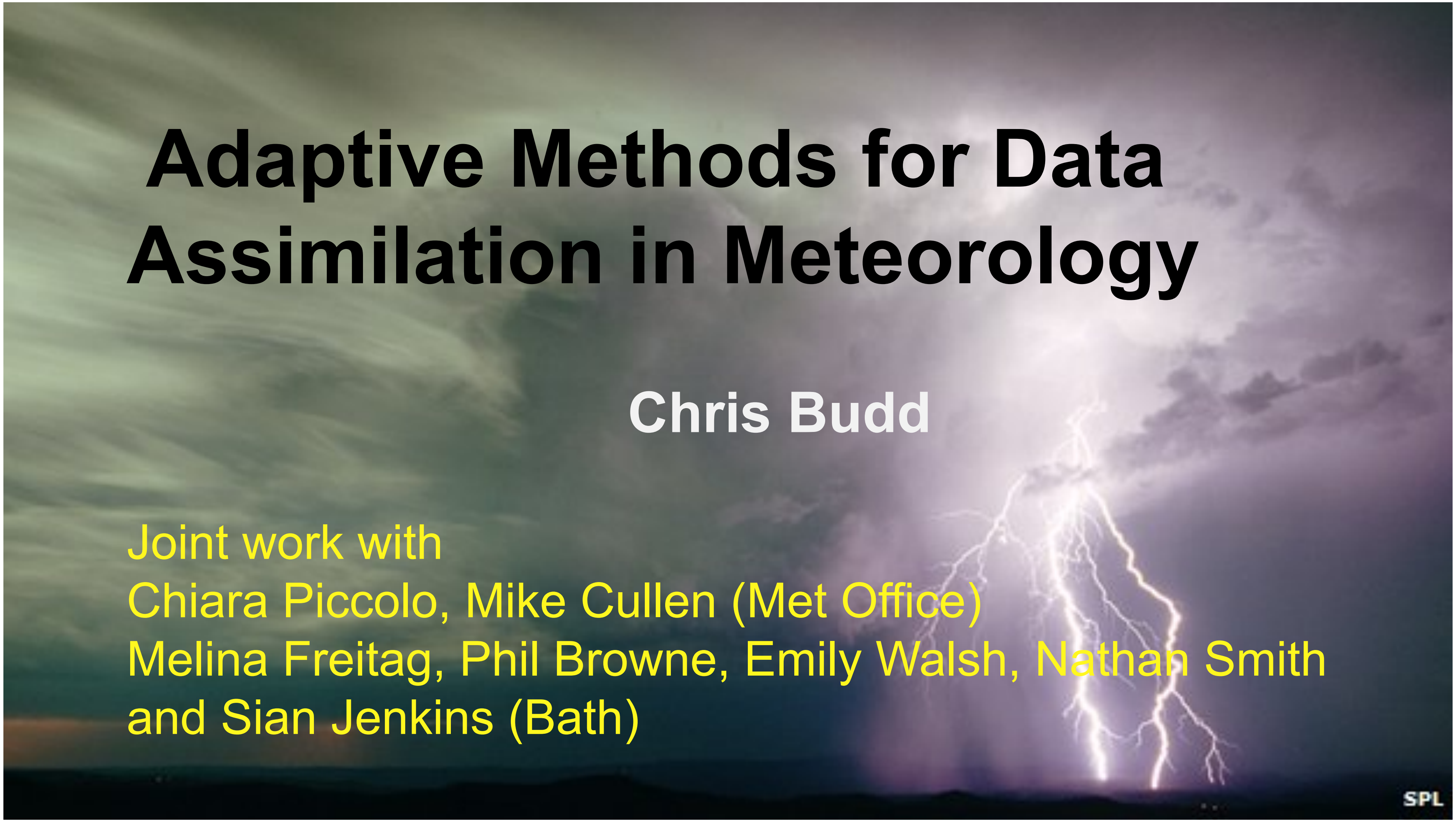
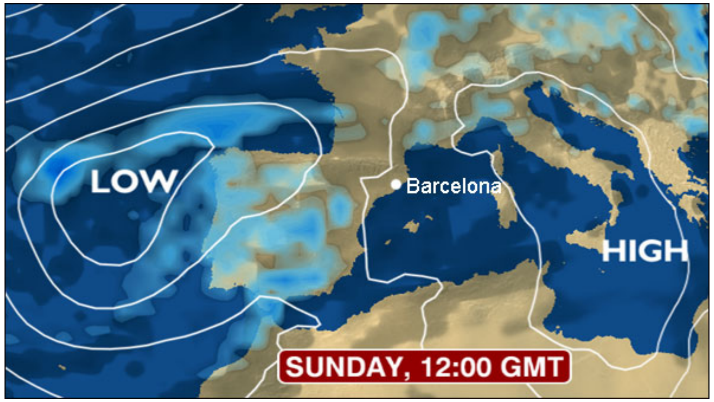


