Isaac Newton Institute for Mathematical Sciences

A Course of Three Sessions on

Reasoning via Formal Models in Economics

Theme 1: Models as Parables

Lecture 1: Exploitation as Cooperation

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Readings:

P. Dasgupta (2012), "Dark Matters: Exploitation as Cooperation," *Journal of Theoretical Biology*, 299(21 April), 180-187.

D. Fudenberg and E. Maskin (1986), "The Folk Theorem in Repeated Games With Discounting or With Incomplete Information," *Econometrica*, 54(3), 533-556.

G. Hardin (1968), "The Tragedy of the Commons," *Science*, 162(3859), 1243-1248.

Common Property Resources

"Picture a pasture open to all. It is to be expected that each herdsman will try to keep as many cattle as possible on the commons... As a rational being, each herdsman seeks to maximize his gain. Explicitly or implictly, more or less consciously, he asks, "What is the utility to me of adding one more animal to my herd?"... Adding together the component partial utilities, the rational herdsman concludes that the only sensible course for him to pursue is to add another animal to his herd. And another, and another ... But this is the conclusion reached by each and every herdsman sharing the commons. Therein is the tragedy.... Freedom in a commons brings ruin to all." Garrett Hardin (1968).

Hardin's conclusion has been shown by anthropologists and political scientists to be wrong in many instances, the late Elinor Ostrom's work being among the most prominent. They all pertain to geographically localized commons. That literature has a warm-glow to it. Global commons pose a far more accurate portrait of Hardin's thesis (e.g. the atmosphere as a sink for carbon). In this lecture I model a local commons, show how a community, provided people are far sighted, is able to overcome the tragedy envisaged by Hardin. No appeal will be made to an external enforcer of the restraint community members may have agreed to practise. The mechanism I will invoke imagines herdsmen practising *reciprocity*, under repeated interactions.

I will then argue that one should be circumspect before bathing the practise of reciprocity in warm glow. What appears as cooperation may well be exploitation of some members of others.

The Model

N herdsmen, indexed by *i*. Cattle are private property. The pasture is neither privately owned nor State property, but is communally owned. Outsiders are not permitted to graze their cattle in the pasture: the grazing land is a CPR. The model is timeless.

The size of the pasture is S. Cattle intermingle while grazing, so that on average the cows consume the same amount of grass. If X is the size of the herd in the pasture, total output - of milk - is $H(X,S)$, where H is linear homogeneous in X and S. Assume $H(0,S) = 0$ for all $S \geq 0$; $\partial H/\partial X$, $\partial H/\partial S > 0$ and $\partial^2 H/\partial X^2$, $\partial^2 H/\partial S^2 < 0$.

As S is fixed, we may eliminate S by writing $H(X,S) = SH(X/S,1)$; by letting $S = 1$ without loss of generality; and by defining $F(X) \equiv H(X,1)$. The properties assumed of H imply $F(0) = 0$; $F'(X) > 0$; $F''(X) < 0$; and $F(X)/X > F'(X) > 0$ for $X \geq 0$. See Figure 1.

Herdsmen are interested in the profits they are able to earn from their cattle. We normalise by choosing the market price of product to be 1. The market price of cattle is p (> 0). To have an interesting problem, assume that $F'(0) > p$.

An Unmanaged CPR

We first determine the herd size in the CPR if the latter is unmanaged, that is, if the pasture is free. Let x_i be the size of *i*'s herd (assumed to be a continuous variable). Because cattle intermingle, $x_iF(X)/X$ is output for *i*. *i*'s net profit, π_i, is

$$\pi_i = x_iF(X)/X - px_i. \qquad (1)$$

We now compute the non-cooperative (Nash) equilibrium of the resulting timeless game. Since the model is symmetric, we should expect it to posesses a symmetric equilibrium. (It can be shown that equilibrium in this timeless model is unique.)

Consider herdsman i. If the herd size of each of the other cattlemen is x, equation (1) can be written as

$$\pi_i(x_i,x) = x_i F(x_i+(N\text{-}1)x)/(x_i+(N\text{-}1)x) - px_i. \qquad (2)$$

The profit function $\pi_i(x_i,x)$ reflects the crowding externalities each herdsman inflicts on all others in the unmanaged CPR (π_i is a function not only of x_i, but also of x). Let $\underline{x}$ be the size of each cattleman's herd at a symmetric equilibrium. $\underline{x}$ is the value of x_i that maximizes $\pi_i(x_i,\underline{x})$.

