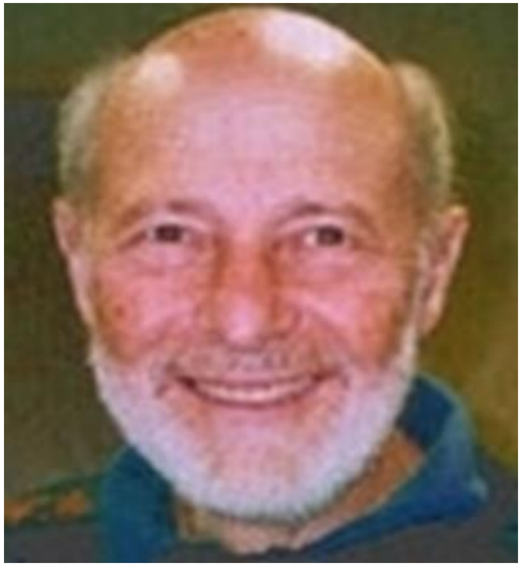
Keller's Influence Behind the "Iron Curtain":

GTD Ideas in the Soviet/Russian Diffraction School



Valery Smyshlyaev (with inputs from V.M. Babich, M.M. Popov, M.A. Lyalinov; Steklov Institute St. Petersburg & UCL London)

Facts-finding mission to St. Petersburg (February 2017):

Leningrad/ St. Petersburg school of Diffraction:

Prof V.M. Babich (Steklov Math Inst St. Petersburg): (official photo; and at International Conference “Days on Diffraction” 2010 official picnic)

Fact #1 found:

“There was no `Iron Curtain’ in Mathematics in the Soviet Union of 1960s and later”

(V.M. Babich, February 2017)

“Western Mathematical journals, eg Courant Inst *Comm Pure Appl Math* were available in Steklov Inst library” (in contrast to NY Times, W. Post and other Western “MSM”)

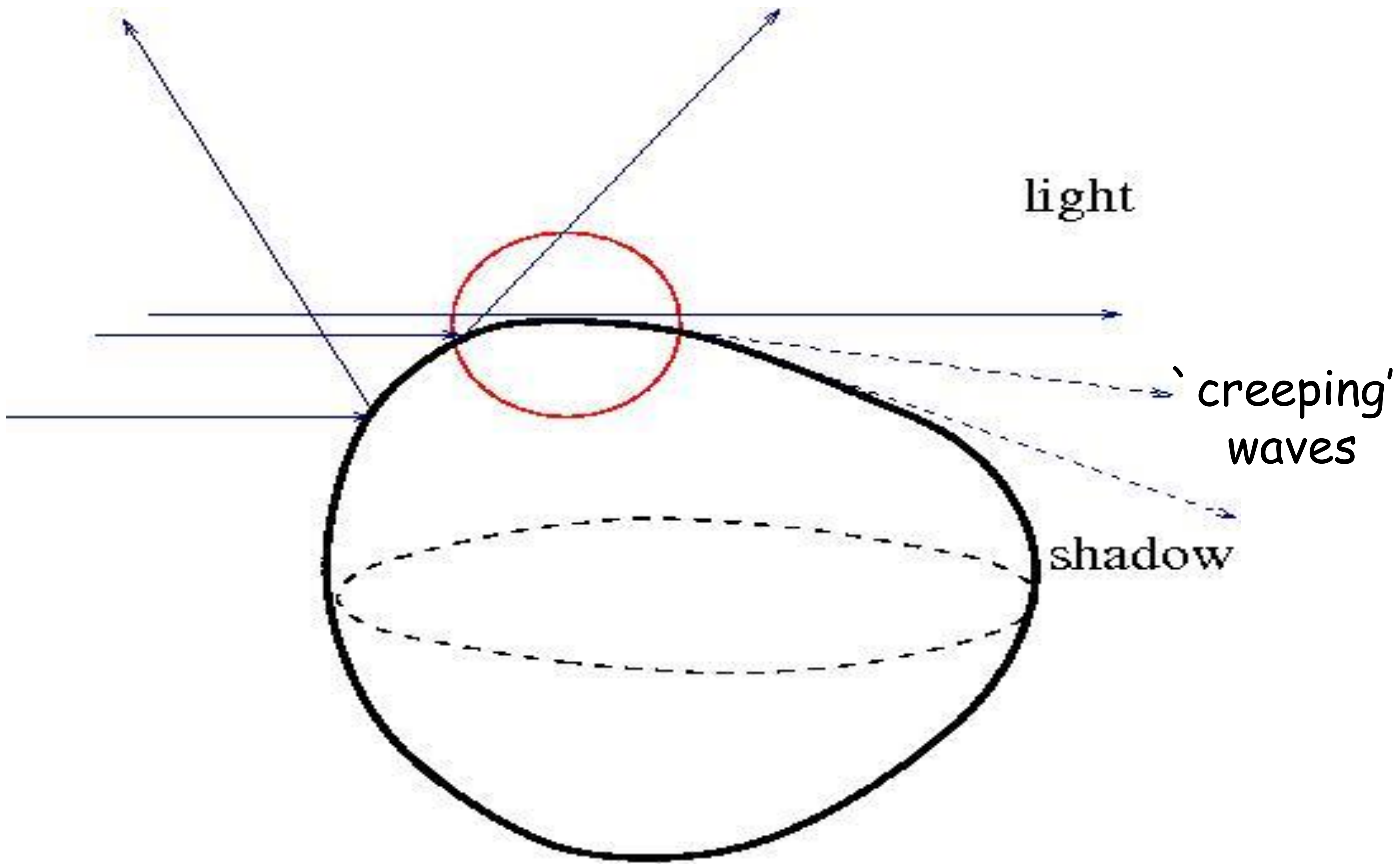
V.A. Fock: one of the founders of the Leningrad/ St. Petersburg Diffraction school

