Radiation by a dielectric wedge

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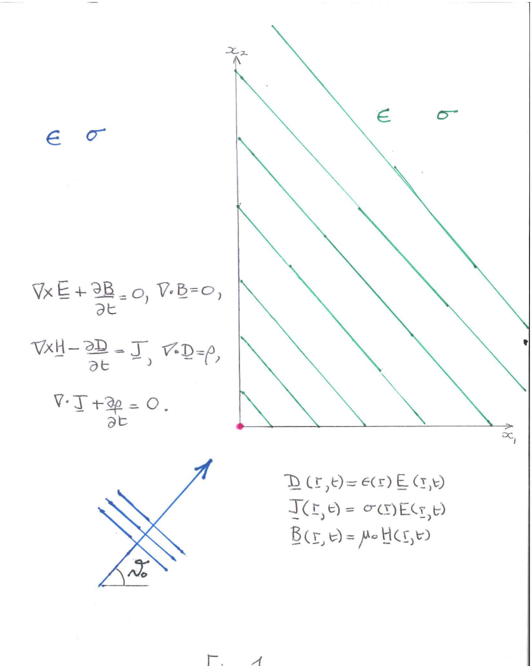
We shall consider the problem of determining the radiated far-field wave amplitude produced when an E-polarized line source is placed at the vertex of a right-angled dielectric wedge. The exact solution for this problem is unknown. We shall use a first-order approximate solution derived from a rigorous analysis of the relevant integral equation. This work was motivated by aspects of scattering by ice crystals in weather prediction.

Figure 1



$\epsilon \quad \sigma$
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x_2
x_1
$\nabla\times \underline{E} + \frac{\partial \underline{B}}{\partial t} = 0, \quad \nabla\cdot\underline{B} = 0,$
$\nabla\times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J}, \quad \nabla\cdot\underline{D} = \rho,$
$\nabla\cdot\underline{J} + \frac{\partial \rho}{\partial t} = 0.$
ϑ_0
$\underline{D}(\underline{r},t) = \epsilon(\underline{r})\,\underline{E}(\underline{r},t)$
$\underline{J}(\underline{r},t) = \sigma(\underline{r})\,\underline{E}(\underline{r},t)$
$\underline{B}(\underline{r},t) = \mu_0\,\underline{H}(\underline{r},t)$

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- Use this approximate solution to derive graphical plots of the far field radiation pattern
- The work ends with conclusions which discuss certain aspects of the work and further work and generalizations

Maxwell's equations in the MKS system are

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}, \qquad \nabla \cdot \mathbf{D} = \rho, \tag{1}$$

with a continuity equation relating the charge density ρ and the current density $\mathbf{J}$ given by

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0. \tag{2}$$

We define piecewise-constant electrical permittivity and conductivity parameters by

$$\begin{aligned} \epsilon(\mathbf{r}) &= \epsilon_d, \quad x_1 \geq 0 \cap x_2 \geq 0 \cap |x_3| \leq \infty, \\ &= \epsilon_v, \quad x_1 \leq 0 \cap x_2 \leq 0 \cap |x_3| \leq \infty, \end{aligned} \tag{3}$$

$$\begin{aligned} \sigma(\mathbf{r}) &= \sigma_d, \quad x_1 \geq 0 \cap x_2 \geq 0 \cap |x_3| \leq \infty, \\ &= \sigma_v, \quad x_1 \leq 0 \cap x_2 \leq 0 \cap |x_3| \leq \infty. \end{aligned} \tag{4}$$

In terms of (3) and (4)the constitutive equations of the medium are

$$\mathbf{D}(\mathbf{r}, t) = \epsilon(\mathbf{r})\mathbf{E}(\mathbf{r}, t), \tag{5}$$

$$\mathbf{J}(\mathbf{r}, t) = \sigma(\mathbf{r})\mathbf{E}(\mathbf{r}, t), \tag{6}$$

$$\mathbf{B}(\mathbf{r}, t) = \mu_0\mathbf{H}(\mathbf{r}, t). \tag{7}$$

For a harmonic time dependance of the form

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \exp(-\imath\omega t),$$

and

$$\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) \exp(-\imath\omega t),$$