To determine $\underline{x}$, differentiate $\pi_i(x_i,\underline{x})$ partially with respect to x_i and equate the result to zero. This yields,

$$F(x_i+(N-1)\underline{x})/[x_i+(N-1)\underline{x}] + x_iF'(x_i+(N-1)\underline{x})/[x_i+(N-1)\underline{x}] - x_iF(x_i+(N-1)\underline{x})/[x_i+(N-1)\underline{x}]^2 = p. \quad (3)$$

At a symmetric equilibrium x_i in equation (3) must equal $\underline{x}$. Now re-arrange terms to confirm that the aggregate herd size in the CPR, which we write as $\underline{X}$, satisfies

$$((N-1)/N)F(\underline{X})/\underline{X} + F'(\underline{X})/N = p, \quad \text{where } \underline{X} = N\underline{x}. \quad (4)$$

Equation (4) says that in equilibrium the price of cattle equals the weighted average of the average product of cattle and the marginal product of cattle, with weights $(N-1)/N$ and $1/N$, respectively. $\underline{X}$ is an increasing function of N. Equation (4) yields
aggregate profit, $\underline{\pi}$, as

$$\underline{\pi} = [F(\underline{X})-\underline{X}F'(\underline{X})]/N > 0, \quad (5)$$

implying that rents are not entirely dissipated. In Figure 1, $\underline{\pi}$ is the area of the rectangle JKLM.

From equation (5) we find that profit per herdsman is

$$\underline{\pi}/N = [F(\underline{X}) - \underline{X}F'(\underline{X})]/N^2 > 0. \qquad (6)$$

In Figure 1, which depicts the case $N = 2$, the equilibrium pair of profits $(\underline{\pi}/2, \underline{\pi}/2)$ is the point A.

Although $\underline{X}/N$ is the equilibrium number of cattle per herdsman, it isn't a dominant strategy for the representative herdsman: *CPRs do not give rise to the Prisoners' Dilemma game. The profit level of each party exceeds his or her min-max profit level, which is* 0.

If N is large, the unmanaged CPR approximates an open access resource and

$$F(\underline{X})/\underline{X} \approx p. \tag{7}$$

The approximate equation (7) says that rents are dissipated almost entirely.

Community Optimum

An unmanaged CPR would be unattractive to the herdsmen: they could increase their profits by managing it together. Imagine that reaching an agreement point over the spoils of the timeless model involves negligible transaction costs. What would be a reasonable agreement among the herdsmen? As the model is symmetric, plausibly they agree to maximise aggregate profit and share that profit equally. Maximising aggregate profit ($F(X)$-pX) yields the condition

$$F'(X) = p. \tag{8}$$

Write X^* as solution of equation (8).

Comparison of equations (4) and (8) shows that $\underline{X} > X^*$. In Figure 1, which depicts the case $N = 2$, the pair of profits $(\pi^*/2, \pi^*/2)$ at the community optimum is the point B.

Reciprocity in Repeated Interactions

Suppose the non-cooperative game we have studied repeats itself indefinitely. Here we explore mutual enforcement of the agreed-upon choice x^* in each period via a long-term relationship among the N agents. The basic idea is this: A credible threat by members of a community that stiff sanctions would be imposed on anyone who broke an agreement could deter everyone from breaking it. The problem then is to make the threat of stiff sanctions credible. The solution to the credibility problem in this case is achieved by recourse to social norms of behaviour.

By a social norm in the present context we mean an equilibrium strategy for each player that supports x^*. How would the idea of social norms apply to groups wishing to cooperate over the use of CPRs? To answer, denote time by t, where $t = 0,1,2,...$ We assume that the herdsmen discount their future profits at the rate, r. Intuitively, we expect that if r is not too large, the agreement can be realized.

Imagine that at $t = 0$ the herdsmen agree to limit each of their herds to X^*/N cattle. If the agreement is carried out, each herdsman's profit in each period is π^*/N (equation (8)). The question arises as to how the agreement can be enforced.

Consider the following strategy for the representative herdsman: Begin by introducing X^*/N cattle into the pasture and continue to bring in X^*/N so long as no herdsman has broken the agreement; but introduce $\underline{X}/N$ cattle into the pasture in every period following the first violation of the agreement by someone ($\underline{X}/N$ being the herd size per cattleman in the unmanaged CPR (equation (4)).

