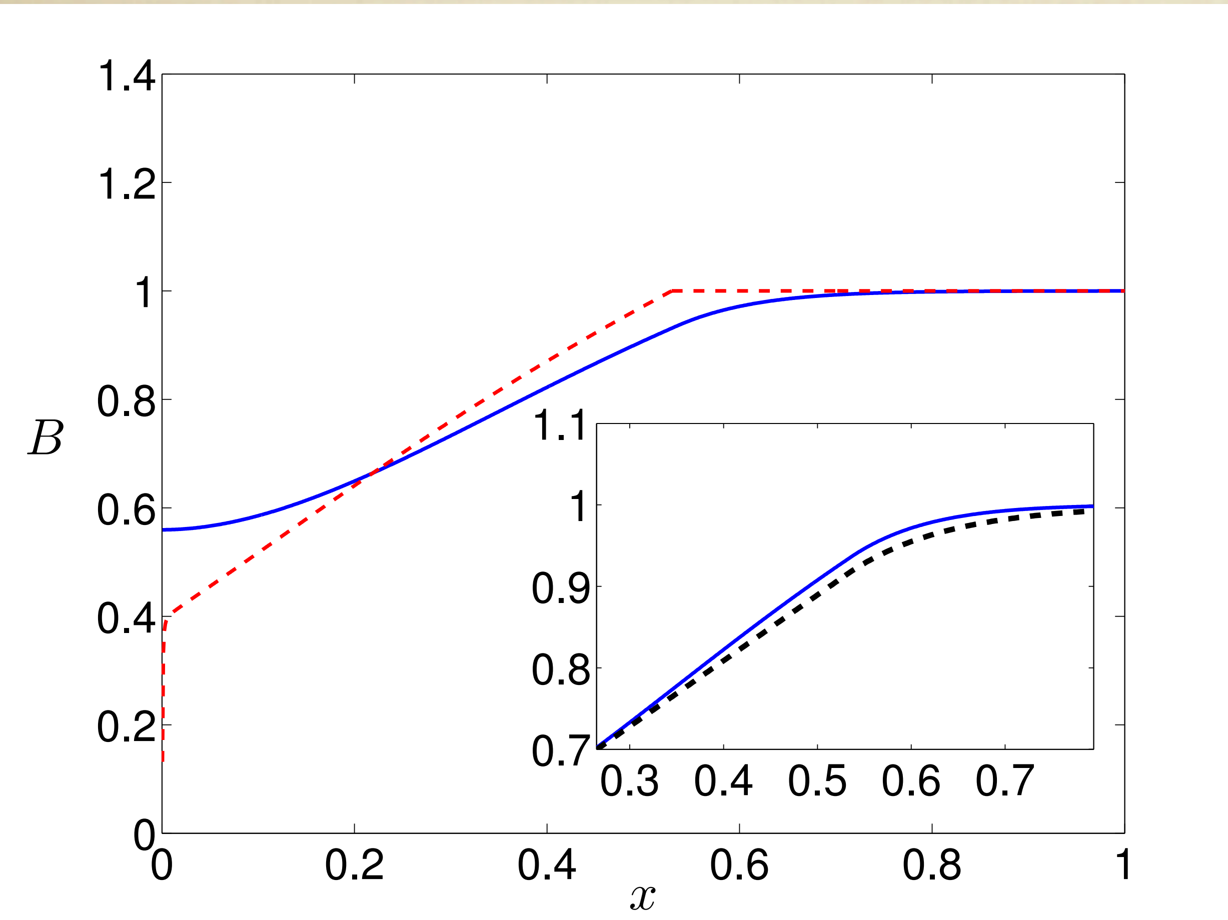
s(t): the first front