V.A. Fock (1898-1974)

- At around mid-1940s, formulated a “principle of locality” for high-frequency diffraction
- Developed, together with M.A. Leontovich, “method of parabolic equation” for local canonical problems

Papers in Russian journals of 1946, 1948, 1949, etc; Some appeared in English in 1965 book: *V.A. Fock, Electromagnetic Diffraction and Propagation Problems, Pergamon Press, 1965*

Grazing diffraction (Fock, c. 1946):



The Fock's principle of locality: look through `magnifying glass' to identify the **canonical problem**.

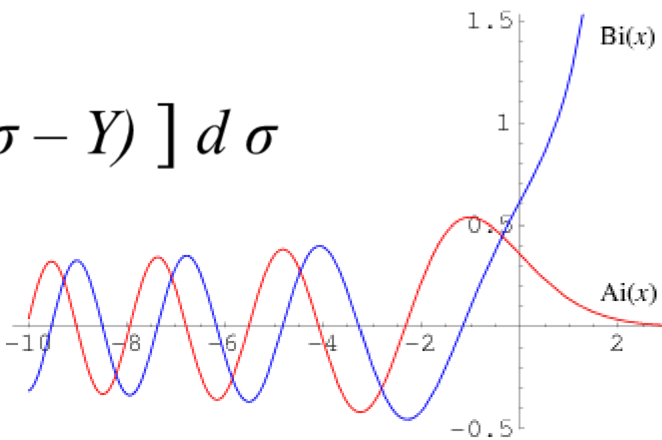
Fock-Leontovich "parabolic equation" method for grazing diffraction (c. 1946):

Canonical problem of grazing diffraction reduced to a separable PDE for "stretched variables" X and Y near the tangency point:

$$iU_X = -U_{YY} - YU,\ \ Y > 0.$$

- Solutions (the "Fock's integral"), e.g. for the Dirichlet b.c.:

$$U(X,Y) = c\int_L e^{i\sigma X}\left[\,\mathrm{Ai}(\sigma - Y) - \frac{\mathrm{Ai}\,(\sigma)}{A_+(\sigma)} A_+(\sigma - Y)\,\right] d\sigma$$