Adaptive Methods for Data Assimilation in Meteorology

Chris Budd

Joint work with
Chiara Piccolo, Mike Cullen (Met Office)
Melina Freitag, Phil Browne, Emily Walsh, Nathan Smith and Sian Jenkins (Bath)

Understanding and forecasting the weather is essential to the future of planet earth and maths place a central role in doing this



Accurate weather forecasting is a mixture of

- Careful modelling of the complex physics of the ocean and atmosphere
- Accurate computations on these models
- Systematic collection of data
- A fusion of data and computation

Data assimilation is the optimal way of combining a complex model with uncertain data

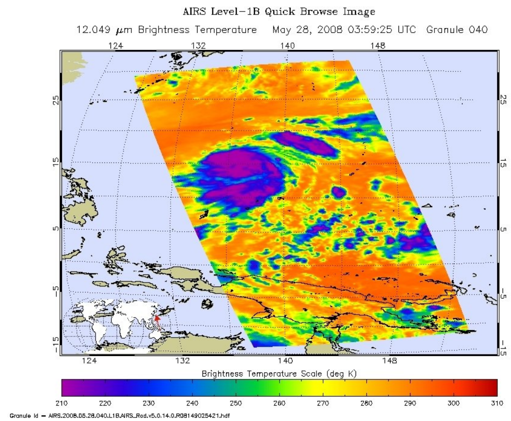
Basic Idea of Data Assimilation

True state of the weather is x_t

Numerical Weather Prediction NWP calculation gives a predicted state x_b with an error

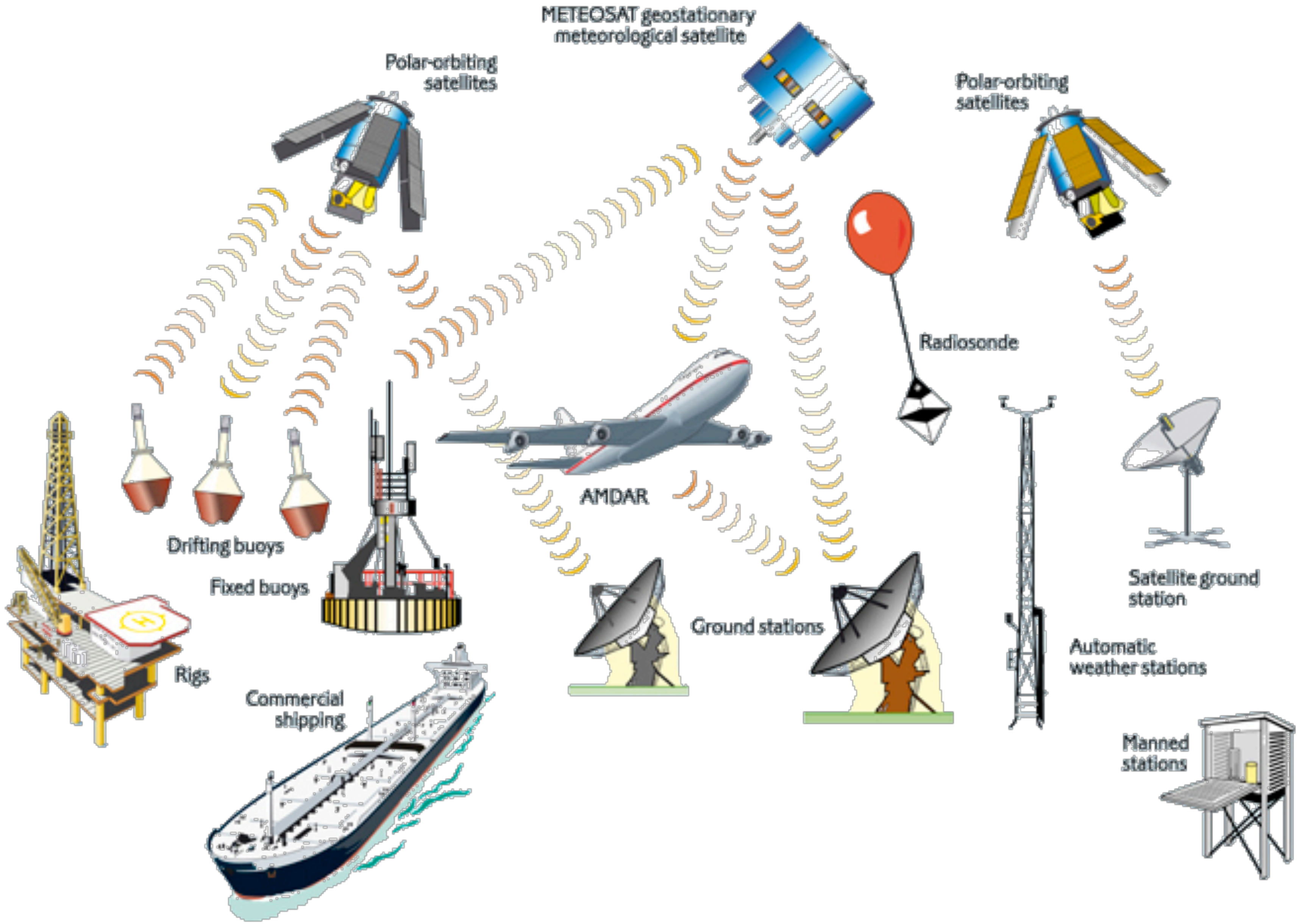
Make a series of observations y of some function $H(x_t)$ of the true state

