An Introduction to Real Options Analysis

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School of Mathematics, The University of Manchester

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- When is it best for a University to go private??????????????
- When should a firm pause their sponsorship of FIFA??????????
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Real Options Analysis

- Seeks to optimise investment decisions under uncertainty,
- Started as a independent subject in the late 70's,
- Is at the intersection between Financial Mathematics and Operations Research,
- The subject has primarily considered irreversible decision making (i.e. lost capital investment), but it can be expanded to consider continuously variable decisions.

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- Is hardly ever (explicitly) used……………

Example: The Option to Abandon

Q. At what point should a factory operating under sale price uncertainty cease production?

The Option to Abandon

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When it starts making a loss?

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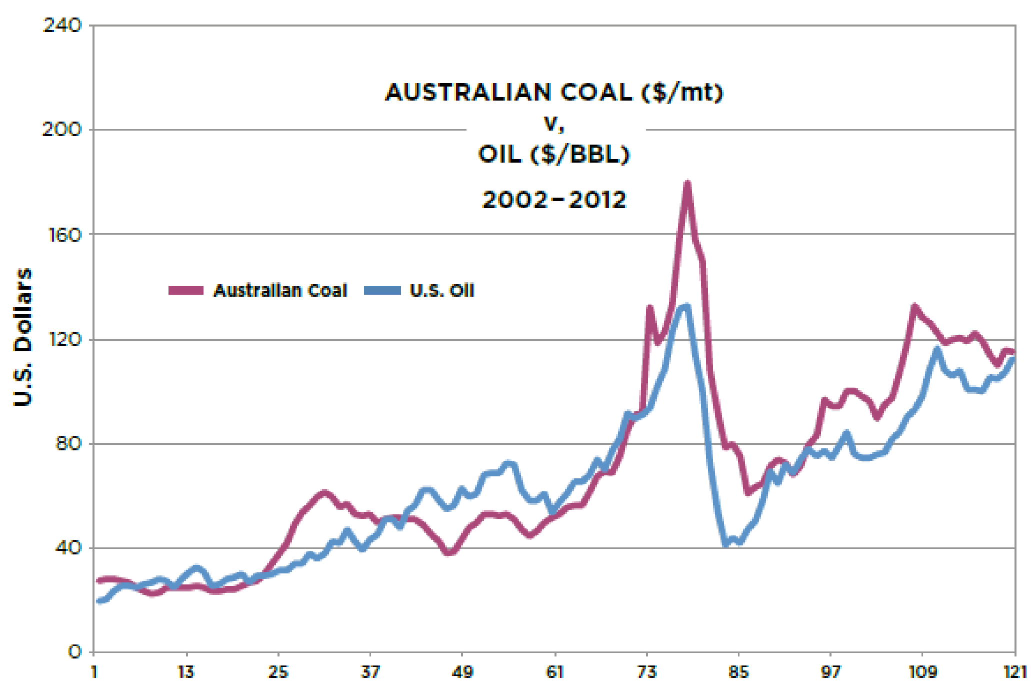
When it starts making a loss?

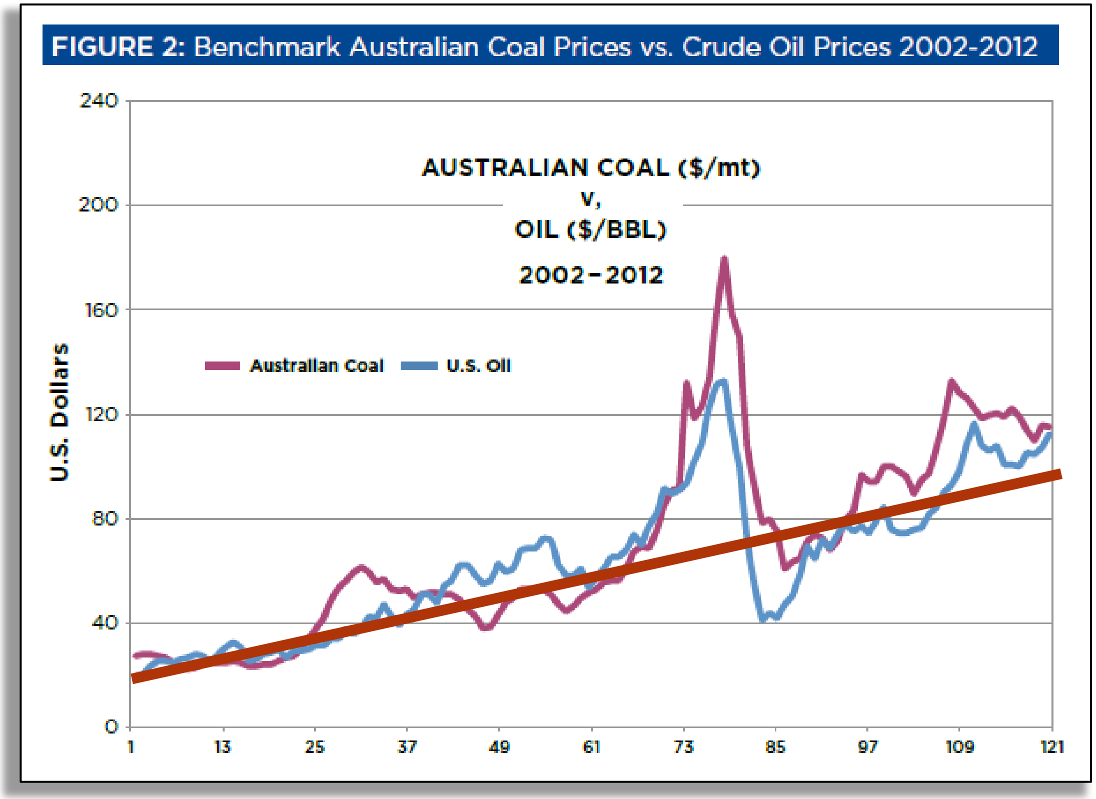
Or, when its expected value falls below the salvage value?

The Option to Abandon

FIGURE 2: Benchmark Australian Coal Prices vs. Crude Oil Prices 2002-2012



The Option to Abandon



Aside: Feynman-Kac Equation

$$w^u(x) = E_x\left[\int_0^\nu e^{-\hat{r}z} g(X_z, u_z)dz + e^{-\hat{r}\nu} h(X_\nu)\right]$$

$$dX_t^u = b(X_t, u_t)dt + \sigma(X_t, u_t)dB_t,$$

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$$\sup_{v \in U}\{LV(x) + g(x,v) - \hat{r}V(x)\} = 0 \quad \text{in} \quad H$$

$$\lim_{x \to y} V(x) = -h(y,v) \quad \text{for} \quad y \in \partial H$$

$$L \equiv \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial}{\partial x_i}$$

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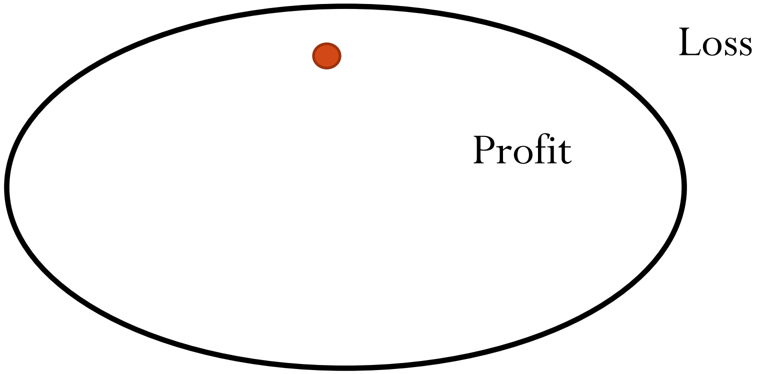

