Multi-Objective Optimisation Techniques in Reservoir Simulation

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Outline

- Introduction
- Stochastic Optimisation
- Model Calibration
- Forecasting
- Reservoir Optimisation
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Mathematics of Flow in Porous Media

- Conservation of Mass
- Conservation of Momentum
  - replaced by Darcy's law

$$\mathbf{v} = -\frac{k(\mathbf{x})}{\mu}\nabla p$$

- Conservation of Energy
  - most processes isothermal
- Equation of State

Equations governing flow

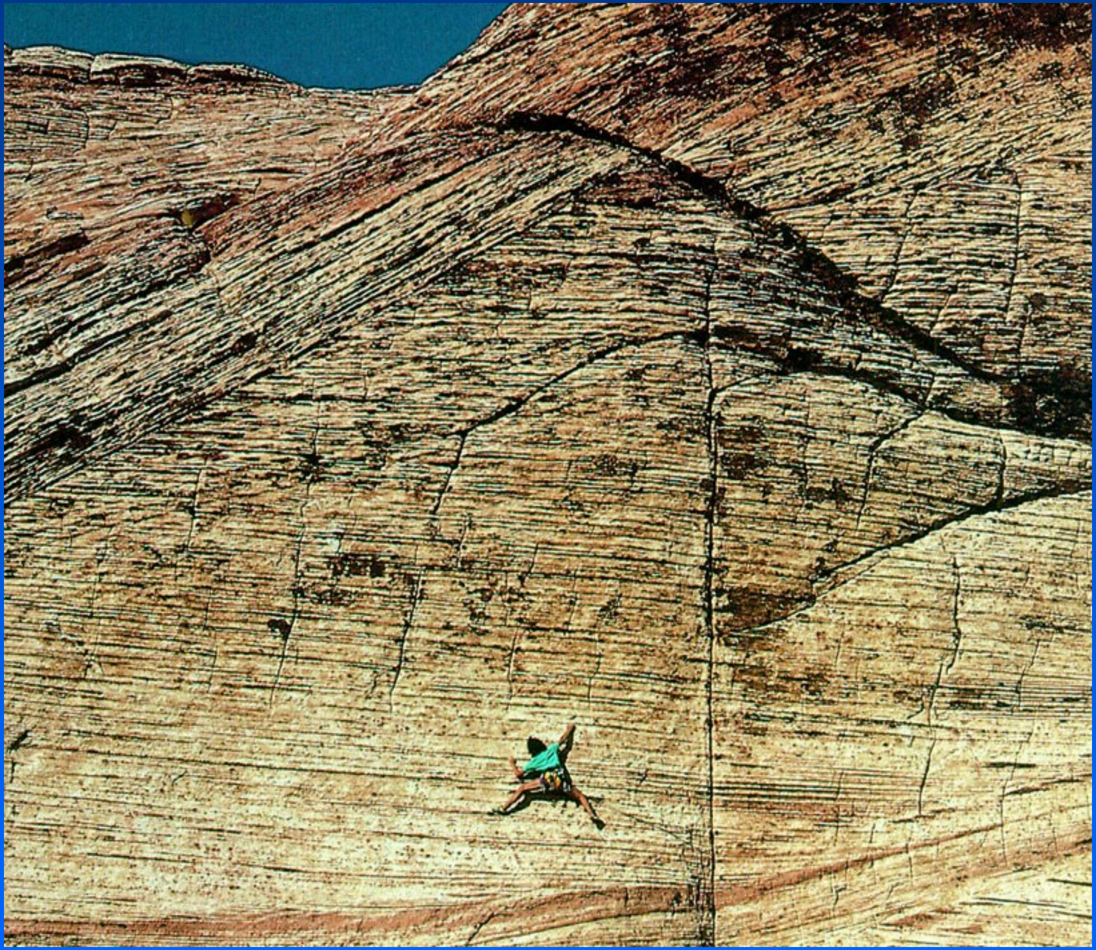
- Parabolic equation for pressure

$$c\frac{\partial p}{\partial t} = \nabla.\left(\boxed{k(\mathbf{x})}\left(\frac{k_{ro}(S)}{\mu_o} + \frac{k_{rw}(S)}{\mu_w}\right)\nabla p\right)$$

- Hyperbolic equation for saturation

$$\boxed{\phi(\mathbf{x})}\frac{\partial\left(\rho_o x_i S_o + \rho_g y_i S_g\right)}{\partial t} + \nabla.\left(\rho_o x_i \mathbf{v}_o + \rho_g y_i \mathbf{v}_g\right) = 0$$

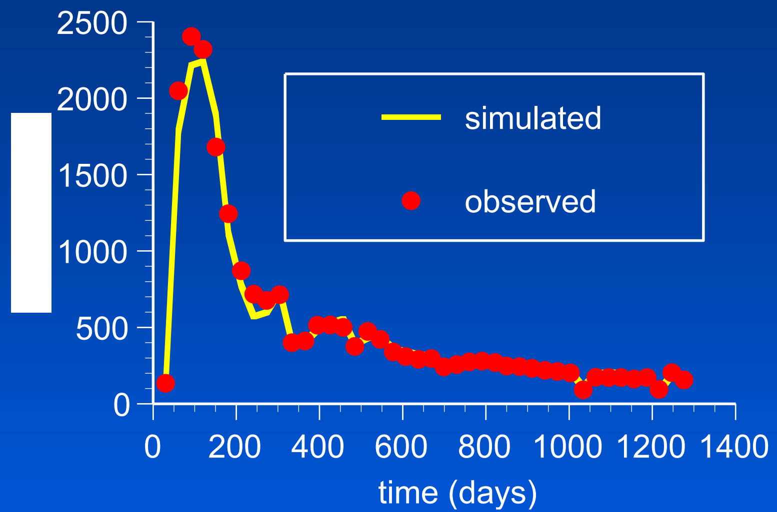
Data Collection

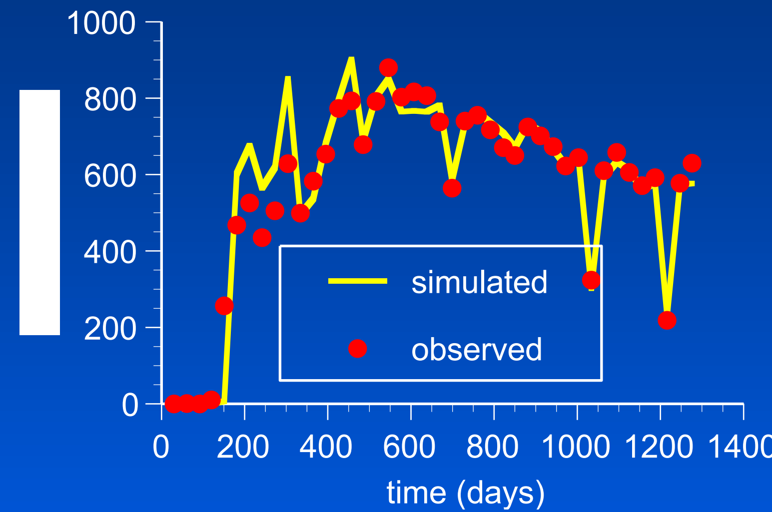


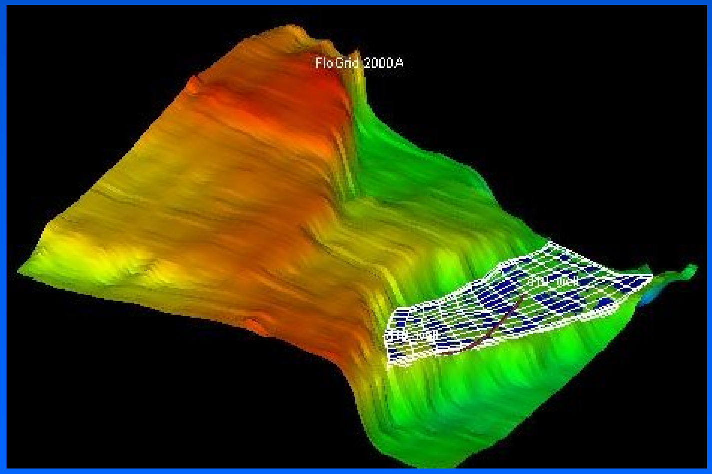
$$\phi(\mathbf{x}) = ?$$

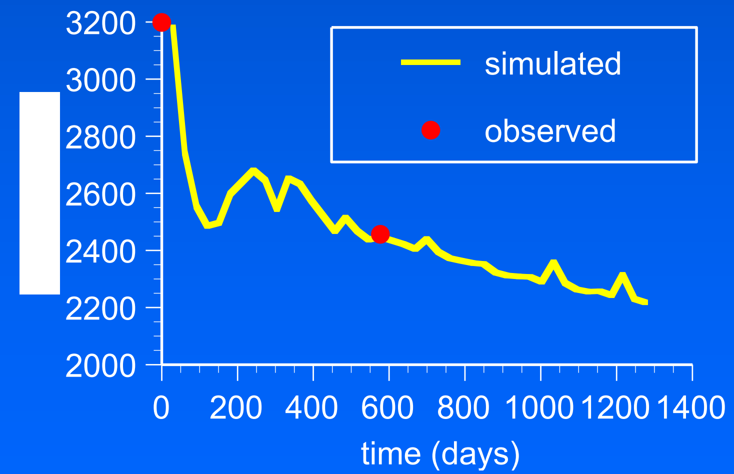
$$k(\mathbf{x}) = ?$$

Model Calibration: Teal South

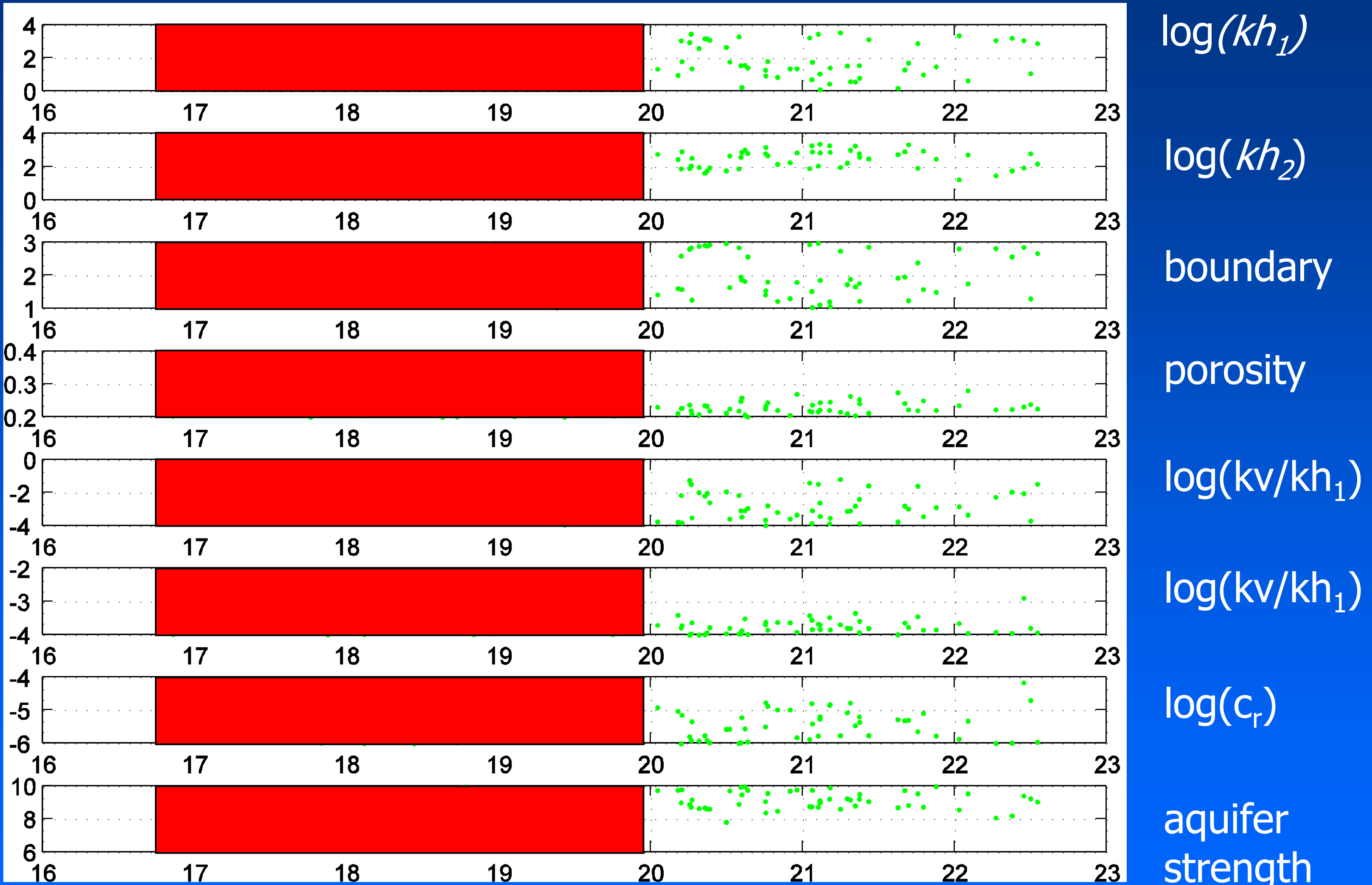








Range of Possible Values for Unknown Parameters



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Particle Swam Optimization (PSO)

