The Statistical Analysis of Sounds and Pictures
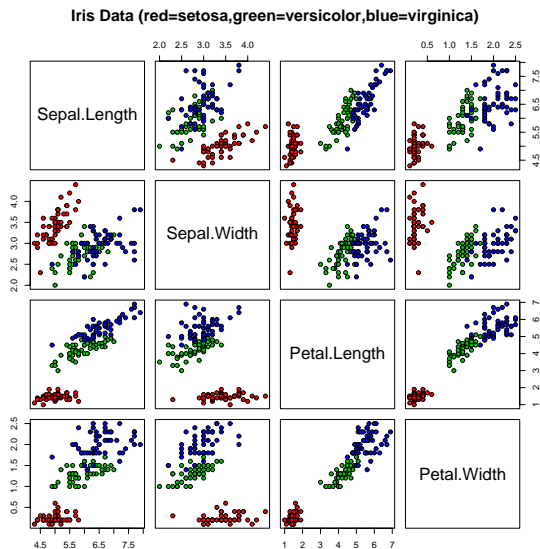
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9 May 2016

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(Cambridge), John Coleman (Oxford)

Traditional Statistical Data Analysis

Iris Data (red=setosa,green=versicolor,blue=virginica)

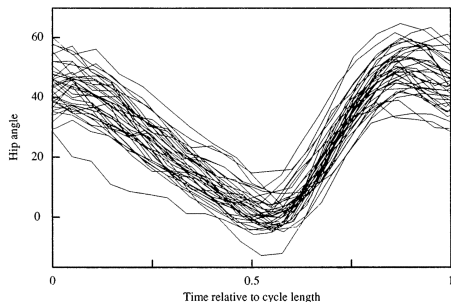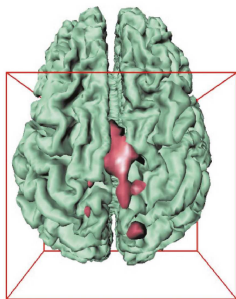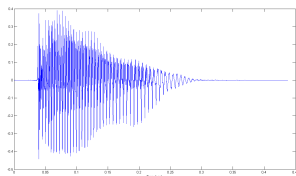Fisher (1936) Iris Data

Functional Data Analysis

Fig. 1. Observed records of angular rotations of the hip in the saggital plane over one gait cycle in each of 39 subjects
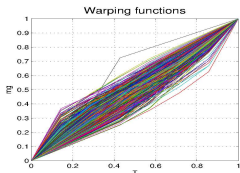
Rice and Silverman (1991) Gait Data
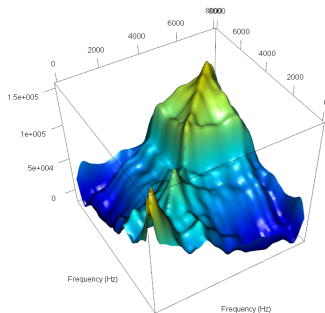
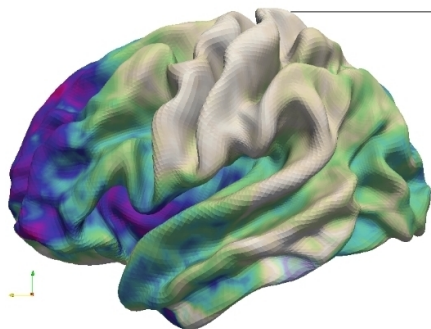# Modern Functional Data Analysis

Images

Sounds

Monotonic Functions

Manifold Valued
Functional Data

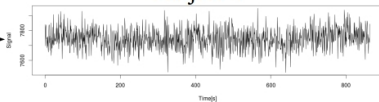# Dynamic Data
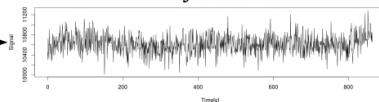

- Resting state fMRI
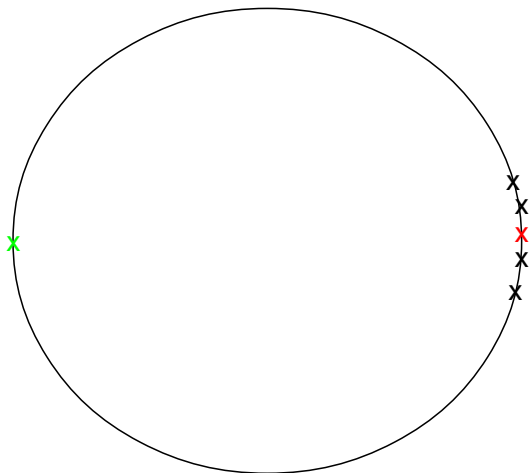- Preprocessing
- Anatomical alignment

Simple Non-Euclidean Statistics

What is the mean of these four numbers: 10, 20, 340, 350?

