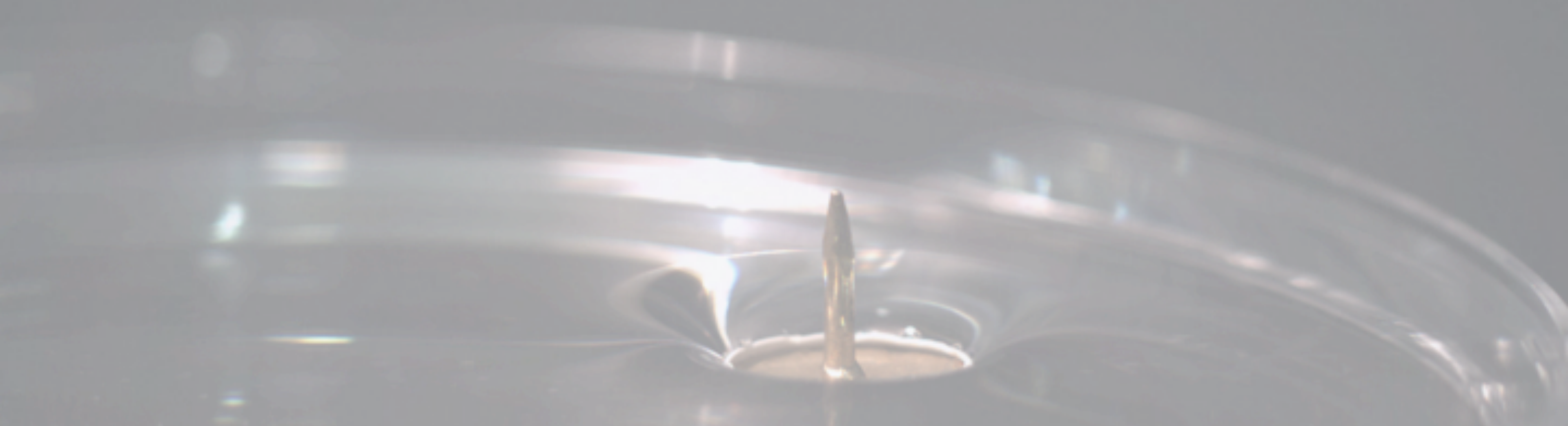
FLOATING WITH SURFACE TENSION

FROM ARCHIMEDES TO KELLER



Dominic Vella

Mathematical Institute, University of Oxford

UNIVERSITY OF OXFORD

erc

JBK @ WHOI

Joe was a regular fixture of the Geophysical Fluid Dynamics programme (run at Woods Hole since 1959 – last attended in 2015)

GFD Photo – 2006

JBK @ WHOI

Joe was a regular fixture of the Geophysical Fluid Dynamics programme (run at Woods Hole since 1959 – last attended in 2015)

GFD Photo – 2006

Chats on the porch of Walsh Cottage are a unique feature of GFD

JBK @ WHOI

PHYSICS FLUIDS VOLUME 10, NUMBER 11 NOVEMBER 1998

BRIEF COMMUNICATIONS

The purpose of this Brief Communications section is to present important research results of more limited scope than regular articles appearing in Physics of Fluids. Submission of material of a peripheral or cursory nature is strongly discouraged. Brief Communications cannot exceed three printed pages in length, including space allowed for title, figures, tables, references, and an abstract limited to about 100 words.

Surface tension force on a partly submerged body

Joseph B. Keller
Departments of Mathematics and Mechanical Engineering, Stanford University, Stanford, California 94305-2125

(Received 26 January 1998; accepted 29 July 1998)

JBK @ WHOI

PHYSICS FLUIDS VOLUME 10, NUMBER 11 NOVEMBER 1998

BRIEF COMMUNICATIONS

The purpose of this Brief Communications section is to present important research results of more limited scope than regular articles appearing in Physics of Fluids. Submission of material of a peripheral or cursory nature is strongly discouraged. Brief Communications cannot exceed three printed pages in length, including space allowed for title, figures, tables, references, and an abstract limited to about 100 words.

Surface tension force on a partly submerged body

Joseph B. Keller
Departments of Mathematics and Mechanical Engineering, Stanford University, Stanford, California 94305-2125

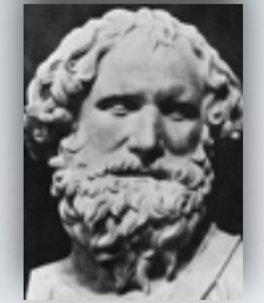
(Received 26 January 1998; accepted 29 July 1998)

Was this paper a result of Walsh Cottage chat?

ACKNOWLEDGMENTS

I thank George Veronis for asking the question that led to this work. This work supported in part by the AFOSR and NSF.

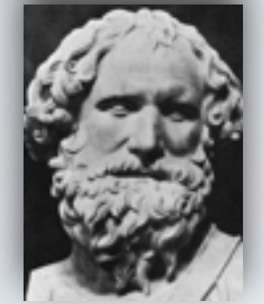
A BRIEF HISTORY OF FLOATING



"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Archimedes, *On Floating Bodies* (c. 250 BC)

A BRIEF HISTORY OF FLOATING

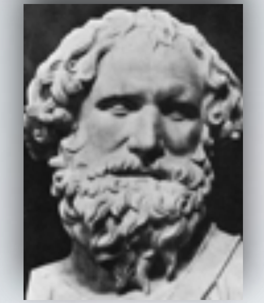


"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Archimedes, *On Floating Bodies* (c. 250 BC)

Archimedes' principle seems to rule out dense objects (e.g. drawing pins) floating on water

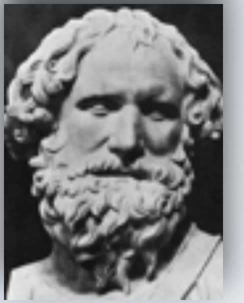
A BRIEF HISTORY OF FLOATING



"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

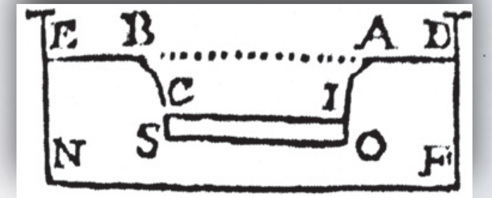
Archimedes, *On Floating Bodies* (c. 250 BC)

A BRIEF HISTORY OF FLOATING

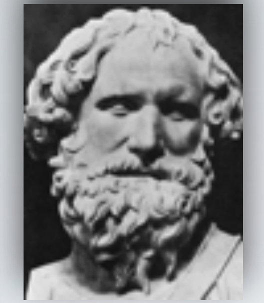


"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Archimedes, *On Floating Bodies* (c. 250 BC)



A BRIEF HISTORY OF FLOATING

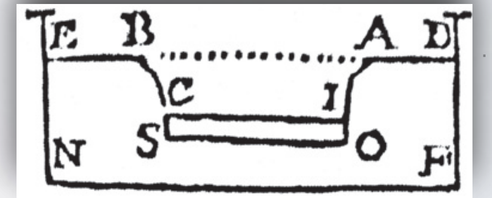


"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

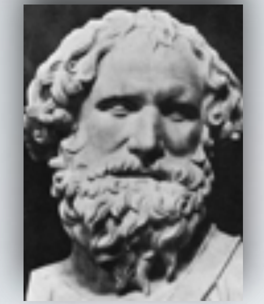
Archimedes, *On Floating Bodies* (c. 250 BC)

"When ever the excess of the Gravity of the Solid above the Gravity of the Water shall have the same proportion to the Gravity of the Water that the Altitude of the Rampart, hath to the thickness of the Solid, that Solid shall not sink, but being never so little thicker it shall."

Galileo Galilei, *Discourse on Floating Bodies* (1663)



A BRIEF HISTORY OF FLOATING



"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

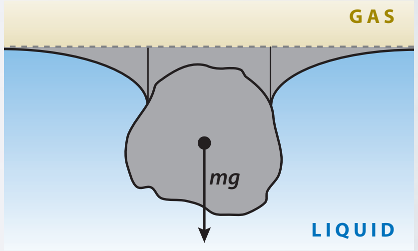
Archimedes, *On Floating Bodies* (c. 250 BC)

"When ever the excess of the Gravity of the Solid above the Gravity of the Water shall have the same proportion to the Gravity of the Water that the Altitude of the Rampart, hath to the thickness of the Solid, that Solid shall not sink, but being never so little thicker it shall."

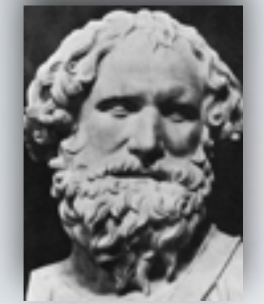
Galileo Galilei, *Discourse on Floating Bodies* (1663)

Galileo's picture

Extra force arises from weight of liquid displaced by meniscus



A BRIEF HISTORY OF FLOATING



"Any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Archimedes, *On Floating Bodies* (c. 250 BC)

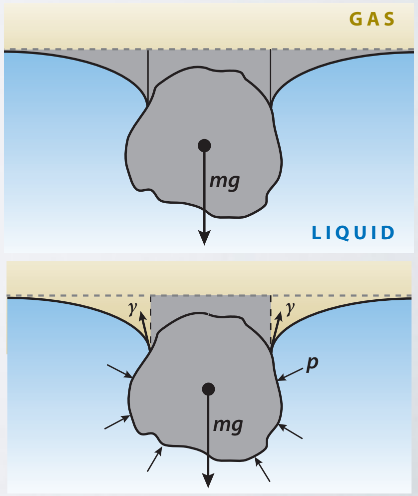
"When ever the excess of the Gravity of the Solid above the Gravity of the Water shall have the same proportion to the Gravity of the Water that the Altitude of the Rampart, hath to the thickness of the Solid, that Solid shall not sink, but being never so little thicker it shall."

Galileo Galilei, *Discourse on Floating Bodies* (1663)

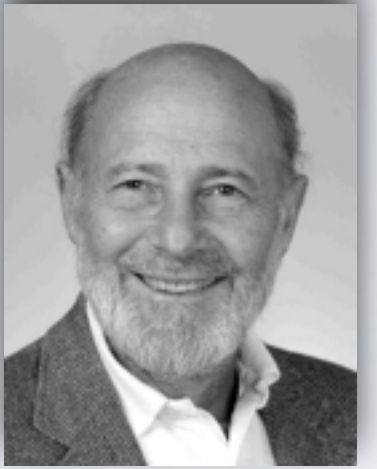
Galileo's picture

Extra force arises from weight of liquid displaced by meniscus

What about the force from surface tension?



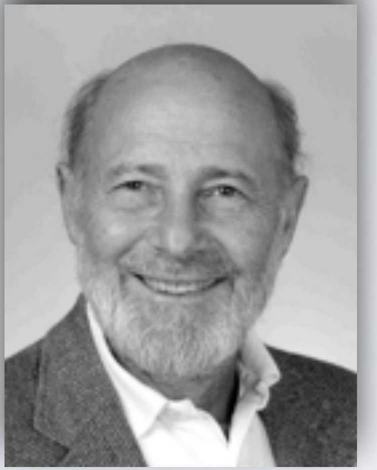
KELLER'S THEOREM



"The vertical component of the surface tension force on a body partly submerged in a liquid is shown to equal the weight of liquid displaced by the meniscus."

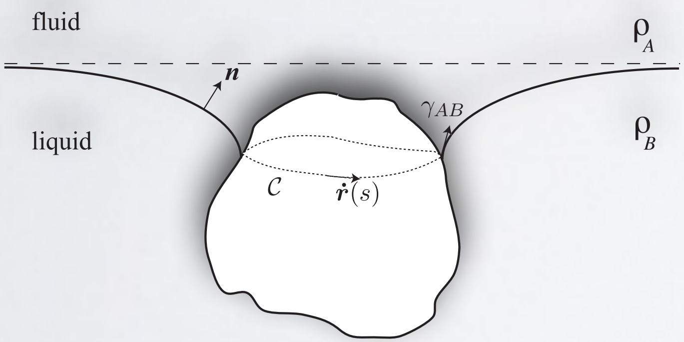
Keller, *Phys. Fluids* (1998)

KELLER'S THEOREM



"The vertical component of the surface tension force on a body partly submerged in a liquid is shown to equal the weight of liquid displaced by the meniscus."