Maxwell's equations give

$$\nabla \times \mathbf{H} + \imath\omega\hat{\epsilon}(\mathbf{r})\mathbf{E} = 0, \tag{8}$$

$$\nabla \times \mathbf{E} - \imath\omega\mu_0\mathbf{H} = 0, \tag{9}$$

where

$$\hat{\epsilon}(\mathbf{r}) = \epsilon(\mathbf{r}) + \imath\sigma(\mathbf{r})\omega^{-1}. \tag{10}$$

We shall restrict our consideration to electric fields which are polarized parallel to the vertex of the wedge, that is, along the x_3 axis, and which do not depend on the x_3 coordinate. For such fields given by

$$\mathbf{E}(x_1, x_2) = [0, 0, E(x_1, x_2)], \tag{11}$$

problem reduces to the solution of the scalar two-dimensional form

$$(\nabla^2 + \omega^2 \mu_0 \hat{\epsilon}_v) E(x_1, x_2) = -\omega^2 \mu_0 \hat{\epsilon}_v)[(\hat{\epsilon}(\mathbf{r})/\hat{\epsilon}_v) - 1] E(x_1, x_2). \tag{12}$$

It is convenient to define the complex wave vector $k(\mathbf{r}) = \omega^2 \mu_0 \hat{\epsilon}(\mathbf{r})$, and wave numbers k_d, and k_v by

$$\begin{aligned} k^2(\mathbf{r}) &= k_d^2, \quad x_1 \geq 0 \cap x_2 \geq 0 \cap |x_3| \leq \infty, \\ &= k_v^2, \quad x_1 \leq 0 \cap x_2 \leq 0 \cap |x_3| \leq \infty, \end{aligned} \qquad (13)$$

in terms of which the equation for the electric field becomes

$$(\nabla^2 + k_v^2) E(x_1, x_2) = -[(k^2(\mathbf{r}) - k_v^2] E(x_1, x_2). \qquad (14)$$

By using Green's theorem the equation (14) can be converted to an integral equation for $E(x_1, x_2)$:

$$E(x_1,x_2) = E^{(0)}(x_1,x_2) + \frac{\imath}{4}(k_d^2 - k_v^2)$$
$$\times \int_0^\infty \int_0^\infty H_0^{(1)}(k_v[(x_1-\xi_1)^2 + (x_2-\xi_2)^2]^{\frac{1}{2}})E(\xi_1,\xi_2)d\xi_1 d\xi_2. \tag{15}$$

Here $E^{(0)}(x_1,x_2)$ represents the electric field incident on the wedge $x_1 \geq 0 \cap x_2 \geq 0 \cap |x_3| \leq \infty$ which we will assume is a line source and therefore satisfies the equation

$$(\nabla^2 + k_v^2)E^{(0)}(x_1,x_2) = -\delta(x_1 - x_1^0)\delta(x_2 - x_2^0), \tag{16}$$

so that $E^{(0)}(x_1,x_2) = \frac{\imath}{4}H_0^{(1)}(k_v\sqrt{(x_1-x_1^0)^2 + (x_2-x_2^0)^2})$, where $x_1^0 \leq 0 \cap |x_2^0| \leq \infty$. The kernel in (15) is a Hankel function of the first kind of order zero., and k_v andk_d are chosen to have positive imaginary parts

$$Im(k_v) > 0, Im(k_d) > 0. \tag{17}$$

.

Laplace transformed singular integral equation.

We now double Laplace transform the integral equation (15) to produce a simpler integral equation. Before this is done we make use of the following representation for the Hankel function Kernel:

$$\frac{\imath}{4}H_0^{(1)}(k_v[(x_1-\xi_1)^2+(x_2-\xi_2)^2]^{\frac{1}{2}}) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\exp\left[\imath z_1(\xi_1-x_1)+\imath z_2(\xi_2-x_2)\right]}{z_1^2+z_2^2-k_v^2}dz_1dz_2, \tag{18}$$

valid for $Im[k_v]>0$. On substituting this expression into the equation (15), and interchanging the order of integration,which is allowed because the resulting expressions are uniformly valid before and after the interchange, we have

$$E(x_1,x_2) = E^{(0)}(x_1,x_2)+(k_d^2-k_v^2) \times\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp[-\imath z_1x_1-\imath z_2x_2]E_{++}(z_1,z_2)dz_1dz_2, \tag{19}$$