- A swarm intellegence algorithm (Kennedy & Eberhart,1995).
- Particles are points in parameter space.
- Particles move based on their own experience and that of the swarm.
- PSO equations

$$v_i^{k+1} = \omega v_i^k + c_1 r_1 \left(p_i^{best} - x_i^k \right) + c_2 r_2 \left(g_{best}^k - x_i^k \right)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1}$$

  - r_1, r_2 are random vectors
  - ω is the inertial weight
  - c_1, c_2 are the cognition and social acceleration components

Dominance and Pareto Optimality

Solution $\mathbf{x}_1$ dominates solution $\mathbf{x}_2$, if :

1. $\mathbf{x}_1$ is no worse than $\mathbf{x}_2$ in all objectives, and
2. $\mathbf{x}_1$ is strictly better than $\mathbf{x}_2$ in at least one objective

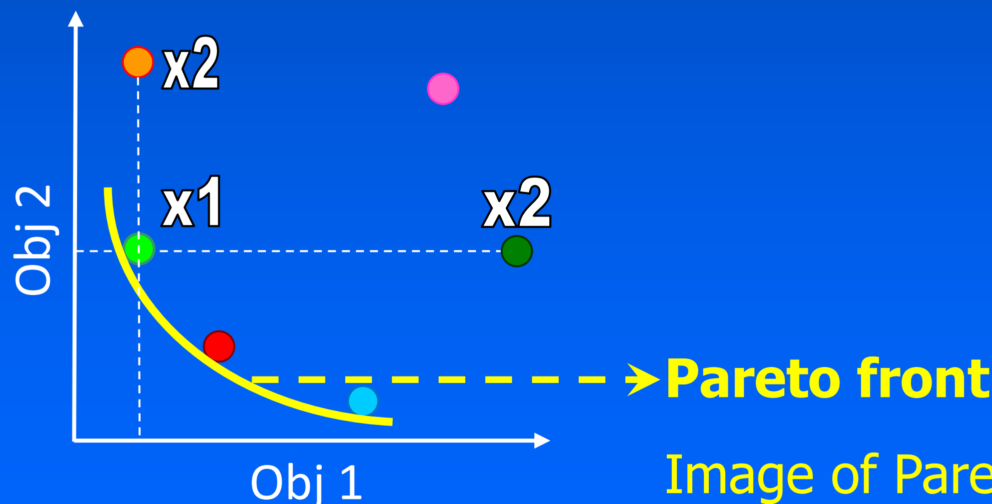


Image of Pareto optimal set in objective space

MO Particle Swam Optimization (PSO)

- MOPSO equations

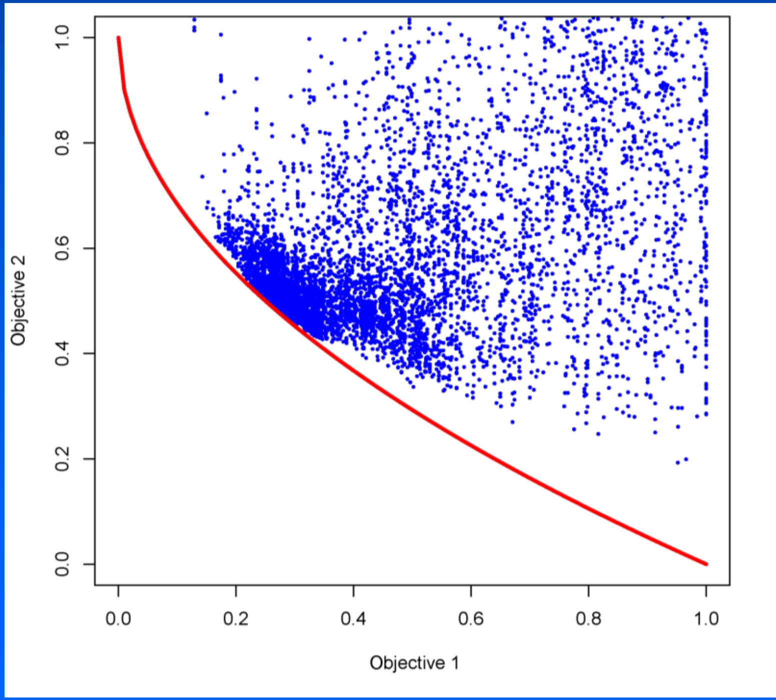
$$v_i^{k+1} = \omega v_i^k + c_1 r_1 \left(\boxed{p_i^{best}} - x_i^k \right) + c_2 r_2 \left(\boxed{g_{best}^k} - x_i^k \right)$$

$$x_i^{k+1} = x_i^k + v_i^{k+1}$$

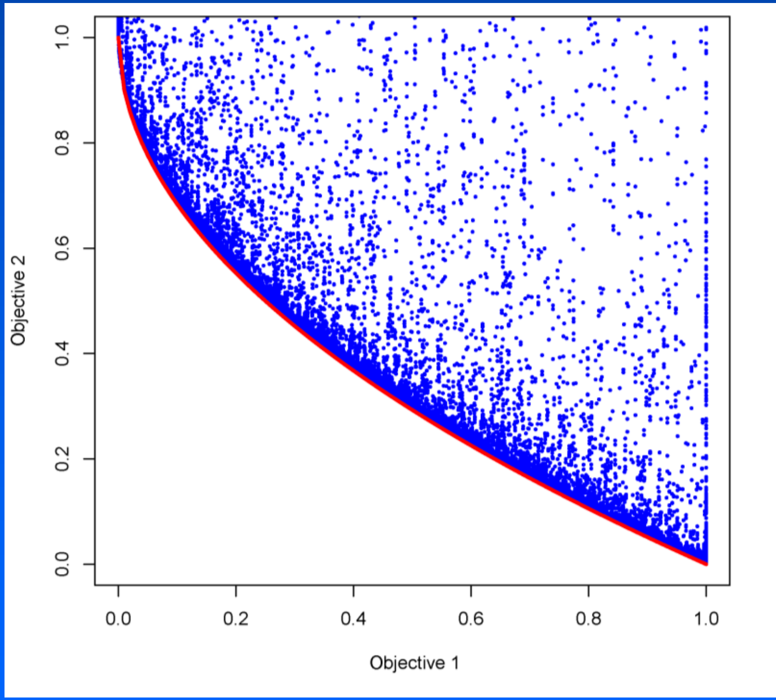
  - r_1, r_2 are random vectors
  - ω is the inertial weight
  - c_1, c_2 are the cognition and social acceleration components
- Pbest and Gbest now sampled from Pareto archive

Why Use Multi-Objective?

- Sum of objectives – limited exploration of Pareto front
  - Example minimising two objectives



Objective sum



Multi-Objective

Figure from Hajizadeh, PhD Thesis (Chapter 6, Figure12), Heriot-Watt, 2011

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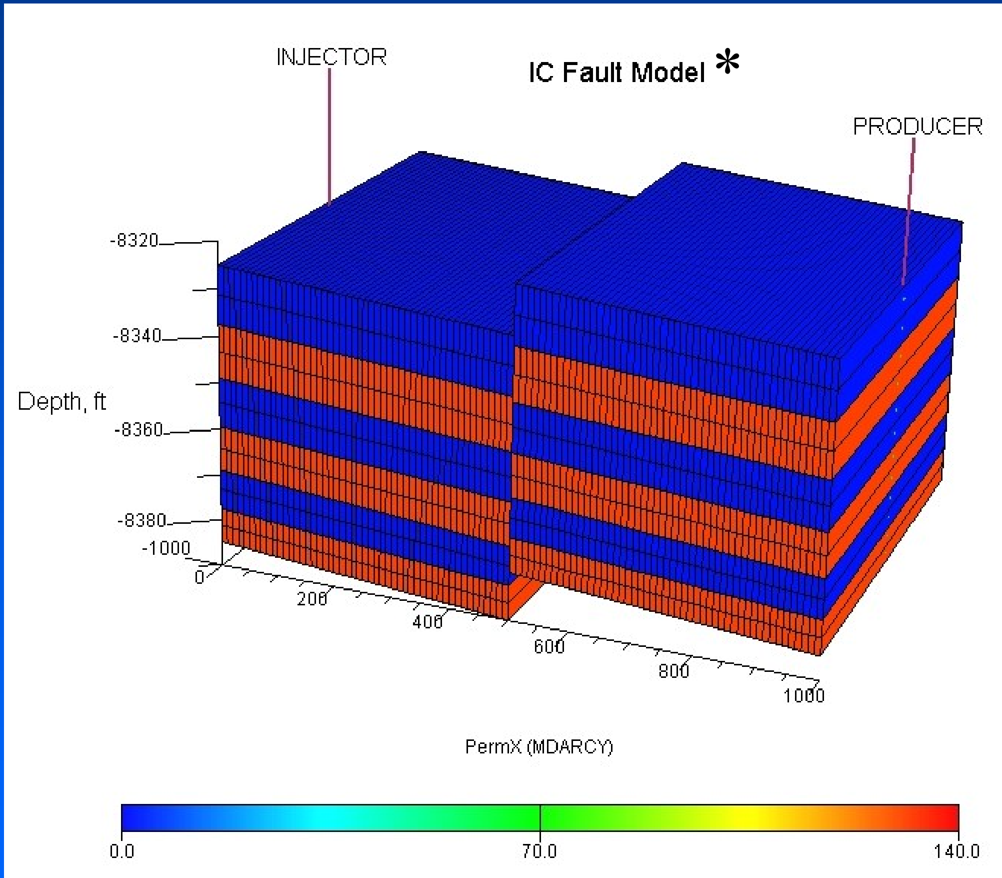
Model Calibration

- Called ‘History Matching’ in oil business
- Synthetic example
  - IC Fault Model
- Real example
  - Zagadka Field

IC Fault Model

- Synthetic 2D Model
- 2 Wells: 1 Inj, 1 Prod
- 6 Layers
- 1,3,5 Poor Sand (blue)
- 2,4,6 Good Sand (red)
- 1 Fault

- 3 Uncertain Inputs:
  1. k_{high} = [100,200] mD
  2. k_{low} = [0,50] mD
  3. throw = [0,60] ft



* Data from Z. Tavassoli, Jonathan N. Carter, and Peter R. King, Imperial College, London

