SYNCHRONISATION:
STICK-SLIP INSTABILITIES, AVALANCHES & CRISES

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with: **S. Gualdi, M. Tarzia, F. Zamponi**

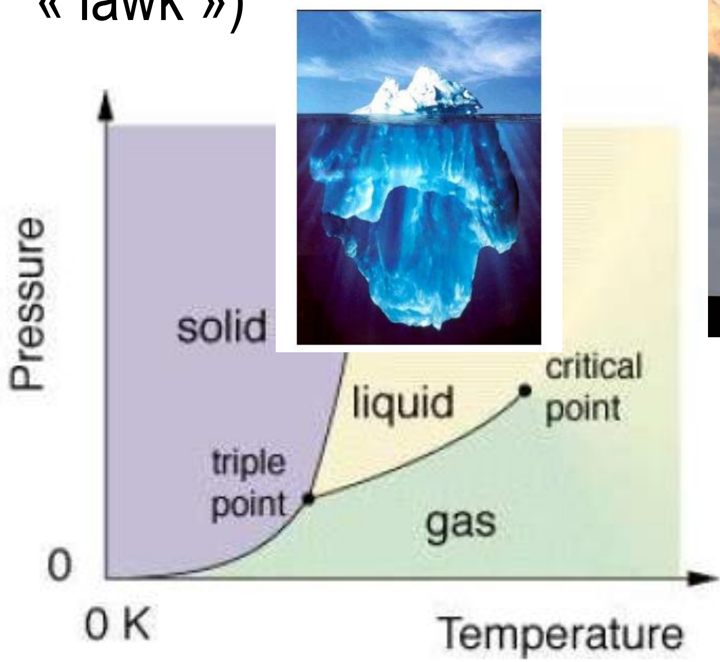
Many thanks to: J. Bonart, M. Wyart

« More is different »

Adapted from Phil Anderson's piece

The behavior of large assemblies of **interacting** individuals (particles) cannot be understood as a simple extrapolation of the properties of isolated individuals. Instead, entirely new, **unanticipated behaviors** may appear and their understanding requires new ideas and methods.

→ The over-arching concept of transitions and phase diagrams (but not always « lawk »)



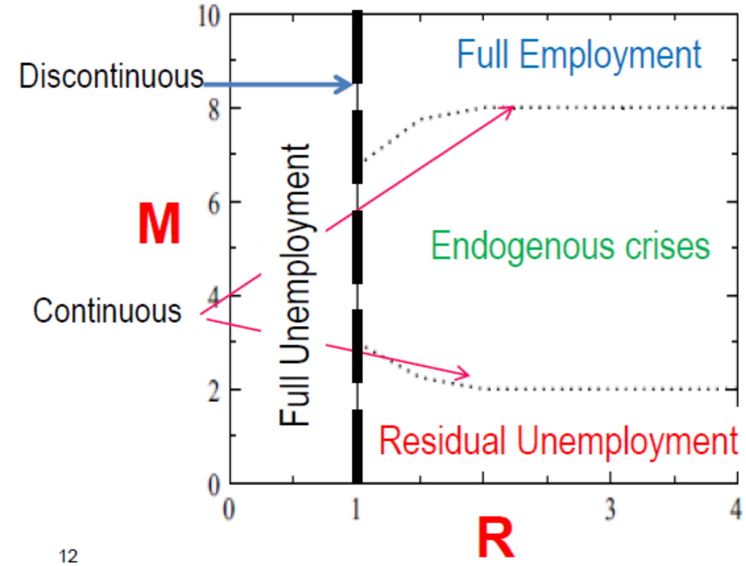
« We are all individuals »
(Life of Brian)

« **Dark Corners »…** (aka « Black Swans »)

We in the field did think of the economy as roughly linear, constantly subject to different shocks, constantly fluctuating, but naturally returning to its steady state over time. […] The main lesson of the 2008 crisis is that we were much closer to **"dark corners"** — situations in which the economy could badly malfunction — than we thought.

Now that we are more aware of nonlinearities and the dangers they pose, we should explore them further theoretically and empirically. […] Trying to create a model that describes crises may be beyond the profession's conceptual and technical reach at this stage.

◅ *From Olivier Blanchard's remarkable recent piece « Where Danger Lurks » (2014)*



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CRISES IN A STYLIZED ABM

- S. Gualdi, M. Tarzia, F. Zamponi, J.-P. Bouchaud, *Tipping points in macroeconomic agent-based models*, Journal of Economic Dynamics and Control (2014)

« Telescopes of the mind »

Collective effects are often counterintuitive and defeat our imagination

→ **We need « telescopes of the mind » to anticipate dark corners / tipping points / Black Swans**

Done properly, computer simulation represents a kind of "telescope for the mind," multiplying human powers of analysis and insight just as a telescope does our powers of vision. With simulations, we can discover relationships that the unaided human mind, or even the human mind aided with the best mathematical analysis, would never grasp. (M. Buchanan, "This Economy Does Not Compute", NYT, October 2008)

→ Surprisingly, numerical simulations are still not well accepted in the economics community (see Farmer & Foley, « The Economy needs Agent Based Modelling », Nature 2009).

<u>Crises in a stylized agent based model</u>

- Beyond classical economics (rational agents, equilibrium): Agent Based Models with heuristic behavioral rules

- Recently (< 10 years): comprehensive macroeconomic ABMs with firms, households, central banks, prices, wages, rates… ("EURACE", "Lagom", "Mark I")

- Problem: these are <u>huge</u> models, with *dozens of parameters*

  → one may easily get lost in the "wilderness" of high dimensional spaces
  → does one *explain* anything?