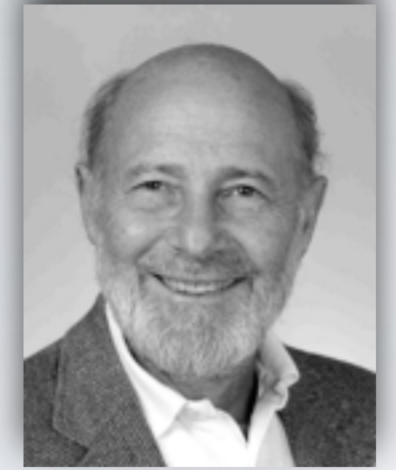
Keller, *Phys. Fluids* (1998)



Surface tension force:

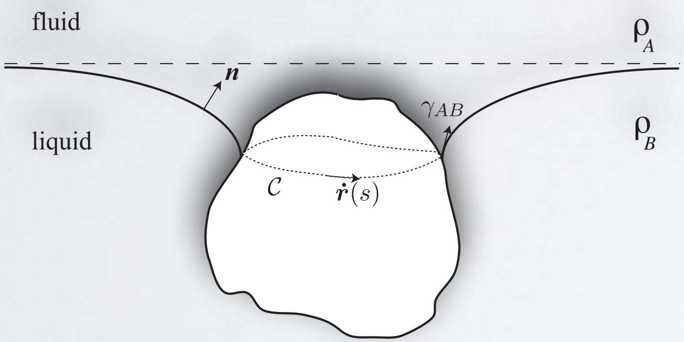
$$\boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}} \dot{\boldsymbol{r}}(s) \times \boldsymbol{n} \, \mathrm{d}s$$

KELLER'S THEOREM



"The vertical component of the surface tension force on a body partly submerged in a liquid is shown to equal the weight of liquid displaced by the meniscus."

Keller, *Phys. Fluids* (1998)



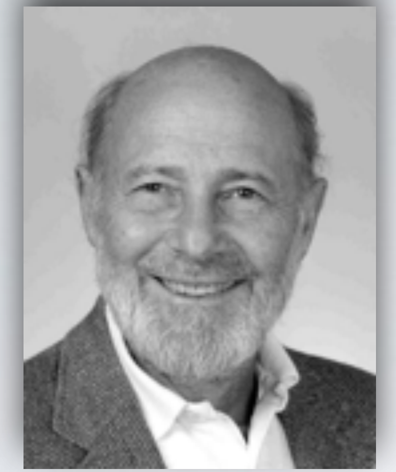
Surface tension force:

$$\boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}} \dot{\boldsymbol{r}}(s) \times \boldsymbol{n} \, \mathrm{d}s$$

Looking at the vertical component:

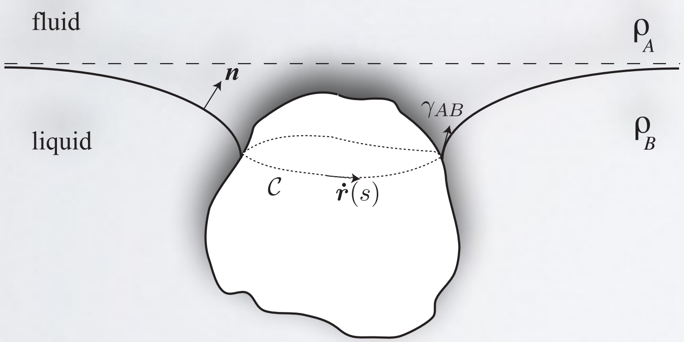
$$\boldsymbol{k} \cdot \boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}_\pi} \boldsymbol{n} \cdot \boldsymbol{\nu} \, \mathrm{d}s' = \gamma_{AB} \int_{\mathbb{R}^2 \setminus W_\pi} \boldsymbol{\nabla} \cdot \boldsymbol{n} \, \mathrm{d}A$$

KELLER'S THEOREM



"The vertical component of the surface tension force on a body partly submerged in a liquid is shown to equal the weight of liquid displaced by the meniscus."

Keller, *Phys. Fluids* (1998)



Surface tension force:

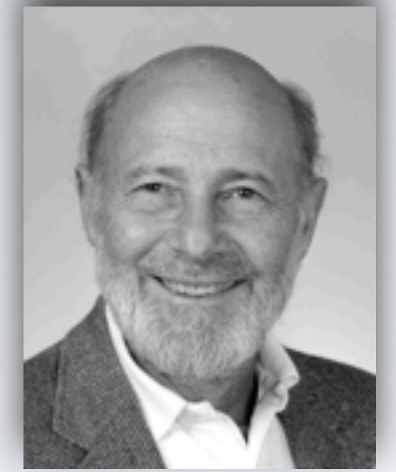
$$\boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}} \dot{\boldsymbol{r}}(s) \times \boldsymbol{n} \, \mathrm{d}s$$

Looking at the vertical component:

$$\boldsymbol{k} \cdot \boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}_\pi} \boldsymbol{n} \cdot \boldsymbol{\nu} \, \mathrm{d}s' = \gamma_{AB} \int_{\mathbb{R}^2 \setminus W_\pi} \boldsymbol{\nabla} \cdot \boldsymbol{n} \, \mathrm{d}A$$

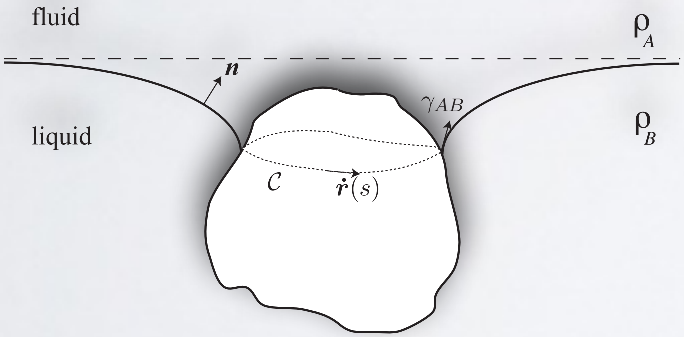
projection of contact line to 2-d plane

KELLER'S THEOREM



"The vertical component of the surface tension force on a body partly submerged in a liquid is shown to equal the weight of liquid displaced by the meniscus."

Keller, *Phys. Fluids* (1998)



Surface tension force:

$$\boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}} \dot{\boldsymbol{r}}(s) \times \boldsymbol{n} \, \mathrm{d}s$$

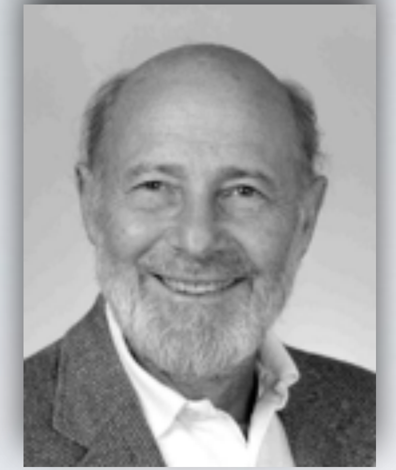
Looking at the vertical component:

$$\boldsymbol{k} \cdot \boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}_\pi} \boldsymbol{n} \cdot \boldsymbol{\nu} \, \mathrm{d}s' = \gamma_{AB} \int_{\mathbb{R}^2 \setminus W_\pi} \boldsymbol{\nabla} \cdot \boldsymbol{n} \, \mathrm{d}A$$

projection of contact line to 2-d plane

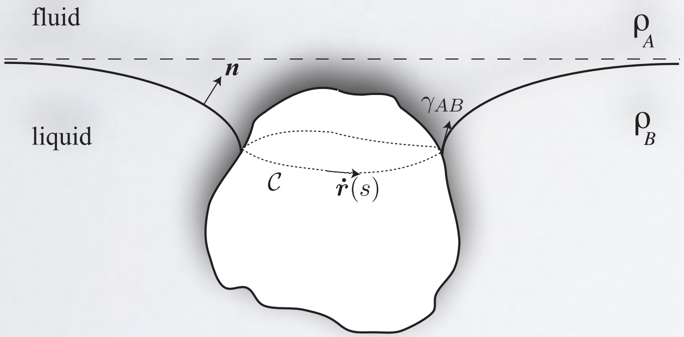
normal to contact line

KELLER'S THEOREM



"The vertical component of the surface tension force on a body partly submerged in a liquid is shown to equal the weight of liquid displaced by the meniscus."

Keller, *Phys. Fluids* (1998)



Surface tension force:

$$\boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}} \dot{\boldsymbol{r}}(s) \times \boldsymbol{n}\, \mathrm{d}s$$

Looking at the vertical component:

$$\boldsymbol{k} \cdot \boldsymbol{F}_{st} = \gamma_{AB} \int_{\mathcal{C}_\pi} \boldsymbol{n} \cdot \boldsymbol{\nu}\, \mathrm{d}s' = \gamma_{AB} \int_{\mathbb{R}^2 \backslash W_\pi} \boldsymbol{\nabla} \cdot \boldsymbol{n}\, \mathrm{d}A$$

normal to contact line

projection of contact line to 2-d plane

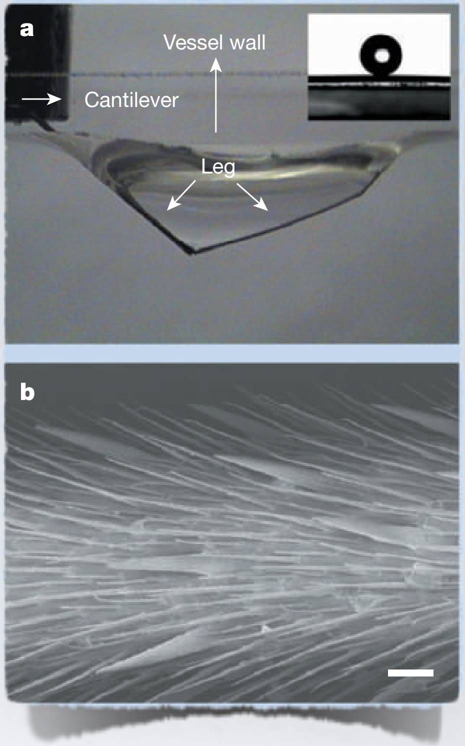
To eliminate curvature, use Laplace-Young equation: $(\rho_B - \rho_A) g h = -\gamma_{AB} \boldsymbol{\nabla} \cdot \boldsymbol{n}$

ROUGH SURFACES

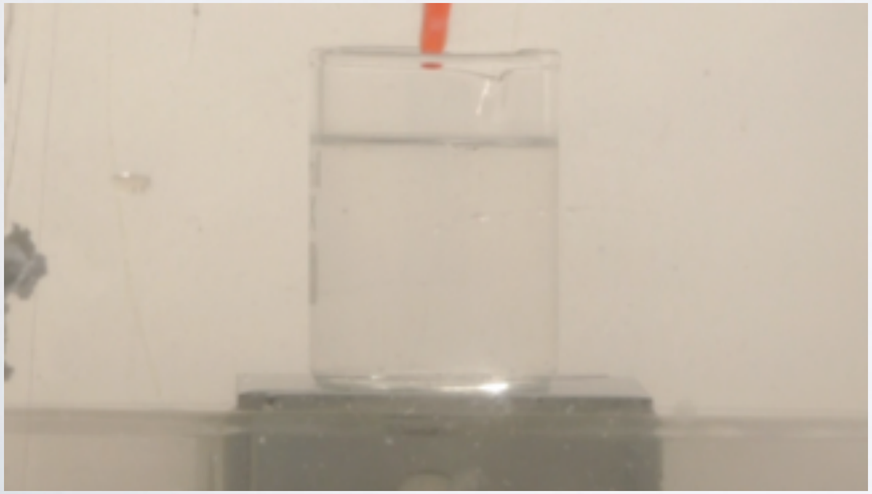
Often think of surface tension as providing a force per unit length – does this explain why water walking insects have hairy legs?

