Logical Foundations for Classical Encryption and Quantum Teleportation

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Post-Quantum Research: Identifying Future Challenges and Directions
Isaac Newton Institute, University of Cambridge
9 May 2014

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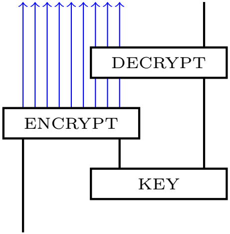

Encrypted communication

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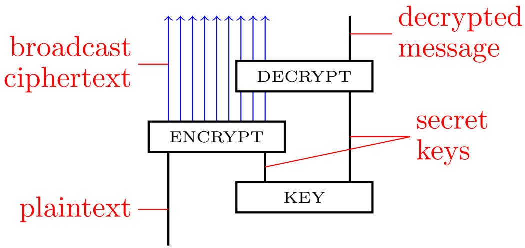

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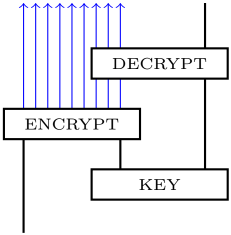

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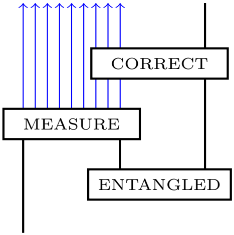

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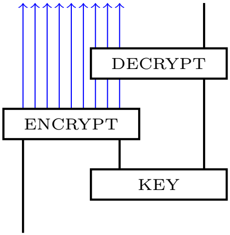

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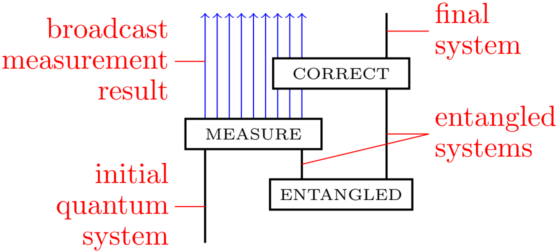

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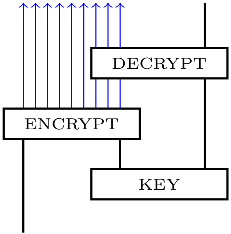

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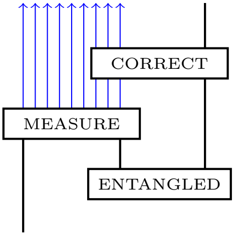

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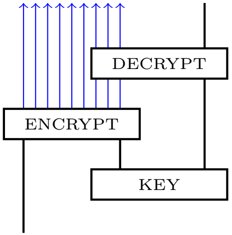

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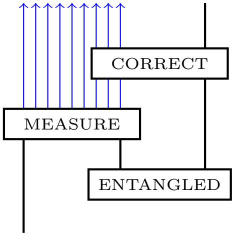

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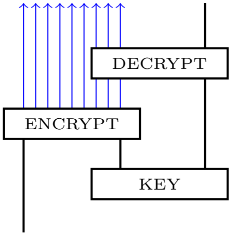

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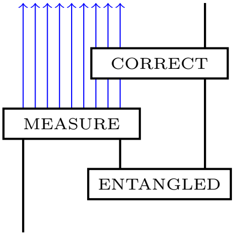

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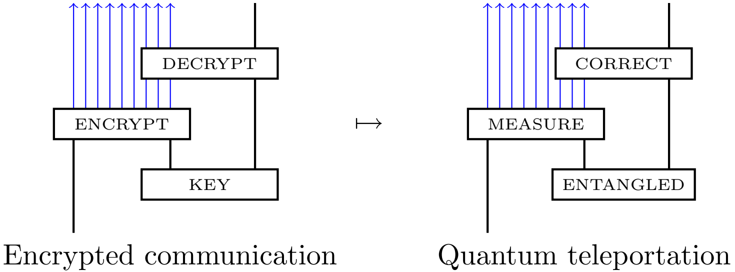

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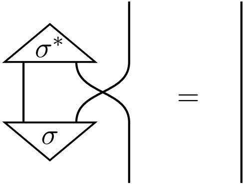
Nice result. There is a general classical-to-quantum construction.

Part of the *categorical quantum computing* programme launched by Abramsky and Coecke in 2004.

Strings and correlation

Consider the following equation, where σ is a bipartite state preparation and σ^* is the corresponding bipartite postselection:



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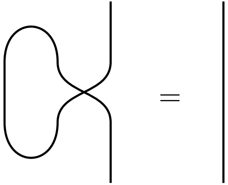

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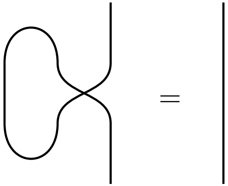

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We can investigate consequences of this equation in different settings.