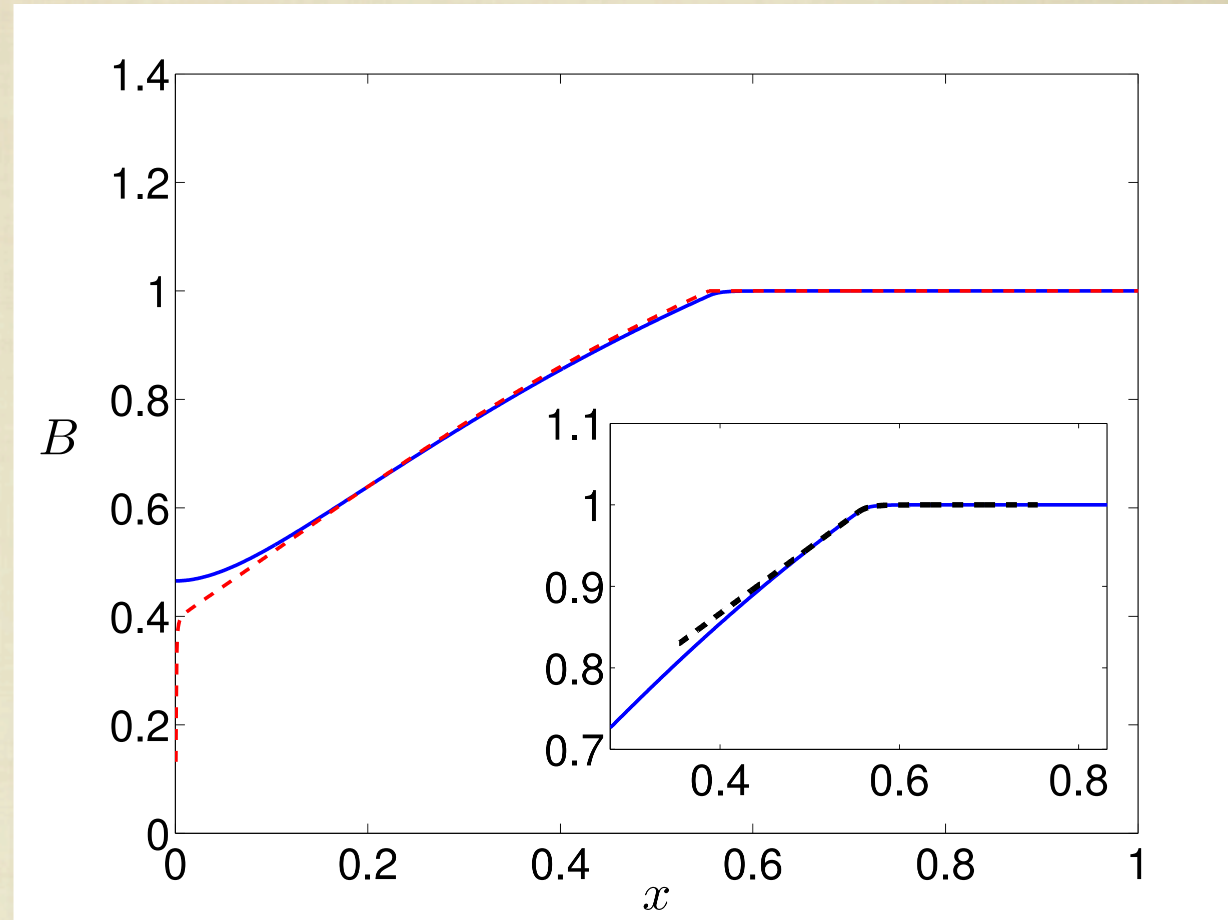
(a) $\delta = 0.1$, $t_n = 0.873$, $t_a = 1.129$