Keller's Theorem shows that this cannot be the case – a hairy surface does not significantly alter the volume of liquid displaced

Key point: contact line length is increased **but** presence of nearby hairs causes mini-menisci to flatten (and hence provide less force)



Gao & Jiang (2004)

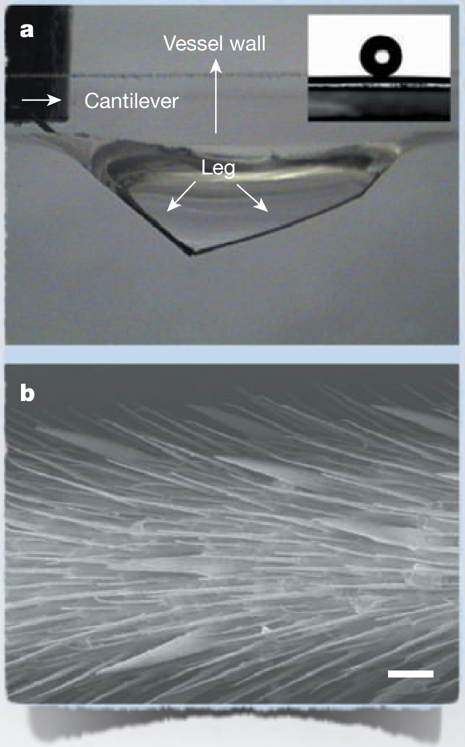


ROUGH SURFACES

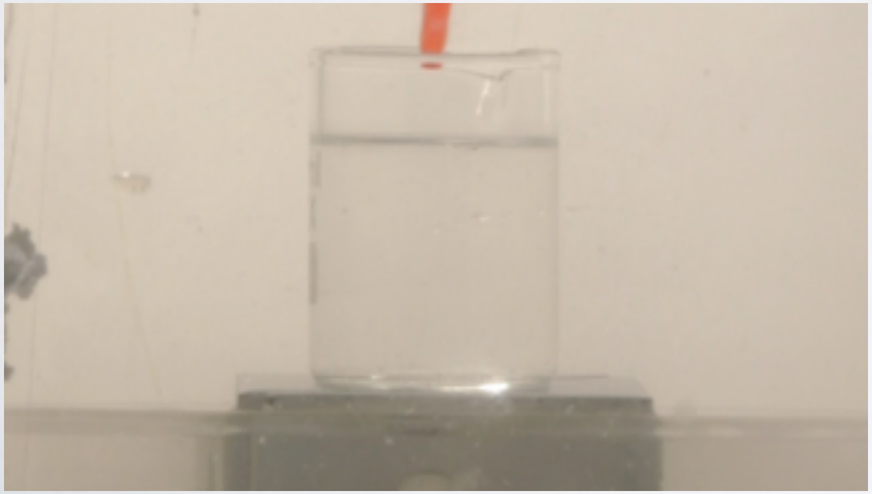
Often think of surface tension as providing a force per unit length – does this explain why water walking insects have hairy legs?

Keller's Theorem shows that this cannot be the case – a hairy surface does not significantly alter the volume of liquid displaced

Key point: contact line length is increased **but** presence of nearby hairs causes mini-menisci to flatten (and hence provide less force)



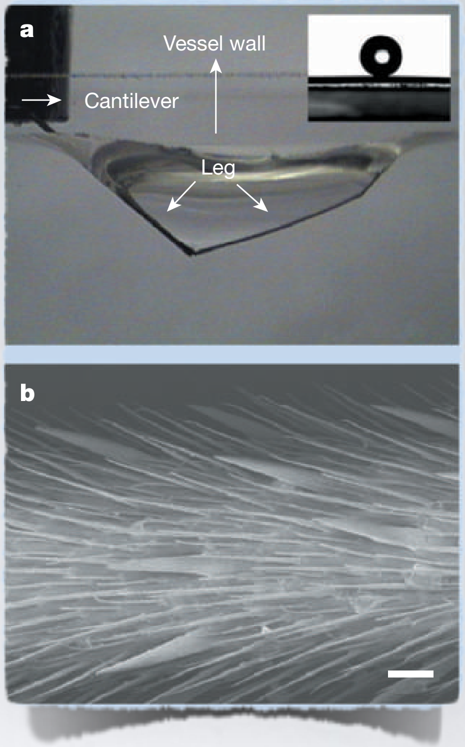
Gao & Jiang (2004)



Also allows us to understand floating of 'inverse Leidenfrost drops' (water on liquid Nitrogen):

ROUGH SURFACES

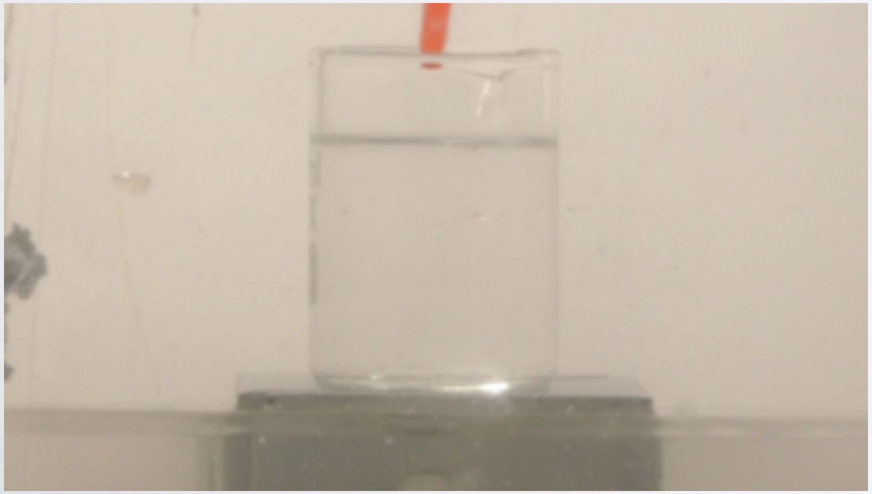
Often think of surface tension as providing a force per unit length – does this explain why water walking insects have hairy legs?



Gao & Jiang (2004)

Keller's Theorem shows that this cannot be the case – a hairy surface does not significantly alter the volume of liquid displaced

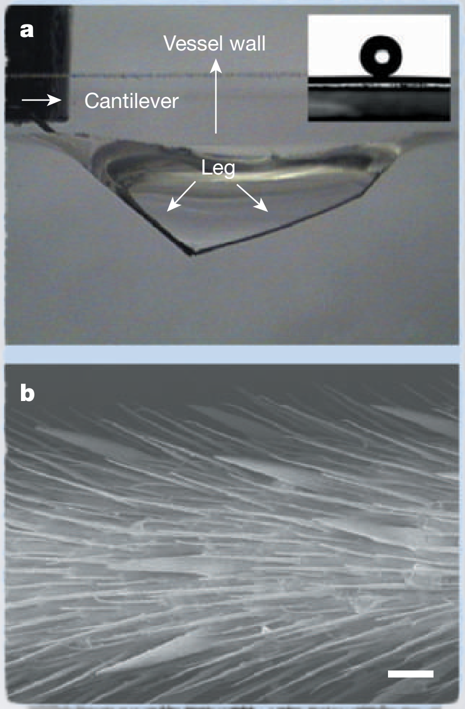
Key point: contact line length is increased **but** presence of nearby hairs causes mini-menisci to flatten (and hence provide less force)



Also allows us to understand floating of 'inverse Leidenfrost drops' (water on liquid Nitrogen):

ROUGH SURFACES

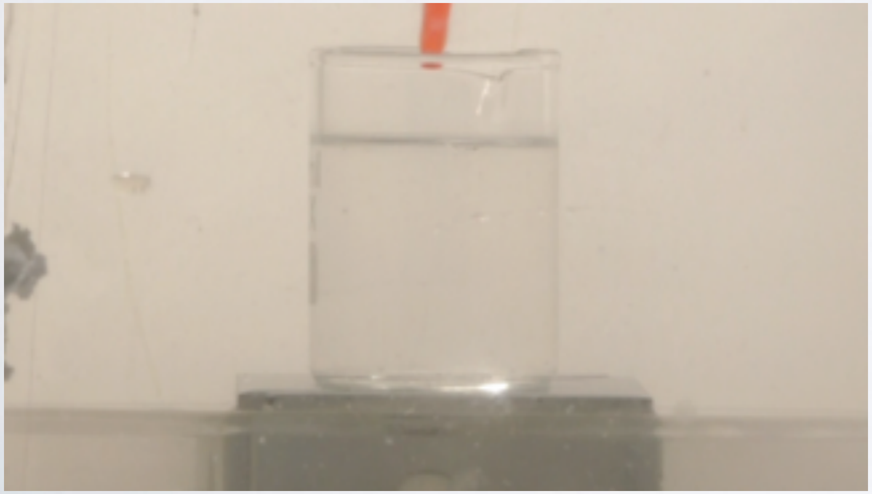
Often think of surface tension as providing a force per unit length – does this explain why water walking insects have hairy legs?



Gao & Jiang (2004)

Keller's Theorem shows that this cannot be the case – a hairy surface does not significantly alter the volume of liquid displaced

Key point: contact line length is increased **but** presence of nearby hairs causes mini-menisci to flatten (and hence provide less force)



Also allows us to understand floating of 'inverse Leidenfrost drops' (water on liquid Nitrogen):

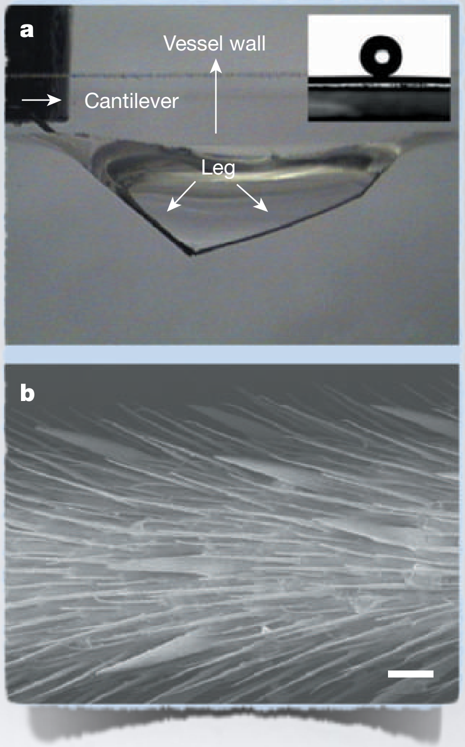
- While levitating, drop is effectively non-wetting sphere

ROUGH SURFACES

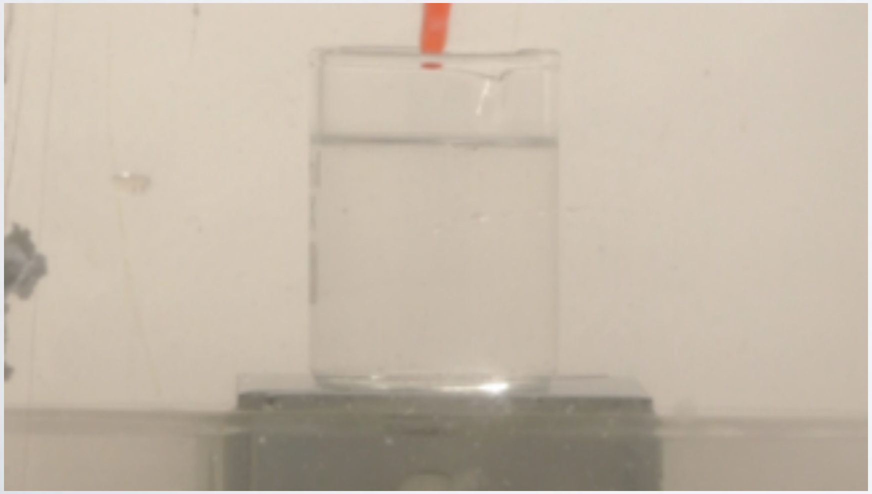
Often think of surface tension as providing a force per unit length – does this explain why water walking insects have hairy legs?