where $E_{++}(z_1,z_2)$ is the double Laplace transform of $E(\xi_1,\xi_2)$ and is given by

$$E_{++}(z_1, z_2) = \int_0^\infty \int_0^\infty \exp[\imath z_1 \xi_1 + \imath z_2 \xi_2] E(\xi_1, \xi_2) d\xi_1 d\xi_2, \quad (20)$$

with $Im[z_1] \geq 0, Im[z_2] \geq 0$. The function $E_{++}(z_1, z_2)$ is a regular analytic function of the two complex variables in the complex domain $Im[z_1] \geq 0, Im[z_2] \geq 0$. Now we take the double Laplace transform of (19) by operating across the equation by $\int_0^\infty \int_0^\infty \exp[\imath k_1 x_1 + \imath k_2 x_2] \cdots dx_1 dx_2$ giving the double Laplace transformed integral equation

$$\begin{aligned} E_{++}(k_1, k_2) &= E_{++}^{(0)}(k_1, k_2) + (k_d^2 - k_v^2) \\ &\times \frac{1}{(2\pi\imath)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{E_{++}(z_1, z_2) dz_1 dz_2}{(z_1^2 + z_2^2 - k_v^2)(z_1 - k_1)(z_2 - k_2)}, \end{aligned} \quad (21)$$

where $Im[k_1] \geq 0$ and $Im[k_2] \geq 0$.

This integral equation is identical to that derived by Radlow and Kraut and Lehman by a more complicated Wiener-Hopf method involving four unknown functions of two complex variables. The unkown sectionally analytic function $E_{++}(z_1, z_2)$ being in effect, the double Laplace transform of the original field quantity. The transformed integral equation involves a much simpler algebraic kernel than the untransformed integral equation which involves Bessel functions.Thus the resulting integrals that will occur in a perturbation solution will presumably be easier to evaluate.

The integral equation (21) has been analyzed by Kraut and Lehman and they show that provided the L_2 norm of $E_{++}^{(0)}$ is bounded, that is

$$||E_{++}^{(0)}||_2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_{++}^{(0)}|(k_1, k_2)|^2 dk_1 dk_2 < \infty, \qquad (22)$$

and that the complex wave numbers satisfy

$$|k_d^2 - k_v^2| < 2|Im(k_v)Re(k_v)|, \qquad (23)$$

then the solution of the integral equation is the limit of a sequence of successive approximations converging in L_2 norm to E_{++}.

That is, the successive approximations to the solution

$$E_{++}^{(0)}(k_1,k_2), E_{++}^{(1)}(k_1,k_2), E_{++}^{(2)}(k_1,k_2), \cdots, E_{++}^{(m)}(k_1,k_2), \qquad (24)$$

take the form

$$E_{++}^{(m+1)}(k_1,k_2) = E_{++}^{(0)}(k_1,k_2) + (k_d^2 - k_v^2) \times \frac{1}{(2\pi\imath)^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{E_{++}^{(m)}(z_1,z_2)dz_1dz_2}{(z_1^2+z_2^2-k_v^2)(z_1-k_1)(z_2-k_2)}, \qquad (25)$$

where $m = 0,1,2,\cdots,$ $Im(k_1) = 0, Im(k_2) = 0$ and $(k_d^2 - k_v^2)$ satisfies (23). The solution of (21) for real k_1 and k_2 is the limit of the sequence (25):

$$E_{++}(k_1,k_2) = \lim_{m\to\infty} E_{++}^{(m)}(k_1,k_2). \qquad (26)$$

.

Approximate solution of the singular integral equation.

We shall now derive an approximate solution of the integral equation for the situation where the line source is at the vertex of the right-angled wedge or at infinity; and the refractive index is such that $1 < |n| < \sqrt{2}$. From the reciprocity theorem the field is the same in both these situations since inherent in Maxwell's equations, the solution for a line source is unaffected if the location of the source is interchanged with that of the point of observation. We may therefore consider a line source at infinity and calculate the field at the origin. This will correspond to a plane wave of a certain magnitude incident on the dielectric right-angled wedge.

