Global Optimisation in Space Applications: a Review

Joerg Fliege & Walton Coutinho

University of Southampton

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Overview

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2. Current State of the Art in Global Optimisation
3. Optimisation Problems in Space Applications
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Introduction & Motivation

Introduction & Motivation

- Large-scale sparse NLP solvers have increased capacity and capability in various areas (trajectory opt., design opt., maneuvre opt., materials & structures opt.)
- Examples: IPOPT, KNITRO, SNOPT, WORHP, etc.
- But all these solvers are *local* optimisers ‘only’!
- Therefore: investigate and assess the use of *global* optimisation methods for space engineering problems.

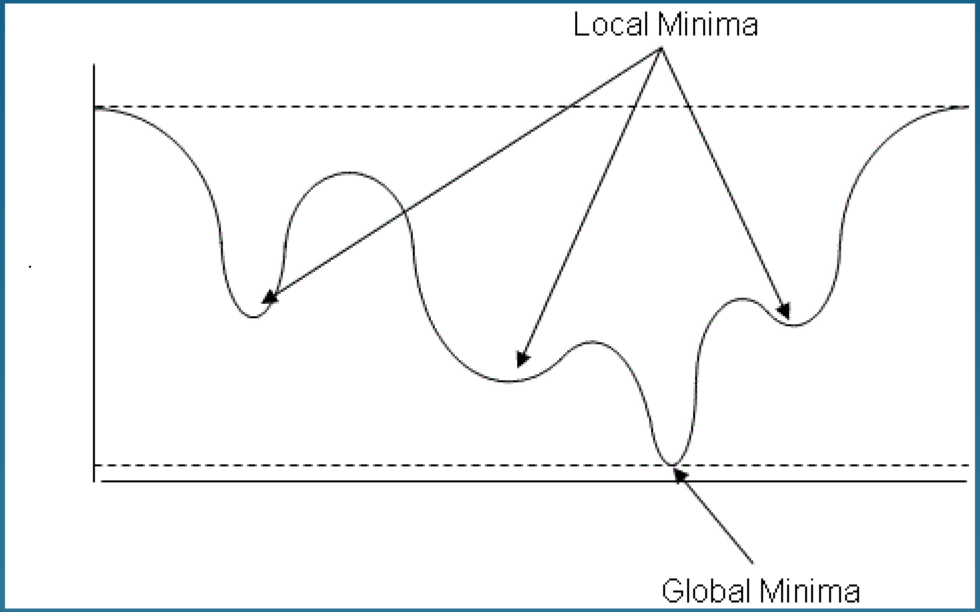
Current State of the Art in Mathematical Methods for Global Optimisation

Mathematical methods for global optimisation: state of the art

Global vs local optimisation:

Local optimisation means finding one local minimum, but not being able to control which one.

Global optimisation means finding the global minimum, or possibly a list of *k* best minima.

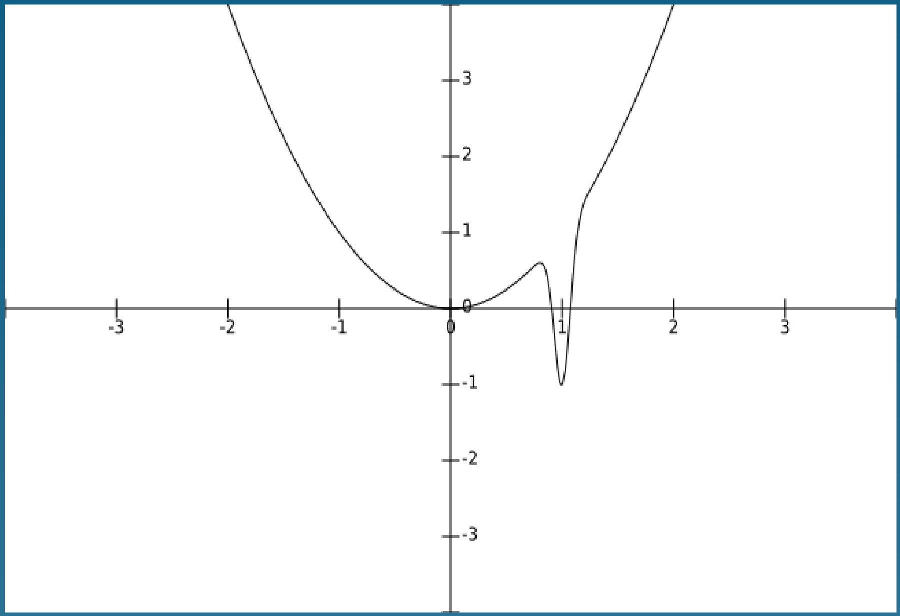


From a theoretical point of view, global optimisation is **hard** (as in “hopelessly hard”): easy to construct problems with exponential number of solutions.