- **Quantum theory.**
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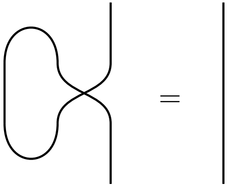

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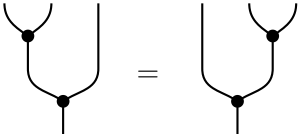
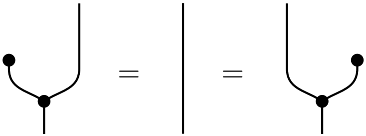
- **Quantum theory.**
  The state σ is *maximally entangled*: $|\sigma\rangle = |00\rangle + |11\rangle$
- **Classical computation.**
  The state σ is *perfectly correlated*: $\sigma = \{00\} \cup \{11\}$.

Surfaces and logic

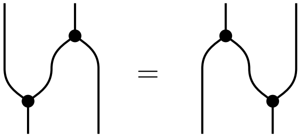
We now think about basic properties of copying, comparing and deleting classical information:



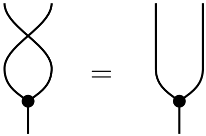
Associativity



Unit



Frobenius law



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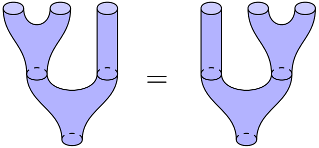

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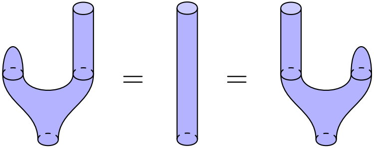

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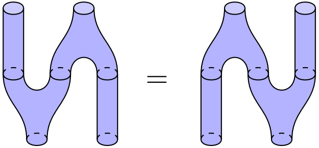

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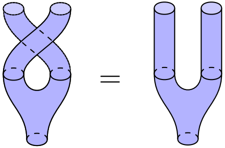

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These are the laws obeyed by surfaces up to deformation!

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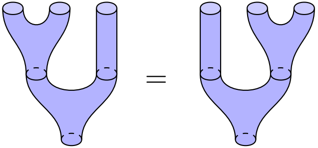

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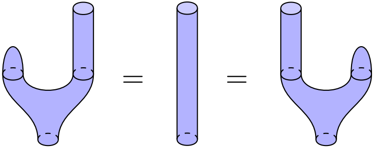

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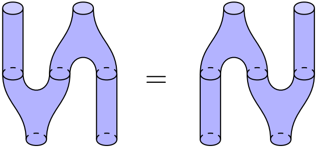

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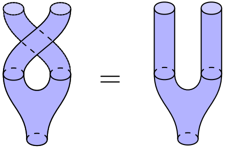
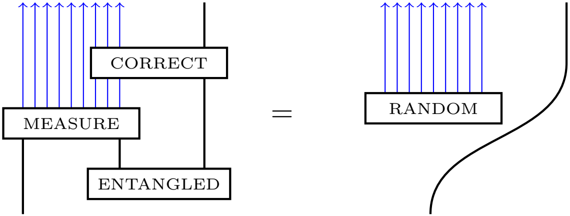


Commutativity

These are the laws obeyed by surfaces up to deformation!
So we change notation and use **topological surfaces**.

Geometrical structure

Here is ordinary teleportation:



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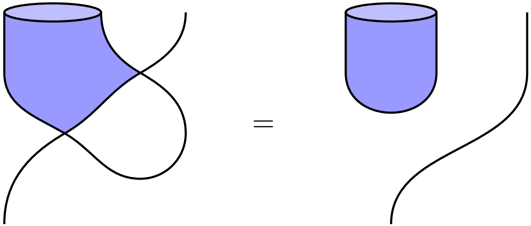

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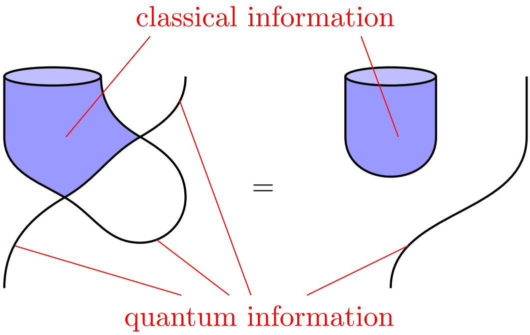