IC Fault Model

Observed

k_{high} = 131.6 mD
k_{low} = 1.3 mD
throw = 10.4 ft

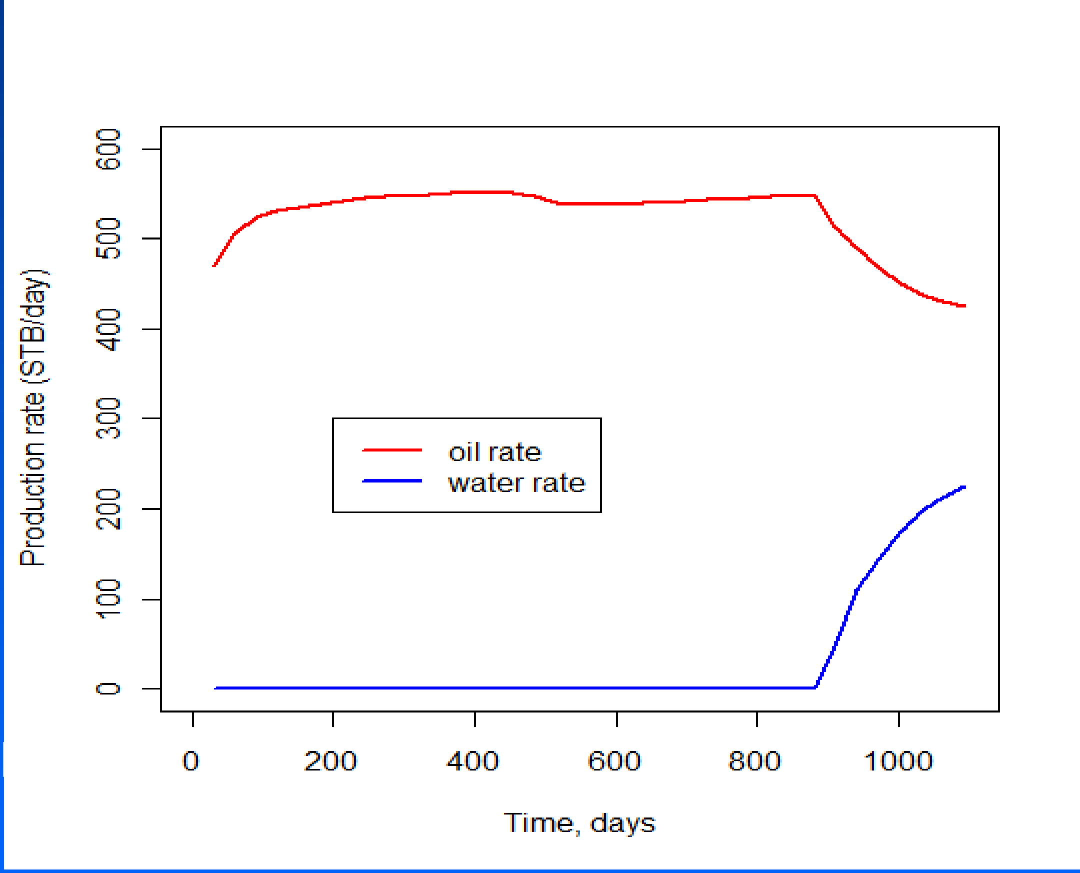
Misfit Definition:

$$M = \frac{1}{n}\sum_{p}\sum_{t}\frac{(sim - obs)^2}{2\sigma^2}$$

$$\sigma = 0.03 \times obs$$

p : oil/water rates

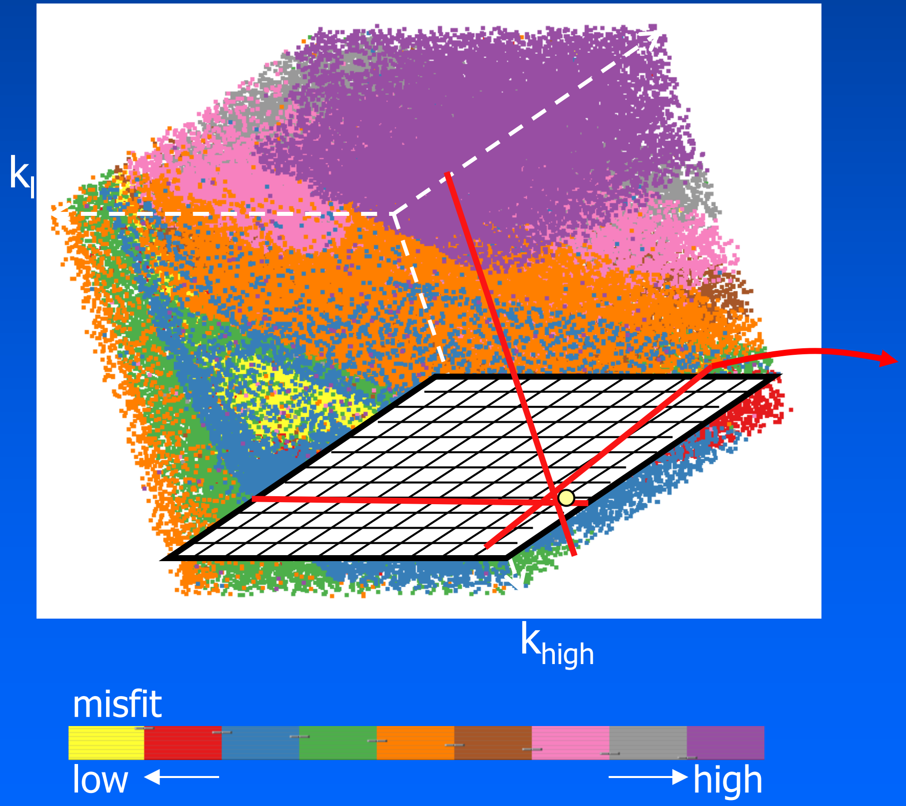
Truth Profile (Observed)