The electric field at the vertex of the wedge is given by

$$E(0,0) = \lim_{m\to\infty} E^{(m)}(0,0), \tag{27}$$

where

$$E^{(m)}(0,0) = \lim_{x_1\to 0^+} \lim_{x_2\to 0^+} \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_{++}^{(m)}(k_1, k_2) \exp\left(-\imath k_1 x_1 - \imath k_2 x_2\right) dk_1 dk_2. \tag{28}$$

To carry out this limiting process it is convenient to use the initial value theorem for the double Laplace transform, that is

$$\lim_{x_1 \to 0^+} \lim_{x_2 \to 0^+} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_{++}^{(m)}(k_1, k_2) \exp\left(-\imath k_1 x_1 - \imath k_2 x_2\right) dk_1 dk_2$$
$$= \lim_{Im(k_1) \to \infty} \lim_{Im(k_2) \to \infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{k_1 k_2 E_{++}^{(m)}(z_1, z_2) dz_1 dz_2}{(z_1 - k_1)(z_2 - k_2)}. \tag{29}$$

It follows from (28) and (29) that

$$E^{(m)}(0,0) = -\lim_{Im(k_1) \to \infty} \lim_{Im(k_2) \to \infty} k_1 k_2 E_{++}^{(m)}(k_1, k_2), \tag{30}$$

for the $m-th$ order approximation to the electric field at the vertex of the wedge.

To first order in $k_d - k_v$, by using (25), (21) and (26) we get

$$E^{(1)}_{++}(k_1, k_2) = E^{(0)}_{++}(k_1, k_2) + (k_d^2 - k_v^2) \times \frac{1}{(2\pi\imath)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{E^{(0)}_{++}(z_1,z_2) dz_1 dz_2}{(z_1^2+z_2^2-k_v^2)(z_1-k_1)(z_2-k_2)}, \tag{31}$$

and, from (30),

$$E^{(1)}(0,0) = E_0(0,0) - (k_d^2 - k_v^2) \times \frac{1}{(2\pi\imath)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{E^{(0)}_{++}(z_1,z_2) dz_1 dz_2}{(z_1^2+z_2^2-k_v^2)}, \tag{32}$$

When the line source tends to infinity with $x_1^0 \leq 0, x_2^0 \leq 0$, the asymptotic form of the Hankel function line source (16) for large argument $r_0 = \sqrt{(x_1^0)^2 + (x_2^0)^2} \to \infty$, gives

$$E^{(0)}(x_1, x_2) = E_0 \exp \imath(a_1 x_1 + a_2 x_2) \tag{33}$$

where $a_1 = k_v \cos\theta_0, a_2 = k_v \sin\theta_0, 0 < \theta_0 < 3\pi/2$; and $E_0 = \frac{1}{2\sqrt{2\pi r_0}} \exp \imath(k_v r_0 + \pi/4)$. The double Laplace transform of (33) gives

$$E_{++}^{(0)}(z_1, z_2) = -\frac{E_0}{(z_1 + a_1)(z_2 + a_2)}. \tag{34}$$

Hence (32) becomes

$$E^{(1)}(0,0) = E_0(1 + \frac{(k_d^2 - k_v^2)}{(2\pi\imath)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dz_1 dz_2}{(z_1^2 + z_2^2 - k_v^2)(z_1 + a_1)(z_2 + a_2)}), \tag{35}$$

where $Im(k_v + a_1) > 0, Im(k_v + a_2) > 0$, must be taken for the integrals to exist for all $0 < \theta_0 < 3\pi/2$.

In an analogous manner it can be shown that the second order field at the edge is given by

$$E^{(2)}(0,0) = E^{(1)}(0,0) + \frac{(k_d^2 - k_v^2)^2}{(2\pi\imath)^4} E_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dz_1 dz_2}{(z_1^2 + z_2^2 - k_v^2)}$$
$$\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{d\zeta_1 d\zeta_2}{(\zeta_1^2 + \zeta_2^2 - k_v^2)(\zeta_1 + a_1)(\zeta_2 + a_2)(\zeta_1 - z_1)(\zeta_2 - z_2)}, \qquad (36)$$

where $Im(k_v) > 0, Im(\zeta_1 + a_1) > 0, Im(\zeta_2 + a_2) > 0$ must be taken for the integrals to exist.