- Even highly stylized models ("Mark 0" – see below) still have many parameters (~10) – at least by physics standards

Crises in a stylized agent based model

- Even highly stylized models still have many parameters (~ 10)

- **A methodological manifesto:** Establish the phase diagram to capture the model's essential phenomenology – using numerical simulations and analytical arguments to navigate in high dimensions

- At this stage of ABM development, qualitative understanding may be more important than quantitative calibration (economists are obsessed by "calibration", before we are even sure the model makes real sense)

→ It turns out that many parameters/rules change quantitatively, but not qualitatively, the aggregate properties of large economies – as often in physical systems

Crises in a stylized agent based model

- "Mark 0" – a stylized ABM with plausible behavioural/*tatônnement* rules:

1) Firms adjust workforce (= production), prices, and possibly wages in reaction to sales. Hiring/firing adjustments maybe different, with ratio **R**
2) Households' consumption budget = a fraction of their wealth. They favour firms with lower prices with some "intensity of choice"
3) Firms default when debt exceeds a multiple **M** of total sales, and are replaced by new firms with some rate [Debt is shared between households & surviving firms]
4) Fully "stock-flow" consistent (i.e. accounting is done correctly (!))

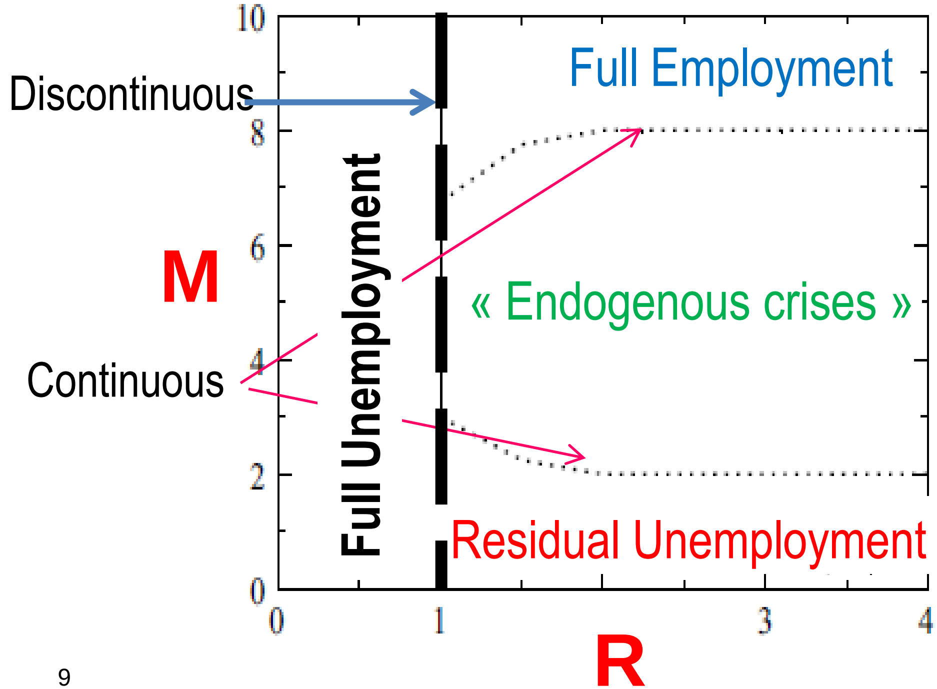
→ Min: 7 parameters; 5 turn out to be "innocuous" but 2 are crucial:

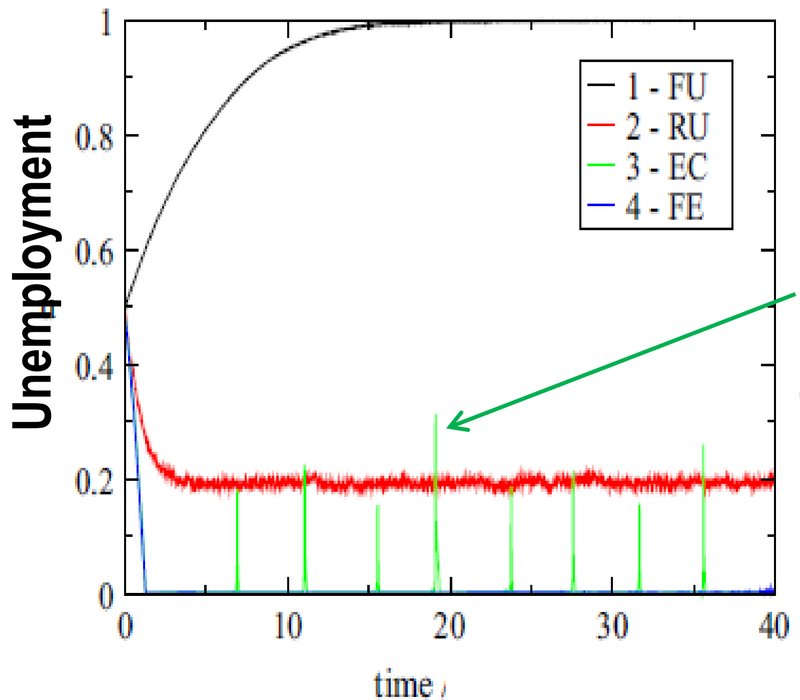
Ratio of hiring/firing adjustment rates (in reaction to sales)
Maximum debt-to-sales ratio before bankruptcy (i.e. leverage)

Crises in stylized agent based models

⇒ A robust topology of the phase diagram

(w.r.t. many behavioural rules and parameters, e.g. fixed/variable wages, etc…)



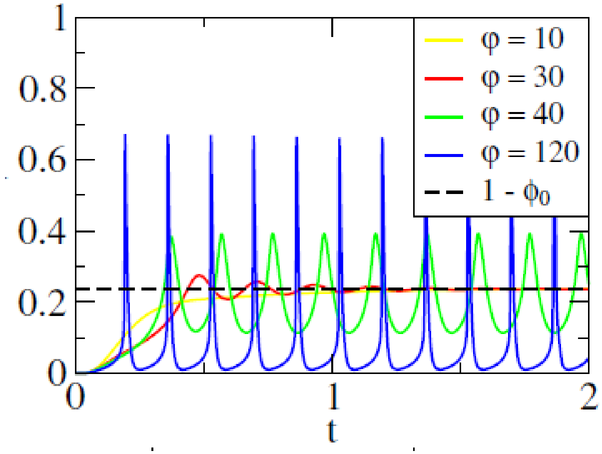


Quasi-periodic unemployment spikes **without external shocks**

True or bug??