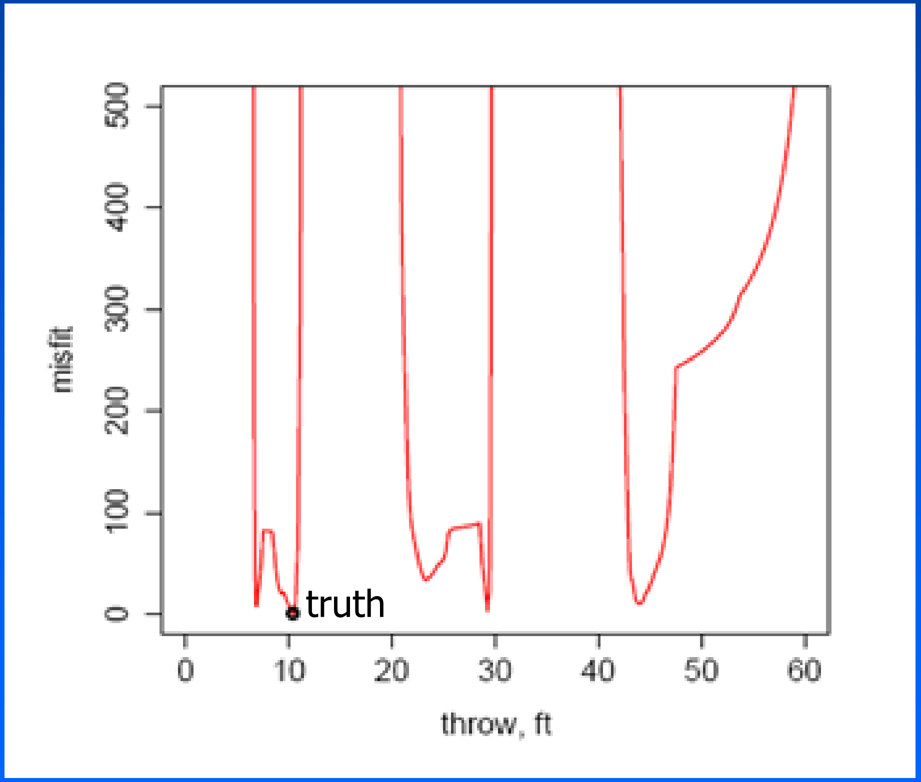


• Simulator controlled to match Bl
the wells

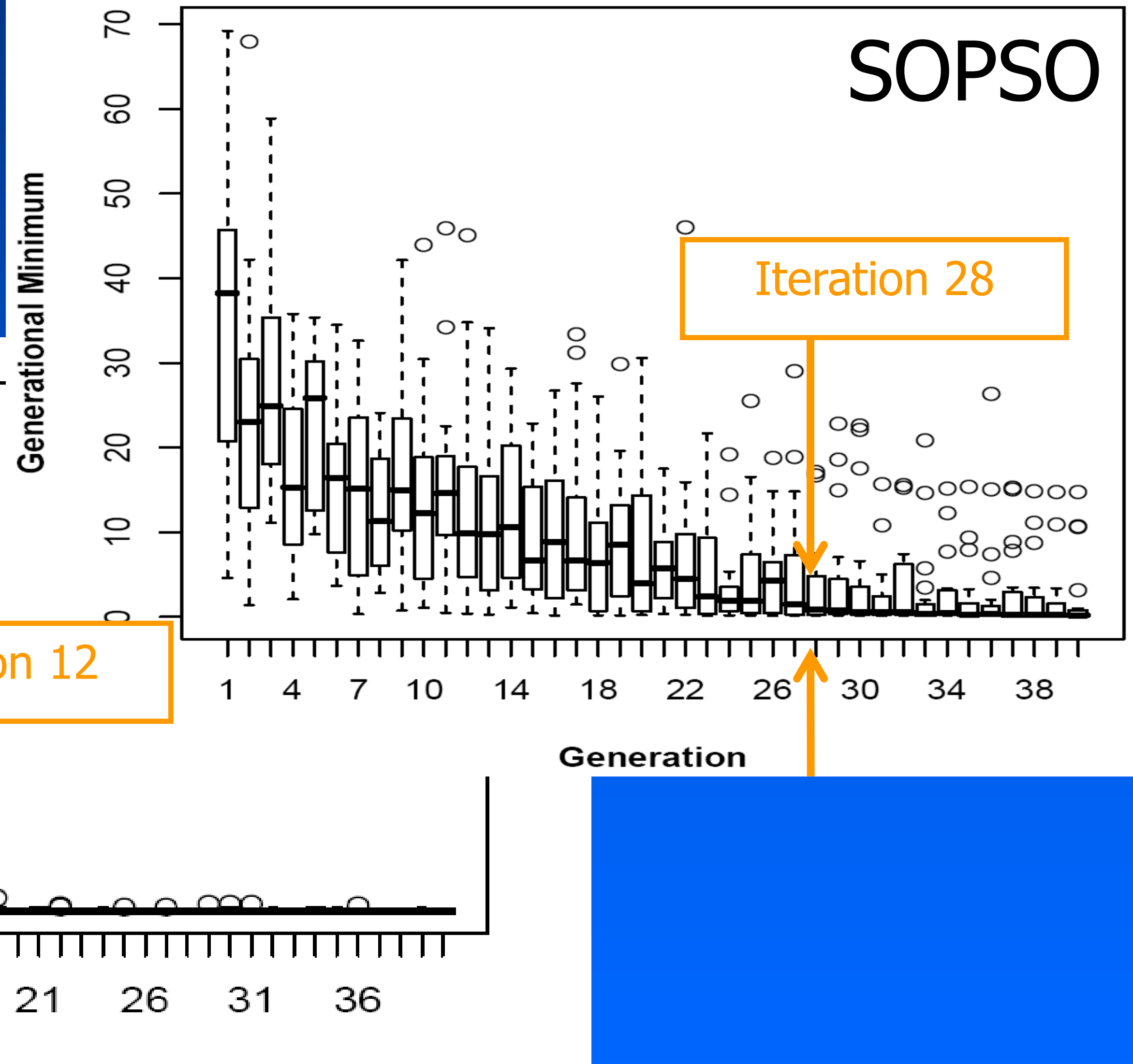
IC Fault Model Misfit Surface

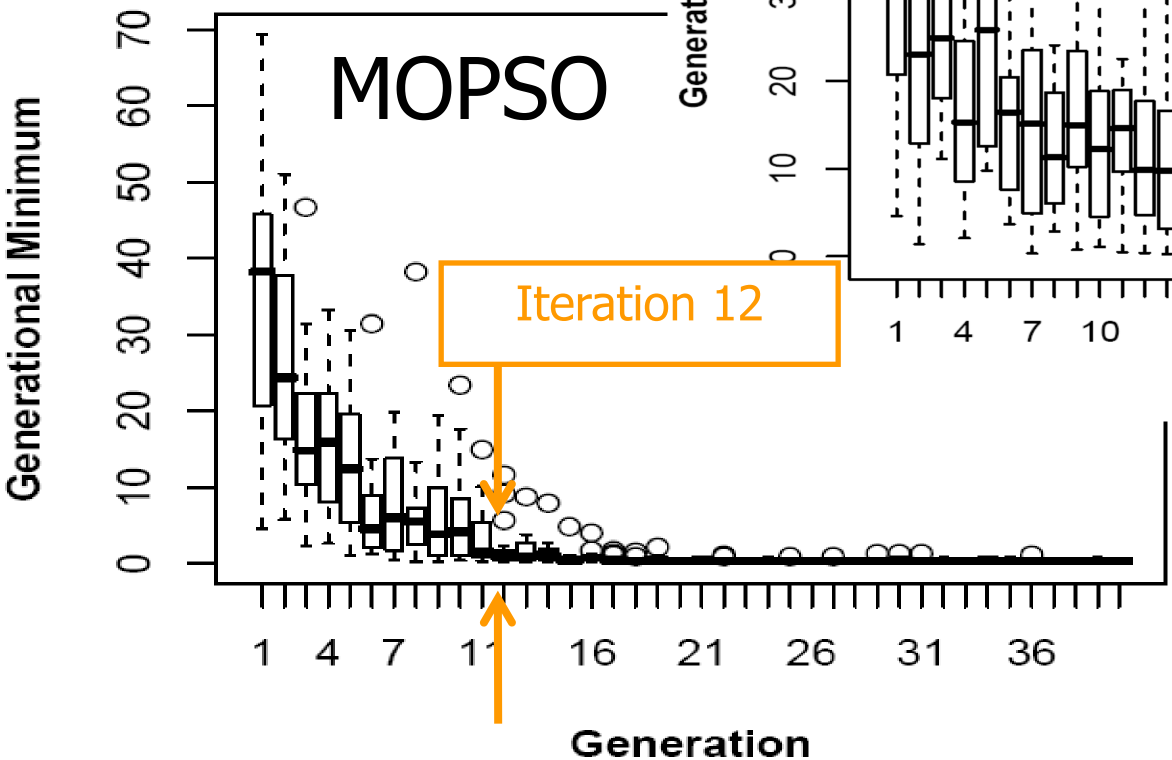
Database (DB)
159,661 uniformly generated models





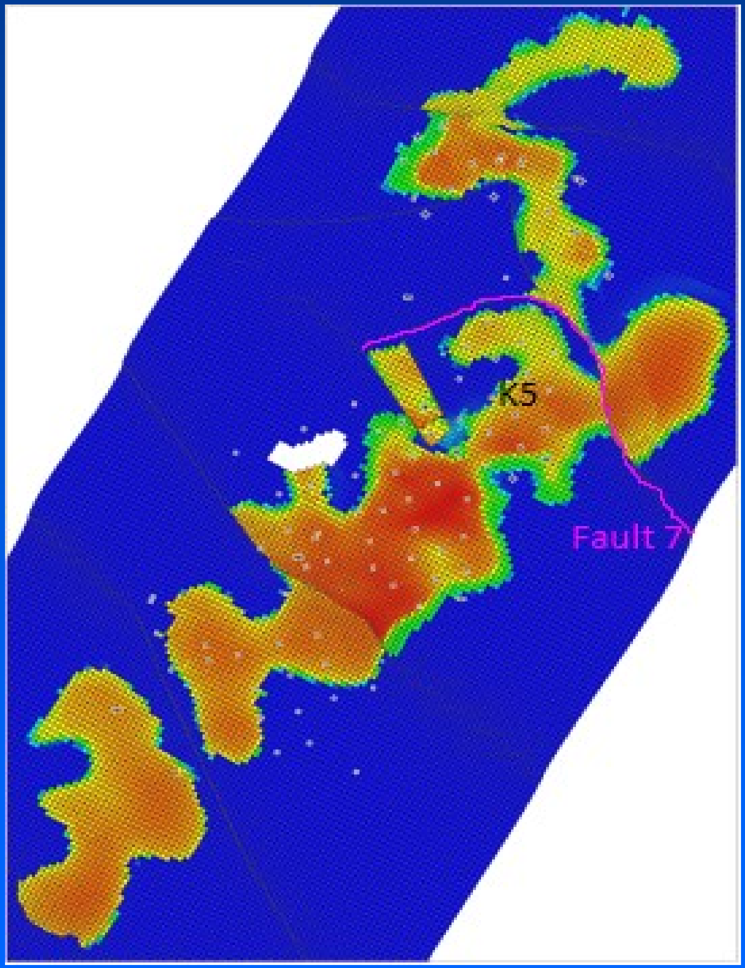
Convergence Speed – 20 Runs



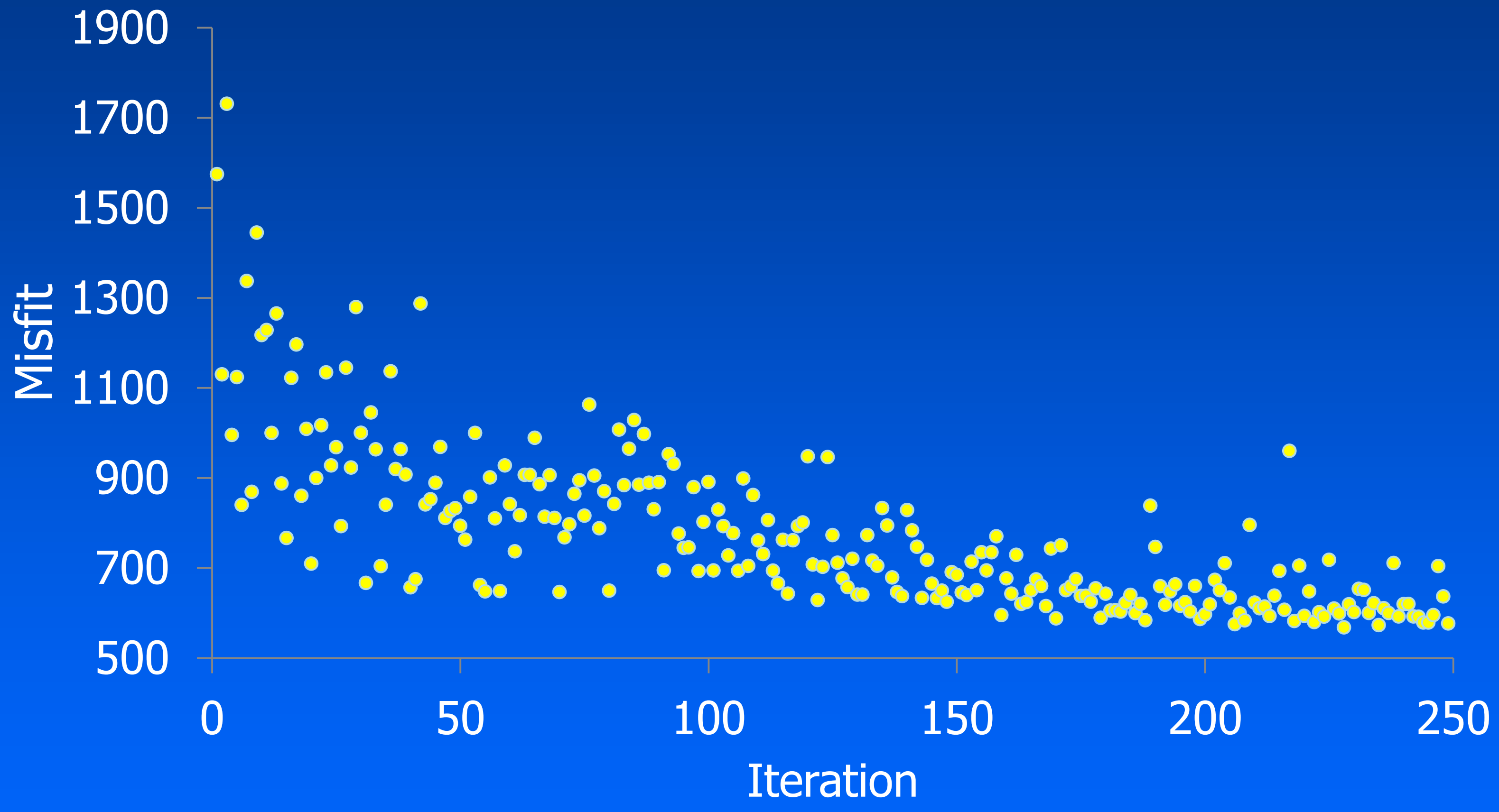


Zagadka Field

- Waterflood/aquifer support
- 95 wells in 10 groups
- 15 years history
- Compartmentalized

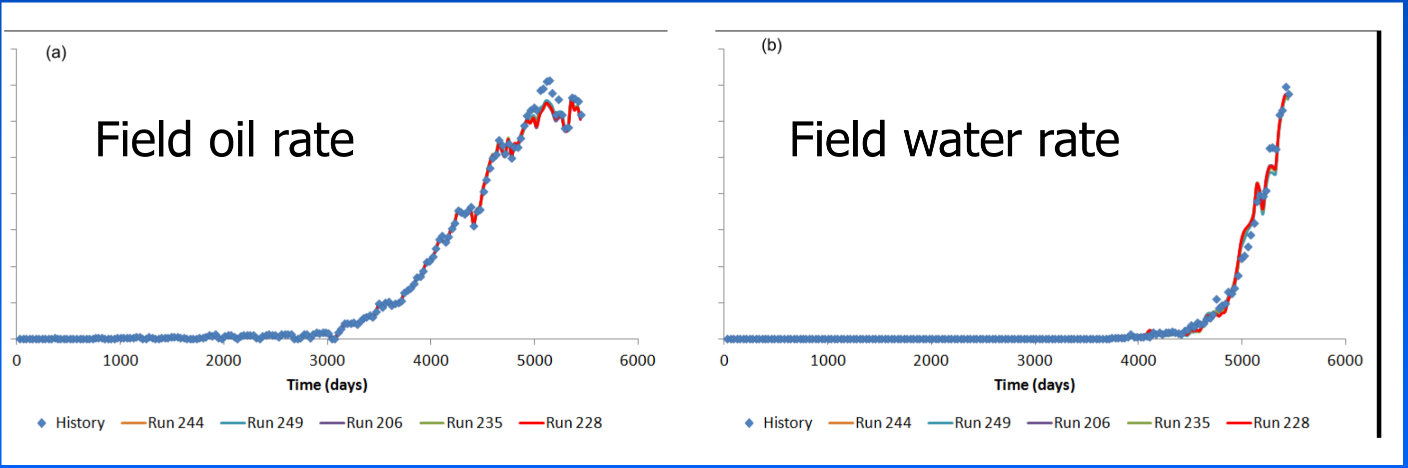


Model 4 Convergence (PSO)