In order to calculate $E^{(1)}(0,0)$ we must evaluate the double integral occurring in (35) for $Im(k_v) > 0$, that is

$$I(a_1,a_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dz_1 dz_2}{(z_1^2+z_2^2-k_v^2)(z_1+a_1)(z_2+a_2)}. \tag{37}$$

Let $z_1 = r\cos\theta, z_2 = r\sin\theta \quad r > 0, 0 < \theta < 2\pi$ then

$$I(a_1,a_2) = \int_0^{\infty}\int_0^{2\pi} \frac{rdrd\theta}{(r^2-k_v^2)(r\cos\theta+a_1)(r\sin\theta+a_2)}; \tag{38}$$

then using the easily proved result

$$\int_0^{2\pi} \frac{d\theta}{(r\cos\theta+a_1)(r\sin\theta+a_2)} = 2\pi\imath\left(\frac{a_2}{(r^2-k_v^2)\sqrt{r^2-k_v^2}} + \frac{a_1}{(r^2-k_v^2)\sqrt{r^2-k_v^2}}\right), \tag{39}$$

we have

$$I(a_1,a_2) = 2\pi\imath a_1 \int_0^\infty \frac{rdr}{(r^2-k_v^2)^2\sqrt{r^2-a_2^2}} + 2\pi\imath a_2 \int_0^\infty \frac{rdr}{(r^2-k_v^2)^2\sqrt{r^2-a_1^2}} \quad . \tag{40}$$

The square root function $\kappa(z)=\sqrt{a^2-z^2}$ is such that $\kappa(z)=a$ for $z=0$, and $Im(\kappa(z))>0$. The only singularities of these integrals in the complex r-plane occur in the first and third quadrant.Thus we can use analytic continuation to put these integrals into a more convenient form by rotating clockwise the path of integration to run from $(0,-\imath\infty)$.

These integrals can now be evaluated by analytic continuation for real k_v giving:

$$I(a_1, a_2) = \\ -\frac{|\sin\theta_0|}{k_v^2\cos\theta_0} + \frac{\arccos|\sin\theta_0|}{k_v^2\cos\theta_0|\cos\theta_0|} - \frac{|\cos\theta_0|}{k_v^2\sin\theta_0} + \frac{\arccos|\cos\theta_0|}{k_v^2\sin\theta_0|\sin\theta_0|}. \tag{41}$$

Hence substituting this result (41) into the expression (35) we have

$$E^{(1)}(0,0) = E_0(1 - \frac{(n^2-1)}{(4\pi)}(\sin\theta_0[\frac{(\pi/2-|\arcsin(\cos\theta_0)|)}{|\sin^3\theta_0|} - \frac{|\cos\theta_0|}{\sin^2\theta_0}] \\ + \cos\theta_0[\frac{(\pi/2-|\arcsin(\sin\theta_0)|)}{|\cos^3\theta_0|} - \frac{|\sin\theta_0|}{\cos^2\theta_0}])). \tag{42}$$

Radiated far field.

Since the incident plane wave $e^{\imath k_v r \cos(\theta-\theta_0)} = e^{\imath k_v (x_1 \cos\theta_0 + x_2 \sin\theta_0)}$, where $0 < \theta_0 < 3\pi/2$, produces the field $E(0,0) = E_z(\theta_0)$ at the origin $r = 0$. For a line source at the edge of the wedge we have the integral representation

$$H_0^{(1)}(k_v r) = \frac{1}{\pi} \int_{S(\theta)} e^{\imath k_v rcos(\theta-\theta_0)} d\theta_0, \tag{43}$$

where $S(\theta)$ is the path of steepest descent. Thus the field radiated by a line source at the origin is given by integrating over the incident angle of the plane wave solution by $\frac{1}{\pi} \int_{S(\theta)} e^{\imath k_v rcos(\theta-\theta_0)} d\theta_0$, giving

$$E_z(r,\theta) = \frac{1}{\pi} \int_{S(\theta)} E_z(\theta_0) e^{\imath k_v rcos(\theta-\theta_0)} d\theta_0. \tag{44}$$

This result also follows directly from an application of the reciprocity theorem.