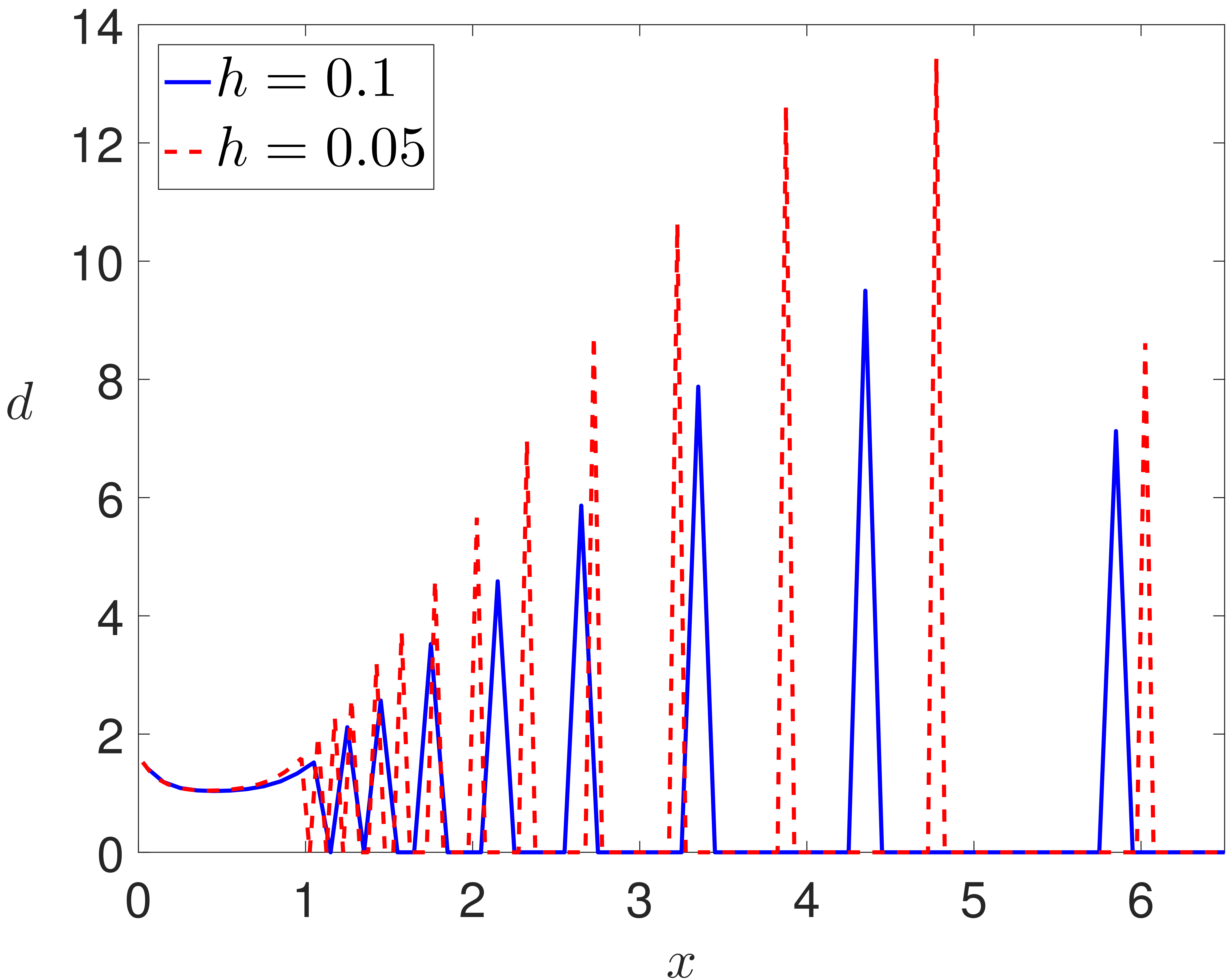
(b) $\delta = 0.01$, $t_n = 1.1170$, $t_a = 1.129$