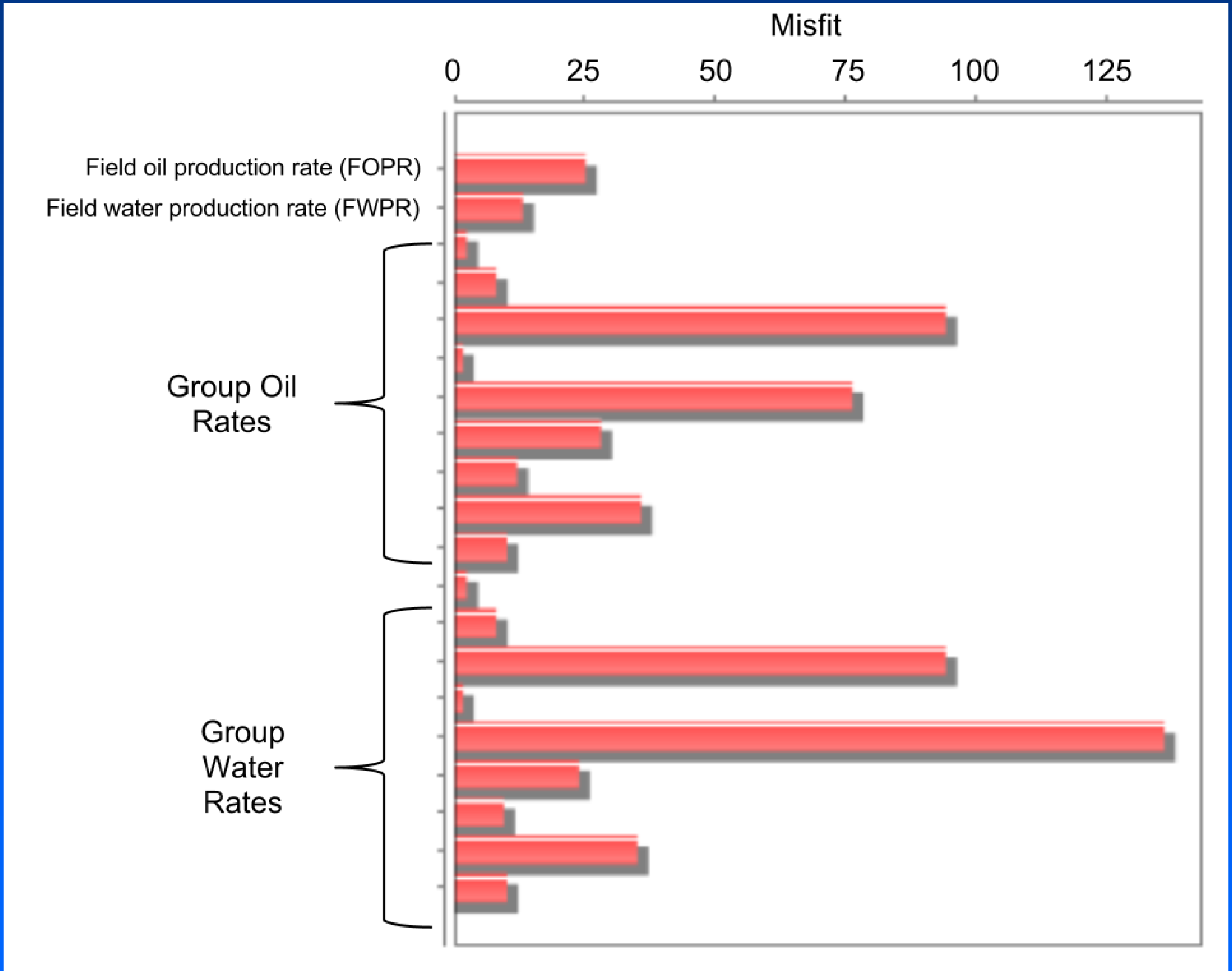


Model 4 Field Level History Match

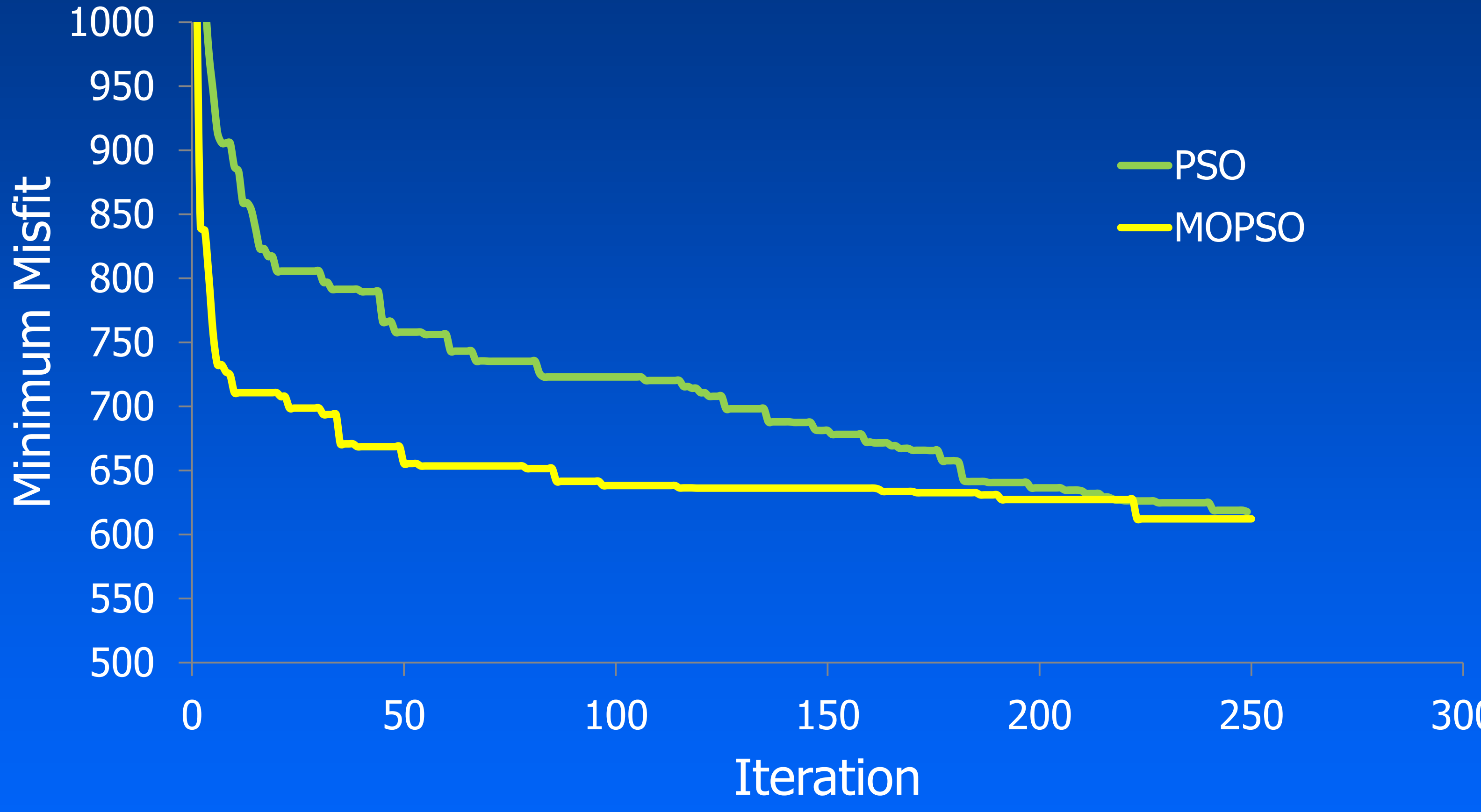
- Match with PSO (single objective)
- 250 simulations (overnight, 12 core workstation)