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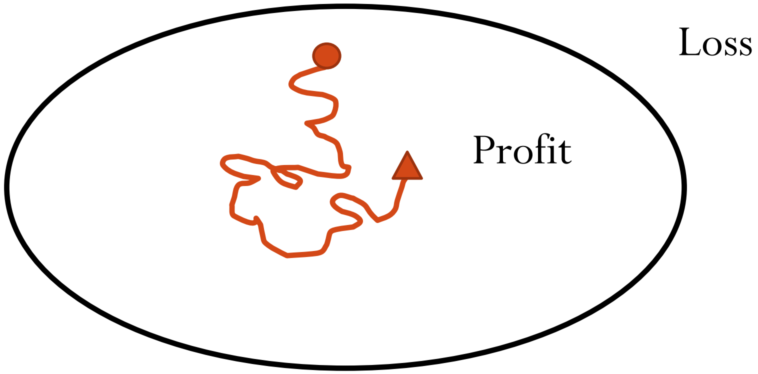

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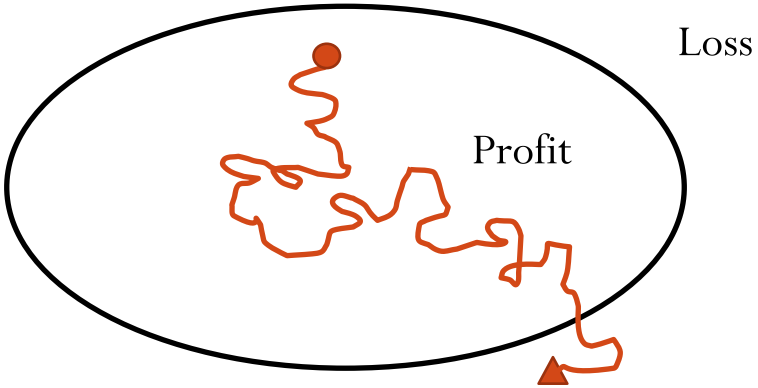

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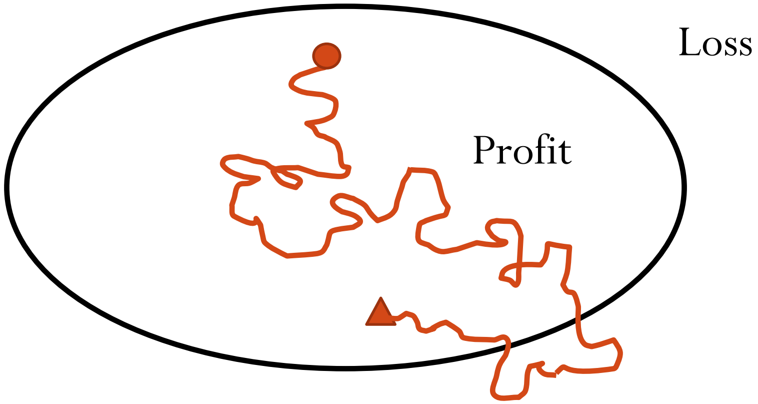

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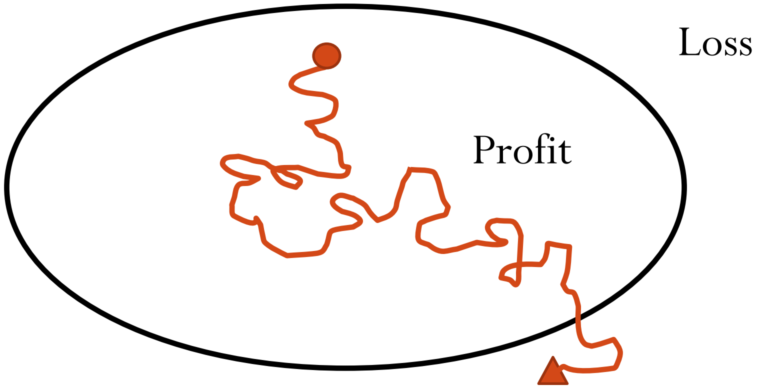

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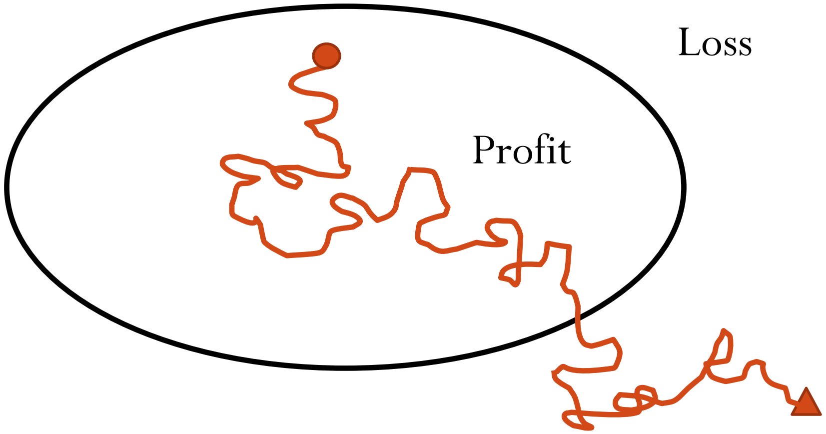

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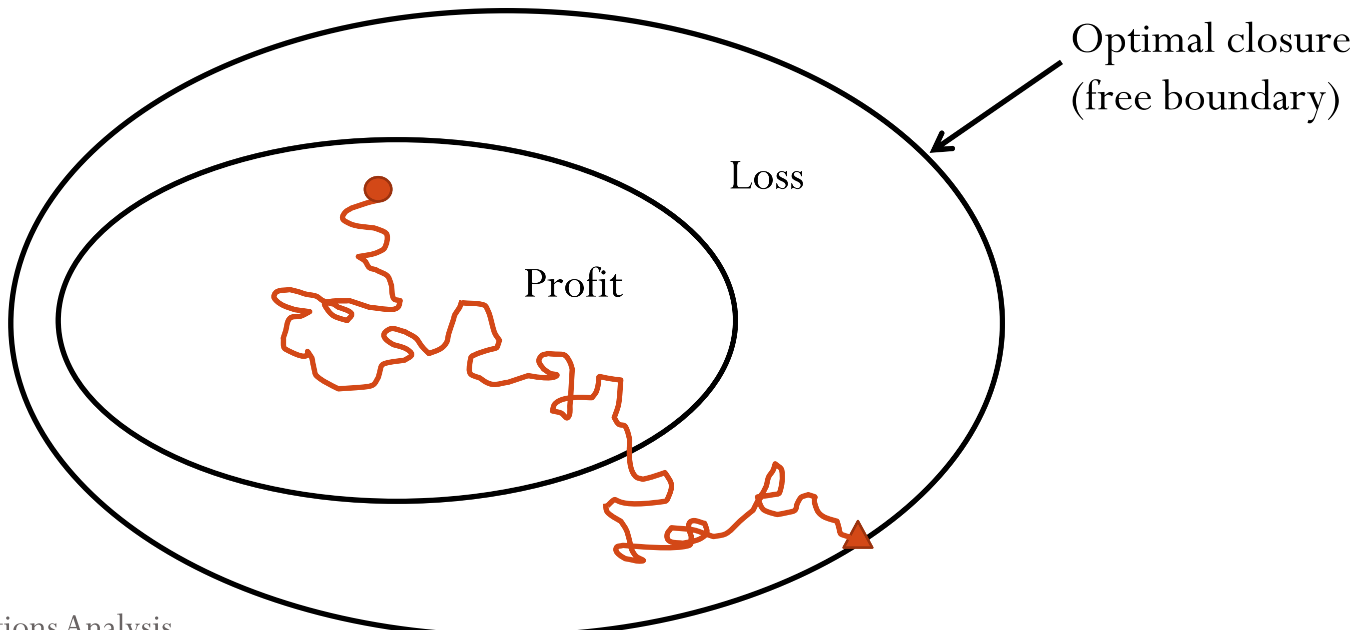