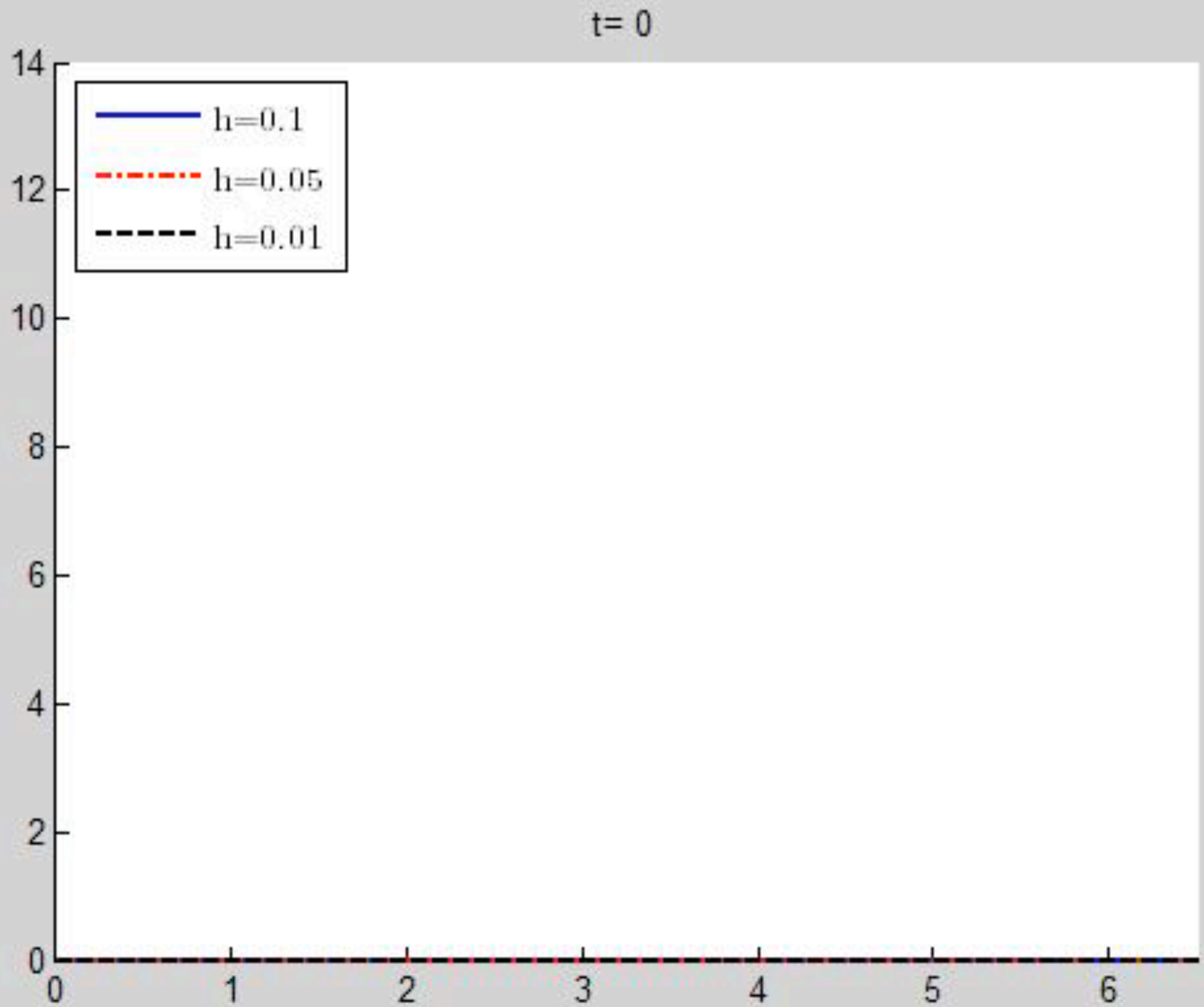
(a) $\delta = 0.1$



(b) $\delta = 0.01$

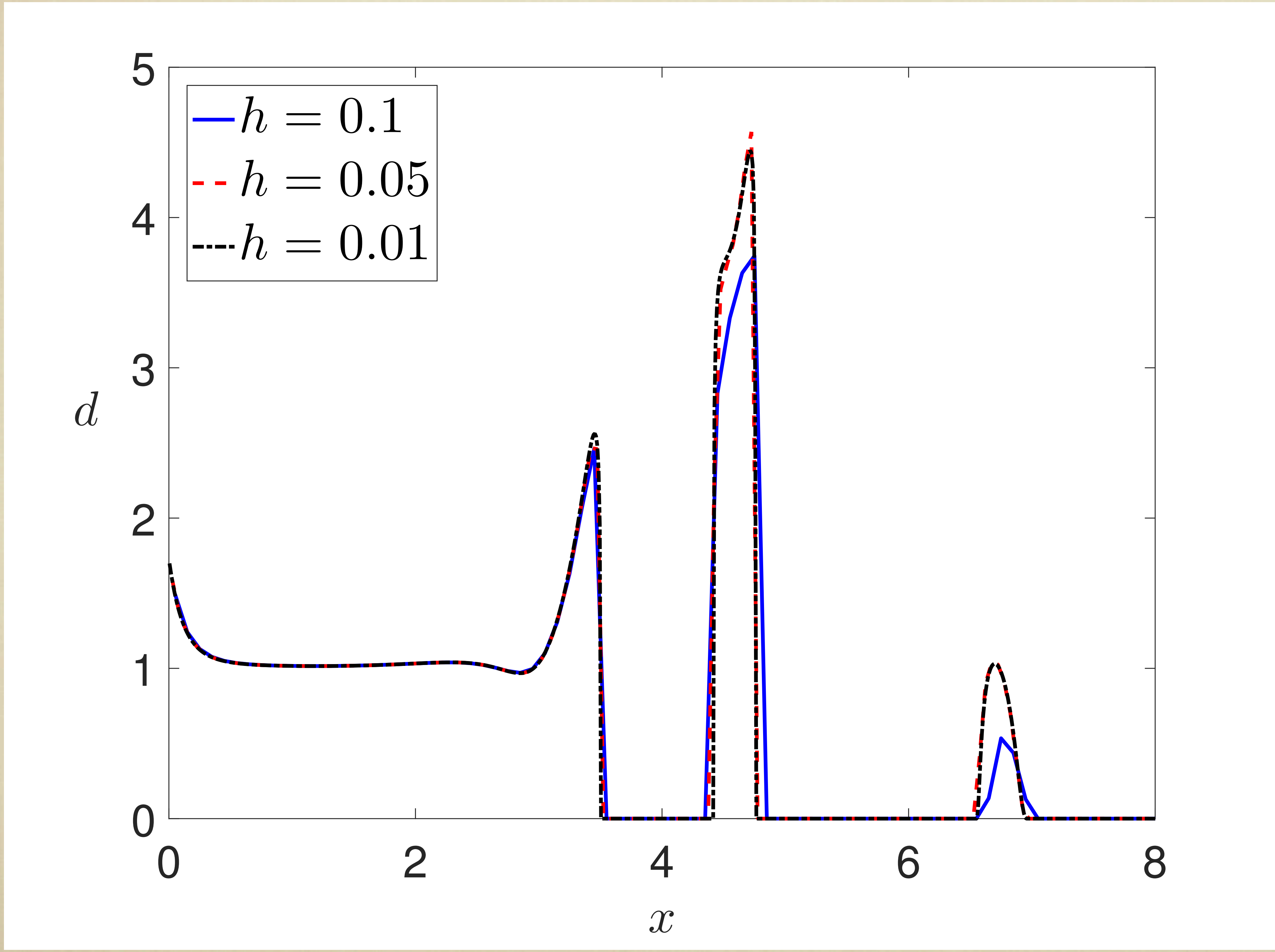


$h = 0.1$
$h = 0.05$
d
x
0
2
4
6
8
10
12
14
0
1
2
3
4
5
6



t= 0
h=0.1
h=0.05
h=0.01
0
2
4
6
8
10
12
14
0