Keller's Theorem shows that this cannot be the case – a hairy surface does not significantly alter the volume of liquid displaced

Key point: contact line length is increased **but** presence of nearby hairs causes mini-menisci to flatten (and hence provide less force)



Gao & Jiang (2004)



Also allows us to understand floating of 'inverse Leidenfrost drops' (water on liquid Nitrogen):

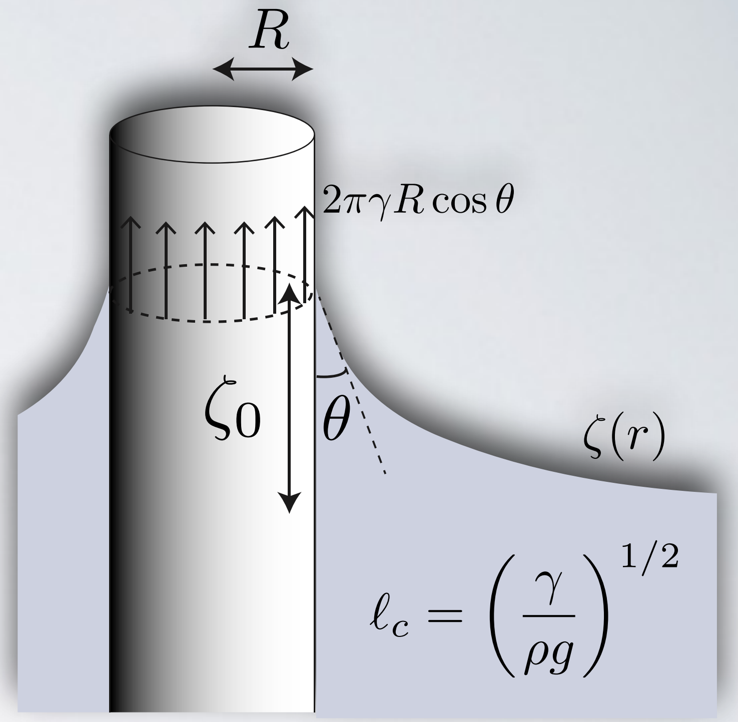
- While levitating, drop is effectively non-wetting sphere
- Once frozen, stops levitating and roughness causes sinking

A SIMPLE APPLICATION

The static meniscus around a small vertical cylinder can be understood using Keller's Theorem too

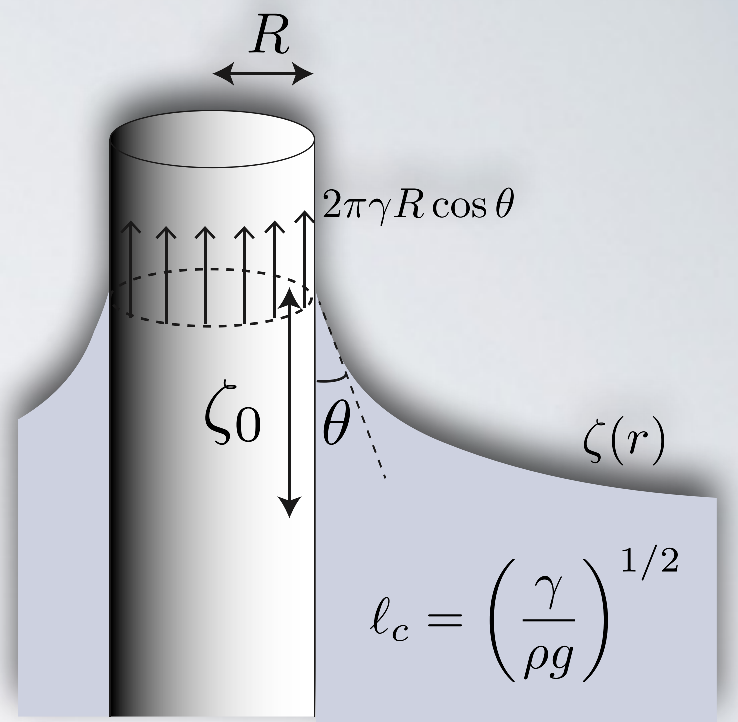
For small cylinders, $\epsilon = R/\ell_c \ll 1$, can perform matched asymptotic analysis of the Laplace–Young equation to find meniscus height ζ_0

(see e.g. James 1974, Lo 1983).



A SIMPLE APPLICATION

The static meniscus around a small vertical cylinder can be understood using Keller's Theorem too



For small cylinders, $\epsilon = R/\ell_c \ll 1$, can perform matched asymptotic analysis of the Laplace–Young equation to find meniscus height ζ_0

(see e.g. James 1974, Lo 1983).

Far away from the cylinder, find the linearized Laplace-Young equation

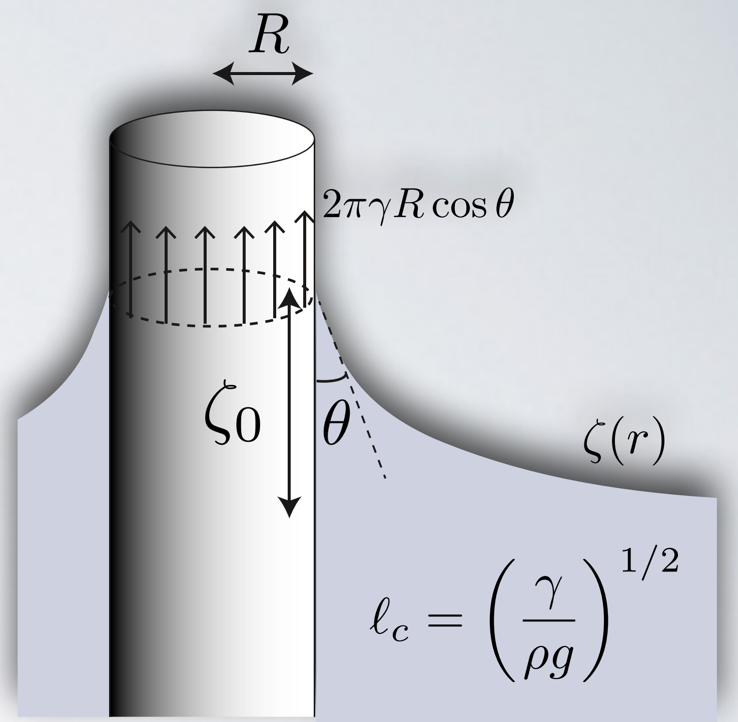
$$\zeta = \frac{\ell_c^2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\zeta}{\mathrm{d}r}\right)$$

A SIMPLE APPLICATION

The static meniscus around a small vertical cylinder can be understood using Keller's Theorem too

For small cylinders, $\epsilon = R/\ell_c \ll 1$, can perform matched asymptotic analysis of the Laplace–Young equation to find meniscus height ζ_0

(see e.g. James 1974, Lo 1983).



Far away from the cylinder, find the linearized Laplace-Young equation

$$\zeta = \frac{\ell_c^2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\zeta}{\mathrm{d}r}\right) \implies \zeta = AK_0(r/\ell_c)$$

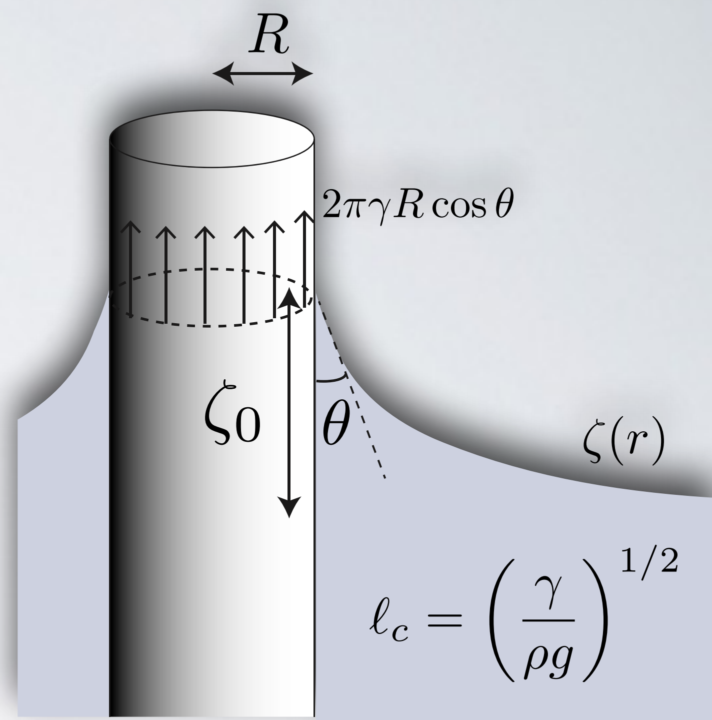
A SIMPLE APPLICATION

The static meniscus around a small vertical cylinder can be understood using Keller's Theorem too

For small cylinders, $\epsilon = R/\ell_c \ll 1$, can perform matched asymptotic analysis of the Laplace–Young equation to find meniscus height ζ_0

(see e.g. James 1974, Lo 1983).

Far away from the cylinder, find the linearized Laplace-Young equation



$$\zeta = \frac{\ell_c^2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\zeta}{\mathrm{d}r}\right) \implies \zeta = AK_0(r/\ell_c)$$

Balancing weight of liquid displaced in outer meniscus with vertical force from surface tension find that

$$\frac{\zeta_0}{\ell_c} \sim \epsilon\log(1/\epsilon)$$

A SECOND PROBLEM

SIAM J. APPL. MATH.
Vol. 43, No. 2, April 1983

0036–1399/83/4302–0004 $01.25/0

SURFACE TENSION DRIVEN FLOWS*

JOSEPH B. KELLER† AND MICHAEL J. MIKSIS‡

Keller & Miksis (KM83) consider inviscid retraction of fluid wedge due to surface tension

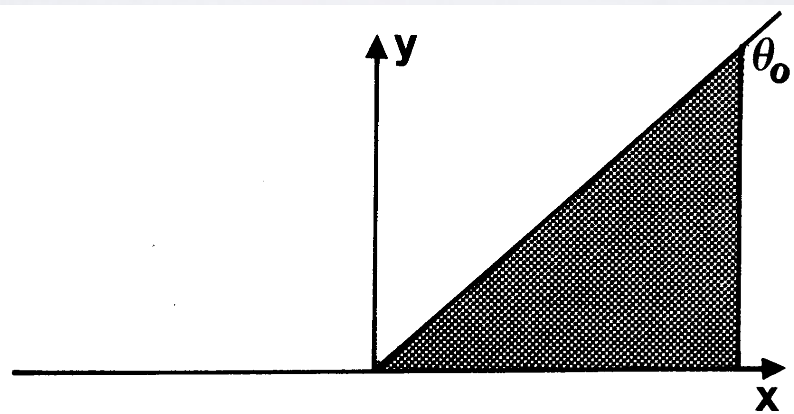


FIG. 1b. *Cross-section of a wedge-shaped layer of liquid bounded by a free surface* $y = x \tan\theta_0$ *and a rigid surface* $y = 0$.

A SECOND PROBLEM

SIAM J. APPL. MATH.
Vol. 43, No. 2, April 1983

0036-1399/83/4302-0004 $01.25/0

SURFACE TENSION DRIVEN FLOWS*

JOSEPH B. KELLER† AND MICHAEL J. MIKSIS‡

Keller & Miksis (KM83) consider inviscid retraction of fluid wedge due to surface tension

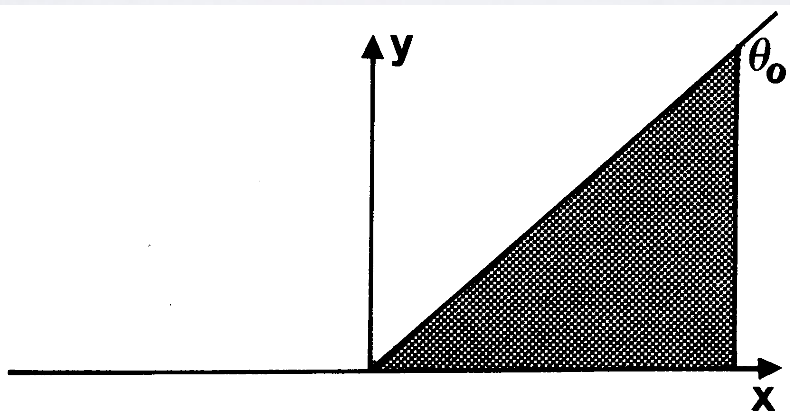


FIG. 1b. *Cross-section of a wedge-shaped layer of liquid bounded by a free surface* $y = x \tan \theta_0$ *and a rigid surface* $y = 0$.