The Option to Abandon

Uncertain future sale price: $dS_t = \mu S_t dt + \sigma S_t dB$

Profit function: $g(S_t, t) = qS_t - \epsilon$

The Option to Abandon

$$\frac{\partial V}{\partial t} + \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \mu S \frac{\partial V}{\partial S} - rV + S - \epsilon = 0.$$

$$V = 0 \quad \text{when} \quad t = T,$$

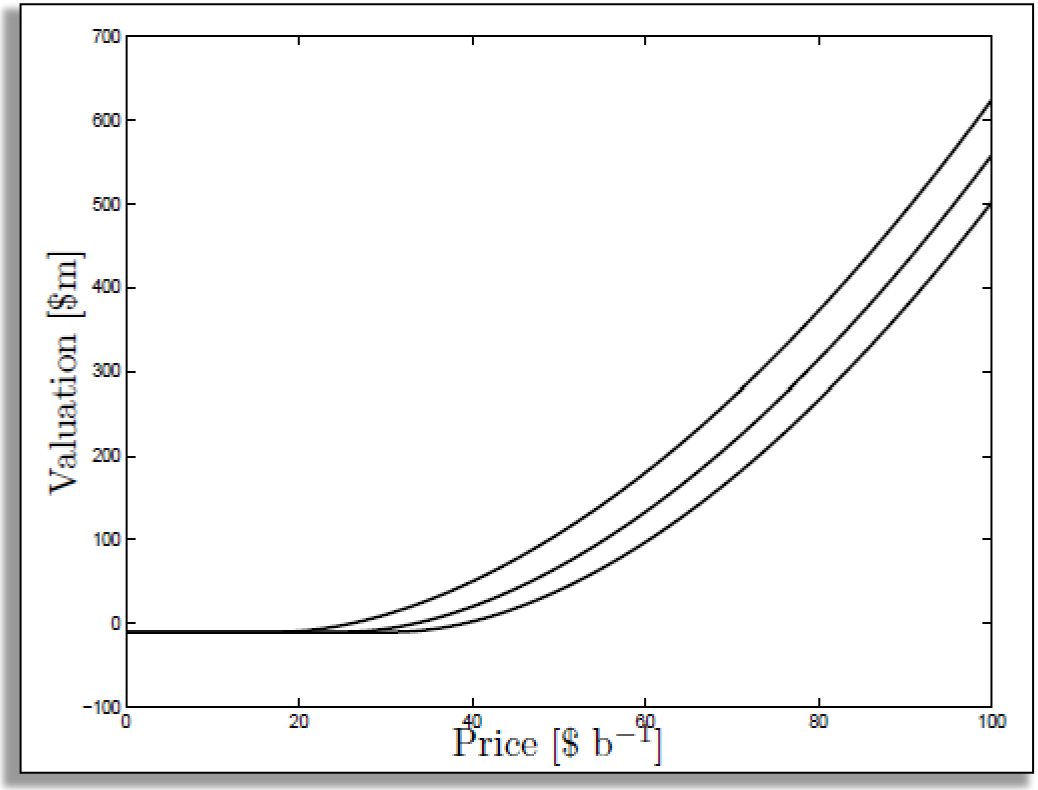
$$\frac{\partial^2 V}{\partial S^2} \to 0 \quad \text{as} \quad S \to \infty,$$

$$V = -C \quad \text{on} \quad S = S^*,$$

Optimality condition (free boundary):

$$\frac{\partial V}{\partial S} = 0 \quad \text{on} \quad S = S^*,$$

The Option to Abandon



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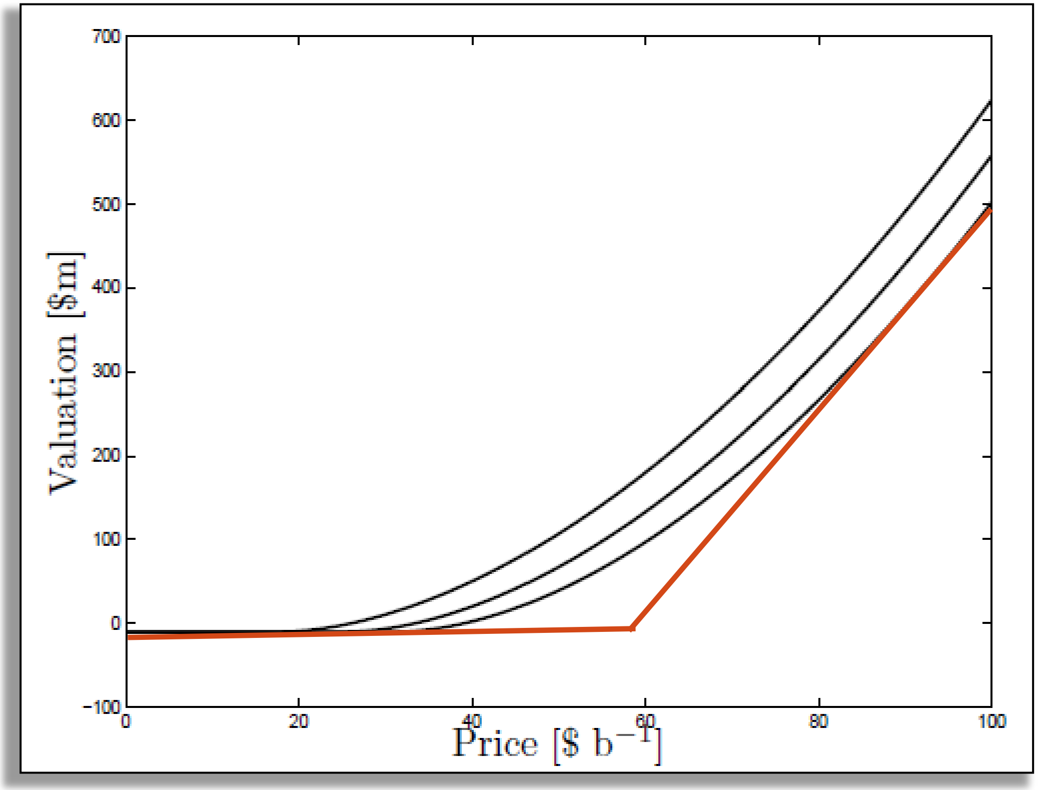

The Options to Abandon, Expand, or Contract

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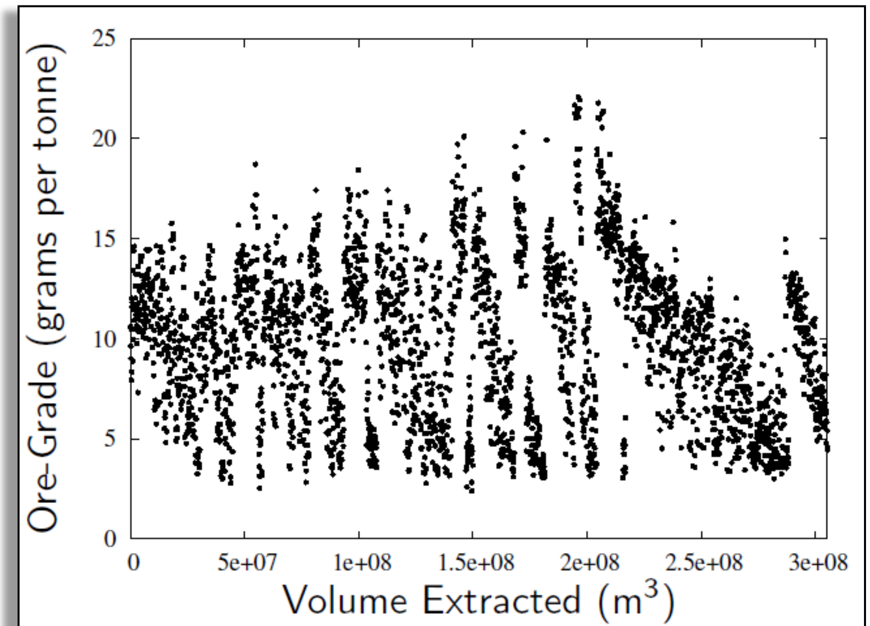