→ **R**atio of hiring/firing adjustment rates: if firms fire faster than they hire, the economy collapses

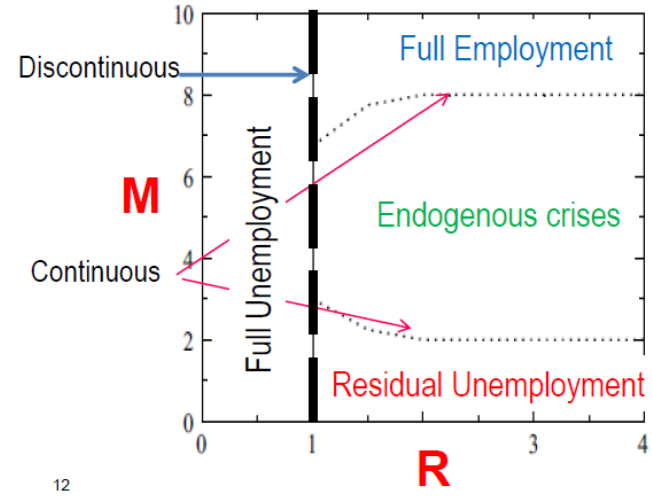
→ **M**aximum level of indebtedness before bankruptcy: if too small, firms default “accidentally” leading to residual unemployment. As **M** grows, a curious phase with **unemployment spikes** sets in



A GENERIC SYNCHRONISATION TRANSITION

- S. Gualdi, J.-P. Bouchaud, G. Cencetti, M. Tarzia and F. Zamponi, "*Endogenous crisis waves: a stochastic model with synchronized collective behavior*", Phys.Rev.Lett. 114, 088701 (2015)

The Endogeneous Crisis Phase



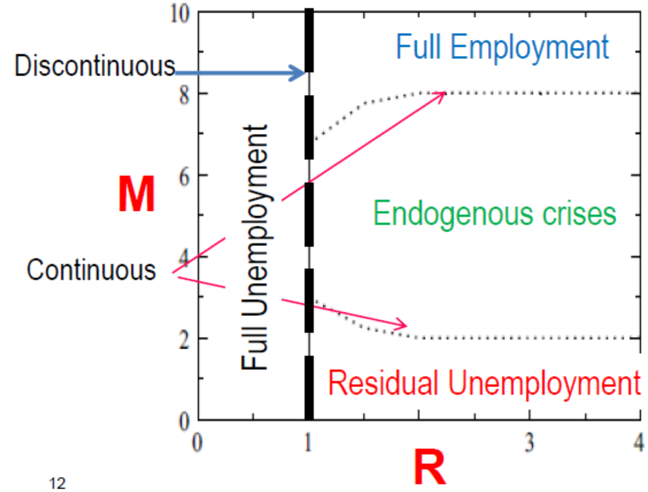
Model intuition:

Waves of collective defaults, mediated by a feedback loop: 12

Default → Increased unemployement and decreased consumption → Fragilisation of other firms → More defaults → Synchronisation ?

- A stochastic **toy model** of the ABM: firms are described by their debt to sales level x
- x follows a (biased) random walk
- When x reaches -Θ the firm is declared bankrupt, employees are made redundant, loans are not repaid
- This fragilizes other firms → all other x's get a kick towards -Θ
- New firms are created with some Poisson rate φ with a clean book

The Endogeneous Crisis Phase



- A purely stochastic model: **Coupled random walks**

$$\dot{P}(x,t) = DP''(x,t) + V(t)P'(x,t) + J(t)\delta(x-\Theta)$$

$$V(t) = b + \beta D P'(0,t)\Theta \qquad P(0,t) = 0 \ \forall\, t$$

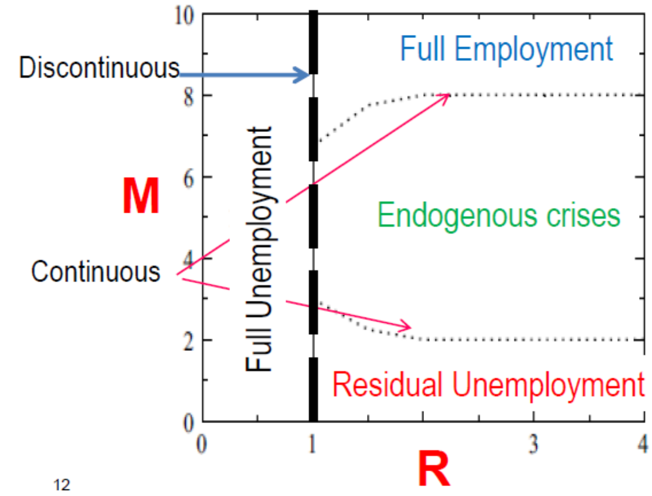
(Note: $x \rightarrow x + \Theta$)

Feedback strength

- Looks too simple for anything interesting to happen
- There is a stationary « boring » state with P_0 ,V_0 and J_0 independent of time
- One can compute the time evolution of a small perturbation around P_0
- This perturbation becomes unstable and oscillates for large enough feedback, induced by bankrupcy avalanches