1
2
3
4
5
6

$$p \approx [AB - \alpha]_+ \tanh\left(\frac{[AB-1]_+ + d}{\sigma}\right)$$



The switching equation

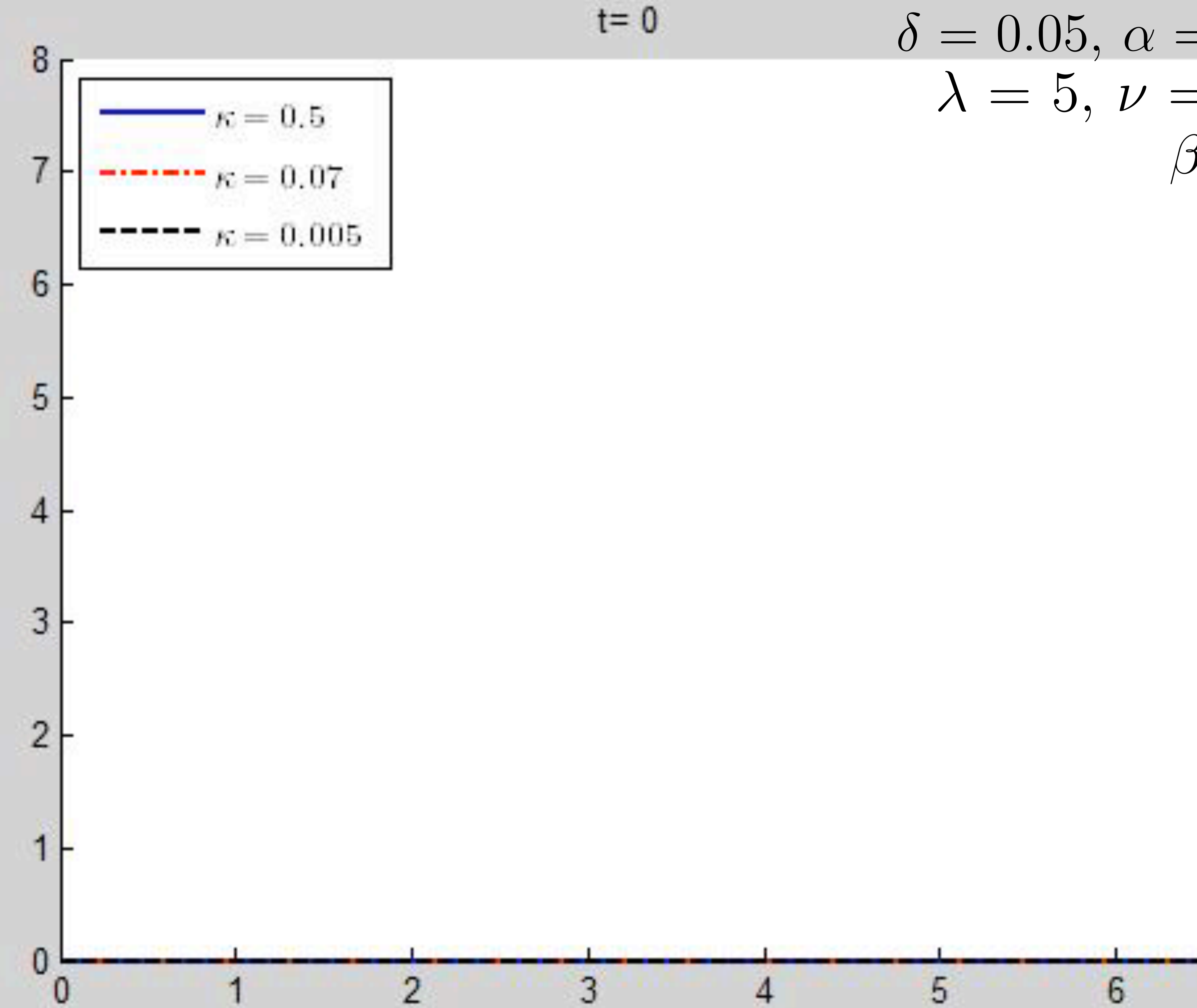
$$p = fq[c - c_s]_+$$

$$t_n f_t = G(f, c) + t_n D_I f_{xx}$$