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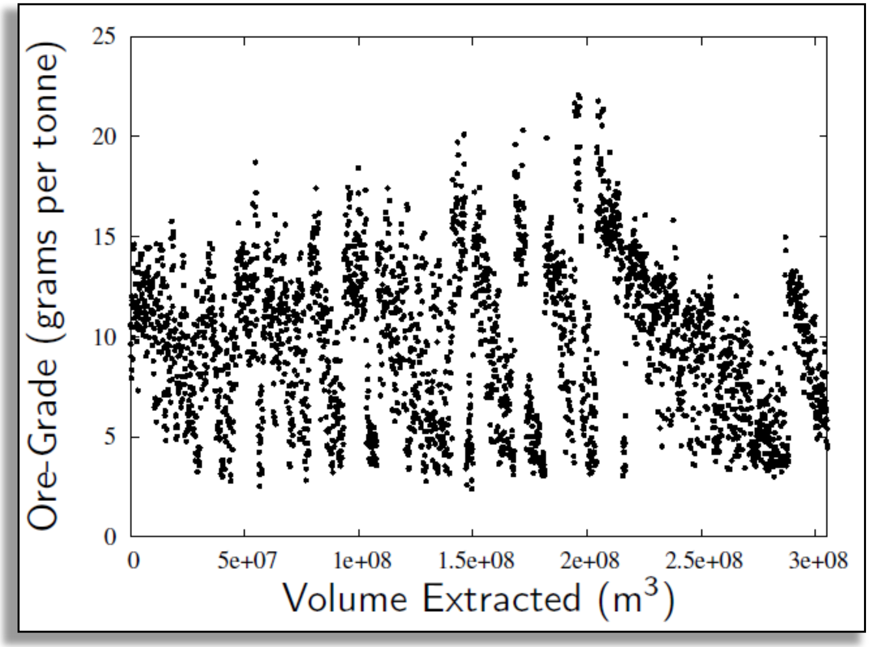

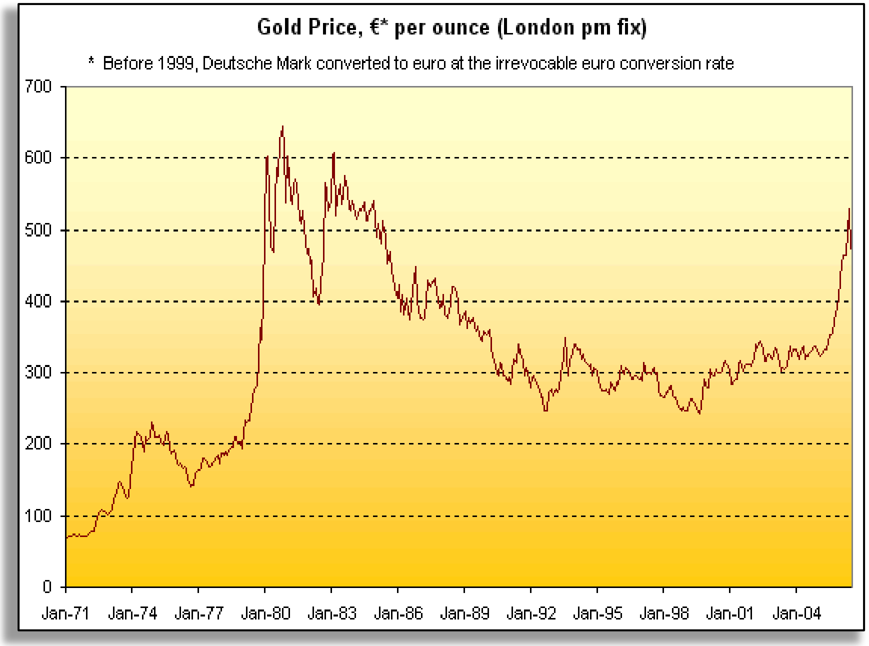

The Options to Abandon, Expand, or Contract

- The firm can operate in one of three states: normal, expanded, or abandoned.
- The firm has to pay a sunk cost for changing state.

Commodity price uncertainty: $dS_t = \mu S_t dt + \sigma S_t dB, \quad dQ_t = -q dt.$

Profit function: $g_i(S,t,Q) = q_i G(Q) S - \varepsilon_i.$

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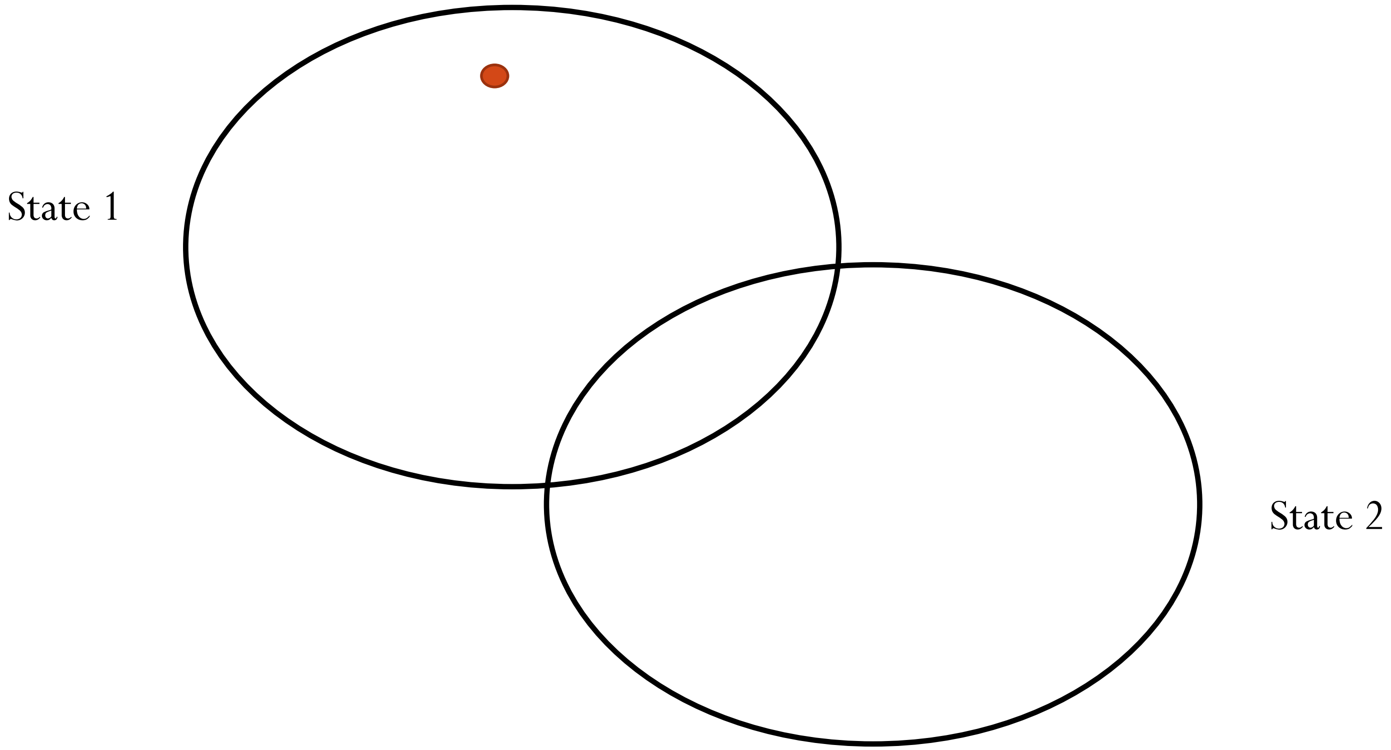