An application of the saddle point method then gives

$$E_z(r,\theta) \sim E_z(\theta) H_0^{(1)}(k_v r) \sim \sqrt{\frac{2}{\pi k_v r}} E_z(\theta) e^{(k_v r - \frac{\imath\pi}{4})}, \tag{45}$$

provided the function $E_z(\theta_0)$ in the integrand has no pole singularities at $\theta_0 = \theta$. Nonetheless there is a mathematical inconsistency in that the exponential wave-number cannot match on the boundaries of the dielectric wedge and inside the dielectric wedge. However the solution is approximate in terms of powers of $(k_d^2 - k_v^2)$, and therefore for mathematical consistency we should expand k_v in terms of $(k_d^2 - k_v^2)$. To this end we note that

$$\begin{aligned} k_v &= \sqrt{\frac{k_v^2 + k_d^2}{2} + \frac{k_v^2 - k_d^2}{2}} = \sqrt{\frac{k_v^2 + k_d^2}{2}} (1 + \frac{k_v^2 - k_d^2}{k_d^2 + k_v^2})^{\frac{1}{2}} \\ &= k + \frac{1}{4k}(k_v^2 - k_d^2) - \frac{1}{32k^3}(k_v^2 - k_d^2)^2 + O[(k_d^2 - k_v^2)^3]. \end{aligned} \tag{46}$$

where $k = \sqrt{\frac{k_v^2 + k_d^2}{2}}$.

Then the expression (45)becomes

$$E_z(r,\theta) \sim \sqrt{\frac{2}{\pi kr}} E_z(\theta) e^{(kr - \frac{\imath\pi}{4})}, \tag{47}$$

so that the first order field everywhere is given by

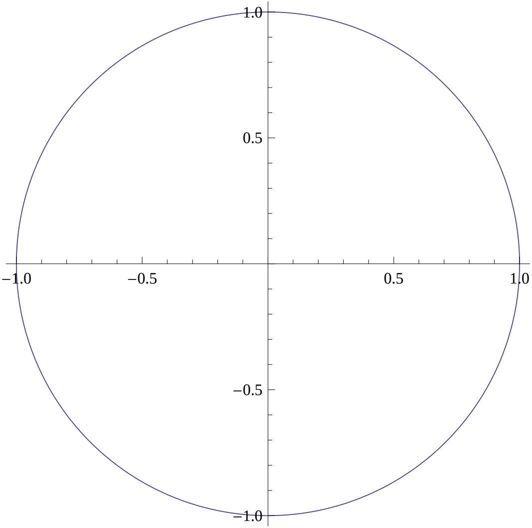
$$E_z(r,\theta) \sim \sqrt{\frac{2}{\pi kr}} E_z^{(1)}(\theta) e^{(kr - \frac{\imath\pi}{4})}. \tag{48}$$

where $E_z^{(1)}(\theta) = E^{(1)}(0,0)$ with θ_0 replaced by θ in (42); and $|(1-2kr)(k_v^2 - k_d^2)| << |4(k_v^2 + k_d^2)|$.

Grapical plots of the radiated field pattern

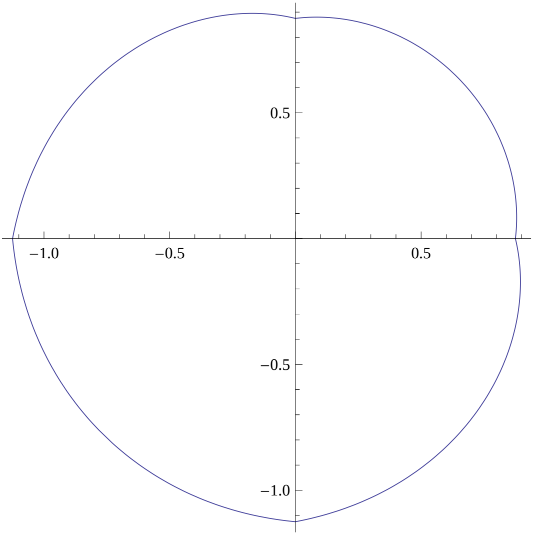
Some graphical plots of $E_z^{(1)}(\theta)/E_0$ for the refractive index taking the values $n = 1, \sqrt{2}, 1.5, 2.5, 3$ are given below. The first two values fall within the range of convergence of the approximation. However, this range of convergence although sufficient, is not necessarily the full range of convergence. It's possible that the approximation is valid for a much larger range of the absolute values of the refractive index.