Match on Group Rates – Single Objective

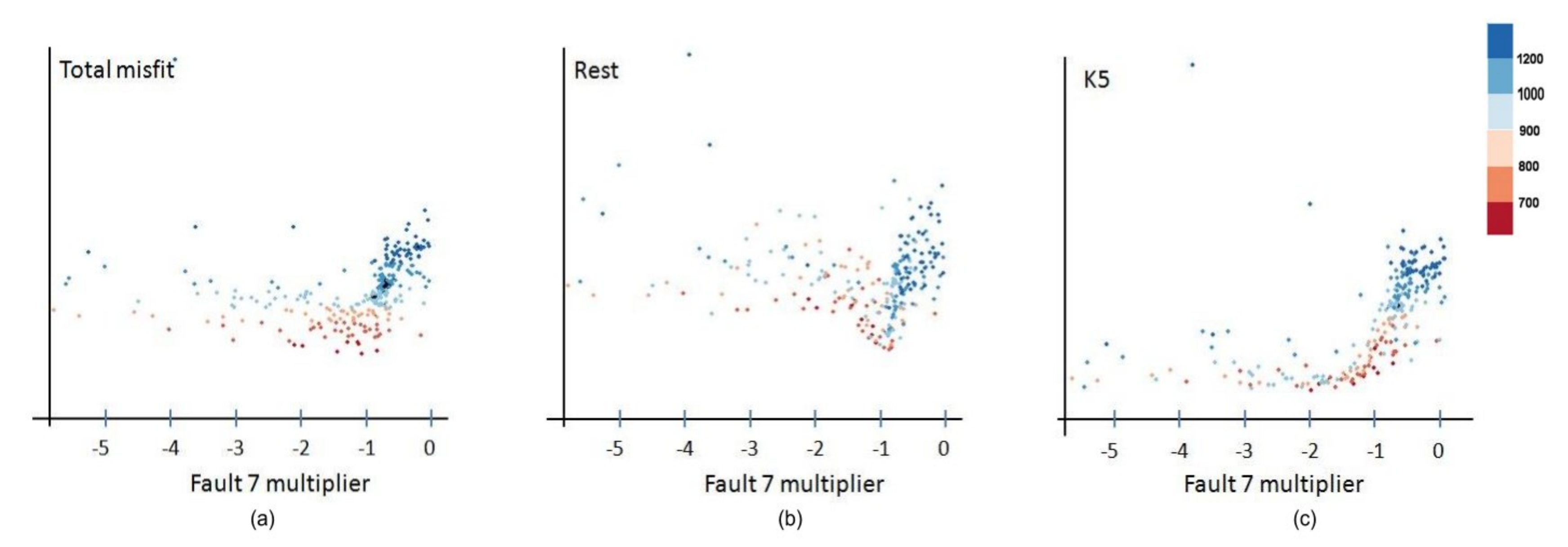


Multi-Objective vs Single Objective



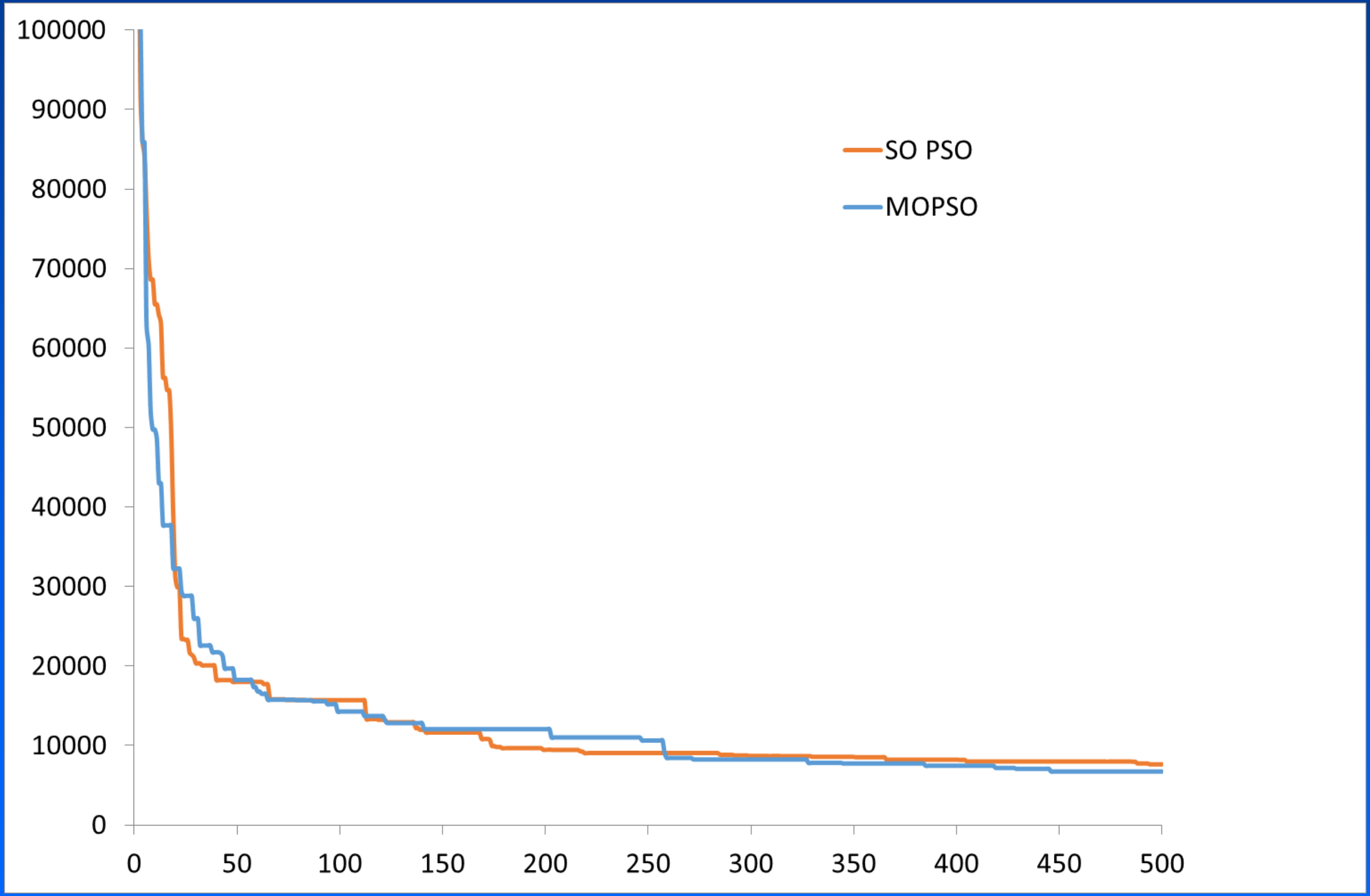
Multi-Objective Trade Off

- MO balances changes to fit one quantity vs another

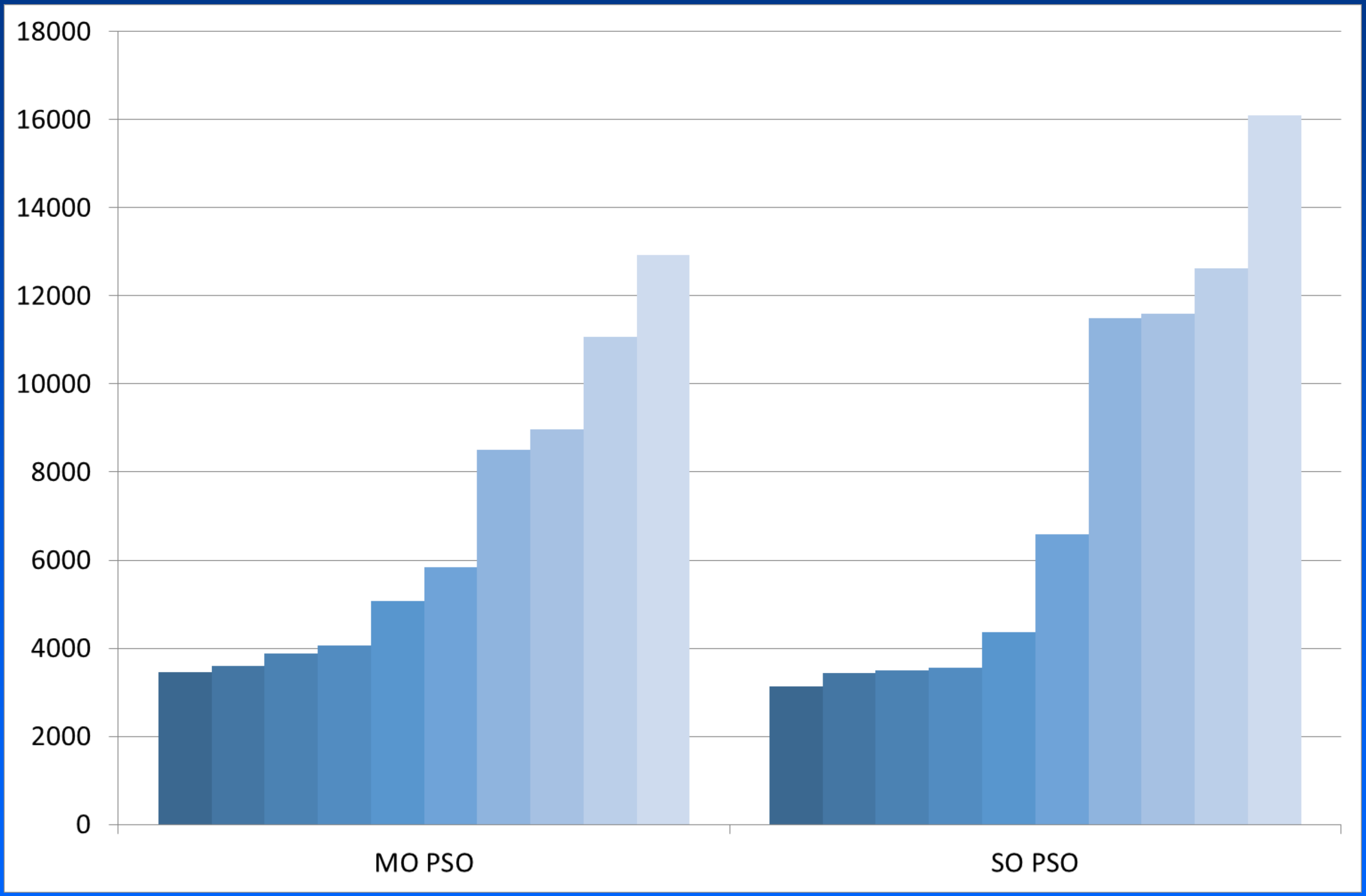


MO Not Always Faster than SO

- Study on EoN field



Histogram of 10 Final Misfits



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Forecasting with MO

- Generic Bayesian integral

$$J = \int g(\mathbf{m}) P(\mathbf{m}) d\mathbf{m}$$

$$\text{eg} \quad \text{mean oil rate} = \int Q_o(\mathbf{m}) P(\mathbf{m}) d\mathbf{m}$$

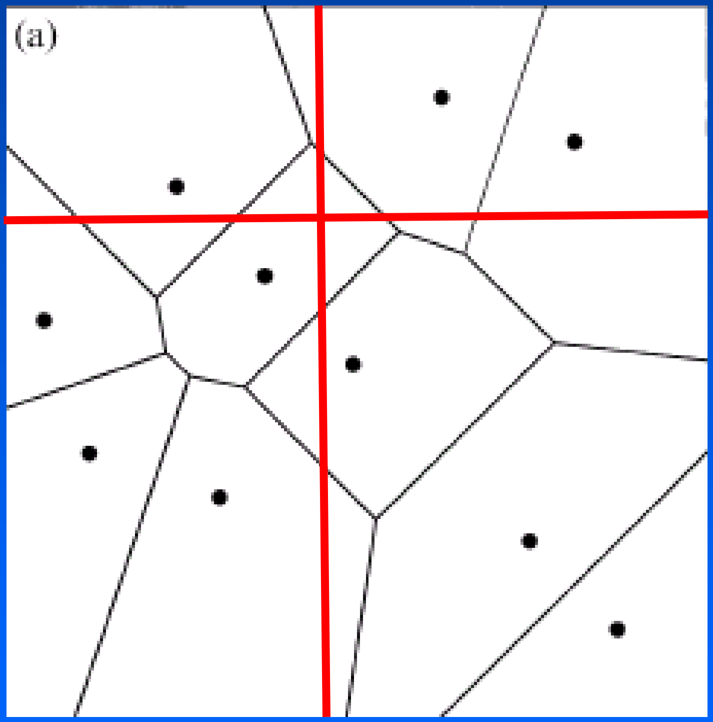
- MC approximation

$$J = \frac{1}{N} \sum_{k=1}^{N} \frac{g(\mathbf{m}_k) P(\mathbf{m}_k)}{h(\mathbf{m}_k)} = \frac{1}{N} \sum_{k=1}^{N} g(\mathbf{m}_k) P(\mathbf{m}_k) dV_k$$

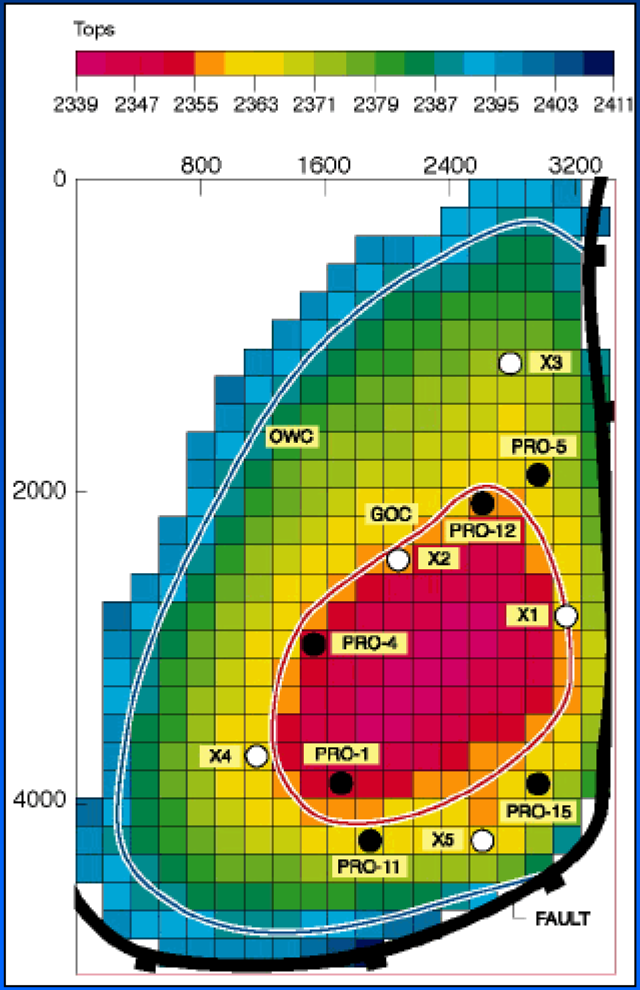
- Resample to calculate $P(\mathbf{m}_k) dV_k$