Mathematical methods for global optimisation: state of the art

Further issues:

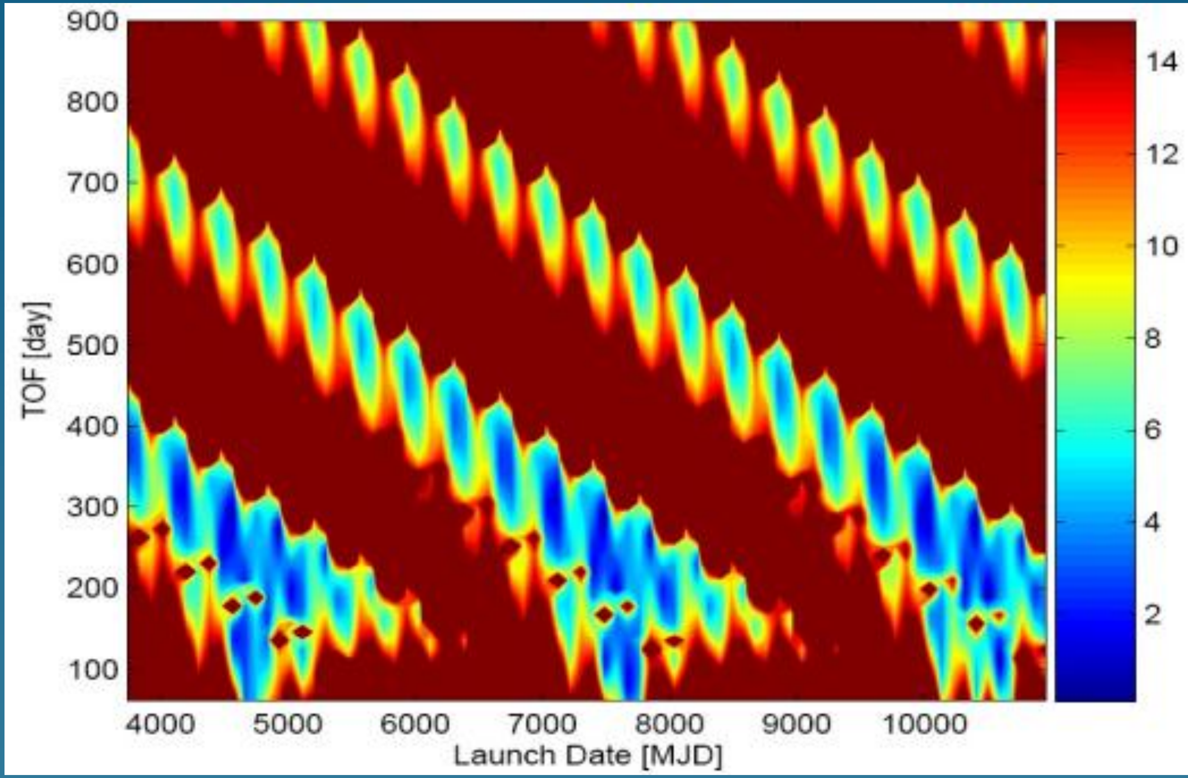
Minima might be susceptible to noise, unknown (badly estimated parameters, etc.)



- **Local optimisation**: can solve *structured smooth* problems with 1.000.000.000 variables & constraints.
- **Local optimisation**: can solve *nonsmooth* with 1.000 variables & constraints.
- **Global optimisation**: can solve unstructured problems with 20 variables?

Mathematical methods for global optimisation: state of the art

Objective function (Delta-V) for an Earth-Apophis transfer problem with two design variables. Note the various "valleys" with different local optima



(Vasile et al 2011)

However: many problems in space engineering appear to have exploitable structures.

Mathematical methods for global optimisation: Algorithms

Summary of algorithms state of the art:

- Mathematical theory sufficient mature and well developed.
- Branch and bound most promising approach for deterministic global optimisation. (Because: lower & upper bounds, optimality certificates, can handle nonlinear constraints, can exploit problem structure.)
- D.C. Programming might be useful due to its versatility.
- Stochastic methods cannot provide proofs of optimality or certificates of accuracy.