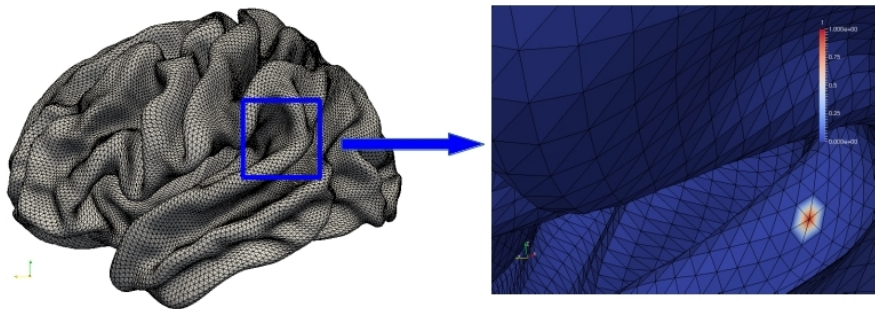# Simple Non-Euclidean Statistics

What is the mean of these four numbers: 10°, 20°, 340°, 350°?

The cortical surface
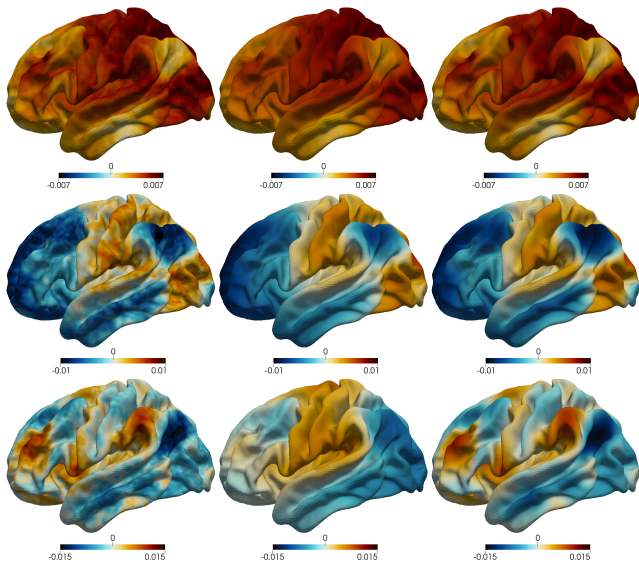
Surface Finite Element

- Finite Element Basis (from Applied Mathematics)
- Highly Computationally Efficient
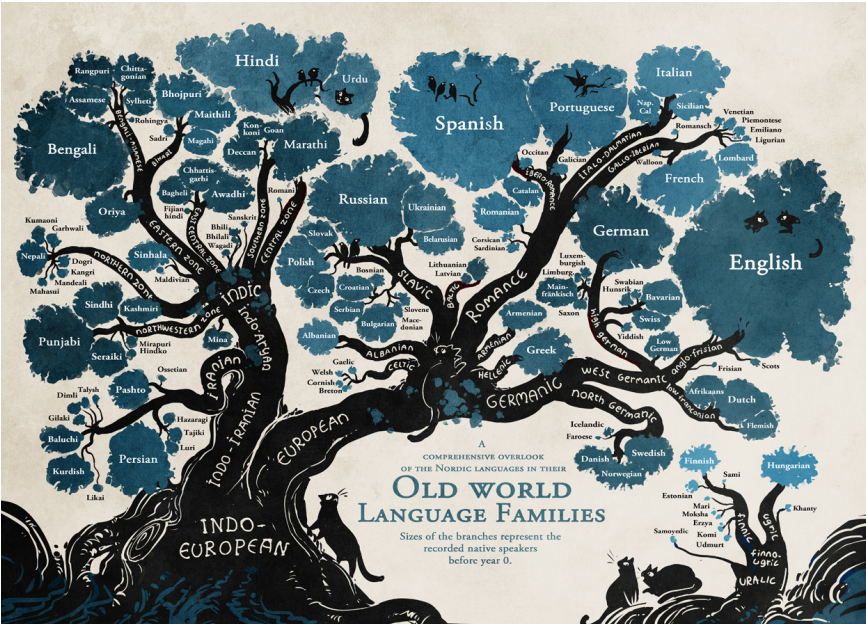- Allows manifold nature to be incorporated into the statistics
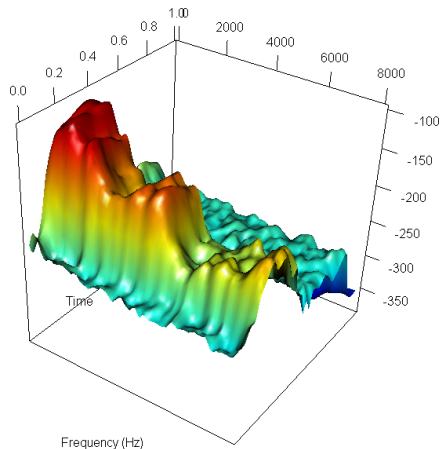
Smooth-Manifold FPCA
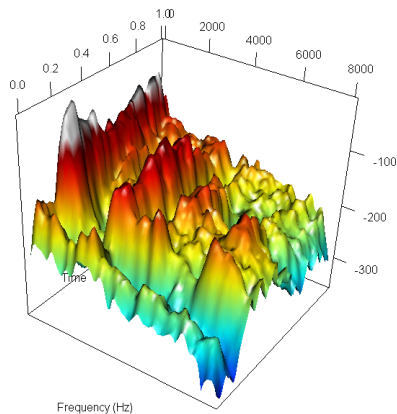
Linguistic Relationships
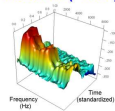
Acoustic Phonetic Data