Calculating Weights for Sample

- Gibbs sampler
  - Assume misfit constant in Voronoi cell



Single & Multi Objective HM for PUNQ-S3



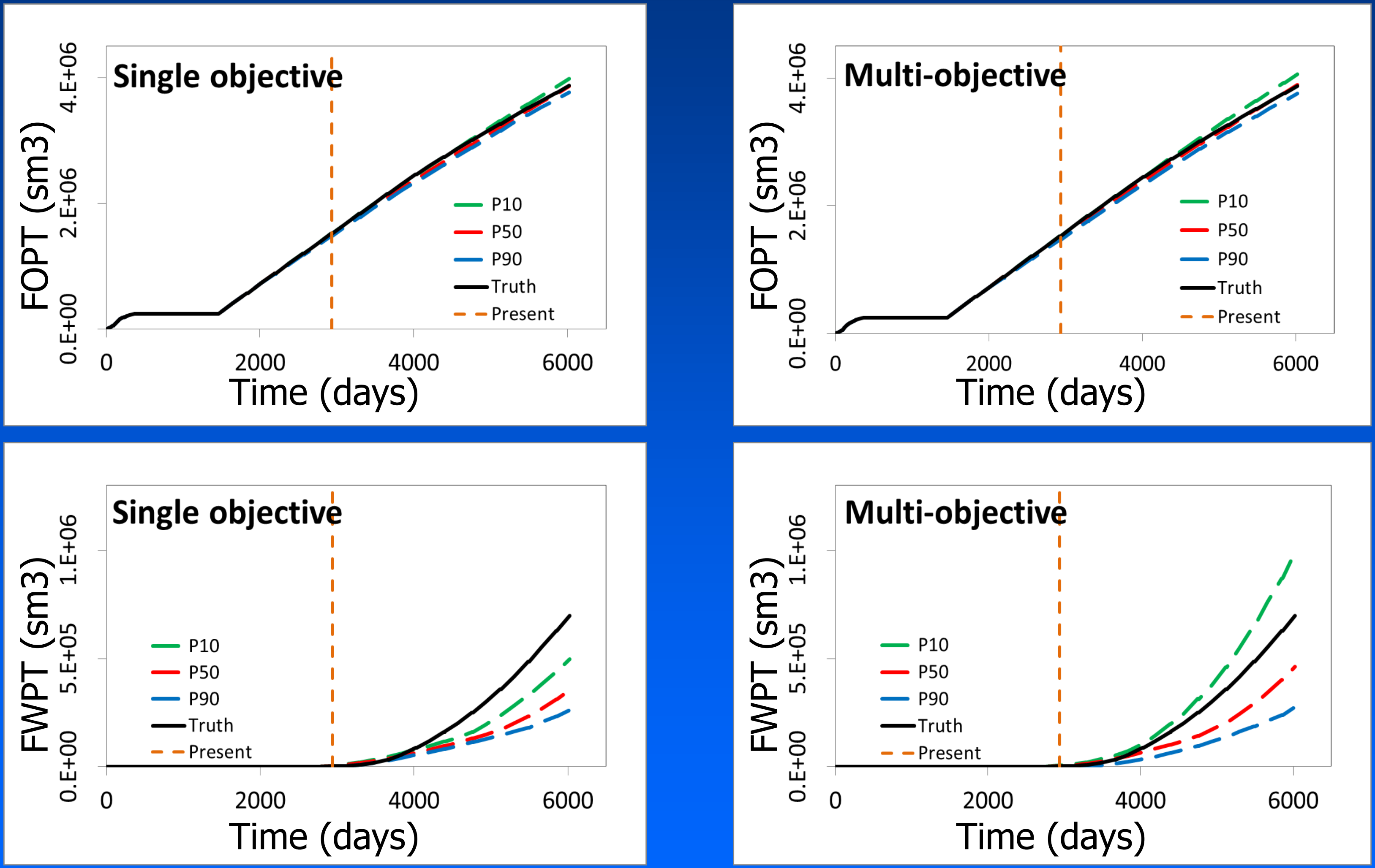
Based on real field

An industry standard benchmark

Multiple wells and production variables

Algorithm used: PSO

SO vs. MO – Forecasting



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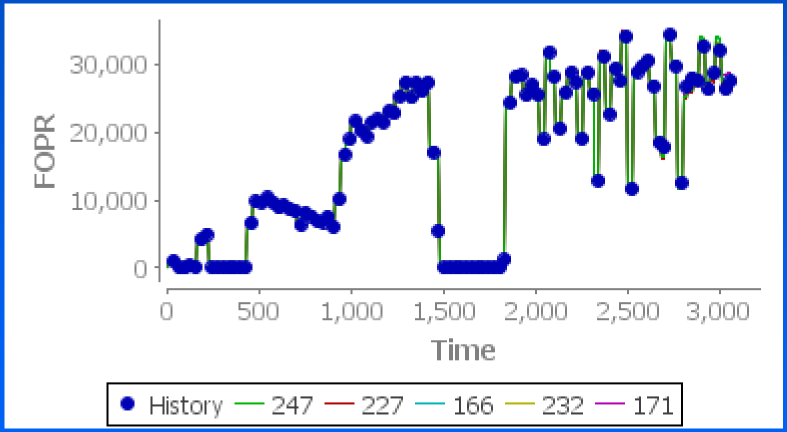
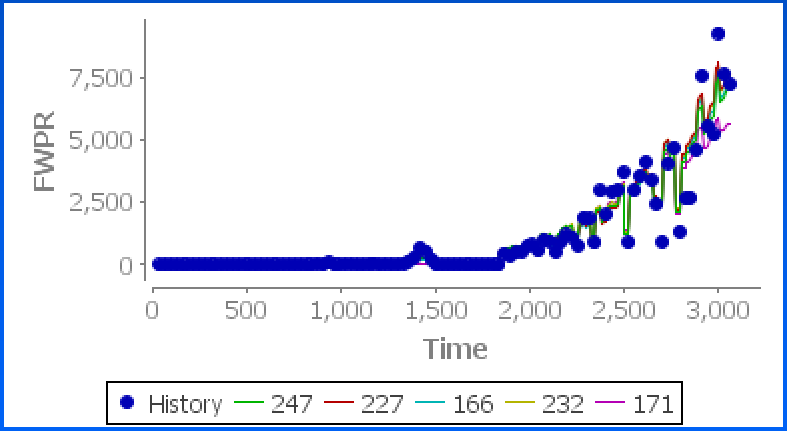
Field Development Optimisation

- Based on Scapa field
  - Original HM done as part of MSc project
  - HM to field rates (from DECC website)
  - Wells are different from real field
    - 4 injectors, 4 producers

Field Level History Match

- Match to first 10 years of history
  - Remaining data used to check forecast

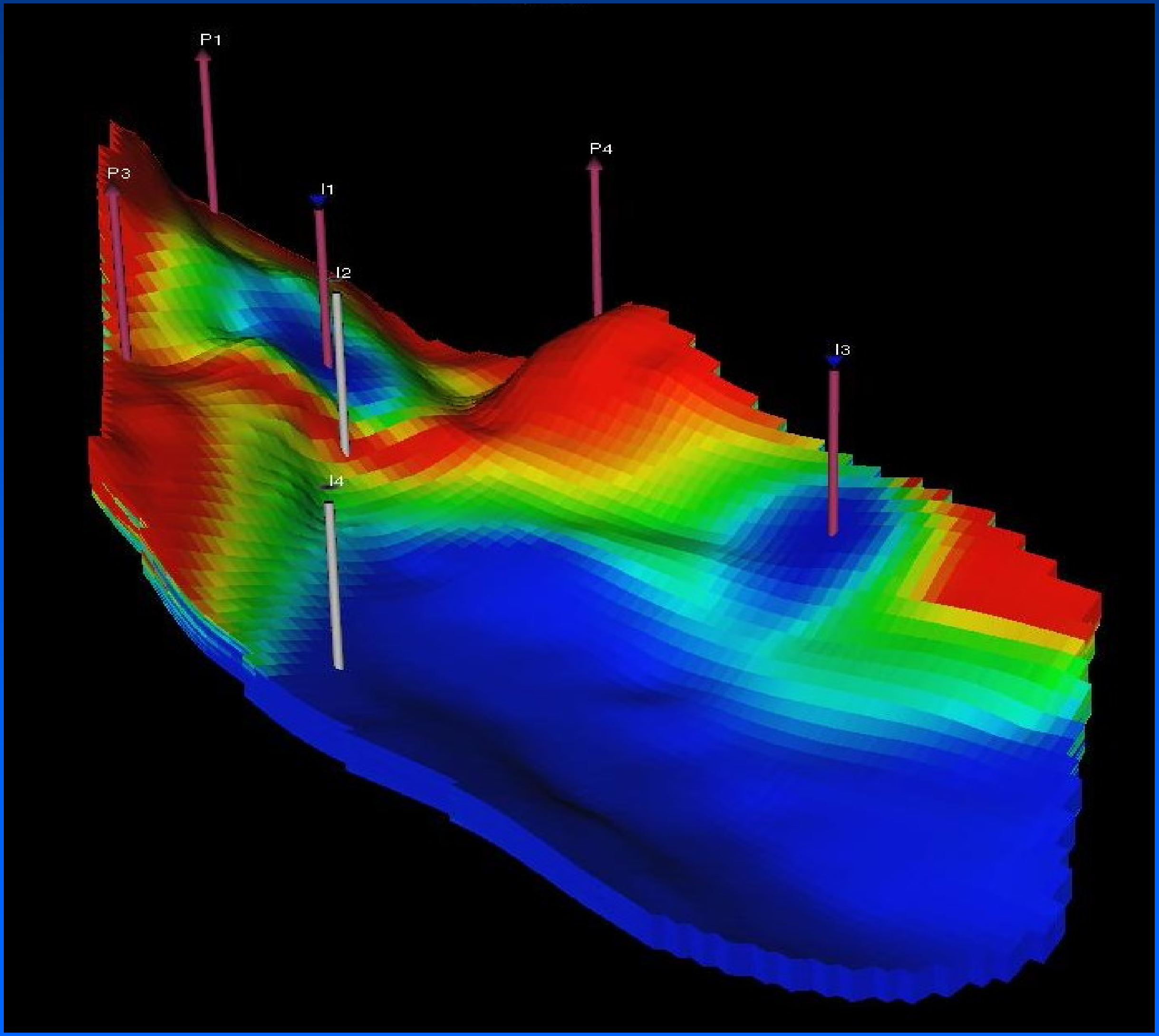




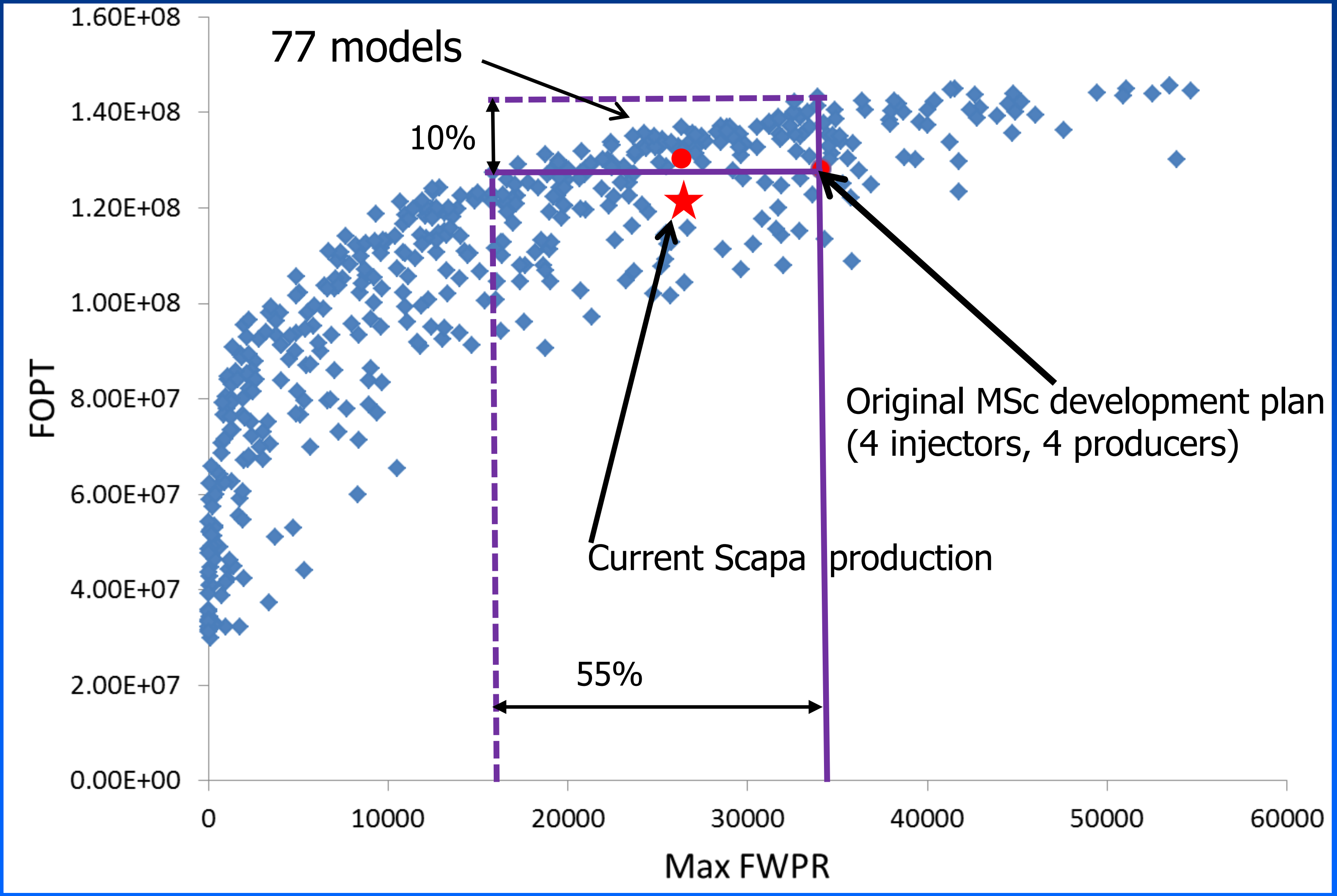
Optimisation

- Multi-Objective
  - Maximise cumulative oil (FOPT)
  - Minimise maximum water rate (FWPR)
- Optimisation variables
  - Well locations (16 variables – i, j for each well)
  - Injection well rates