Figure 2



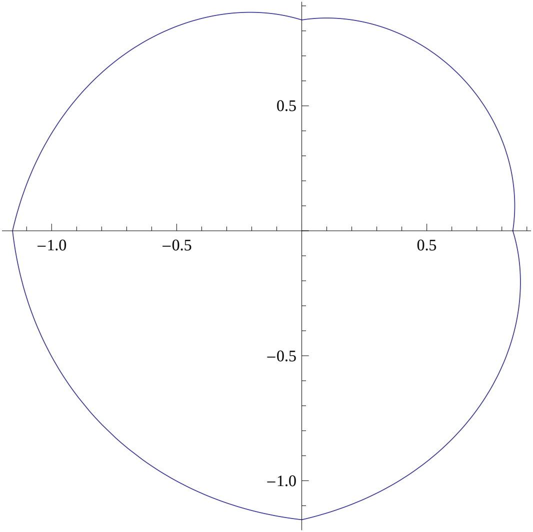
refractive index $n = 1$

Figure 3



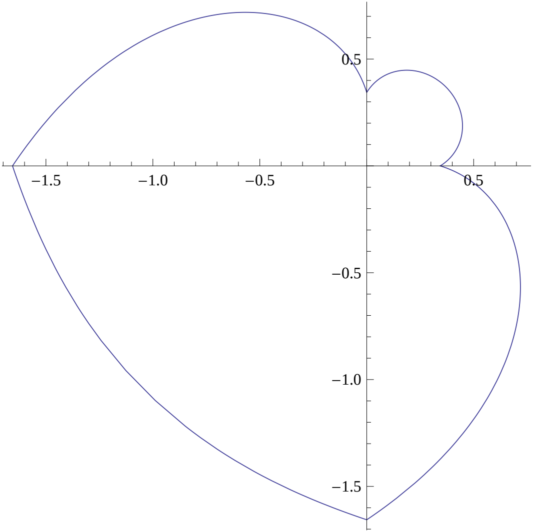
refractive index $n = \sqrt{2}$

Figure 4



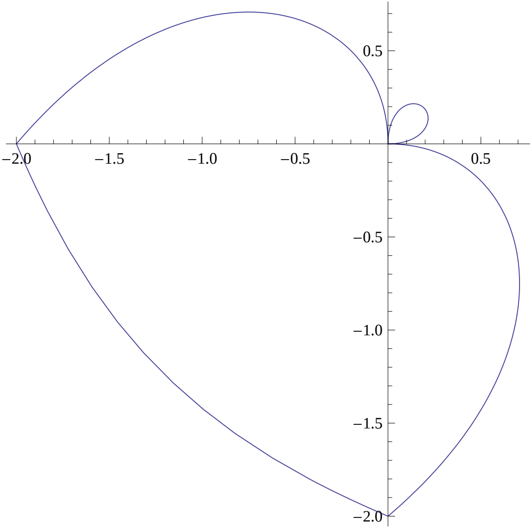
refractive index $n = 1.5$

Figure 5



refractive index $n = 2.5$

Figure 6



refractive index $n = 3$

The graphical results show the radiation pattern symmetry one would expect about the diagonal line $x_1 = x_2$. If we let n increase beyond $n = \sqrt{2}$ the field inside the dielectric wedge gets smaller and most of the field is radiated outside the wedge. The structure is behaving like an impedance wedge as one would expect. We could go further and evaluate the higher order perturbation effect. This would involve the evaluation of a multiple integral. Its the authors contention that this is possible, if somewhat laborious to fulfill.

We have shown that the use of the double Wiener-Hopf technique involving the use of two complex variables and the complicated concomitant problems of factorization in the two complex variables is not necessary for the problem of the diffraction by a right-angled dielectric wedge. The simpler approach of using directly the double complex Laplace or Fourier transform is sufficient to derive the basic transformed double integral equation that can be approximately solved as a Neumann series solution.
As an application, the approximate solution for a line source at the edge of the wedge is derived and graphical plots show the symmetric nature of the far field radiation pattern.

Conclusions

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- The method can be extended to deal with the H-polarized line source problem.

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- Finally, we remark that this far field pattern can be refined upon by calculating higher order iterate effects.