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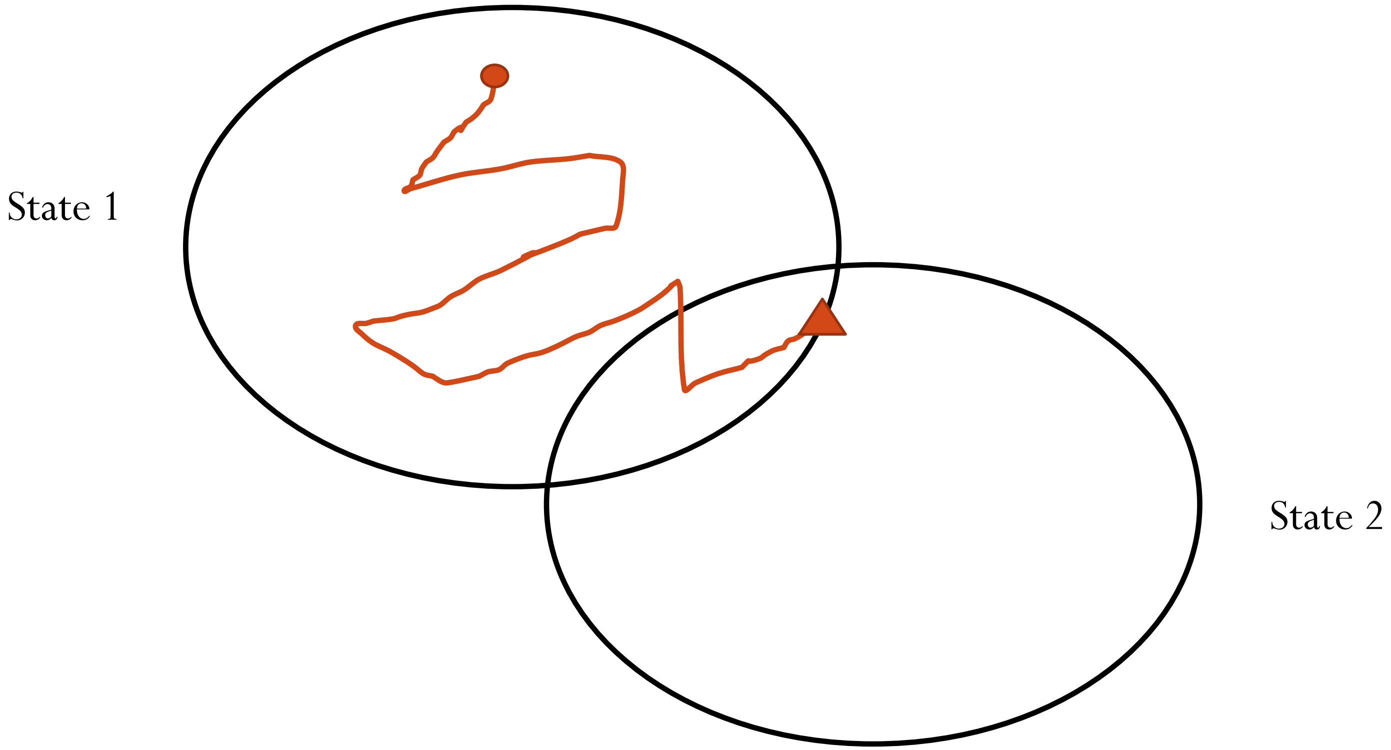

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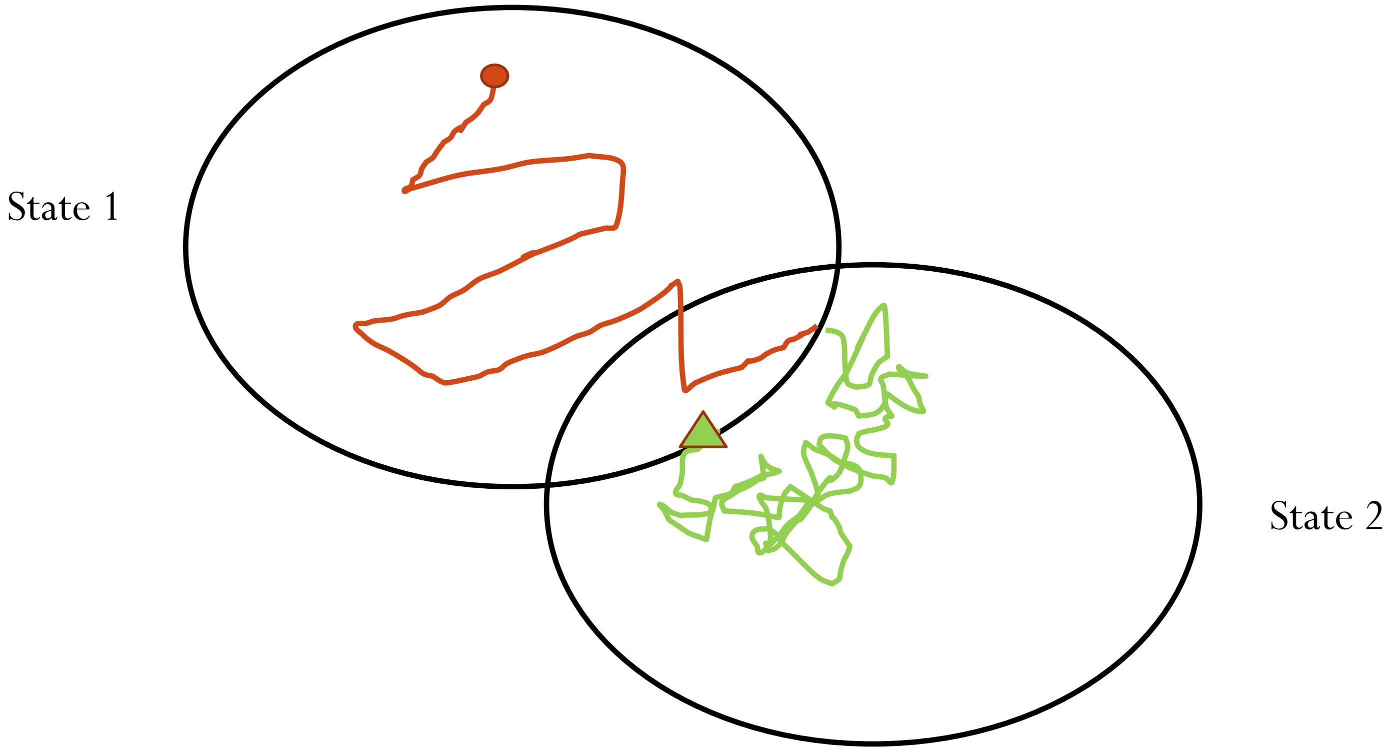

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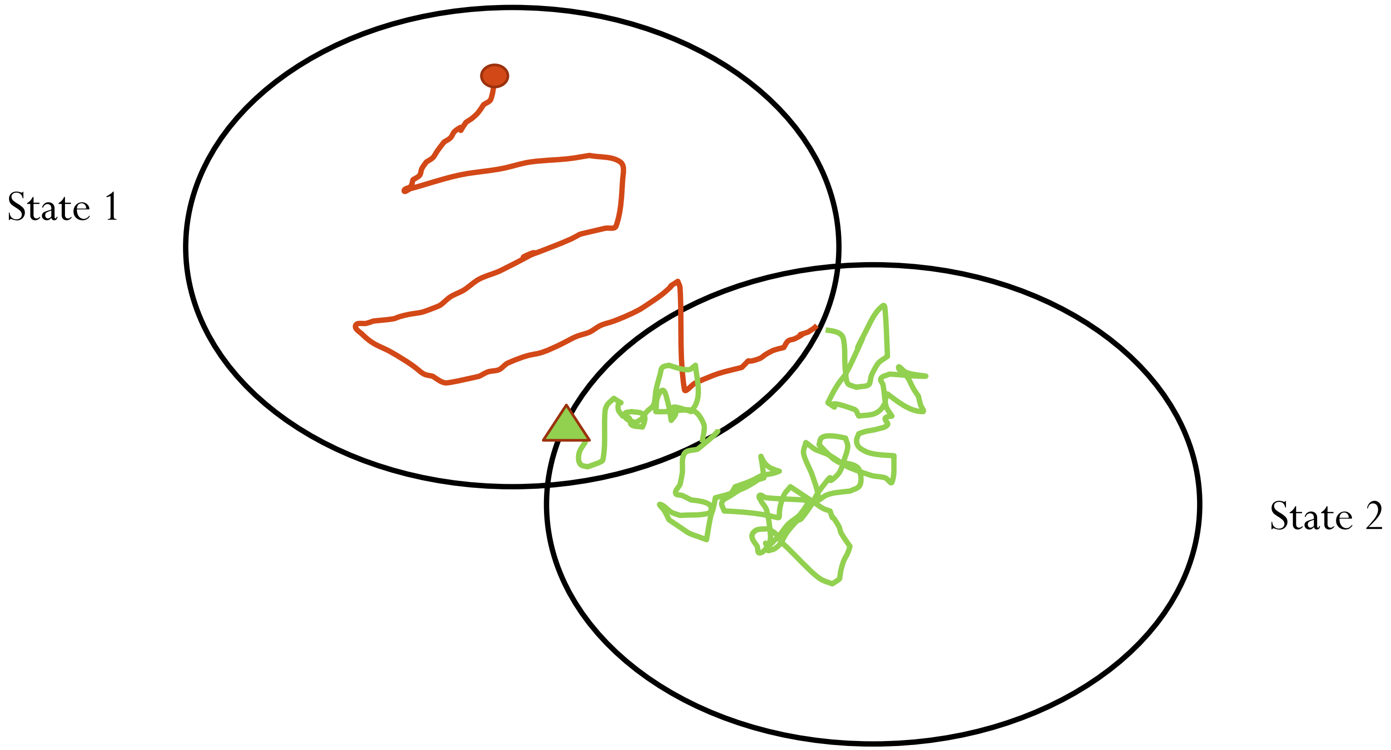