This is called the "grim strategy", or simply *grim*, because of its unforgiving nature.

The claim is that if r is not too large, the threat by someone to switch permanently to $\underline{X}/N$ (which is the equilibrium size in the unmanaged commons) following the first defection by anyone is credible if all other herdsmen play grim. Because the herdsmen discount their respective profits at a low rate, no one can do better than to choose grim if all others choose grim. So, grim is a Nash equilibrium of the repeated game, meaning that it can function as a social norm.

Problem: Determine the maximum discount rate, r, under which grim works. Let that be r_{max}. Show that if $r > r_{max}$ then there is no social norm that can be invoked to support X^*/N.

Answer: Let w be the maximum profit a herdsman can make in one period when all others play cooperatively. Then

$r_{max} = (\pi^* - \underline{\pi})/Nw$

Note: even when cooperation is a possible equilibrium, non-cooperation is an equilibrium too. The failure to cooperate could be due simply to a collection of unfortunate, self-confirming beliefs, nothing else.

The Dark Side of CPRs

There are two pieces of bad news: (1) inequality (C) and (2) exploitation (D). See Figure 2. Either point can be supported by a suitable norm if r is small enough.

How? The interesting case to consider is D. So imagine that the "agreement" is to support the profit allocation, D. Notice that grim won't do. Why? A more subtle norm has to be invoked, if D is to be implemented in each period.

Call a herdsman a *conformist* if he or she cooperates with others who are conformists but pubishes those who are non-conformists (by ensuring that they are pressed to their min-max profit level, which is 0 in the commons problem). This may sound circular, but it isn't, because the norm requires all parties to begin the repeated game by keeping their agreement. It would then possible for fo any herdsman in any period to determine whch herdsman is conformist aand which herdsman is not. The norm requires that punishment be inflicted not only one those who broke the original agreement (1st order violation), but also those who failed to punish those who broke the orginal agreement (2nd order violation), ont those who failed to punish those who failed to punish those who broke the original agreement (3rd order violation), and so on.

Theorem (Fudenberg and Maskin, 1986): *Provided r is small enough, D can be supported by a social norm of the above kind*.

Moral: The unsuspecting anthropologist will come away marvelling at the way people cooperate by following norms of behaviour, even while the allocation being sustained is one where agent 2 is worse off than she would have been had there been no social norm.