$\mathrm{Ai}(z)$ Airy function $(y'' - z\,y = 0)$

$$A_+(\sigma) := Ai(\,e^{2\pi i/3}\ \sigma\,).$$

Matches to Geometrical optics and decays towards shadow

“A very important result of J.B. Keller of 1956 & 1959” (V.M. Babich):

Diffraction by a Convex Cylinder*

JOSEPH B. KELLER†

Summary—**The leading term in the asymptotic expansion for large $k = 2\pi/\lambda$, of the fields reflected and diffracted by any convex cylinder are constructed. The cross section of the cylinder is assumed to be a smooth curve which may be either closed or open and extending to infinity. The method employed is an extension of geometrical optics in two respects. First, diffracted rays are introduced. Secondly, fields are associated with the rays in a simple way. The results are applicable when the wavelength is small compared to the cylinder dimensions.**

I. Introduction

As an application of a general method which has been devised for the asymptotic solution of diffraction problems,[1] we will treat the two-dimensional problem of diffraction of an arbitrary wave by a cylinder. For simplicity we will first consider an incident cylindrical wave. The field u will be assumed to be a scalar satisfying the reduced wave equation $(\nabla^2 + k^2)u = 0$ and vanishing on the cylinder. Later we will consider the vanishing of $\partial u/\partial \nu$ on the cylinder, and also the impedance boundary condition. Finally we will treat an arbitrary incident wave. The cross section of the cylinder will be assumed to be a smooth convex curve. If it extends to infinity (like a

In fact this comparison is used to determine a certain factor in the present method. In Section VI the result is applied to diffraction by a thick screen and by a wedge terminated by part of a circular cylinder. In Section VII diffraction by a parabolic cylinder is considered and exact agreement is found with the results of Rice[4] which are based upon asymptotic expansion of the exact solution. In Section IX the total cross section of the cylinder and related matters are discussed.

The method of this paper is closely related to the procedure used by Franz and Depperman[5] in their approximate treatment of diffraction by a circular cylinder.

II. Ray Tracing

Incident Rays

Since the medium is homogeneous, the incident rays are straight lines emanating from the source P of the incident cylindrical wave. Since no rays can penetrate the cylinder, there is a region called the geometrical shadow, which is devoid of incident rays.

Reflected Rays

Each incident ray which hits the cylinder gives rise to

(Trans. I.R.E. AP-4 1956 312–321),

And the follow-up ..

COMMUNICATIONS ON PURE AND APPLIED MATHEMATICS, VOL. XII, 159–209 (1959)

Diffraction by a Smooth Object*

BERTRAM R. LEVY and JOSEPH B. KELLER

PART I

APPLICATION OF THE GEOMETRICAL THEORY OF DIFFRACTION

1. Introduction

This paper, on the diffraction of a wave by a smooth convex opaque object of any shape, consists of two parts. In Part I the "geometrical theory of diffraction" [1], a new theory of wave propagation, is explained and applied to the problem. In Part II the diffraction problem for various objects of special shape is formulated as a boundary value problem and solved by the usual separation of variables method. The solutions are then expanded asymptotically for large $ka = 2\pi a/\lambda$ (a = obstacle dimension, λ = wave-

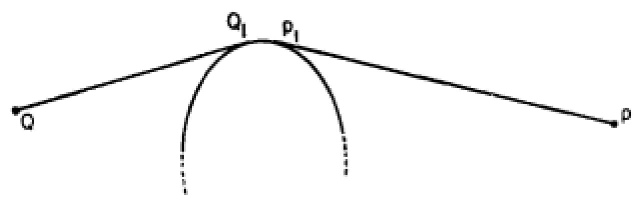


Figure 1
A single diffracted ray from Q to P. The ray consists of a straight line tangent to the surface at Q_1, a geodesic arc on the surface from Q_1 to P_1, and a straight diffracted ray from P_1 to P.