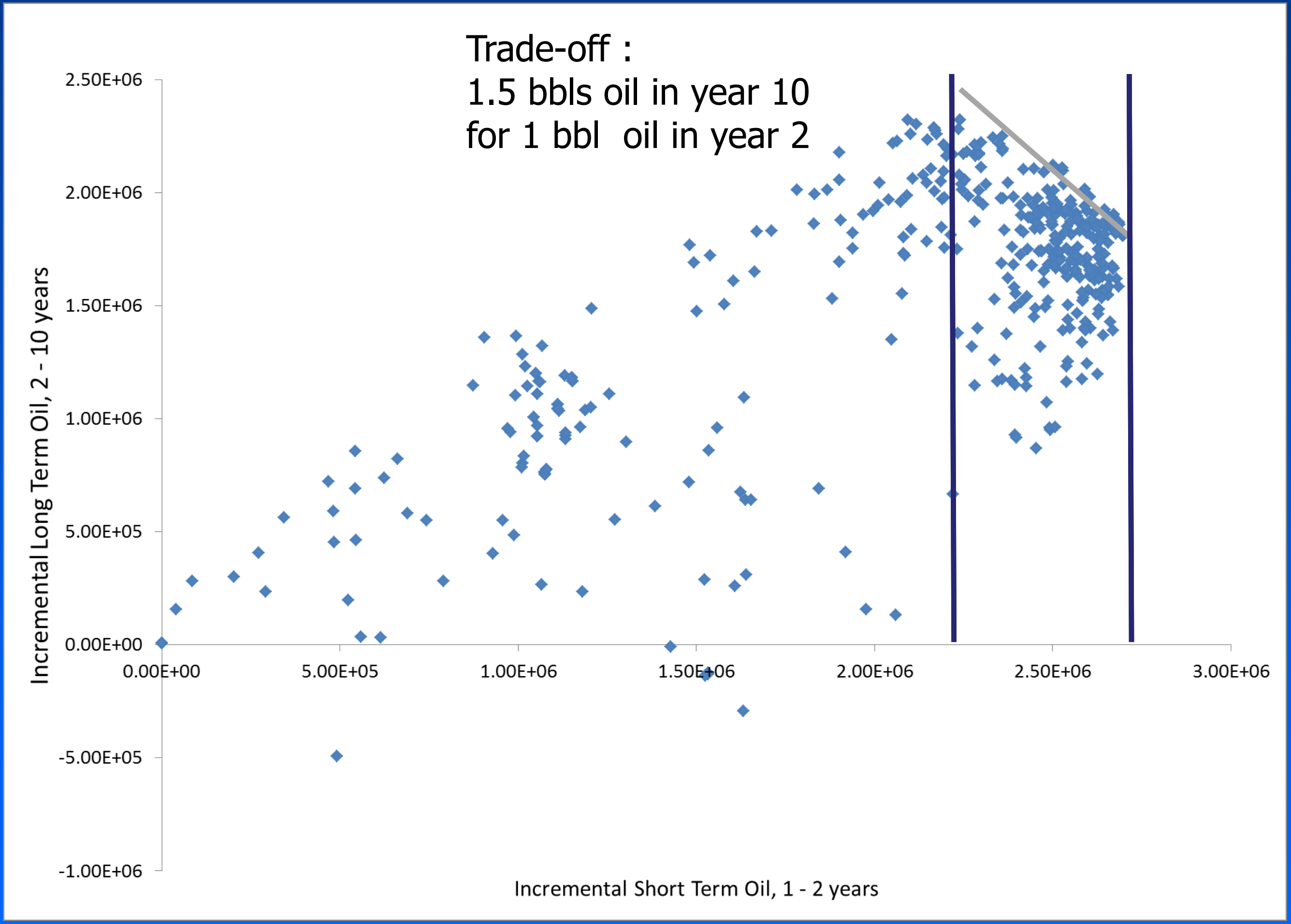
Field Development Optimisation (Scapa)



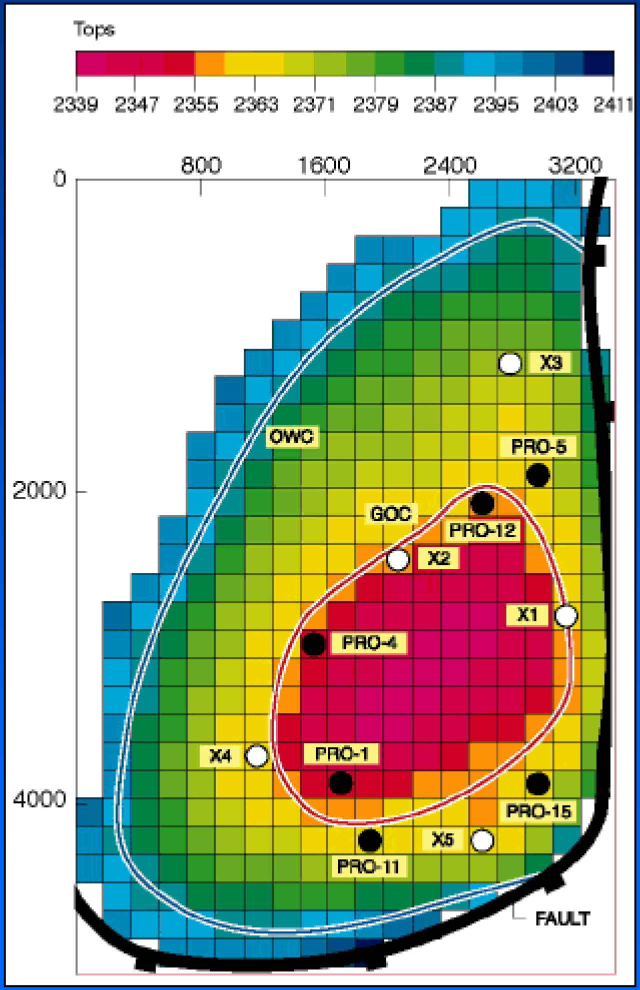
Optimise Field Development Plan



Scapa: Optimise Mid-Life Infill Wells

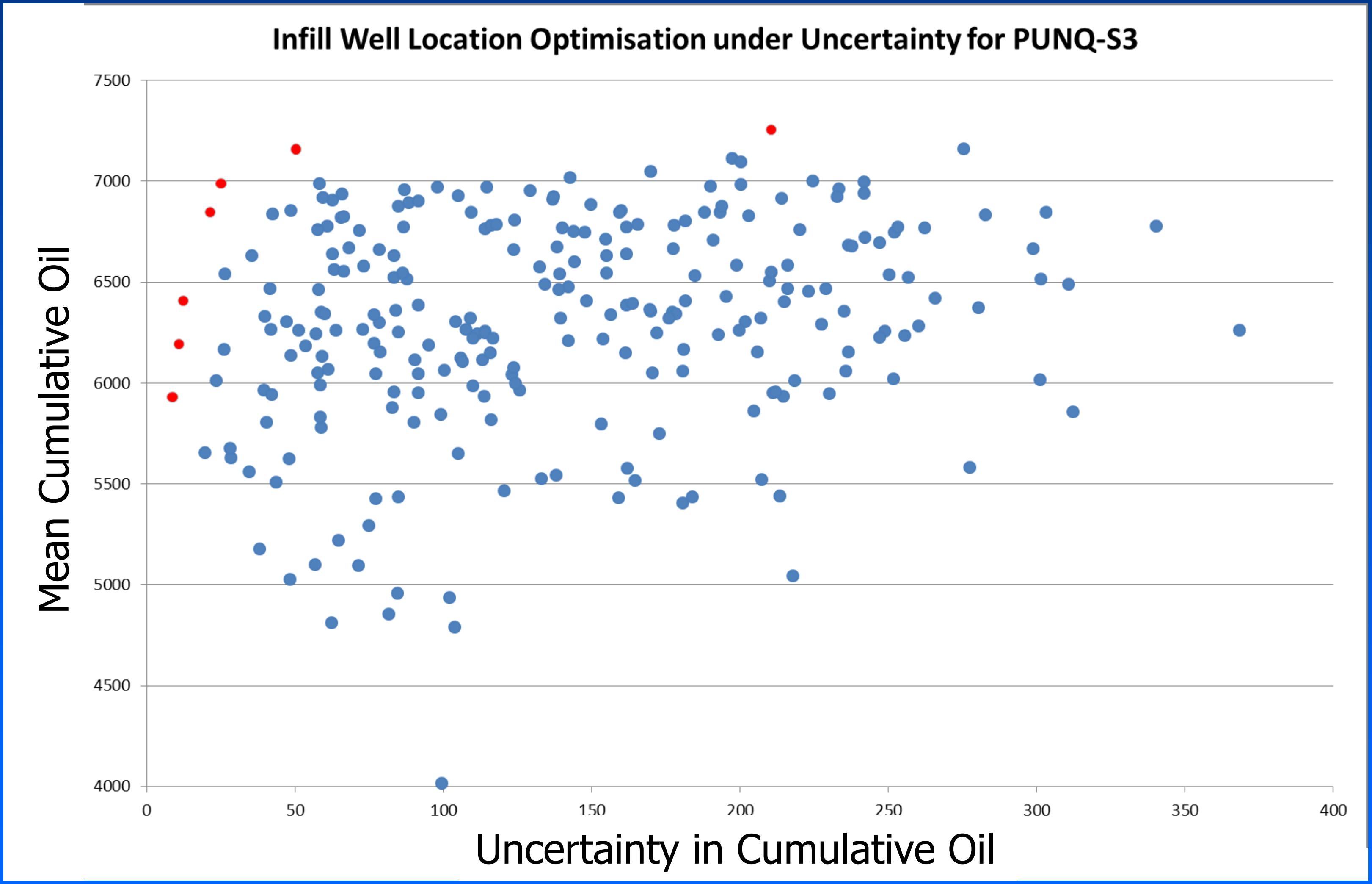


Optimisation Under Uncertainty



- PUNQ-S3
- Multiple HM models
- Optimise infill wells

Optimisation Under Uncertainty



Summary

- History Matching
  - Multi-objective may increase convergence speed, but not always
- Forecasting
  - Multi-objective provides greater coverage of pareto front; can lead to better forecasts
- Optimisation
  - Explore the full range of trade-offs
  - Can include uncertainty

Acknowledgements

- EoN for funding the SO vs MO comparison
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- RFD for licences of tNavigator