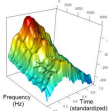Un (one in French)   Um (one in Portuguese)

Example of Language Transformation

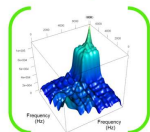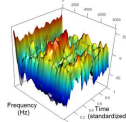**Sound surface for a French speaker pronouncing the word *un* ("one")** = **Average sound surface for *un* ("one")** + **Frequency cova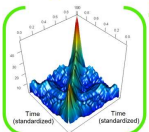riance structure for French s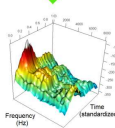peakers** $^{\frac{1}{2}}$ × **Speaker-specific residual** × **Time covariance structure for French speakers** $^{\frac{1}{2}}$
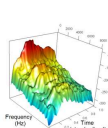
$$S_1^{F \to P} = \overline{S_1^P} + [(C_\omega^P)^{1/2} \otimes (C_t^P)^{1/2}][(C_\omega^F)^{-1/2} \otimes (C_t^F)^{-1/2}](S_1^F - \overline{S_1^F})$$

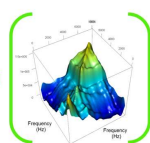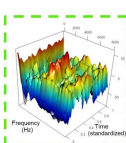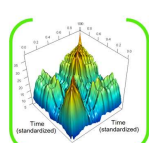The speaker-dependent information is transposed in the sound generation process for the Portuguese language.

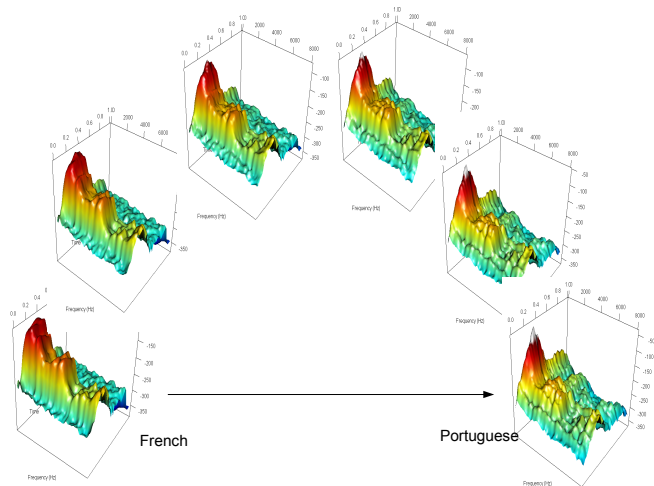**Projection of the French speaker in the population of Portuguese sound** = **Average sound surface for *um* ("one")** + **Frequency covariance structure for Portuguese speakers** $^{\frac{1}{2}}$ × **Speaker-specific residual** × **Time covariance structure for Portuguese speakers** $^{\frac{1}{2}}$
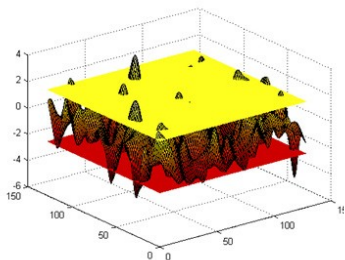
Changing a speaker from one language to another

(Pigoli et al, *arXiv*, 2015)
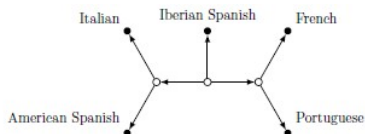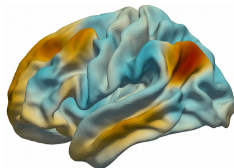

French

Portuguese

Number Seven

The Future - Joining Pure Maths, Applied Maths and Statistics

Topological Data Analysis

Geometry and Statistics

Linking Applied Computational Analysis and Statistics

$$(\hat{\mathbf{u}}, \hat{f}) = \underset{\mathbf{u}, f}{\operatorname{argmin}} \sum_{i=1}^{n} \sum_{j=1}^{s} (x_i(p_j) - u_i f(p_j))^2 + \lambda \mathbf{u}^T \mathbf{u} \int_{\mathcal{M}} \Delta_{\mathcal{M}}^2 f$$

The Future