Existing software for global optimisation

Key findings, part I:

- The majority of existing global optimisation solvers are sandboxes for academic research.
- Maintenance, bug fixes, enhancements, licensing etc., depend primarily on academic interests and/or personal passions.
- Industrial grade software products (coding standards, support, documentation) are very scarce.
- Solver performance characteristics are often driven by particular academic interests.

Existing software for global optimisation

Key findings, part II:

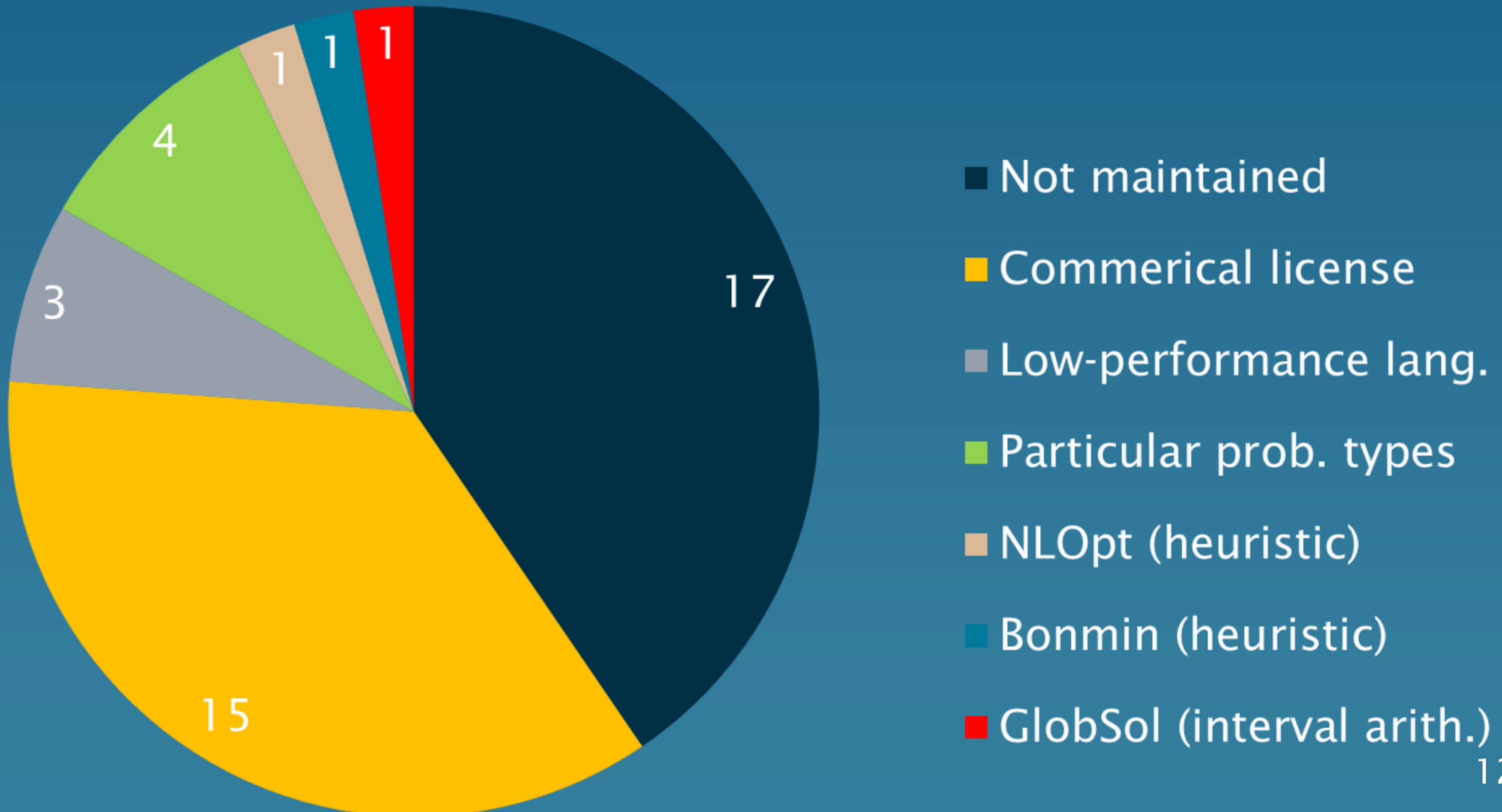
- Almost all solvers implement only one solution strategy instead of a flexible framework of methods.