$$u_d(P) = A_i(Q_1)\, D(P_1)\, D(Q_1) \left[\frac{d\sigma(Q_1)}{d\sigma(P_1)}\right]^{1/2} \left[\frac{\rho_1}{s(\rho_1+s)}\right]^{1/2} \exp\left\{ik[\phi_i(Q_1)+t+s]-\int_0^t \alpha(t)\,dt\right\}.$$

$$\alpha(t) = \frac{-i\,k^{1/3}\xi_n}{2^{1/3}\rho^{2/3}(t)}$$

“A very important result describing the filed in the shadow behind an obstacle. On the basis of heuristic considerations, this formula was first derived by Keller (1956) for the two-dimensional case, and was later obtained by Levy & Keller (1959) for the three-dimensional case” (Babich & Byldurev, 1972; Engl. Transl. 1991)

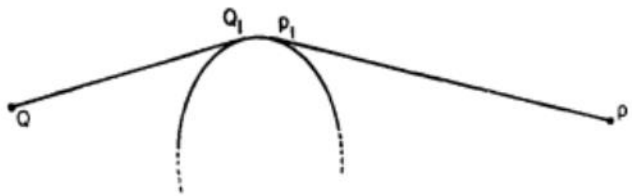


Figure 1
A single diffracted ray from Q to P. The ray consists of a straight line tangent to the surface at Q_1, a geodesic arc on the surface from Q_1 to P_1, and a straight diffracted ray from P_1 to P.

Keller’s important contribution (according to Babich, Feb 2017):

Locality arguments in deep shadow, qualitative arguments on the creeping vs shed energy splitting, reciprocity arguments.

This Keller’s work was followed by:

- Boundary layer/ matched asymptotics extensions from the “Fock’s zone” (cf Babich & Kirpichnikova 1974);

- Uniform asymptotics near the tangency point (D. Ludwig, Babich& Buldyrev 1972);

- Error bounds in the shadow (very hard!), V. Filippov & others.

Other Keller's works that "significantly influenced Russian diffraction" (V. Babich):

- Friedlander, F. G.; Keller, J. B. Asymptotic expansions of solutions of $(\nabla 2+k2)u=0$. *Comm. Pure Appl. Math. 8, (1955)*
- Keller, J. B.; Karal, Frank C., Jr. Surface wave excitation and propagation. *J. Appl. Phys. 31 (1960).*

"Complex eikonals and surface rays in connection with the theory of skin effect were considered earlier in Keller & Karal (1960)"

(From V.M. Babich & N.Ya Rusakova, The Propagation of Rayleigh Waves Over the Surface of a Non-Homogeneous Elastic Body with an Arbitrary Shape" *USSR Computational Mathematics and Mathematical Physics, 1963, 2:4, 719–735 (English translation of Zh. Vych. Mat 2, No. 4, 652-665. 1962.)*

THE PROPAGATION OF RAYLEIGH WAVES OVER THE SURFACE OF A NON-HOMOGENEOUS ELASTIC BODY WITH AN ARBITRARY FORM*

V.M. BABICH and N.Ya. RUSAKOVA
(Leningrad)

(Received 5 October 1961)

In this paper solutions are derived for dynamical equations of the theory of elasticity with a discontinuity only at the boundary. (The boundary is assumed to be free from stresses). These solutions generalize the well-known "plane" Rayleigh waves for the case of a non-homogeneous elastic body of arbitrary form (see [1], chapter XII).

"Generalized" Rayleigh waves propagate over the surface in accordance with Fermat's principle (i.e. along surface rays), and for them an analogue of the well-known ray method of calculating the intensity of wave fronts can be developed (see [2], [3]). A detailed account is given here of the results of note [4] and their generalization for the case of a non-homogeneous body.