The Endogeneous Crisis Phase

- A purely stochastic model: ($x \rightarrow x + \Theta$)



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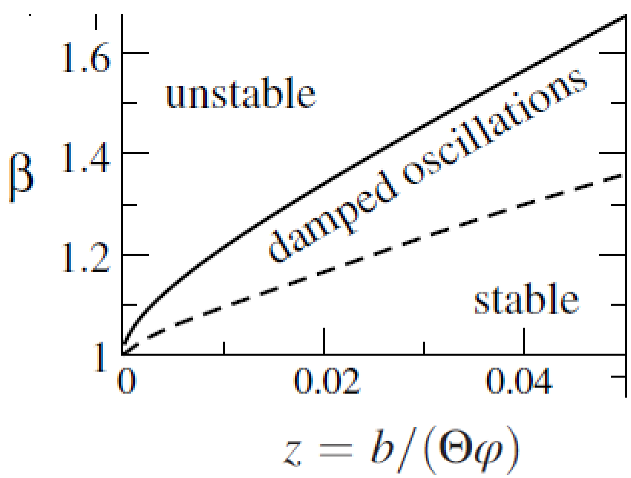
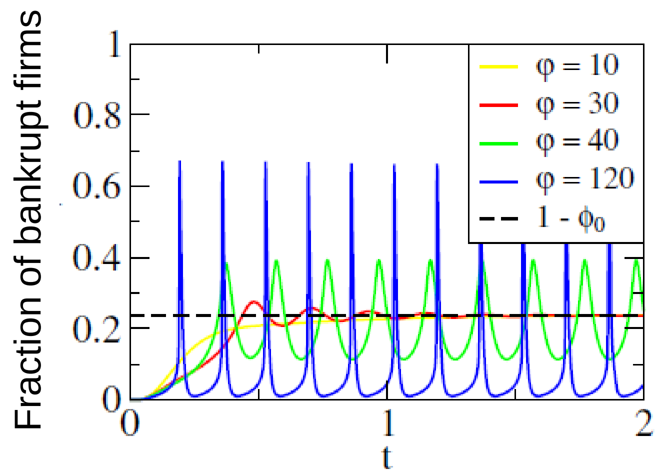




- Synchronisation transition akin to the Kuramoto model, but no « oscillators » here
- Transition **robust** to noise (Poisson reinjection rate, random Θ's, etc.)

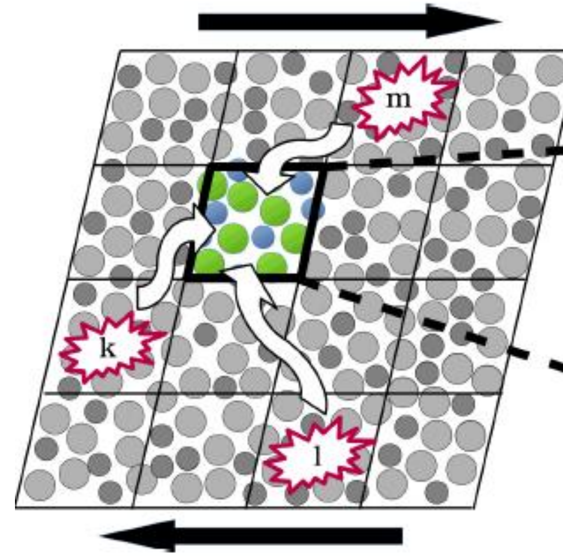
Fireflies, neurons, epidemia

- Synchronisation of fireflies

*"For 300 years, Western travellers to Southeast Asia had been returning with tales of enormous congregations of fireflies blinking on and off in unison, in displays that stretched for miles along the riverbanks. How could thousands of fireflies orchestrate their flashing so precisely and on such vast scales? For decades, no one could come up with a plausible theory. A few believed there must be **a maestro**, a firefly that cues all the rest. Only by the late 60's did the pieces of the puzzle begin to fall into place…"* S. H. Strogatz, *SYNC: The Emerging Science of Spontaneous Order* (Hyperion, New York, 2003).

- Other situations*:*

→ *Interbank default contagion (divergence of the default rate at some t^* when $J=0$)*
→ *Epidemics: waves of outbreaks (the Great Plague)*
→ *Mean-field description of depinning of interfaces in random media (fracture front, contact line): transition between jerky and steady motion*
→ *Stick-slip transition in jammed systems (the Hebraud-Lequeux model)*



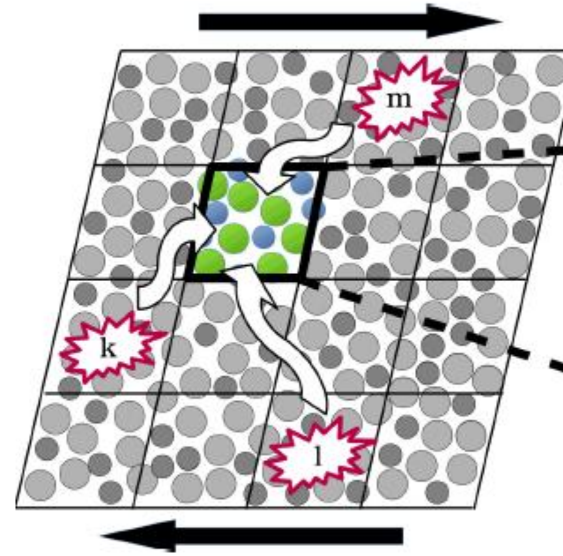
Puosi, Olivier, Martens

A STYLIZED MODEL FOR JAMMED MATERIALS

Synchronisation in (generalized) Hebraud-Lequeux models

- J.-P. Bouchaud, S. Gualdi, M. Tarzia and F. Zamponi, "*Spontaneous Instabilities and Stick-Slip motion in a generalized Hébraud-Lequeux model*", Soft Matter, 12, 1230 (2016)

