Multi-object filtering methods for multi-target tracking

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What is Target Tracking?

Target tracking algorithms are methods for determining the positions and velocities of moving objects. These are fundamental for applications in

- Aerospace defence
- Maritime surveillance
- Space situational awareness

Figure : Tracking ships in Rotterdam harbour (courtesy of TNO).

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(Loading video)

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Bayes Filtering/ single-object tracking



Bayes filter

$\begin{array}{l} p_{k}(x_{k} \mid z_{1:k}) \xrightarrow{\text{prediction}} p_{k+1|k}(x_{k+1|k} \mid z_{1:k}) \xrightarrow{\text{data-update}} p_{k+1}(x_{k+1} \mid z_{1:k+1}) \\ \text{Kalman filter (Swerling / Stratonovich/ Kalman, late 1950s)} \\ N(x_{k}; m_{k}, P_{k}) \xrightarrow{\text{prediction}} N(x_{k+1|k}; m_{k+1|k}, P_{k+1|k}) \xrightarrow{\text{data-update}} N(x_{k+1}; m_{k+1}, P_{k+1}) \\ \text{Particle filter (Handschin & Mayne, Imperial 1966/ N. Gordon, Imperial 1993)} \end{array}$

$$\{x_k^{(i)}, w_k^{(i)}\}_{i=1}^N \xrightarrow{\text{prediction}} \{x_{k+1|k}^{(i)}, w_{k+1|k}^{(i)}\}_{i=1}^N \xrightarrow{\text{data-update}} \{x_{k+1}^{(i)}, w_{k+1}^{(i)}\}_{i=1}^N \xrightarrow{\text{data-update}} x_{k+1}^{(i)} \xrightarrow{\text$$

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Bayes Filtering: Prediction



Prediction $p_{k+1|k}(x_{k+1} \mid z_{1:k}) =$

$$\int \underbrace{f_{k+1|k}(x_{k+1} \mid x_k)}_{\downarrow} p_k(x_k \mid z_{1:k}) dx_k$$

Markov transition density

Bayes Filtering: Data Update



Update $p_{k+1}(x_{k+1} | z_{1:k+1}) =$ $\frac{g_{k+1}(z_{k+1} | x_{k+1}) p_{k+1|k}(x_{k+1} | z_{1:k})}{\int \underbrace{g_{k+1}(z_{k+1} | x_{k+1})}_{\downarrow} p_{k+1|k}(x_{k+1} | z_{1:k+1}) dx_{k+1}}$

observation likelihood

Multi-Target Tracking



The objective in multi-target tracking is to

- Jointly estimate **both** the number of targets and their states.
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Bayesian Multi-Object Filtering



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A spatial point pattern gives the locations of a set of objects $\mathbf{Y} = \{\mathbf{y}_1, \dots, \mathbf{y}_n\}$ in some region of the space e.g. $\mathbf{y}_i \in \mathscr{Y}$, where $i = 1, \dots, n$ and $\mathscr{Y} \subseteq \mathbb{R}^{n_y}$

- 2-dimensional positions of objects in an image from a sensor (i.e. observation space),
- 3-dimensional positions and velocities of objects in some real-world environment (i.e. state space).



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Point Process Representation

A spatial point process is therefore characterised with the following

Table : Point Process Representation

Number of objects	Cardinality probability	Joint spatial density
0	ho(0)	-
1	ho(1)	$p_1(x_1)$
2	$\rho(2)$	$p_2(x_1, x_2)$
3	$\rho(3)$	$p_3(x_1, x_2, x_3)$
4	$\rho(4)$	$p_2(x_1, x_2, x_3, x_4)$
n	$\rho(n)$	$p_n(x_1, x_2, x_3, x_4, \ldots, x_n)$

Point processes

Definition (Probability generating functional - Moyal 1962)

The probability generating functional G_{Φ} of a process Φ can be written for $w \in \mathscr{U}(X)$ as

$$G_{\Phi}(w) = J_{\Phi}^{(0)} + \sum_{n \ge 1} \frac{1}{n!} \int w(x_1) \dots w(x_n) J_{\Phi}^{(n)}(x_1, \dots, x_n) d^n x_n$$

where $J^{(0)}_{\Phi}$ is the probability for the population to be empty and where $d^n x$ stands for $dx_1 \dots dx_n$.