1. Introduction

The solutions of the equations of the theory of elasticity

$$(\lambda + \mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \mu\Delta\mathbf{u} = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1.1}$$

having the form

$$\mathbf{u}_a = \varphi\left(\frac{1}{c},\ -\sqrt{\frac{1}{a^2}-\frac{1}{c^2}}\right) f\left(t - \frac{x}{c} + y\sqrt{\frac{1}{a^2}-\frac{1}{c^2}}\right), \qquad a = \sqrt{\frac{\lambda+2\mu}{\rho}},$$

$$\mathbf{u}_b = \psi\left(-\sqrt{\frac{1}{b^2}-\frac{1}{c^2}},\ \frac{1}{c}\right) f\left(t - \frac{x}{c} + y\sqrt{\frac{1}{b^2}-\frac{1}{c^2}}\right), \qquad b = \sqrt{\frac{\mu}{\rho}}$$

are called longitudinal and transverse plane waves respectively. φ and

* *Zh. vych. mat* 2, No. 4, 652-665, 1962.

1. Ray solutions with complex eikonals τ_a and τ_b*

Henceforth we shall assume that the surface S is analytical, the Lamé parameters $\lambda(x, y, z)$, $\mu(x, y, z)$ and the density of the medium $\rho = \rho(x, y, z)$ are also analytical functions of x, y, z. On the surface S the required discontinuous solutions have a singularity only for $t = \tau$ $(\tau = \int ds/c)$. It is therefore natural to assume that on the

* Complex eikonals and surface rays in connection with the theory of the skin effect were considered earlier in [11].

(From Babich & Rusakova, 1962)

We shall write out the expressions for the remaining coefficients in the appendix. Here we shall only point out that the coefficients C and D are purely imaginary, the coefficient E is a linear combination of $\partial\lambda/\partial\nu$, $\partial\mu/\partial\nu$ and $\partial\rho/\partial\nu$ and also purely imaginary, F is a linear combination of $\partial\lambda/\partial\tau$, $\partial\mu/\partial\tau$, $\partial\rho/\partial\tau$ and F is a real coefficient. Solving equation (6.2) we obtain

$$\chi_0(\tau, \alpha) = \chi(0, \lambda)\frac{1}{\sqrt[4]{g_{\alpha\alpha}}}\exp\int\frac{C}{A}\frac{b_{\tau\tau}}{c^2}\,d\tau\exp\int\frac{D}{A}\frac{b_{\alpha\alpha}}{g_{\alpha\alpha}}\,d\tau\exp\int\frac{E}{A}\,d\tau\exp\int\frac{F}{A}\,d\tau, \tag{6.4}$$

i.e. the complex intensity is expressed in the form of a product of six factors, each of which characterizes the effect of one or other factor on the Rayleigh wave. The first factor characterizes the initial form of the Rayleigh wave, the second depends only on the internal geometry of the rays and shows that the amplitude of the Rayleigh wave varies with the divergence of the surface rays in the same way as in the case of the propagation of volume waves. The following three factors show that the curvature of the surface along and across the ray and the rate of change of λ, μ and ρ with the depth affect only the phase of the Rayleigh wave, but not its amplitude. The rate of change of λ, μ and ρ along the ray, on the other hand, affects the amplitude only.

11. Keller, J.B. and Karal, C., Surface wave excitation and propagation. *J. Appl. Phys.*, 31, No. 6, 1039-1046, 1960.

Another Example of Keller's significant influence on Soviet/ Russian Mathematics (again, according to V. Babich):

- JB Keller, SI Rubinow, Asymptotic solution of eigenvalue problems, Annals of Physics, 1960.

(Deriving asymptotic formulas for eigenvalues and eigenfunctions for Laplace/ elliptic operators in domains/ on manifolds via geometrical optics and "quantization method", cf `quasi-classics' in quantum mechanics.)

- V.F. Lazutkin (St. Petersburg)
- V.P. Maslov (Moscow)

(Re `Maslov index'/ `Maslov-Arnold index')

The Keller's Geometric Theory of Diffraction (GTD):

JOURNAL OF THE OPTICAL SOCIETY OF AMERICA VOLUME 52, NUMBER 2 FEBRUARY, 1962

Geometrical Theory of Diffraction*

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(Received September 13, 1961)