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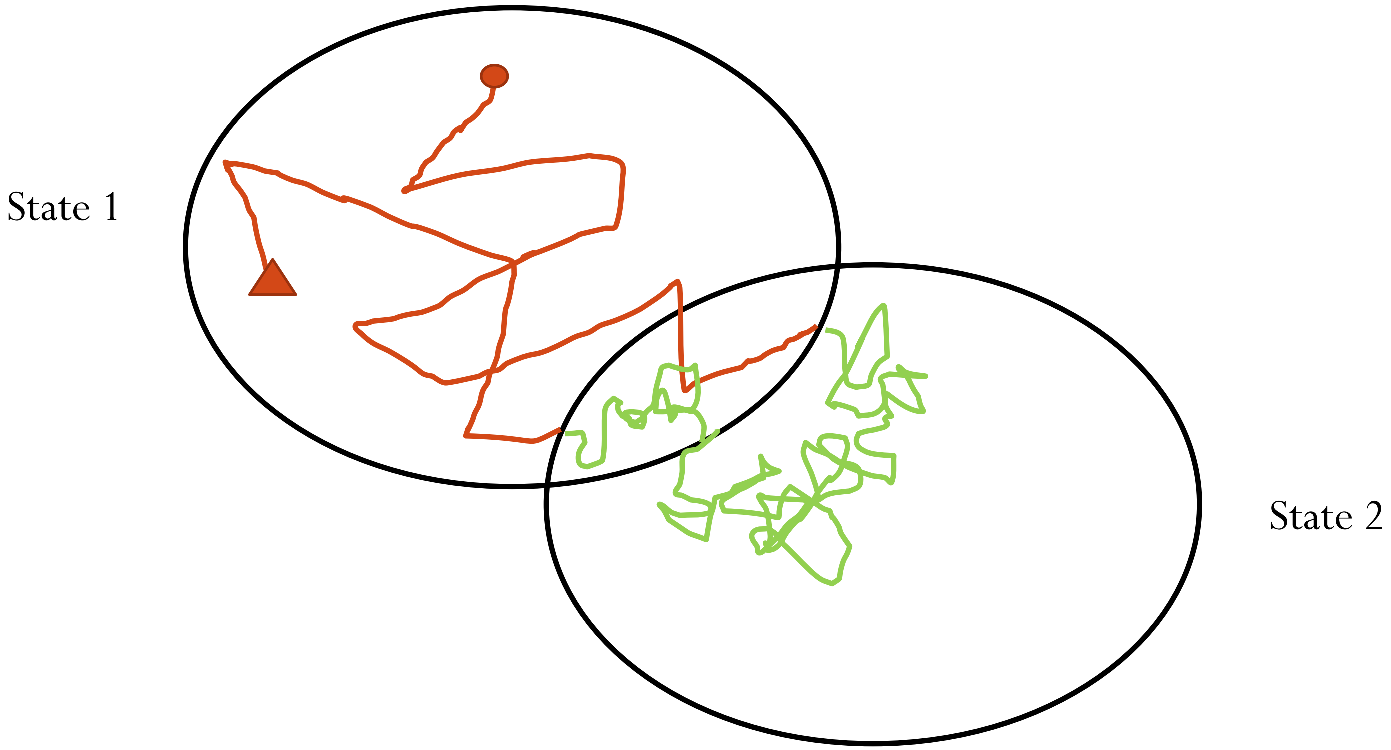


The Options to Abandon, Expand, or Contract

$$\frac{1}{2}\sigma^2 S\frac{\partial^2 V_1}{\partial S^2} - \frac{\partial V_1}{\partial \tau} - q_1\frac{\partial V_1}{\partial Q} + \kappa(\mu - S)\frac{\partial V_1}{\partial S} - rV_1 + q_1 GS - \varepsilon_1 = 0,$$

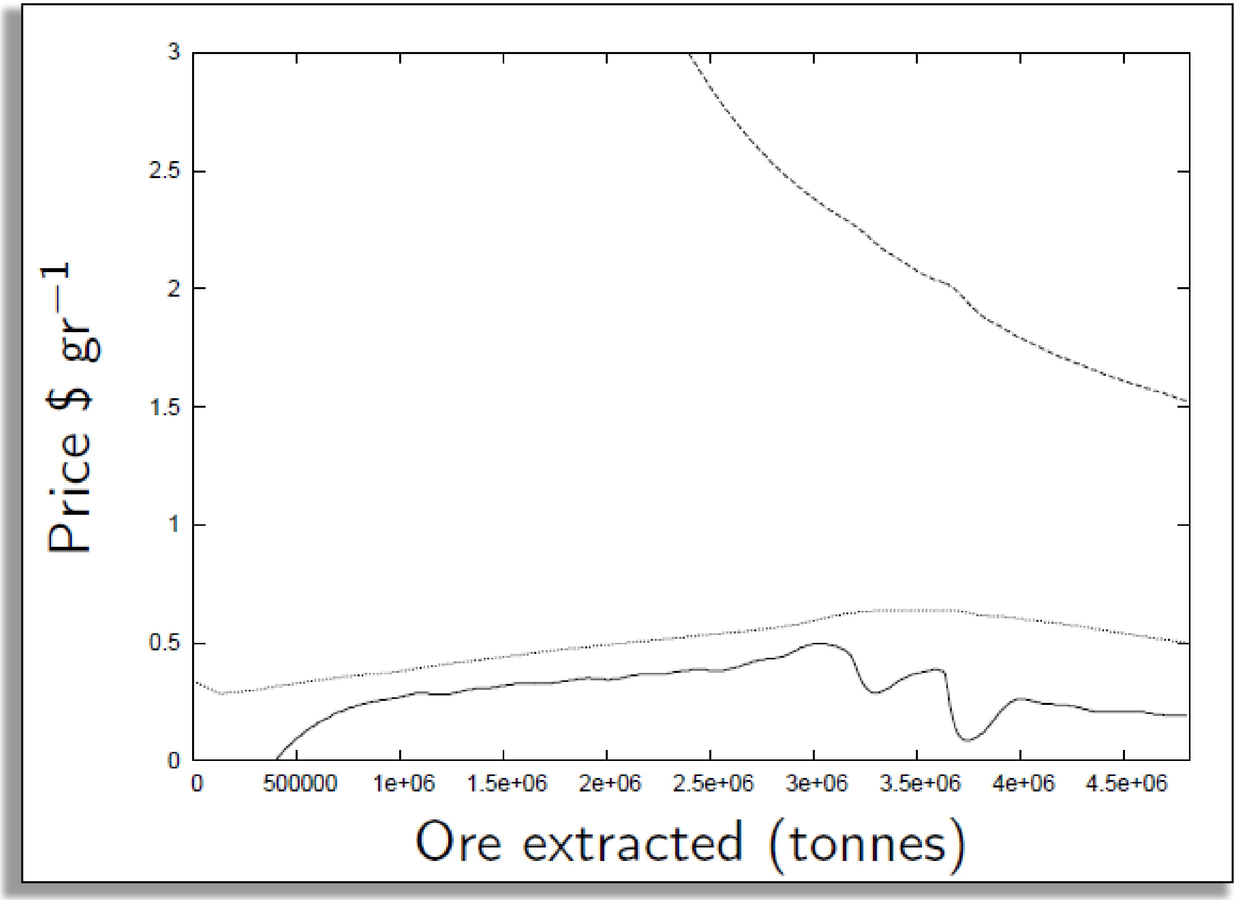
$$\begin{gathered} V_1 = 0 \quad \text{when} \quad \min\{\tau, Q\} = 0, \\ V_1 = -C_{1a} - K \quad \text{when} \quad S = S^*_{1a} \\ V_1 = V_2 - C_e \quad \text{on} \quad S = S^*_e, \end{gathered}$$