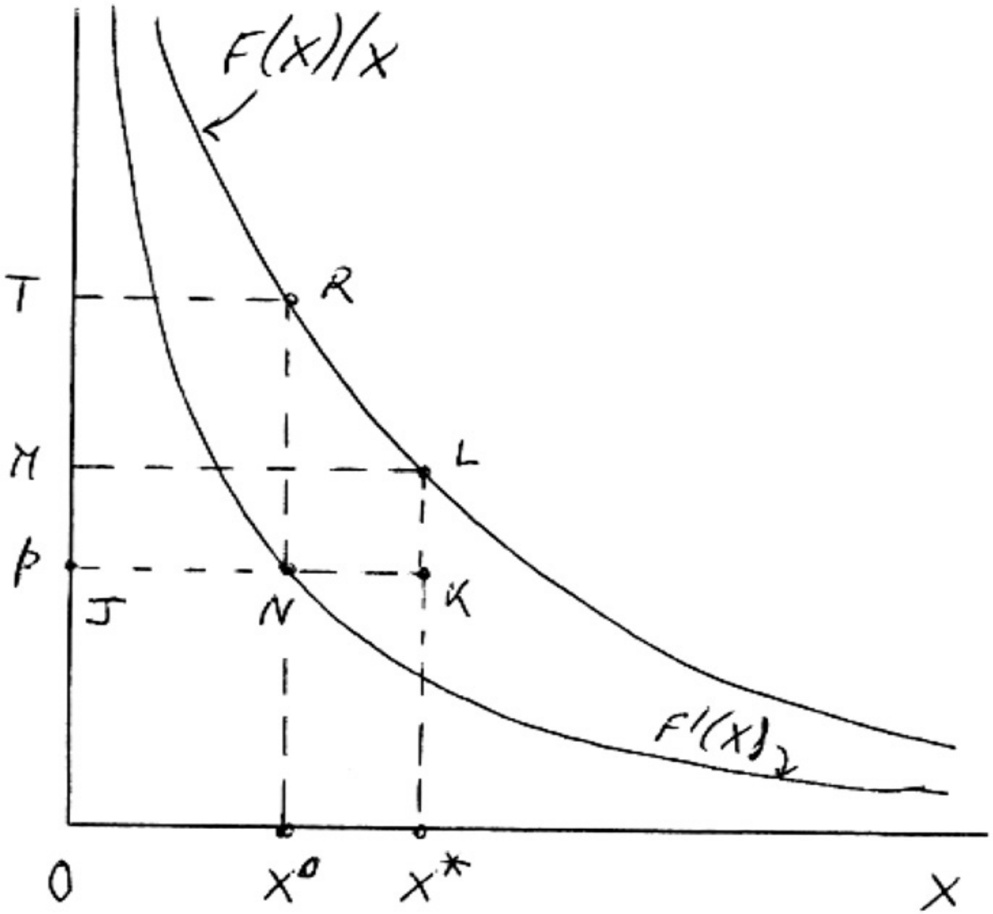

Figure 1

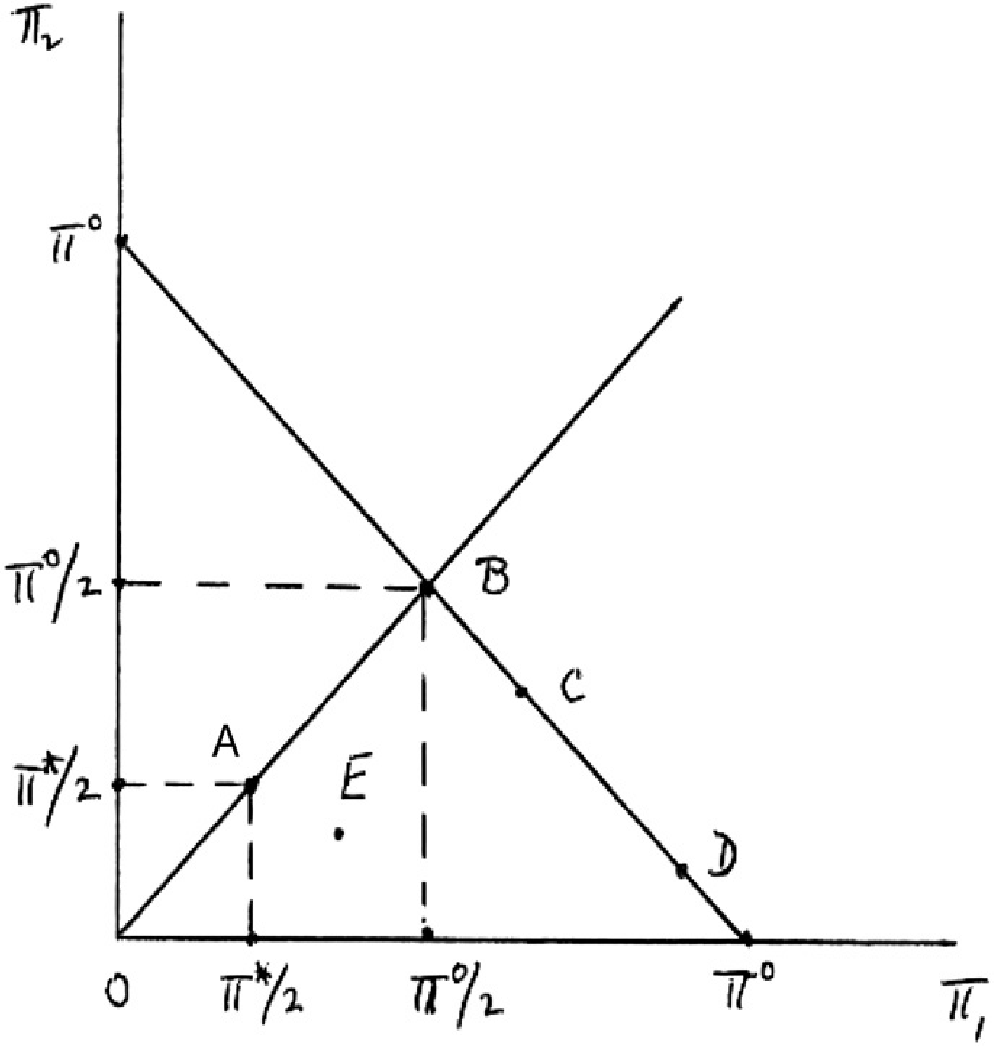

Figure 2