The Hébraud-Lequeux Model

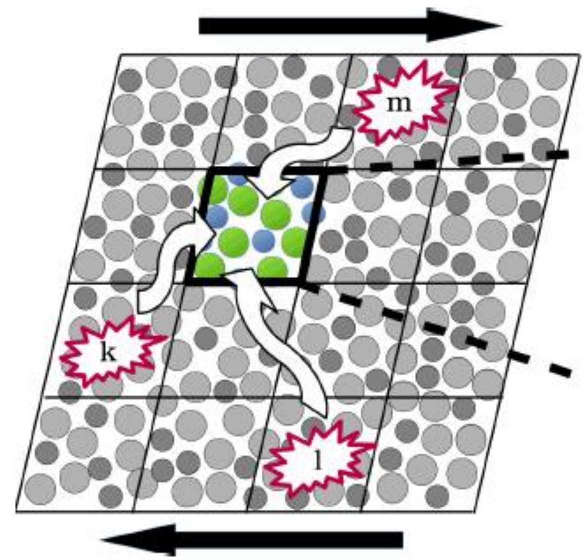


Intuition:

Local plastic yield → Change stress on other elements →
→ More yields → Synchronisation ?

- The **HL (toy) model**: local elements are described by their local stress σ
- σ follows a (biased) random walk, due to the external strain rate
- When $|\sigma|$ exceeds Θ, the element (STZ) yields at a rate $1/\tau_{pl.}$ and starts flowing
- This rearrangement modifies the stress everywhere → all other x's get a random kick
- Flowing elements re-jam (in a zero stress state) at a rate $\varphi = 1/\tau_{fl.}$

The Hébraud-Lequeux Model



Mathematically: (with $\Theta = \sigma_c$)

$$\dot{P}(\sigma,t) = D_t P''(\sigma,t) - G_0\dot{\gamma}P'(\sigma,t) + J_t\delta(\sigma) - \frac{1}{\tau_{\rm pl}}P(\sigma,t)H(|\sigma|-\sigma_{\rm c}),$$

$$J_t = \frac{1}{\tau_{\rm fl}}(1-\phi_t),$$ → Rejamming of the (1-ϕ) flowing elements

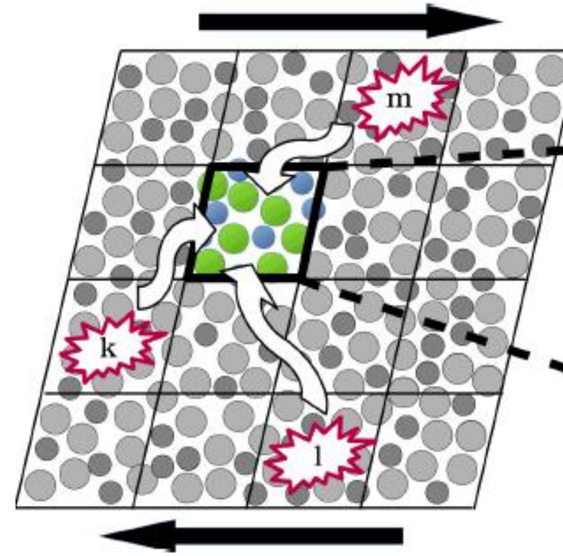
$$\phi_t = \int_{-\infty}^{\infty} \mathrm{d}\sigma P(\sigma,t),$$ → Fraction of jammed elements

$$\Gamma_t = \frac{1}{\tau_{\rm pl}}\int_{|\sigma|\geq\sigma_{\rm c}} \mathrm{d}\sigma|\sigma|P(\sigma,t),$$ → Stress released by the yielding elem.

$$D_t = D_{\rm int} + \alpha\omega\int_{-\infty}^{t} \mathrm{d}s e^{-\omega(t-s)}\Gamma_s.$$ → Noise induced by yielding elements;
smoothed over « retardation time » 1/ω
Feedback coupling: α

The Hébraud-Lequeux Model

$$\dot{P}(\sigma,t) = D_t P''(\sigma,t) - G_0\dot{\gamma}P'(\sigma,t) + J_t\delta(\sigma) - \frac{1}{\tau_{\rm pl}}P(\sigma,t)H(|\sigma|-\sigma_{\rm c}),$$



Remarks:

- « Mean-field » description: global feedback described as a diffusion term, when the Eshelby stress field is very heterogeneous, decaying as r^{-3} (see M. Wyart's talk)

- The original Hebraud-Lequeux model corresponds to $\tau_{fl.} = 0$ (instantaneous re-jamming) and $\omega \rightarrow$ infty (instantaneous transmission of the relaxed stress), but with $\omega\tau_{fl.}$ unspecified

- Still, some crucial assumptions of the model seem to be validated by numerical simulations on realistic systems (Puosi, Olivier, Martens, 2015)

The phase diagram

$$\dot{P}(\sigma,t) = D_t P''(\sigma,t) - G_0\dot{\gamma}P'(\sigma,t)$$
$$+ J_t\delta(\sigma) - \frac{1}{\tau_{\mathrm{pl}}}P(\sigma,t)H(|\sigma|-\sigma_{\mathrm{c}}),$$

$$J_t = \frac{1}{\tau_{\mathrm{fl}}}(1-\phi_t),$$

$$\phi_t = \int_{-\infty}^{\infty} \mathrm{d}\sigma P(\sigma,t),$$

$$\Gamma_t = \frac{1}{\tau_{\mathrm{pl}}}\int_{|\sigma|\geq\sigma_{\mathrm{c}}} \mathrm{d}\sigma|\sigma|P(\sigma,t),$$