THE GENERAL THEORY OF STOCHASTIC POPULATION PROCESSES

BY

J. E. MOYAL

Australian National University, Canberra, Australia (1)

Point processes

Example (Probability generating functional -Moyal 1962)

Taking the k^{th} -order variation of $G_{\Phi}(w)$ in the directions $\{\xi_1, \ldots, \xi_k\}$, we have,

$$\delta^{k} G_{\Phi}(w;\xi_{1},\ldots,\xi_{k}) = \sum_{n \geq k} \frac{1}{(n-k)!} \int \xi_{1}(x_{1}) \ldots \xi_{k}(x_{k}) w(x_{k+1}) \ldots w(x_{n}) J_{\Phi}^{(n)}(x_{1},\ldots,x_{n}) d^{n} x.$$

We can recover the Janossy densities $J^{(n)}_{\Phi}(x_1,...,x_n)$ and factorial moments $M^{(n)}_{\Phi}(x_1,...,x_n)$ with

$$J_{\Phi}^{(n)}(x_1,...,x_n) = \delta^k G_{\Phi}(0;\xi_1,...,\xi_k),$$
$$M_{\Phi}^{(n)}(x_1,...,x_n) = \delta^k G_{\Phi}(1;\xi_1,...,\xi_k).$$

THE GENERAL THEORY OF STOCHASTIC POPULATION PROCESSES

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L E MOYAL

Point Process Intensity

The intensity function of a point process is the average density of points (expected number of points per unit area).

Intensity function \equiv Probability Hypothesis Density (PHD); notation D(x)



Point Process Intensity (Cont'd)

• The intensity may be constant across the region (homogeneous)

• or some regions may have higher intensity than others (inhomogeneous):



(samples from: homogeneous PP - left; inhomogeneous PP - right)

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Point Process Superposition

- Often we observe different (independent) point patterns in the same region originating from different point processes (e.g. false and true target detections).
- We can model this phenomenon as the superposition of independent point processes.







 X_1

 X_2



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 X_1 X_2 $X_1 \cup X_2$

The Poisson point process with Poisson rate $\lambda > 0$ has the following properties

- The expected number of objects in the region is λ .
- The locations of the points are independent and identically distributed according to some spatial distribution s(x).
- Intensity function of a Poisson point process $D(x) = \lambda \cdot s(x)$.



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PHD filtering - Poisson-based multi-target tracker

(Loading video)

Figure : Object tracking in sonar images

The first industrial application of the PHD filter was for oil pipeline tracking for BP (2006) in SeeByte Ltd.

Multi-object modelling for target detection & tracking



Figure :

http://www.nollywoodone.com/latestadditions/9009-the-u-s-military-s-realtime-google-street-view-airborne-spycamera-can-track-an-entire-city-in-1-800mp.html Populations of objects modelled with *point processes*

Multi-object modelling for target detection & tracking



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- Target population;
- Target interactions;
- Target measurements;
- Missed detections;
 - False alarms;
 - Target appearing/disappearing;

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Multi-object Bayes filtering: prediction



The evolution of the multi-object process is specified with the composite functional

$$G(h) = G_k \left(G_{k+1|k}(h|\cdot) \right)$$

Multi-object Bayes filtering: update



The joint functional describing the relation between targets and measurements is specified through the functional

$$G(v,w) = G_k(vG_L(w|\cdot)).$$

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• Exploitation of higher-order statistics on point processes, notably the *region-based variance in target number*.



What is going on inside *B*?

"There are roughly $\mu_{\Phi}(B)$ targets, give or take $\sqrt{var_{\Phi}(B)}$, within *B*".

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"There are roughly $\mu_{\Phi}(B)$ targets, give or take $\sqrt{var_{\Phi}(B)}$, within *B*". Sensor control using variance

Example: Cluster Processes

Cluster Processes are formed with the composition of functionals, eg. $G(\psi) = G_1(G_2(\psi))$.

Multi-Group Multi-Object Bayes Filtering

Group-Target State Variable: $X = \{(c_1, X_1), \dots, (c_n, X_n)\}$



 $p_k(\mathbb{X}_k \mid Z^{(k)}) \xrightarrow{\text{prediction}} p_{k+1|k}(\mathbb{X}_{k+1} \mid Z^{(k)}) \xrightarrow{\text{data-update}} p_{k+1}(\mathbb{X}_{k+1} \mid Z^{(k+1)})$

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Tracking Multiple Group Targets

Measurements from the displaced camera are conditional on the calibration parameters.







Estimating calibration parameters via tracking of paper airplanes





Estimating calibration parameters via tracking of paper airplanes





Estimating calibration parameters via tracking of paper airplanes

- Parent process is the vehicle location
- Daughter process is map, conditioned on vehicle location



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Multi-sensor fusion -maritime surveillance (CDE with BAE Systems, DSTL)

EO/radar data fusion using PHD filter estimation.













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