KM83 has been very influential in drop breakup (though only studied 2D)

ANOTHER RELATED PROBLEM

Analysis performed by KM83 also describes the early stages of meniscus onset

Keller & Miksis calculated similarity solutions for the interface shape with different contact angles

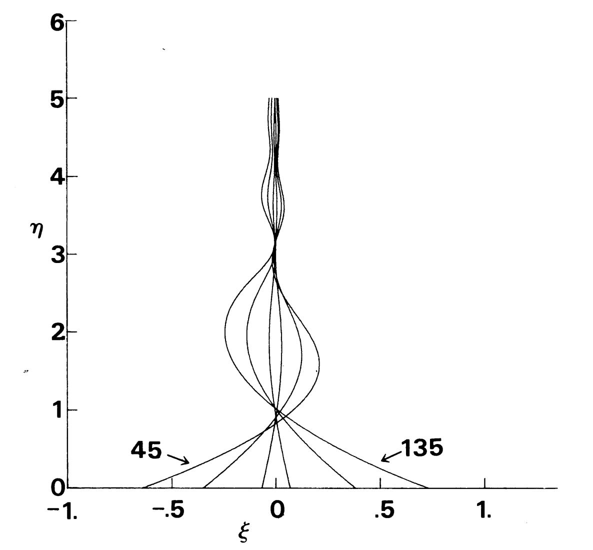
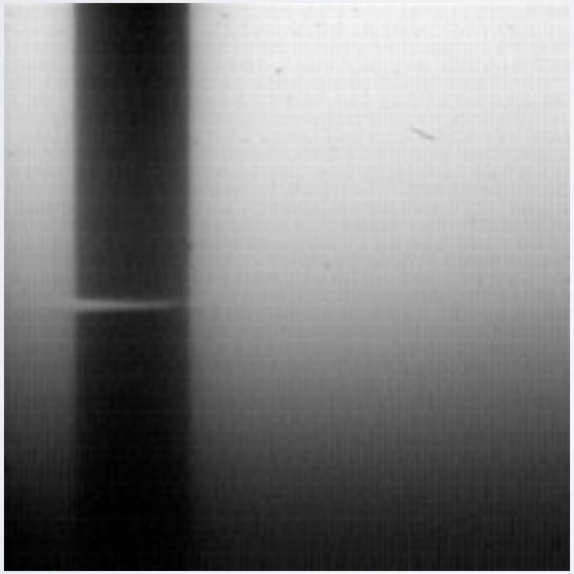


FIG. 4. *Same as Fig.* 3 *with* $\theta_0 = 90°$ *and* $\beta = 45°, 65°, 85°, 95°, 115°, 135°$.

Christophe Clanet (Paris)



R = 0.225 mm

duration ≃ 1 s

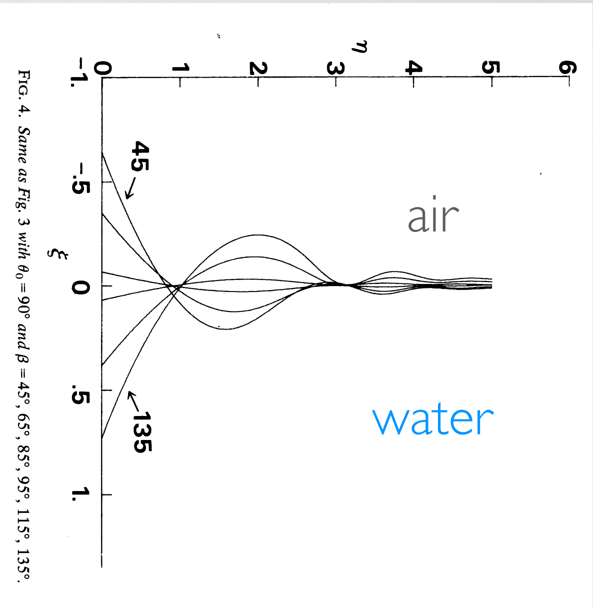
Clanet & Quéré (2002) studied 'onset' of menisci on cylinders of various radii

Key question is: *How does meniscus height evolve with time?*

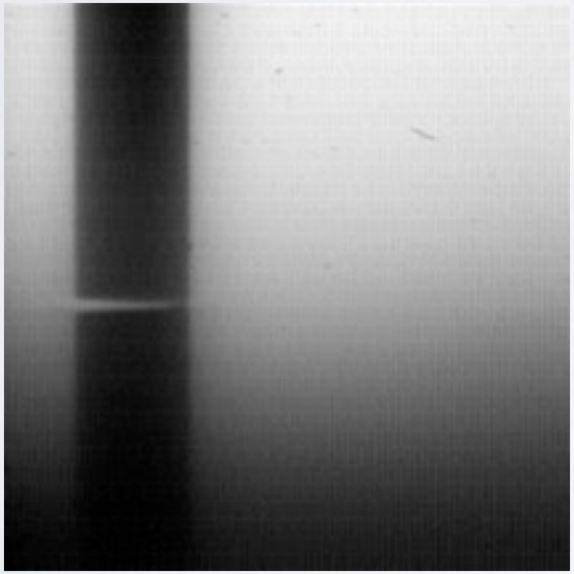
ANOTHER RELATED PROBLEM

Analysis performed by KM83 also describes the early stages of meniscus onset

Keller & Miksis calculated similarity solutions for the interface shape with different contact angles



Christophe Clanet (Paris)



R = 0.225 mm
duration ≃ 1 s

Clanet & Quéré (2002) studied 'onset' of menisci on cylinders of various radii

Key question is: *How does meniscus height evolve with time?*

ANOTHER RELATED PROBLEM

Analysis performed by KM83 also describes the early stages of meniscus onset

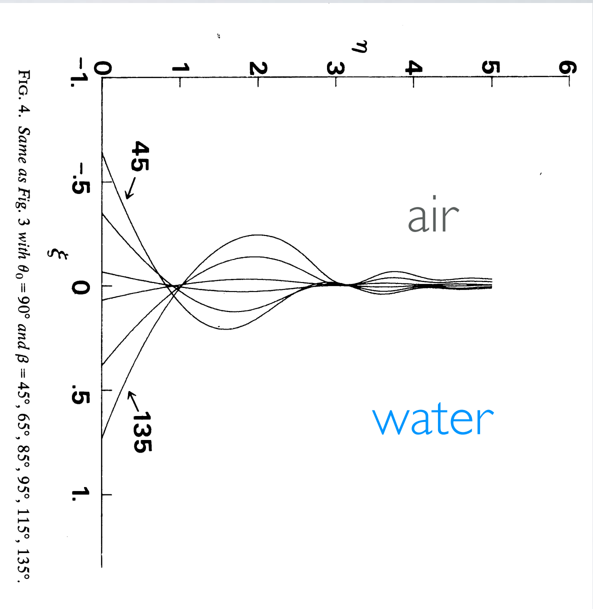
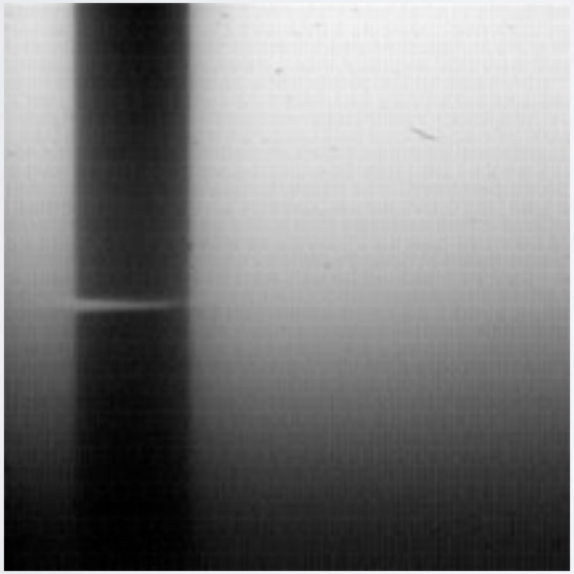


FIG. 4. Same as Fig. 3 with $\theta_0 = 90°$ and $\beta = 45°, 65°, 85°, 95°, 115°, 135°$.

Keller & Miksis calculated similarity solutions for the interface shape with different contact angles

Christophe Clanet (Paris)



R = 0.225 mm
duration ≈ 1 s

Clanet & Quéré (2002) studied 'onset' of menisci on cylinders of various radii

Key question is: *How does meniscus height evolve with time?*

ANOTHER RELATED PROBLEM

Analysis performed by KM83 also describes the early stages of meniscus onset

Keller & Miksis calculated similarity solutions for the interface shape with different contact angles

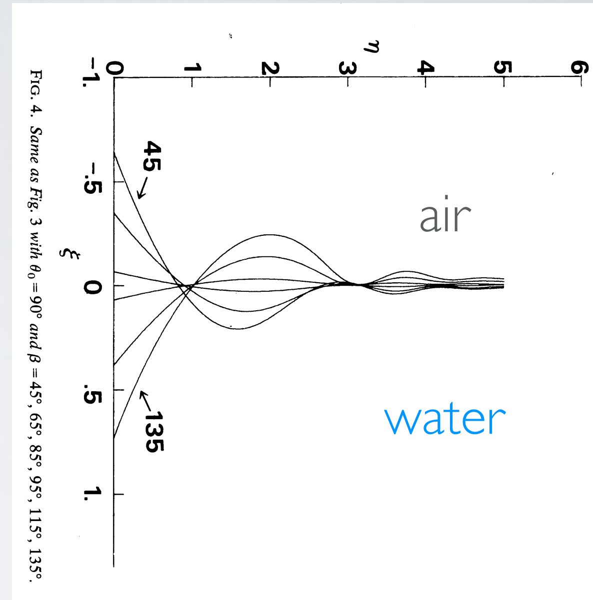
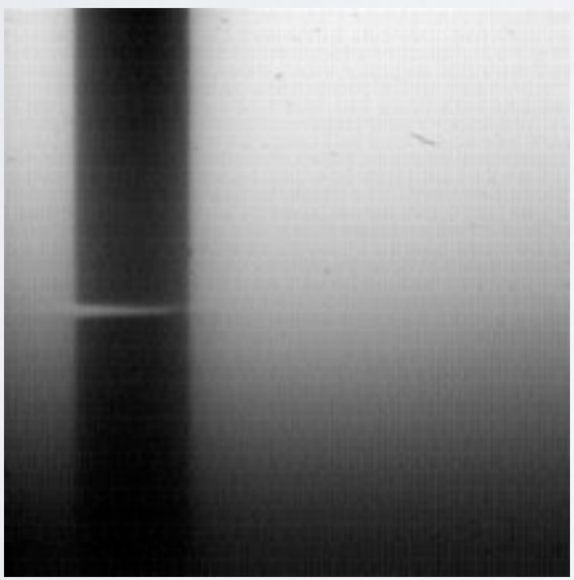


FIG. 4. Same as Fig. 3 with $\theta_0 = 90°$ and $\beta = 45°, 65°, 85°, 95°, 115°, 135°$.