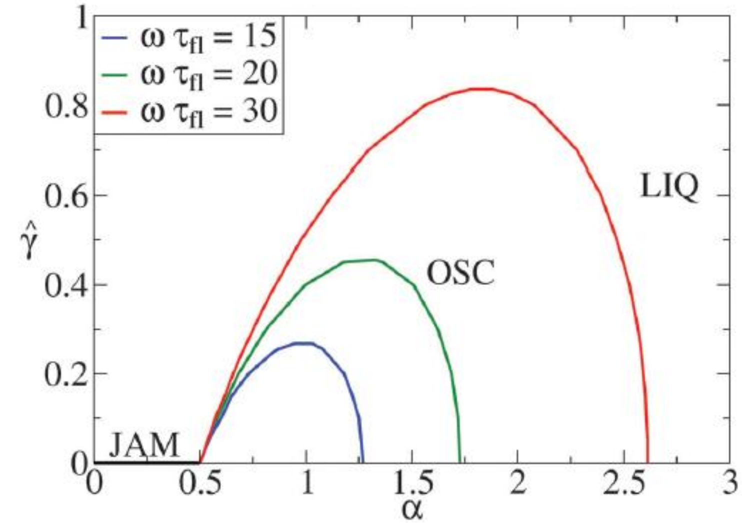
$$D_t = D_{\mathrm{int}} + \alpha\omega\int_{-\infty}^{t} \mathrm{d}s e^{-\omega(t-s)}\Gamma_s.$$

Analytical strategy:

- Find the stationary state of the model with a (small) strain rate
- Compute the corresponding average stress

$$\left.\begin{array}{l}\rightarrow \text{A jamming transition for } \alpha < \tfrac{1}{2} \text{ (i.e. when } D_{\mathrm{int}} \rightarrow 0\text{, activity stops)} \\ \rightarrow \text{Herschel-Bulkley rheology in the jammed phase}\end{array}\right\} \text{as in HL}$$

- Determine the (exact) stability region of the above stationary state



SYNCHRONISATION TRANSITION & PHASE DIAGRAM

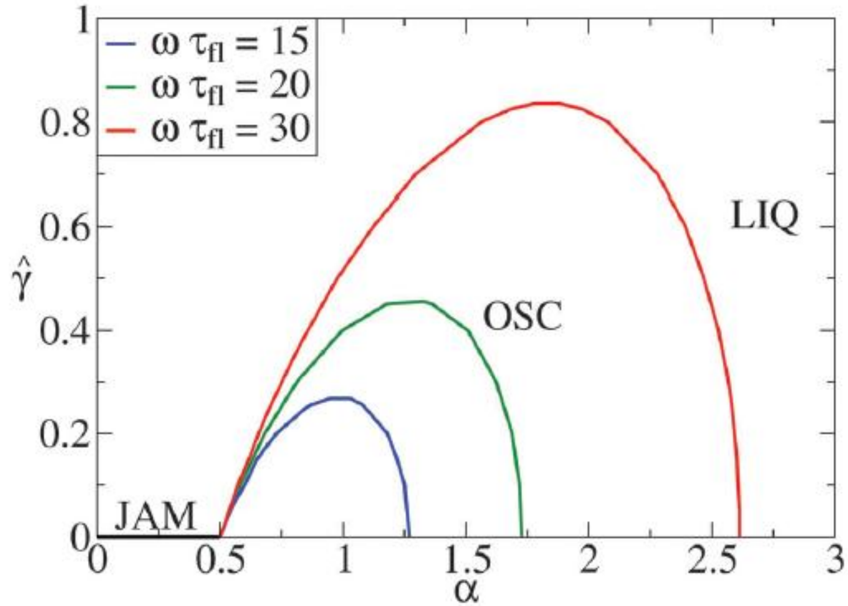
Exact equation for λ (max growth rate of perturbations)

$$\frac{\hat{\lambda}}{\hat{\omega}}\left(1-\frac{\hat{\omega}-1}{\hat{\lambda}+1}\right)=\frac{-\zeta}{e^{-\zeta}+1}\frac{(e^{-\kappa_-}-1)(e^{-\kappa_+}-1)}{\kappa_+\kappa_-}\times\frac{\kappa_+\sinh\kappa_- - \kappa_-\sinh\kappa_+}{\sinh(\kappa_-)-\sinh(\kappa_+)}.$$