- Development competition is often driven by performance on test set libraries of academic nature, rather than practical problems from space engineering.

- Most solvers do not exploit problem structures like sparsity, that the problem is a discretised control problem, or other information.

Existing software for deterministic global optimisation

(Not depicted: implementations of stochastic approaches.)



Current state of the art in global optimisation

Major conclusions:

- Mathematical theory now sufficiently mature to be employed.
- Some mathematical approaches have successfully been translated into prototype codes of varying quality.
- Few to none of the existing software solutions are able to be used in an industrial setting; no appropriate software by a European provider exist.

Optimisation Problems in Space Engineering

Optimisation in space applications

- Over **900** publications analysed, the majority from the years 2001-2014.
- Optimisation requirements (solution ‘quality’, feasibility, maximum computation time, maximum discretisation size, etc.) gathered.
- Particular issues (convergence, computation time exceeded, local minima *vs*. global minima, etc.) identified.
- See FR for ESA study contract no. 5401001690 for details.

Optimisation in space applications

In which application areas is (global) optimisation heavily required?

- Worst case analysis
- Trajectory optimisation
- Launcher design optimisation
- Safety analysis
- Dynamic systems analysis
- Closed loop guidance
- Fault detection analysis
- Control structure interactions

Optimisation in space applications

- Various types of optimisation problems (nonconvex, nonsmooth, noisy, mixed-integer, simulation-based, multiobjective, etc.) widespread in space engineering.
- Widespread use of classical (local) optimisation techniques (SQP, IP, SLP, etc.) and corresponding solvers: IPOPT, KNITRO, SNOPT, WORHP, etc.
- Some use of stochastic global search methods of limited efficiency (evolutionary algorithms, simulated annealing, differential evolution, etc).
- Few efforts to use efficient deterministic global optimisation techniques that find global optima reliably and robustly with mathematical certainty.

Recommendations & Road Map

Recommendations & Road Map

State of the Art I:

Demand: strong demand in space engineering for high performance deterministic global optimisation solvers

Problem requirements: wide variety of problems, no single solver expected to perform well on all.

Algorithms: theory sufficiently mature.

Recommendations & Road Map

State of the Art II:

Algorithms: for deterministic algorithms, branch & bound most promising approach.

Software: existing software mostly sandboxes; no production & maintenance efforts at industrial standards; no exploitation of problem structures.

Problem libraries & test cases: collections small or nonexistent.

Recommendations & Road Map

State of Art & Shortcomings	Recommendation
Problem libraries. Existing problem libraries are small (9 resp. 57 problems) & focus on particular problem types.	Extend libraries so that statistically significant information can be generated. Provide full transcriptor service for problem generation.
Performance indices. Performance indices for stochastic global optimisation solvers are not very well developed	Review and extend existing performance indices for optimisation problems occuring in space engineering.
Model fidelities in trajectory optimisation and elsewhere. Many trajectory optimisation problems use low-fidelity models (<11 variables).	Consider models of higher fidelity and larger number of variables using direct transcription and assess nature of global minima.

(See Section 1.9 of FR 5401001690 for full list.)

Recommendations & Road Map

State of Art & Shortcomings	Recommendation
Deterministic optimisation solvers. No study has been undertaken comparing performance of stochastic optimisation solvers with modern deterministic optimisation problems on space engineering optimisation problems.	Provide thorough benchmark study.
Uncertainties in optimisation. Role & effects of uncertainties in parameter values of optimisation models appears to be unclear, in general.	Assess, analyse, and quantify the role of uncertainties in global trajectory optimisation, in particular w.r.t. robust problem formulations.
Multiobjective optimisation. Efficient techniques are absent in the treatment of multiobjective optimisation problems.	Assess current state-of-the-art and current research on mathematical methods for multiobjective optimisation.

(See Section 1.9 of FR for full list.)

Recommendations & Road Map

Topic	Recommendation
Problem Libraries	Built library of all types of relevant space engineering optimisation problems, spanning difficulties ranging from simple to extremely challenging
Global solvers	Develop a deterministic global optimisation solver based on branch-and-bound, and incorporating several bounding techniques for increased versatility.
Local solvers	Leverage on mature developments in local optimisation by including interfaces to local solvers in order to rapidly improve solution estimates and provide tight bounds for objective function values.

Any questions or comments?