Christophe Clanet (Paris)



$R = 0.225$ mm
duration $\simeq 1$ s

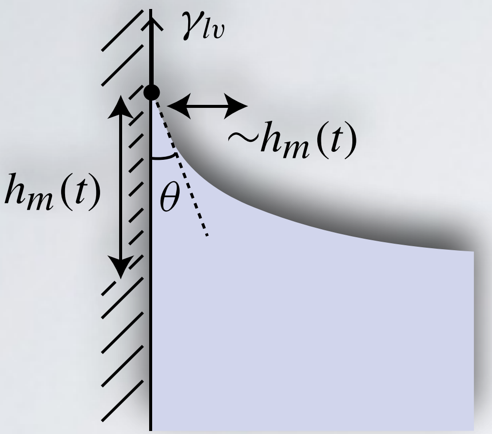
Clanet & Quéré (2002) studied 'onset' of menisci on cylinders of various radii

Key question is: *How does meniscus height evolve with time?*

KM83 analysis is relevant to high Reynolds (or Kapitsa) number and large cylinders

PLANES/LARGE CYLINDERS

'Large' cylinders are effectively planar – consider how meniscus rises up a wall



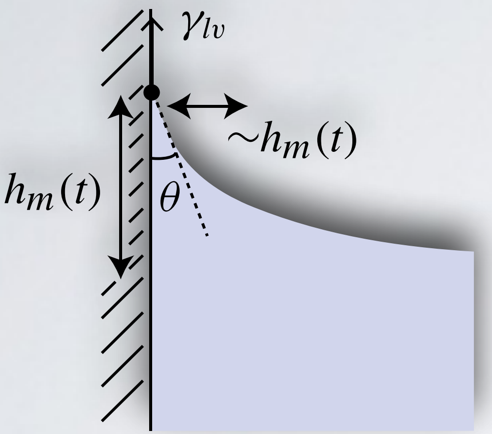
Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma$$

Geometry: $$M \sim \rho_l h_m^2$$

$$\implies h_m \sim \left(\frac{\gamma t^2}{\rho_l}\right)^{1/3}$$

PLANES/LARGE CYLINDERS

'Large' cylinders are effectively planar – consider how meniscus rises up a wall



Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma$$

Geometry: $$M \sim \rho_l h_m^2$$

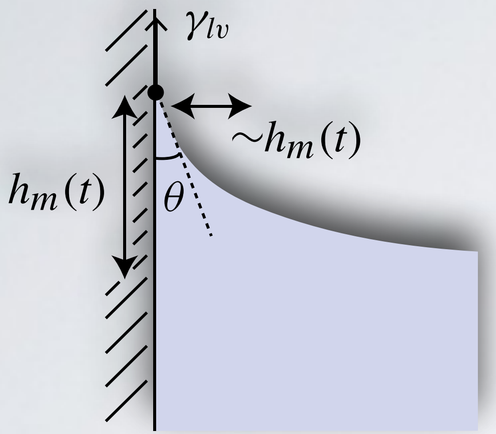
$$\implies h_m \sim \left(\frac{\gamma t^2}{\rho_l}\right)^{1/3}$$

This is the classic "Keller & Miksis scaling"

(though their calculation also provides the pre-factor)

PLANES/LARGE CYLINDERS

'Large' cylinders are effectively planar – consider how meniscus rises up a wall



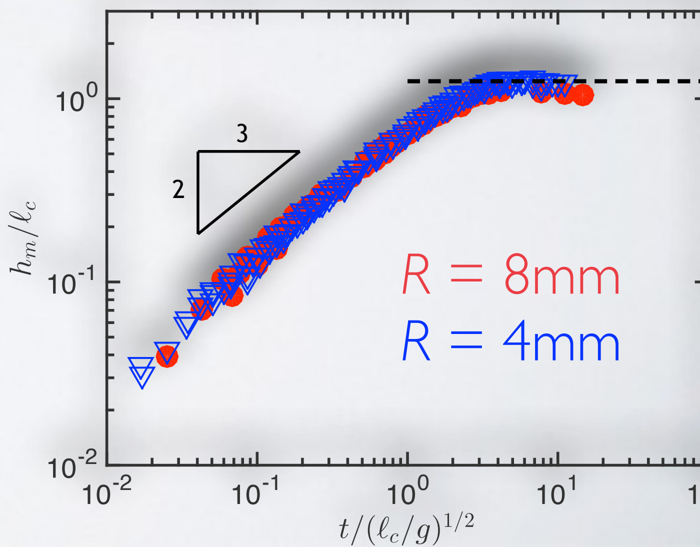
Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma$$

Geometry: $$M \sim \rho_l h_m^2$$

$$\Longrightarrow h_m \sim \left(\frac{\gamma t^2}{\rho_l}\right)^{1/3}$$

This is the classic "Keller & Miksis scaling"

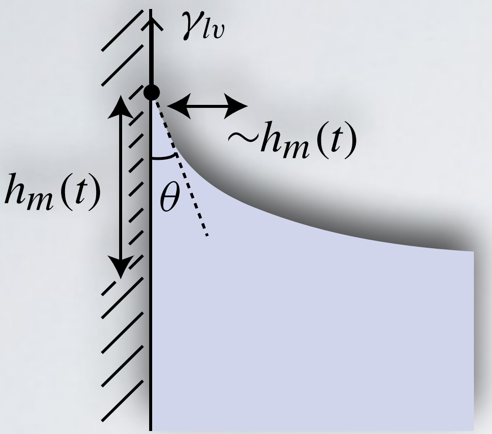
(though their calculation also provides the pre-factor)



Clanet & Quéré (2002)

PLANES/LARGE CYLINDERS

'Large' cylinders are effectively planar – consider how meniscus rises up a wall



Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma$$

Geometry: $$M \sim \rho_l h_m^2$$

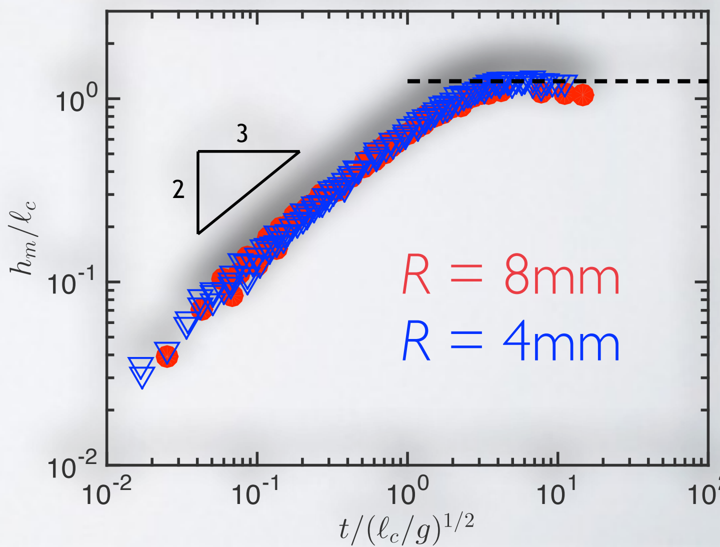
$$\Longrightarrow h_m \sim \left(\frac{\gamma t^2}{\rho_l}\right)^{1/3}$$

This is the classic "Keller & Miksis scaling"

(though their calculation also provides the pre-factor)

Experimentally, observe KM scaling early on (before meniscus height saturates)

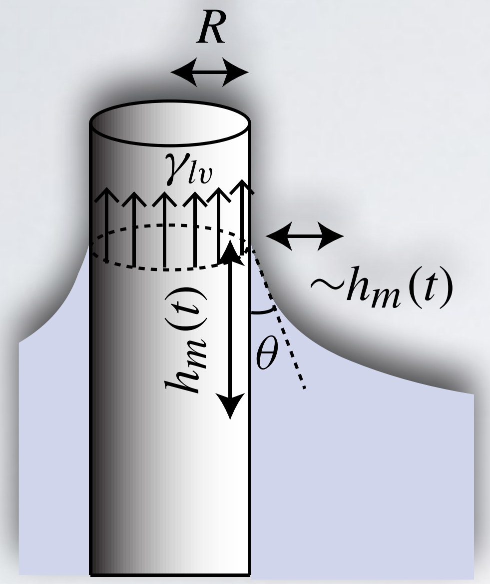
No dependence on cylinder size for large enough cylinders



Clanet & Quéré (2002)

SMALL CYLINDERS

Clanet & Quéré repeat the same scaling argument, but with a finite radius cylinder:



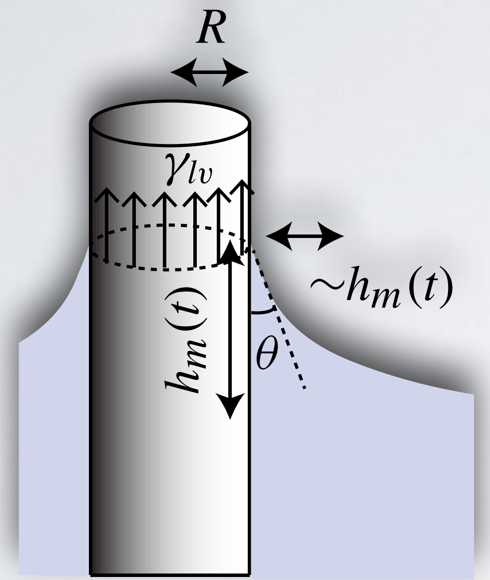
Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma R$$

Geometry: $$M \sim \rho_l h_m^3$$

$$\Longrightarrow \; h_m \sim \left(\frac{\gamma R t^2}{\rho_l}\right)^{1/4}$$

SMALL CYLINDERS

Clanet & Quéré repeat the same scaling argument, but with a finite radius cylinder:

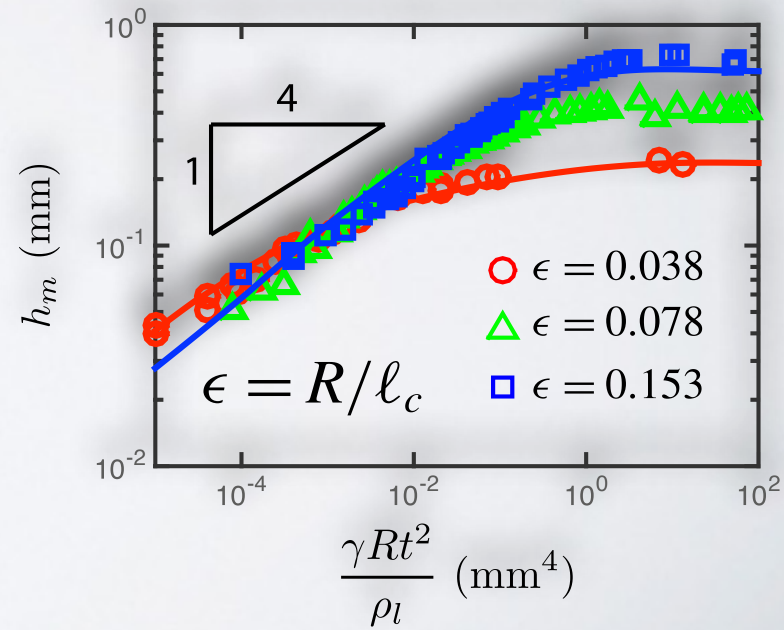


Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma R$$

Geometry: $$M \sim \rho_l h_m^3$$

$$\Longrightarrow \quad h_m \sim \left(\frac{\gamma R t^2}{\rho_l}\right)^{1/4}$$

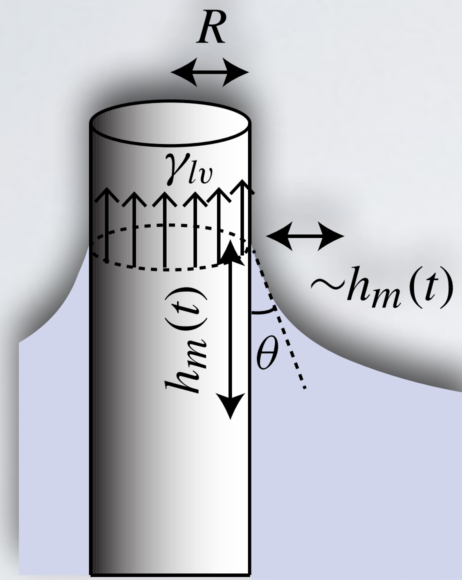
Clanet & Quéré's experiments *seem* to confirm this for different radius cylinders



Clanet & Quéré (2002)

SMALL CYLINDERS

Clanet & Quéré repeat the same scaling argument, but with a finite radius cylinder:

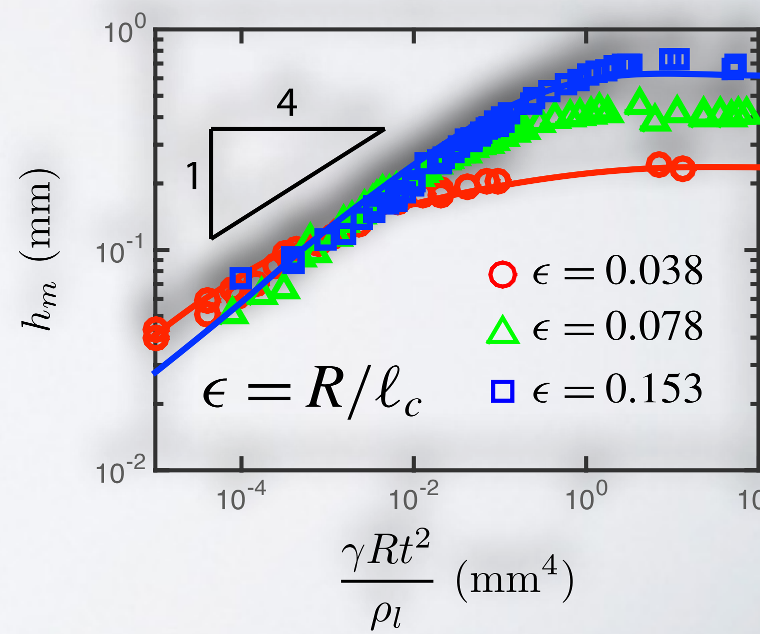


Newton II: $$\frac{\mathrm{d}}{\mathrm{d}t}(M\dot{h}_m) \sim \gamma R$$

Geometry: $$M \sim \rho_l h_m^3$$

$$\implies h_m \sim \left(\frac{\gamma R t^2}{\rho_l}\right)^{1/4}$$

Clanet & Quéré's experiments *seem* to confirm this for different radius cylinders



Clanet & Quéré (2002)

See diffusive growth, rather than $t^{2/3}$ of KM83

A WARNING

Estimation of mass of meniscus region implicitly assumes that $h_m \gg R$

$$M \sim h_m^3 \quad\Downarrow\quad h_m \sim t^{1/2}$$

$$M \sim R h_m^2 \quad\Downarrow\quad h_m \sim t^{2/3}$$

A WARNING

Estimation of mass of meniscus region implicitly assumes that $h_m \gg R$

$$M \sim h_m^3$$
$$\Downarrow$$
$$h_m \sim t^{1/2}$$

$$M \sim R h_m^2$$
$$\Downarrow$$
$$h_m \sim t^{2/3}$$

But earlier application of Keller's Theorem gave final meniscus height:

$$\zeta_0 \sim R \log(\ell_c / R)$$

A WARNING

Estimation of mass of meniscus region implicitly assumes that $h_m \gg R$

$$M \sim h_m^3$$
$$\Downarrow$$
$$h_m \sim t^{1/2}$$

$$M \sim R h_m^2$$
$$\Downarrow$$
$$h_m \sim t^{2/3}$$

But earlier application of Keller's Theorem gave final meniscus height:

$$\zeta_0 \sim R \log(\ell_c / R)$$

So largest value of $\dfrac{h_m}{R} \sim \log \dfrac{\ell_c}{R}$ which is not large (experimentally)

A WARNING

Estimation of mass of meniscus region implicitly assumes that $h_m \gg R$

$$M \sim h_m^3$$
$$\Downarrow$$
$$h_m \sim t^{1/2}$$

$$M \sim R h_m^2$$
$$\Downarrow$$
$$h_m \sim t^{2/3}$$

But earlier application of Keller's Theorem gave final meniscus height:

$$\zeta_0 \sim R \log(\ell_c / R)$$

So largest value of $\dfrac{h_m}{R} \sim \log \dfrac{\ell_c}{R}$ which is not large (experimentally)

For early times should expect KM83 scaling again

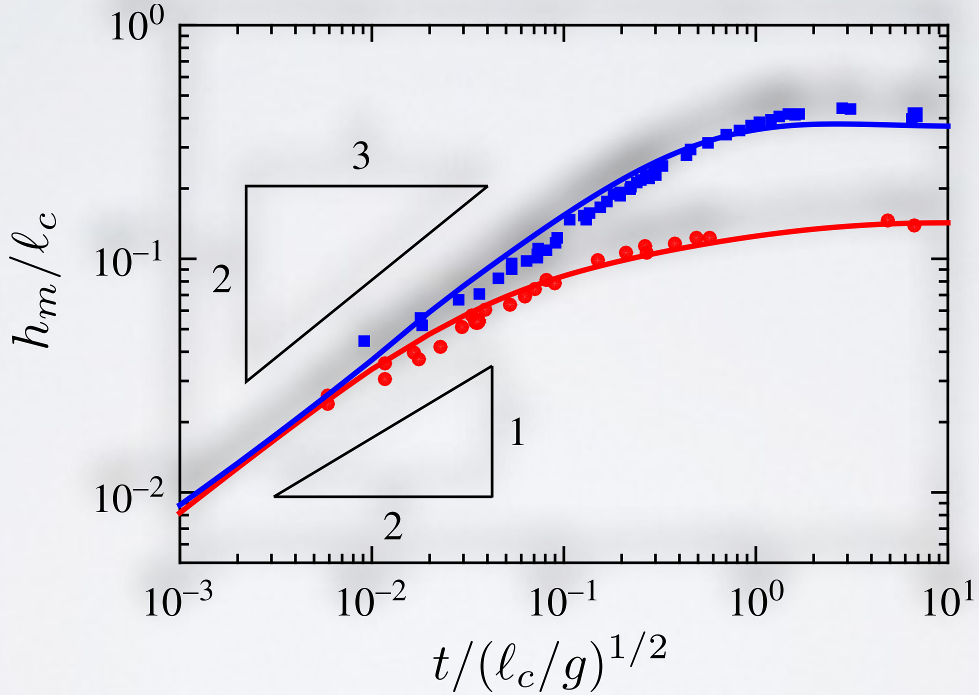
(For early enough times the cylinder appears planar to the meniscus.)

WHAT REALLY HAPPENS?

To provide more complete data (early times) use numerical simulations

Assume:

- Large Reynolds number (inertia dominated)
- Contact angle is constant and equal to measured equilibrium value

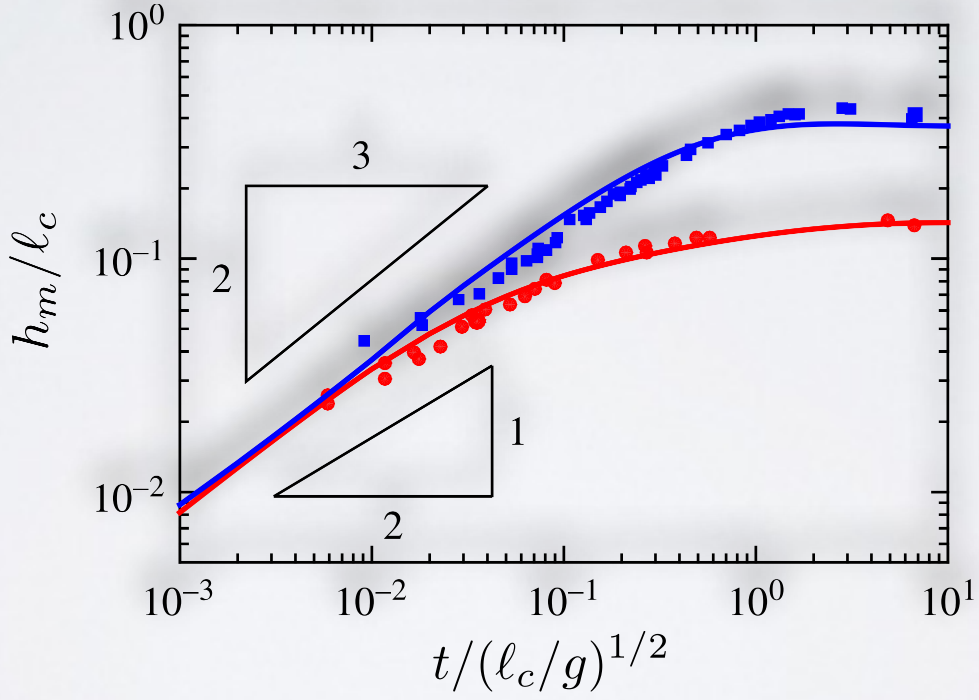


WHAT REALLY HAPPENS?

To provide more complete data (early times) use numerical simulations

Assume:

- Large Reynolds number (inertia dominated)
- Contact angle is constant and equal to measured equilibrium value



Observe:

- (Relatively simple) model reproduces experiments quantitatively
- Apparent $\sqrt{t}$ behaviour seems to be turn around from $t^{2/3}$ to flat

WHAT CAN WE SAY?

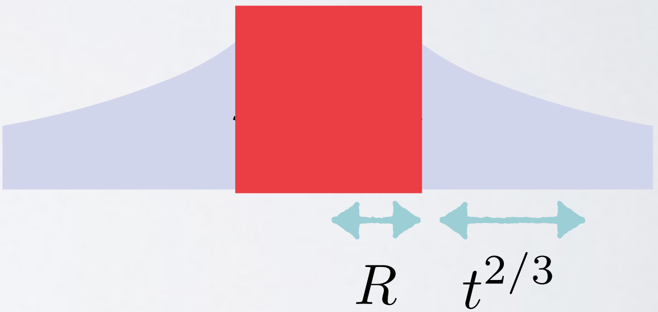
For early times, have seen that vertical scale of meniscus is

$$h_m \sim \left(\frac{\gamma t^2}{\rho_l}\right)^{1/3}$$

Motion self-similar for early times and Laplace equation in fluid (inviscid)

$\Rightarrow$ relevant horizontal scale is also: $\left(\frac{\gamma t^2}{\rho_l}\right)^{1/3}$

Cylinder radius is a length scale
(breaking self-similarity of 2-D problem)



WHAT CAN WE SAY?

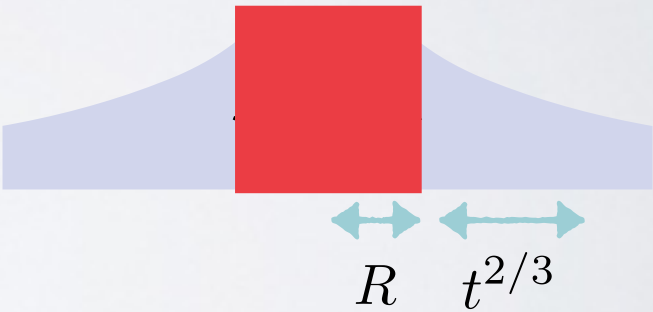
For early times, have seen that vertical scale of meniscus is

$$h_m \sim \left(\frac{\gamma t^2}{\rho_l} \right)^{1/3}$$

Motion self-similar for early times and Laplace equation in fluid (inviscid)

$\Rightarrow$ relevant horizontal scale is also: $\left(\frac{\gamma t^2}{\rho_l} \right)^{1/3}$

Cylinder radius is a length scale
(breaking self-similarity of 2-D problem)



Important parameter in the problem is then the ratio of these lengths

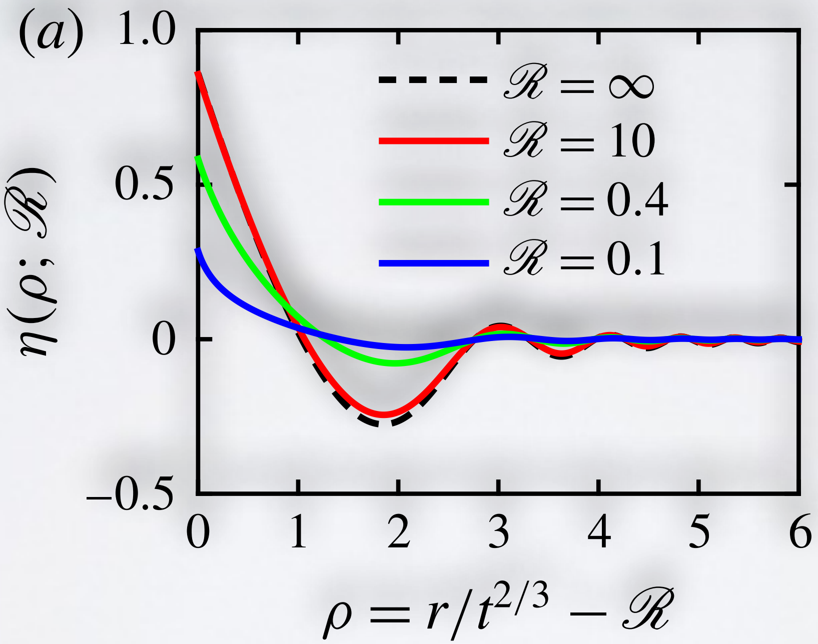
$$\mathcal{R} = \epsilon / t^{2/3}$$

UNIVERSAL BEHAVIOUR

Introduction of the parameter $\mathcal{R}$ gives universal dynamics at early times