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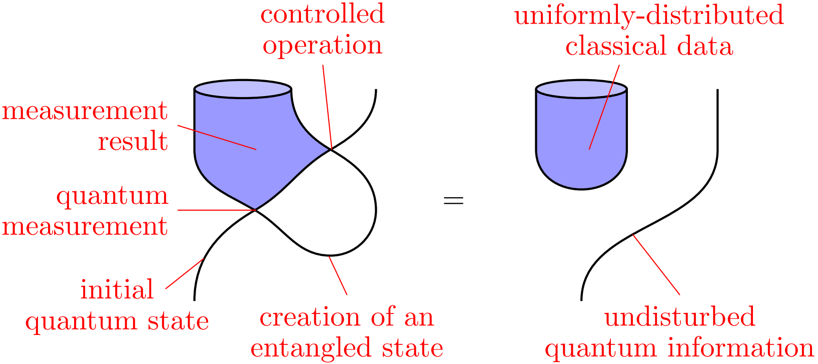

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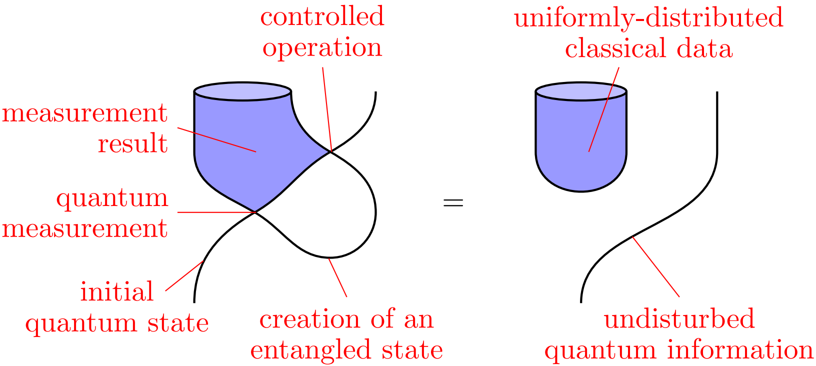

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Theorem. Quantum solutions correspond exactly to implementations of quantum teleportation.

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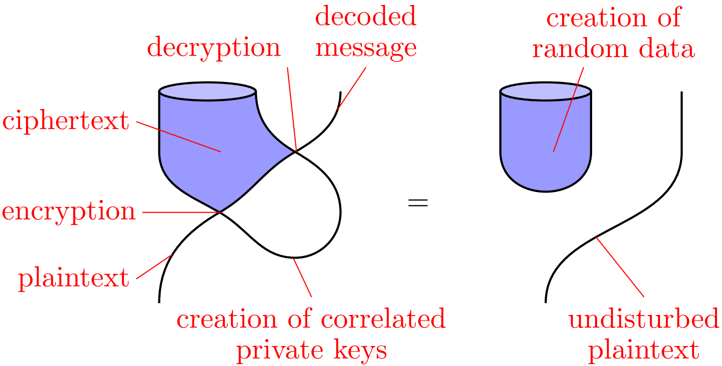

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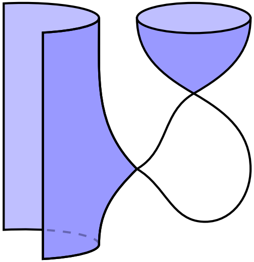
Theorem. Classical solutions correspond exactly to implementations of classical one-time-pad encryption.

Dense coding

This equation describes *dense coding*:



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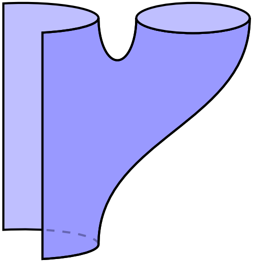

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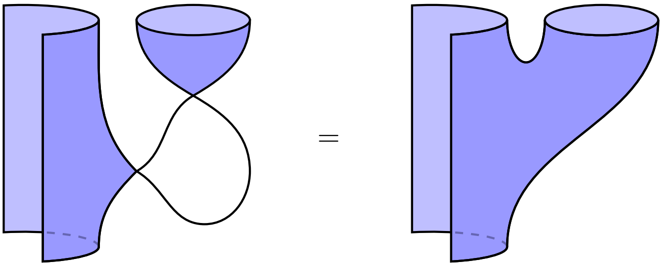

It describes the transmission of data through a channel with only *half* the apparent required capacity!

This is *topologically equivalent* to the teleportation equation.

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Thank you!