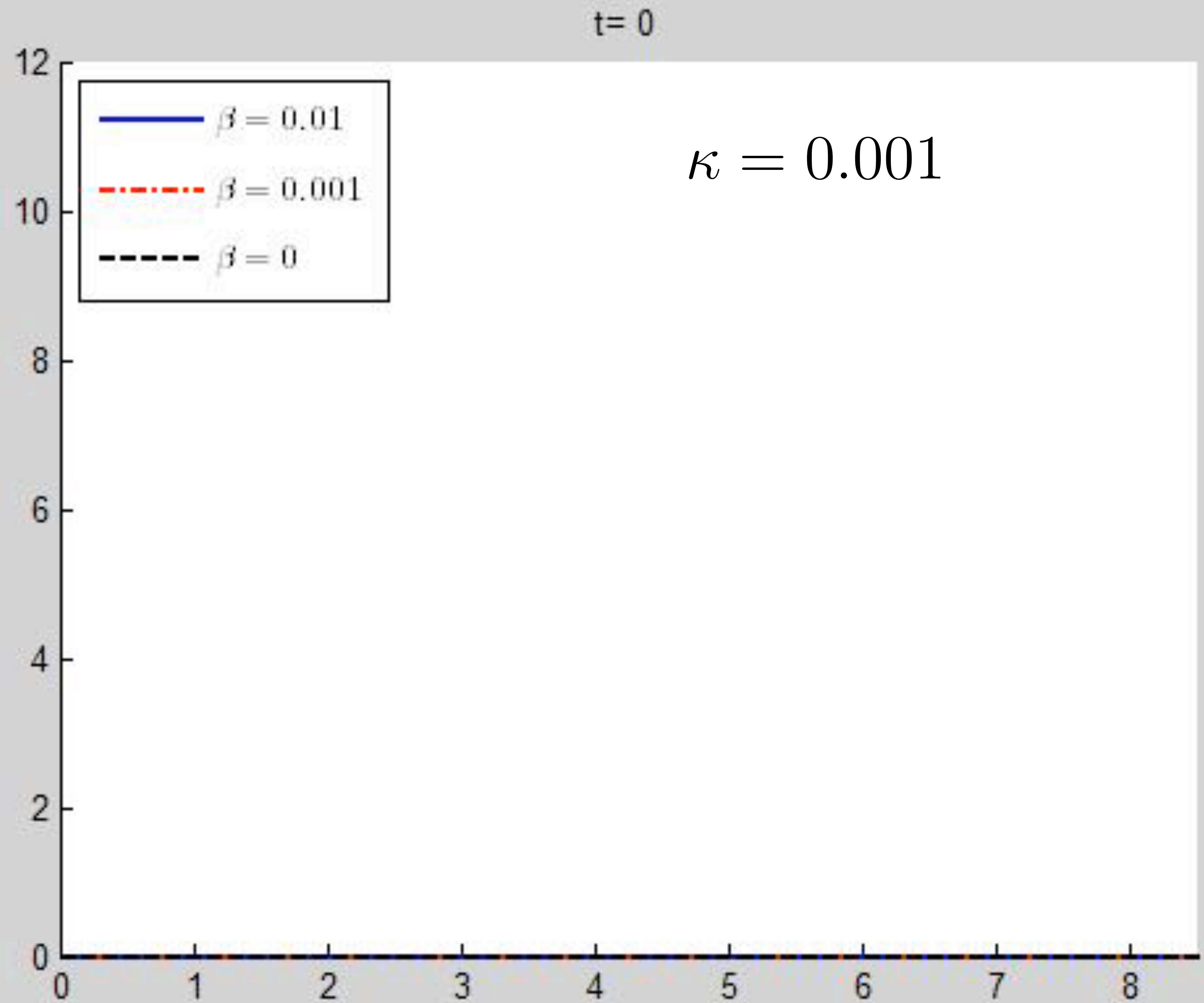
$$G(f, c) = 2(f - f_1)(f - f_2)(f_3 - f)$$

$$f_1 = -\left[\frac{c - c_n}{c_n - c_s}\right]_+^2, \quad f_2 = \frac{c_n - c}{c_n - c_s}, \quad f_3 = 1 + \left[\frac{c_s - c}{c_n - c_s}\right]_+^2$$

$$\rightarrow \qquad \kappa f_t = G(f, c) + \kappa \beta f_{xx}$$



$\delta = 0.05,\ \alpha = 0.25$
$\lambda = 5,\ \nu = 0.1.$
$\beta = 0$


t= 0
12
10
8
6
4
2
0
0
1
2
3
4
5
6
7
8
$\beta = 0.01$
$\beta = 0.001$
$\beta = 0$
$\kappa = 0.001$