(Travel through a family of similarity-like solutions, parametrized by $\mathcal{R}$)

In limit $\mathcal{R} = \infty$ recover (true) similarity solution of KM83

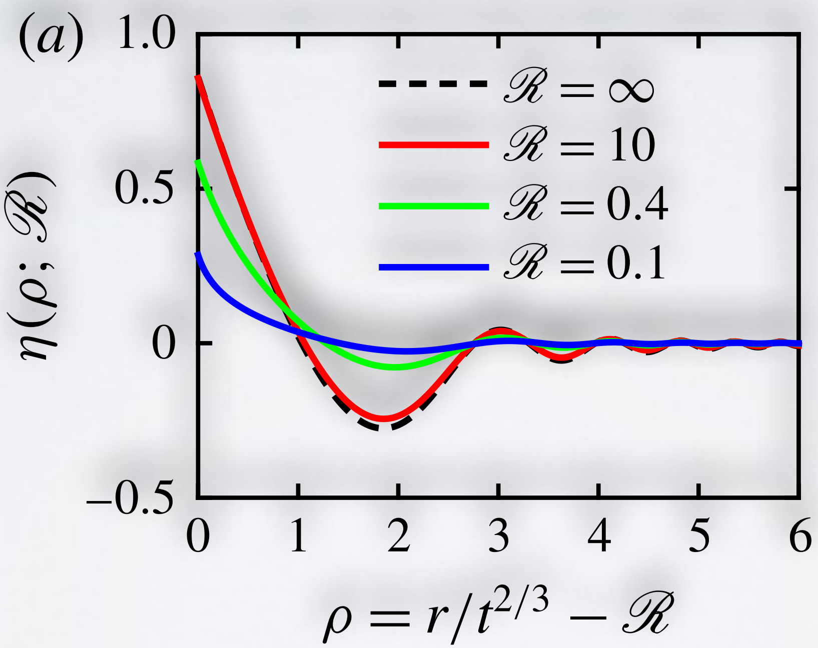


UNIVERSAL BEHAVIOUR

Introduction of the parameter $\mathcal{R}$ gives universal dynamics at early times

(Travel through a family of similarity-like solutions, parametrized by $\mathcal{R}$)

In limit $\mathcal{R} = \infty$ recover (true) similarity solution of KM83



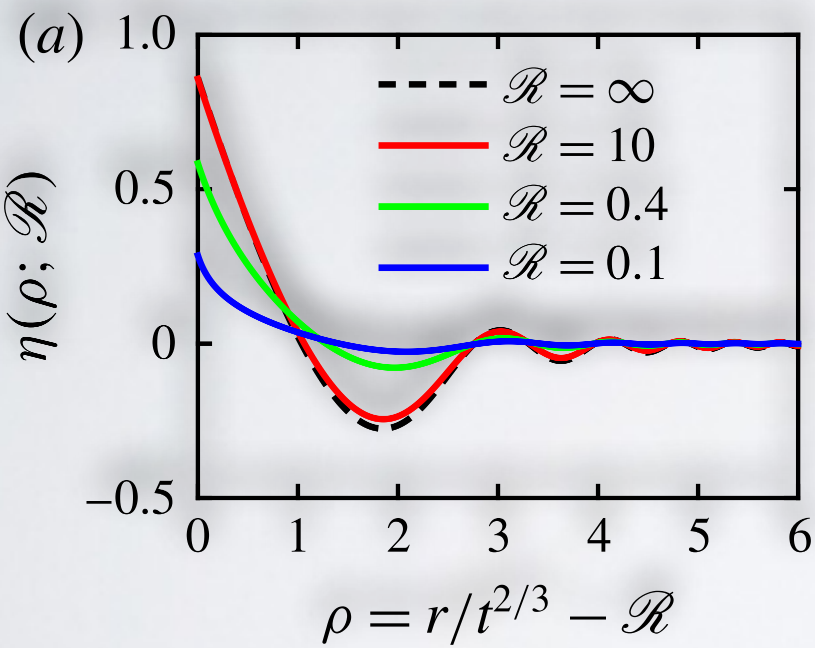
This also suggests that at early times the experimental data (with different radii) should rescale onto a universal curve (for given contact angle)

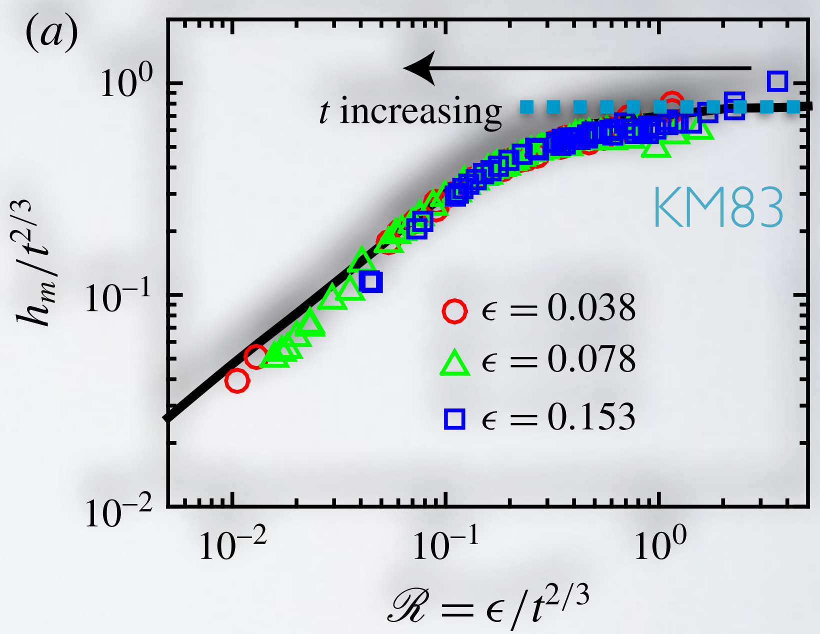
UNIVERSAL BEHAVIOUR

Introduction of the parameter $\mathcal{R}$ gives universal dynamics at early times

(Travel through a family of similarity-like solutions, parametrized by $\mathcal{R}$)

In limit $\mathcal{R} = \infty$ recover (true) similarity solution of KM83





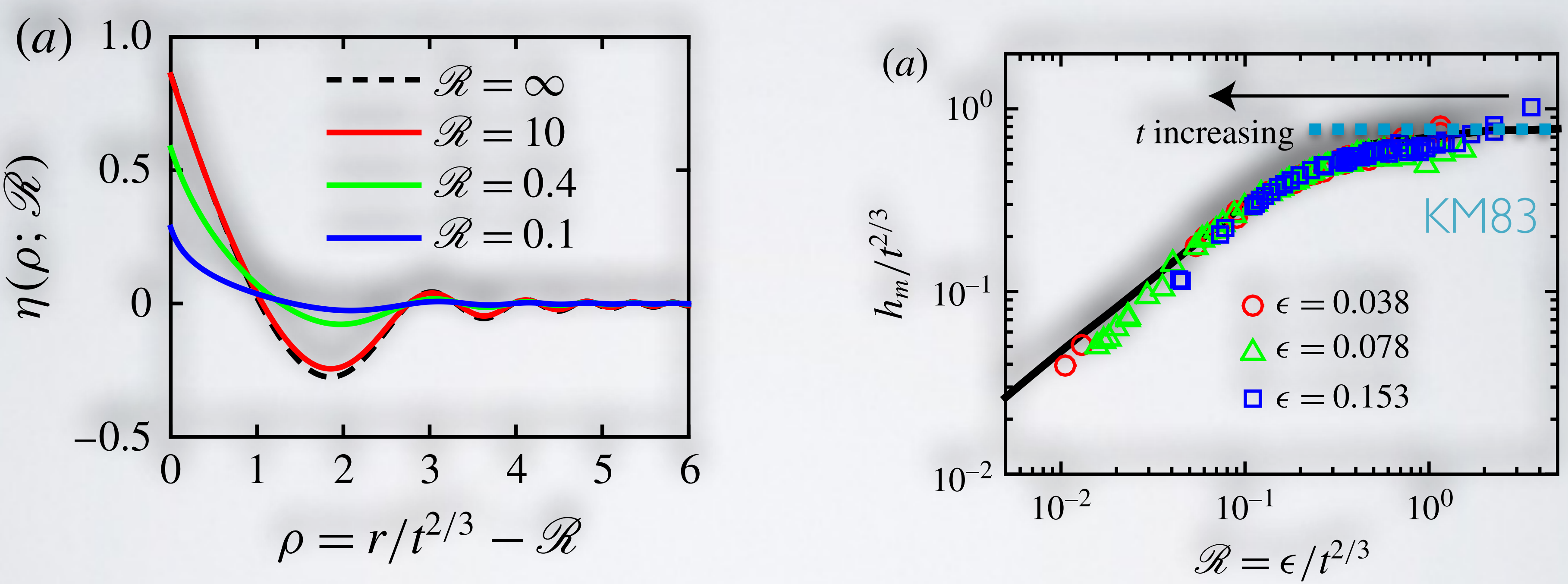
This also suggests that at early times the experimental data (with different radii) should rescale onto a universal curve (for given contact angle)

UNIVERSAL BEHAVIOUR

Introduction of the parameter $\mathcal{R}$ gives universal dynamics at early times

(Travel through a family of similarity-like solutions, parametrized by $\mathcal{R}$)

In limit $\mathcal{R} = \infty$ recover (true) similarity solution of KM83

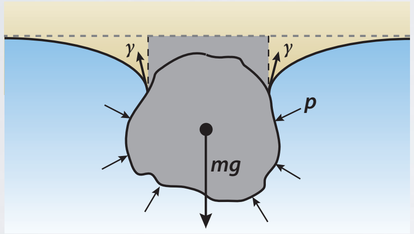


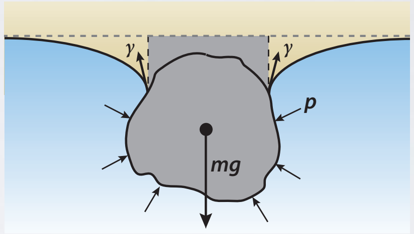
This also suggests that at early times the experimental data (with different radii) should rescale onto a universal curve (for given contact angle)

Finally, do not see any sign of Clanet & Quéré diffusive scaling

CONCLUSIONS

- Keller's Theorem shows the equivalence of two ways of thinking about forces from surface tension

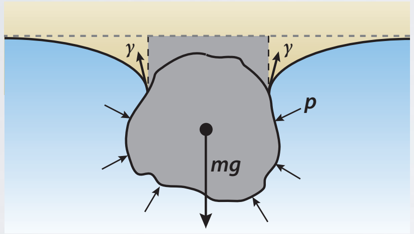


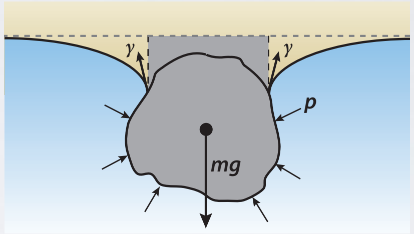


(Can choose whichever is most convenient, provided that you are logically consistent!)

CONCLUSIONS

- Keller's Theorem shows the equivalence of two ways of thinking about forces from surface tension





(Can choose whichever is most convenient, provided that you are logically consistent!)

- Can extend Keller & Miksis similarity solutions to problems with an intrinsic length scale, though it is hard to break the $t^{2/3}$ scaling law