$$\frac{1}{2}\sigma^2 S\frac{\partial^2 V_2}{\partial S^2} - \frac{\partial V_2}{\partial \tau} - q_2\frac{\partial V_2}{\partial Q} + \kappa(\mu - S)\frac{\partial V_2}{\partial S} - rV_2 + q_2 GS - \varepsilon_2 = 0,$$

$$\begin{gathered} V_2 = 0 \quad \text{when} \quad \min\{\tau, Q\} = 0, \\ V_2 = -C_{2a} - K \quad \text{when} \quad S = S^*_{2a} \\ V_2 \sim S \quad \text{as} \quad S \to \infty. \end{gathered}$$

$$\frac{\partial V}{\partial S} = 0 \quad \text{on} \quad S = \{S^*_e, S^*_{1a}, S^*_{2a}\}.$$

The Options to Abandon, Expand, or Contract



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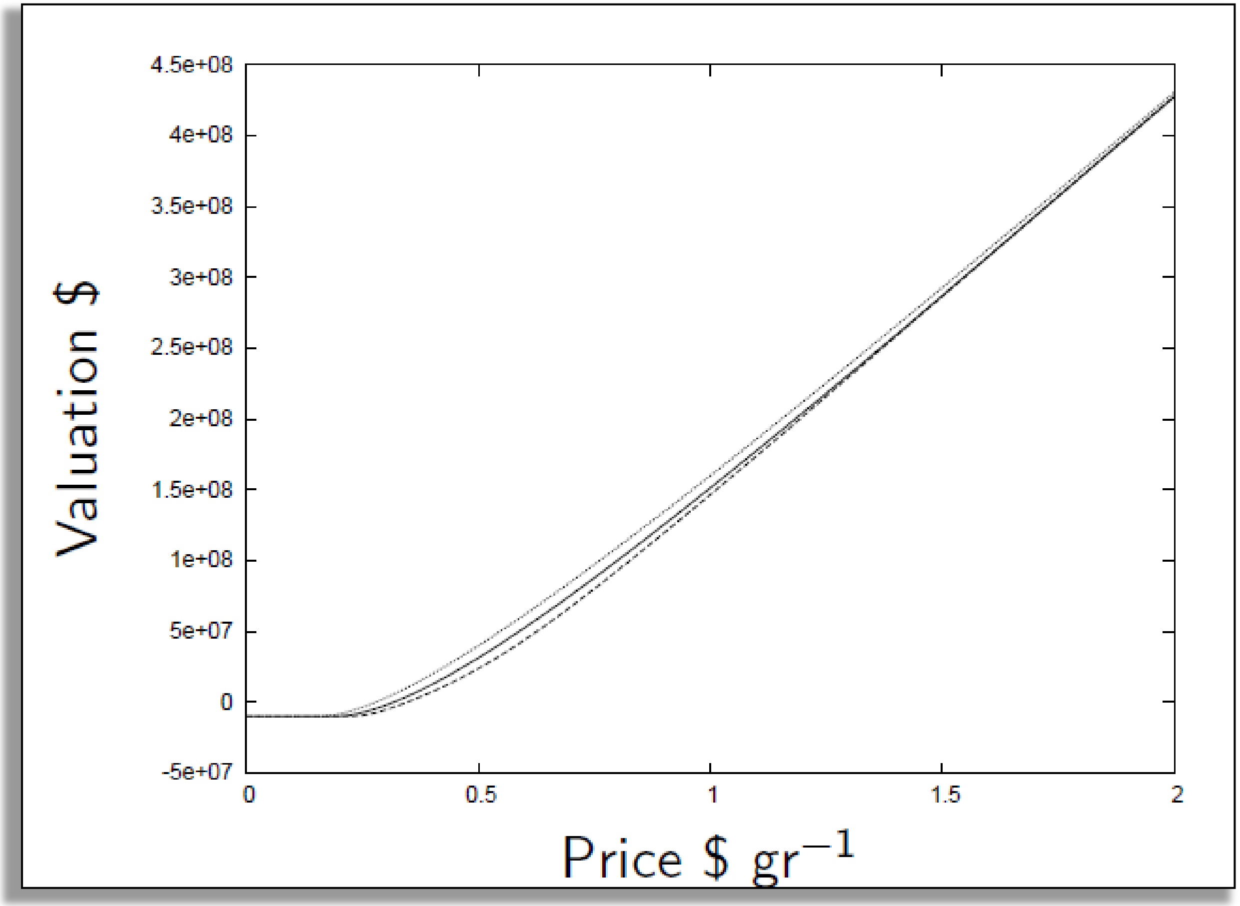

Real Options Limitations

- Discounting dilemmas (mainly an academic thing…)

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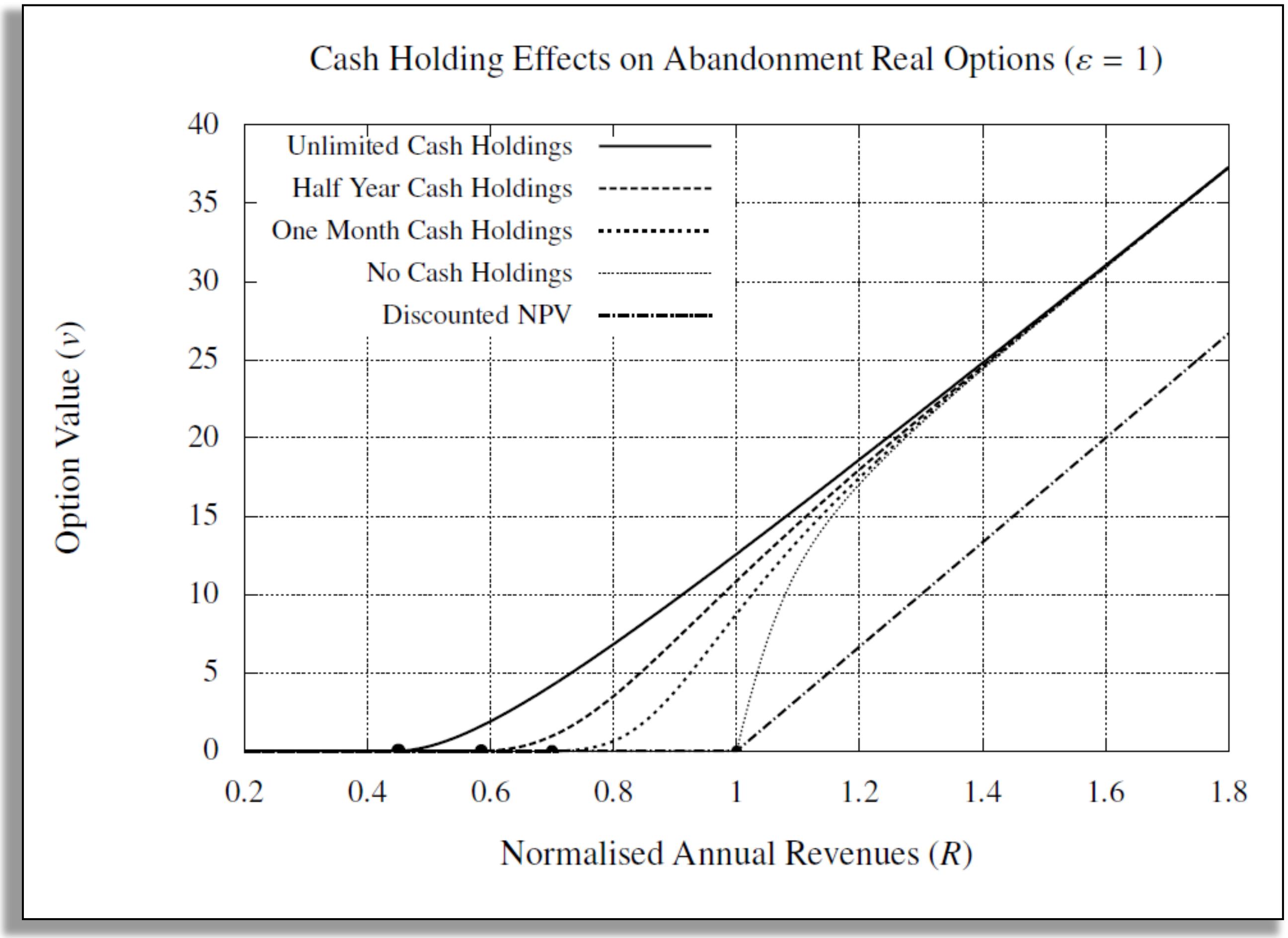
- The uncertainties won't always be nicely behaved!
- Neglects liquidity issues (i.e. assumes that making a loss is fine)