$$D_0\kappa^2 - \dot{\gamma}\kappa - \lambda = 0$$

$$\zeta = \dot{\gamma}\sigma_c/D_0$$

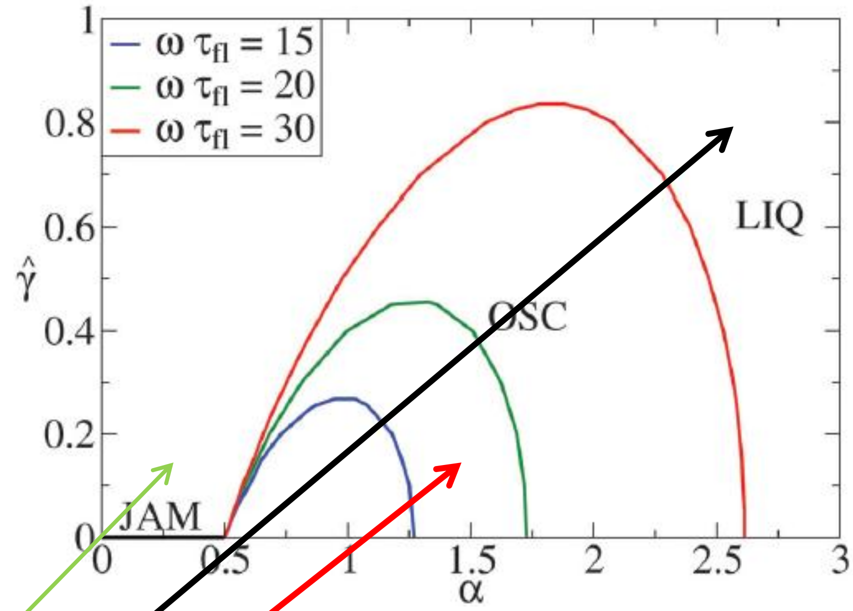
The phase diagram



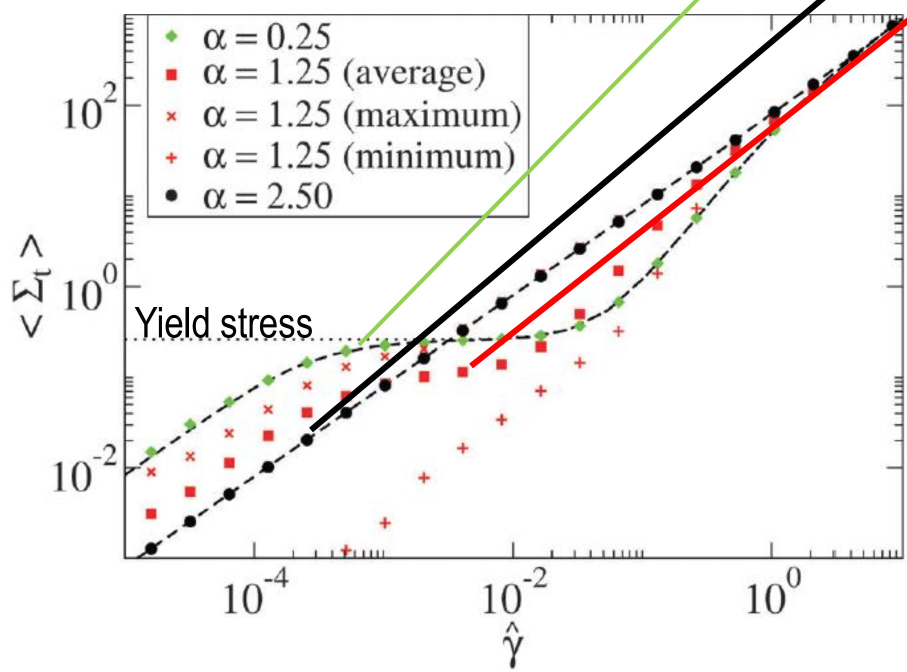
Results:

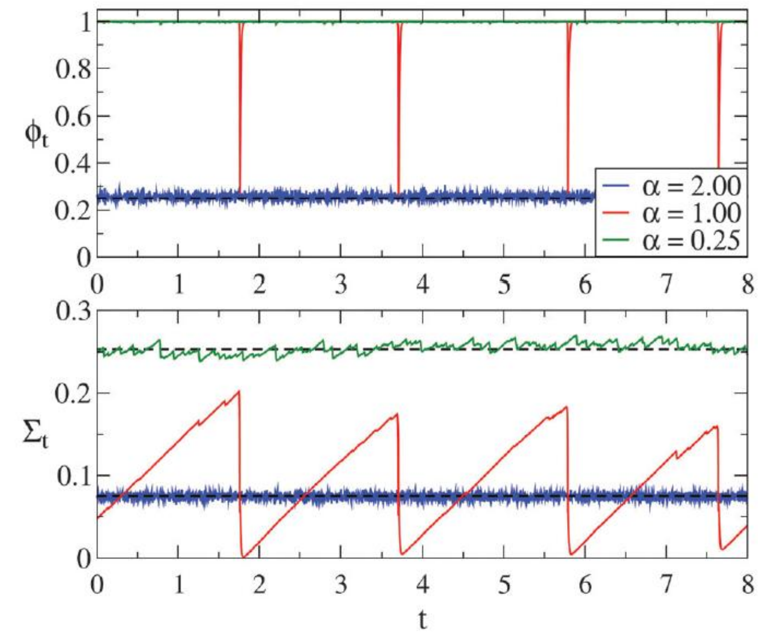
- Existence of a synchronization transition (« stick-slip ») when $\tau_{pl.} < C(\alpha,\omega,\gamma)\ \tau_{fl.}$ (*impossible in the HL limit* where $\tau_{fl.}=0$!)
- Size of the « OSC » phase expands as $\omega\tau_{fl.}$ increases
- Exact analytical solution, very similar to the « bankruptcy » model
- Transition induced by yielding avalanches

The phase diagram



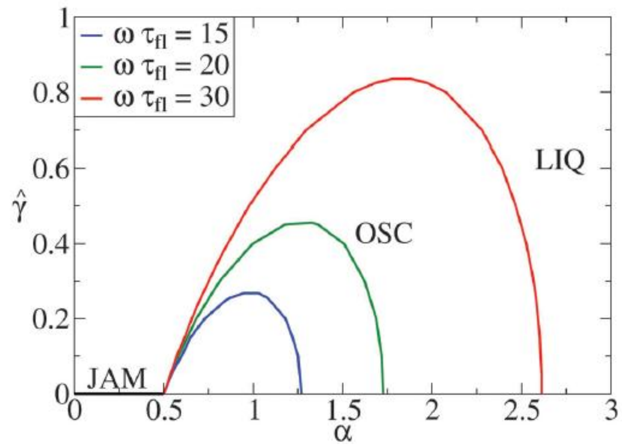
Results:

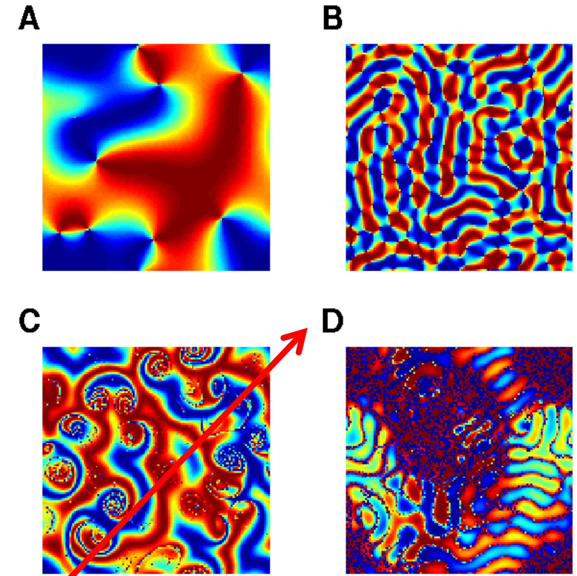




Fraction of jammed elements and stress
Numerical simulation with 10,000 elements

The phase diagram



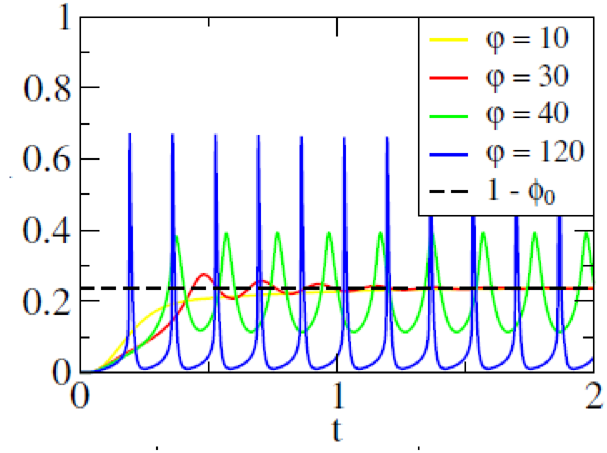


Further remarks:

- The phase diagram is robust to many other changes, i.e. reinitialisation at non zero stress, random thresholds Θ, etc.
- However, the power-law decay of the Eshelby influence kernel induces a power-law distribution of « kick » sizes triggered by yielding events
- This changes the diffusion term in the HL model into a Lévy-like kernel (M. Wyart) – does this kill the synchronisation transition ?
- More generally, finite dimension/local effects induce new phenomena (shear banding?) -- see e.g. the complex space-time dynamics in the Kuramoto model

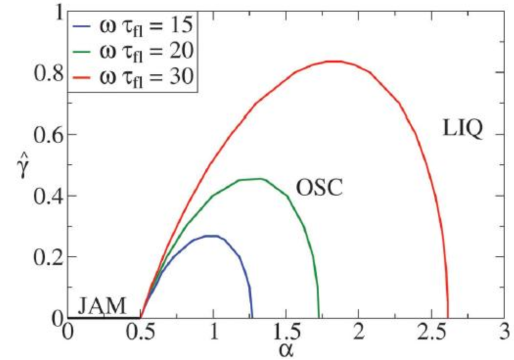
CONCLUSION/EXTENSIONS

The Synchronisation effect



- The synchronisation transition is well known to those who know it well
- But it is <u>not</u> universally known (in particular not in economics circles)
- Its robustness is remarkable and makes its manifestation somewhat surprising (note the resilience to all sorts of noise)
- It tooks us time to realize that our 1d model of « Mark 0 » could synchronize and that the physics was that of the Kuramoto model (anticipated by Winfree 1967, see also Strogatz)

The Hébraud-Lequeux model



- A remarkably simple mean-field model that captures the feedback of yielding events/STZ on the rest of the system (but also bankruptcies, defaults, illnesses, etc.)
- The HL model has a well-defined jamming transition where activity cannot be self-consistently sustained
- The HL model also has a synchronisation transition that corresponds to intermittent flow/stick-slip motion, as often observed experimentally
- Other phenomenological models of soft-glassy rheology predict such oscillatory instabilities, but the underlying microscopic feedback mechanism is less clear (at least to my eyes)

Open issues

- Non mean-field, spatial effects – can synchronisation still occur with power-law decaying elastic coupling?

- Estimation of the various time scales in the extended HL model $\tau_{pl.}, \omega, \tau_{fl.}$

- Direct signature of yielding events/STZ?

→ For a dipolar force (local collapse): anomalous decay of the dynamic structure factor as $\mathbf{S(q,t) = exp\ (-A\ (qt)^{3/2})}$ (Cipelletti et al.; JPB, E. Pitard)
→ Generalisation to quadrupolar STZ: $\mathbf{S(q,t) = exp\ (-B\ (qt))}$ where B depends on $\tau_{fl.}$

- J.-P. Bouchaud, *Crises and collective socio-economic phenomena: Simple models and challenges*, J. Stat. Phys. 151, 567–606 (2013).
- S. Gualdi, M. Tarzia, F. Zamponi, J.-P. Bouchaud, *Tipping points in macroeconomic agent-based models*, Journal of Economic Dynamics and Control (2014)
- S. Gualdi, M. Tarzia, F. Zamponi, J.-P. Bouchaud, *Monetary Policy and Dark Corners in a stylized Agent Based Model,* J Econ Interact Coord (2016)

SOME REFERENCES

- S. Gualdi, J.-P. Bouchaud, G. Cencetti, M. Tarzia and F. Zamponi, "*Endogenous crisis waves: a stochastic model with synchronized collective behavior*", Phys.Rev.Lett. 114, 088701 (2015)
- J.-P. Bouchaud, S. Gualdi, M. Tarzia and F. Zamponi, "*Spontaneous Instabilities and Stick-Slip motion in a generalized Hébraud-Lequeux model*", Soft Matter, 12, 1230 (2016)