The geometrical theory of diffraction is an extension of geometrical optics which accounts for diffraction. It introduces diffracted rays in addition to the usual rays of geometrical optics. These rays are produced by incident rays which hit edges, corners, or vertices of boundary surfaces, or which graze such surfaces. Various laws of diffraction, analogous to the laws of reflection and refraction, are employed to characterize the diffracted rays. A modified form of Fermat's principle, equivalent to these laws, can also be used. Diffracted wave fronts are defined, which can be found by a Huygens wavelet construction. There is an associated phase or eikonal function which satisfies the eikonal equation. In addition complex or imaginary rays are introduced. A field is associated with each ray and the total field at a point is the sum of the fields on all rays through the point. The phase of the field on a ray is proportional to the optical length of the ray from some reference point. The amplitude varies in accordance with the principle of conservation of energy in a narrow tube of rays. The initial value of the field on a diffracted ray is determined from the incident field with the aid of an appropriate diffraction coefficient. These diffraction coefficients are determined from certain canonical problems. They all vanish as the wavelength tends to zero. The theory is applied to diffraction by an aperture in a thin screen diffraction by a disk, etc., to illustrate it. Agreement is shown between the predictions of the theory and various other theoretical analyses of some of these problems. Experimental confirmation of the theory is also presented. The mathematical justification of the theory on the basis of electromagnetic theory is described. Finally, the applicability of this theory, or a modification of it, to other branches of physics is explained.

I. INTRODUCTION

GEOMETRICAL optics, the oldest and most widely used theory of light propagation, fails to account for certain optical phenomena called diffraction. We shall describe an extension of geometrical optics which does account for these phenomena. It is called the geometrical theory of diffraction.[1,2] Like geometrical optics, it assumes that light travels along certain straight or curved lines called rays. But it introduces various new ones, called diffracted rays, in addition to the usual rays. Some of them enter the shadow regions and account for the light there while others go into the illuminated regions.

Diffracted rays are produced by incident rays which hit edges, corners, or vertices of boundary surfaces, or which graze such surfaces. Ordinary geometrical optics does not describe what happens in these cases but the new theory does. It does so by means of several laws of diffraction which are analogous to the laws of reflection and refraction. Like them, the new laws are deducible from Fermat's principle, appropriately modified. Away

these wave fronts and which satisfies the usual eikonal equation. Thus all the fundamental principles of ordinary geometrical optics can be extended to the geometrical theory of diffraction.

Ordinary geometrical optics is often used to determine the distribution of light intensity, polarization, and phase throughout space. This is accomplished by assigning a field value to each ray and letting the total field at a point be the sum of the fields on all the rays through that point. The phase of the field on a ray is assumed to be proportional to the optical length of the ray from some reference point where the phase is zero. The amplitude is assumed to vary in accordance with the principle of conservation of energy in a narrow tube of rays. The direction of the field, when it is a vector, is given by a unit vector perpendicular to the ray. This vector slides parallel to itself along the ray in a homogeneous medium, and rotates around the ray in a specific way as it slides along it in an inhomogeneous medium.

Exactly the same principles as those just described can be used to assign a field to each diffracted ray. The

(3192 citations, according to Google Scholar, 1/3/17)

Keller’s GTD ideas influenced, among many others:

V.A. Borovikov (Moscow):

Borovikov, *Diffraction by polygons and polyhedral, Moscow 1966* (among many other contributions) possibly first use of Fourier Transform for analysing HF-asymptotoics via singularities’ propagation in time-domain (cf later “microlocal analysis”);

Borovikov & Kinber, *Geometric Theory of Diffraction, Moscow 1978* (in English 1994).

A view on the GTD from St. Petersburg (February 2017):

- The underlying ideas (like “principle of locality”, cf Fock) have been in the air, and intuitively understood by the Russian diffraction community;

- The GTD, as a `slogan’, helped to popularise the ideas for a wider community including engineers;

- The GTD helped to crystallize the key “philosophical” ideas, and to raise the importance of the “canonical problems”.

- These are Keller’s (many) *“specific”* results/ ideas which were valued most (and which influenced most).

Conclusion:

“The Influence of Joseph B. Keller’s ideas on the Soviet/ Russian diffraction school was, and remains, very significant.”

(V.M. Babich, St. Petersburg, February 2017)