Real Options Limitations (Liquidity)



Future Direction (my unbiased take…)

- Regulators can use Real Options Analysis to calculate expectations beyond just valuation, such as event probabilities.
- In knowing a firm’s strategy, they can then construct ways to influence them.

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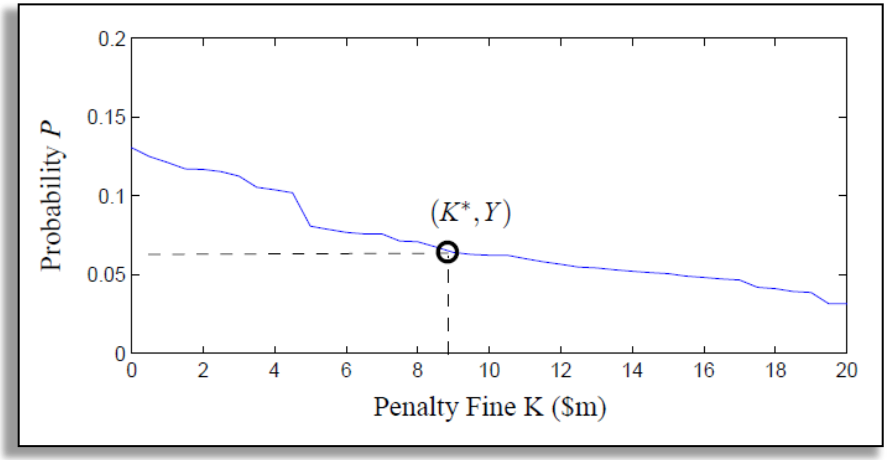

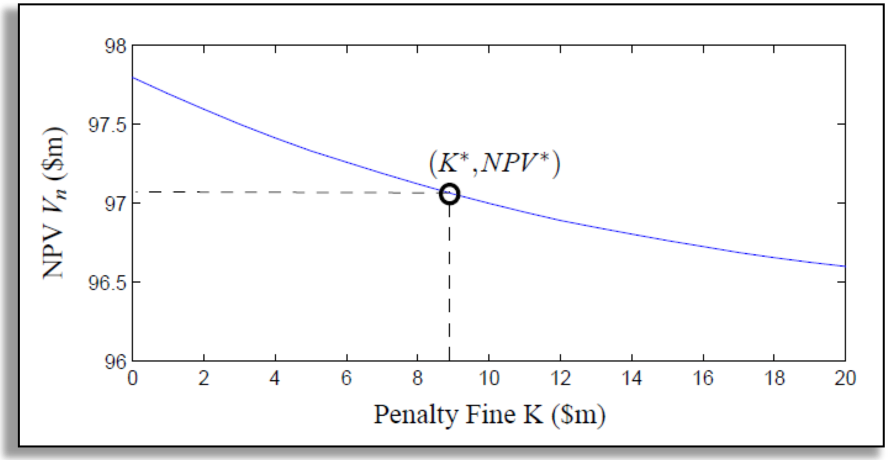

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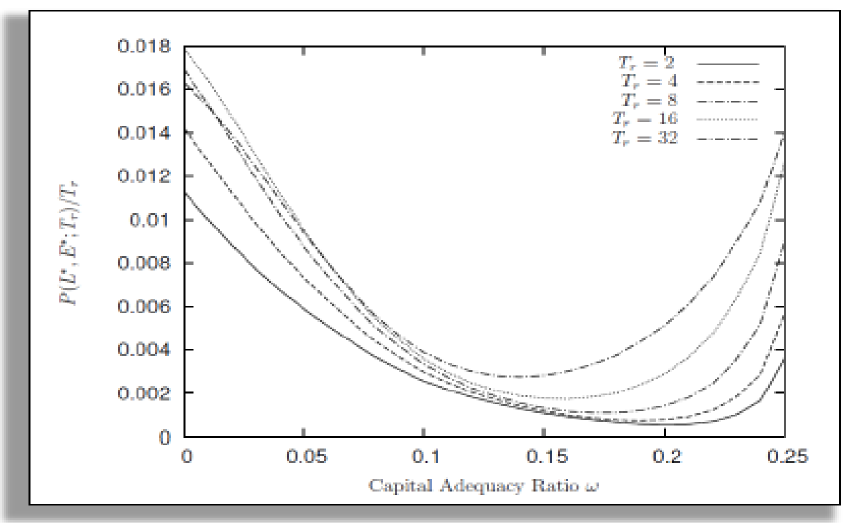
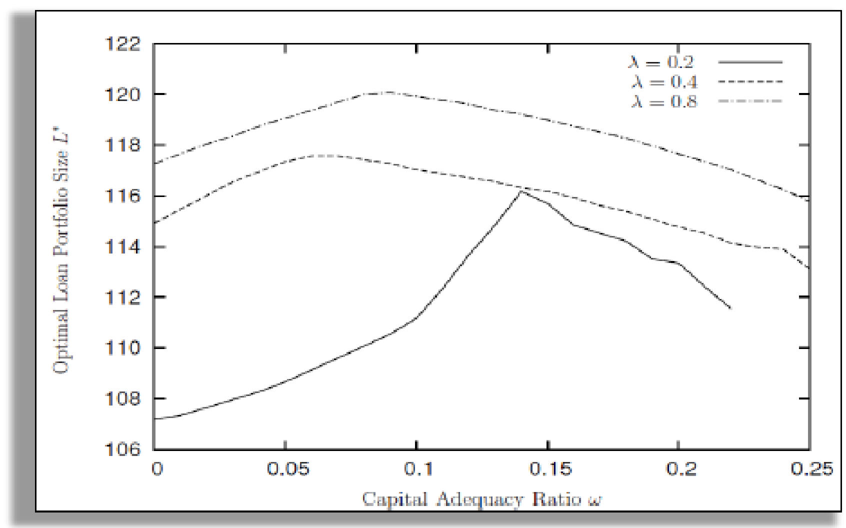
Future Direction (Basel III)

$$\frac{dL_t}{L_t} = (l - a)dt - (\mu + \lambda K)dt - \sigma dW_t - d\hat{Q}_t, \qquad \frac{E_t}{L_t} \geq \omega,$$





Google `Evatt, financial regulation, SSRN' for our working paper on this.

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Summary

- Real Options Analysis concerns the valuation of a organisation's flexible decision making.
- It helps give an idea of how to manage future uncertainties.
- If in doubt, wait (but not forever…).
- The future is regulation?

Recommended reading: Dixit & Pindyke, Investment under Uncertainty, Princeton University Press

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Evatt *et al.*, Proceedings of the Royal Society A, 2010,

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Evatt *et al.*, IMA Journal of Management Mathematics, 2014.

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