Eg. Limited set of temperature measurements with error



Now combine the prediction with the observations

Data: Sources of observation



Both the NWP prediction and the data have errors.

Can we optimally estimate the atmospheric state which is consistent with both the prediction and the data and estimate the resulting error?

NOTE: Approximately

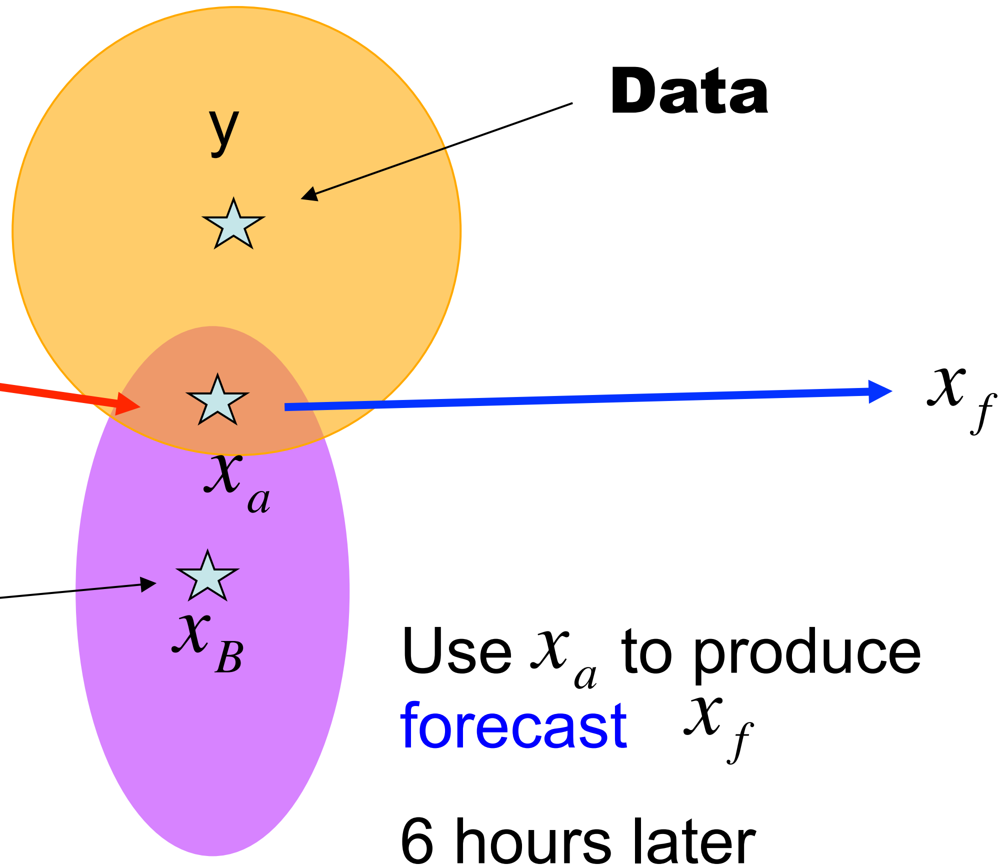
10^9 degrees of freedom

10^6 data points

So significantly underdetermined problem

Best state estimate

analysis



NWP prediction x_B

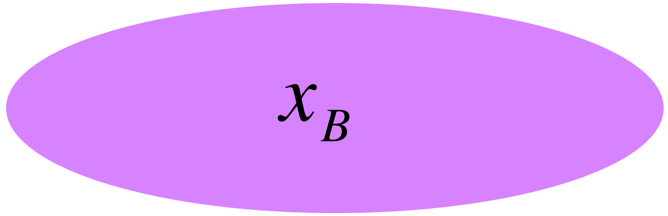
Use x_a to produce forecast x_f

6 hours later

Assume initially:

1. Errors are unbiased Gaussian variables
2. Data and NWP prediction errors are uncorrelated
3. H(x) is a linear operator

Assumptions about the error



Data error: Gaussian, Covariance R

Background (NWP) error: Gaussian, Covariance B

Maximum likelihood of data y given truth x is

$$M = P(x|y)/P(x) = e^{-J(x)}$$

$$\boxed{J(x_a) = \frac{1}{2}\left(x_a - x_b\right)^T B^{-1}\left(x_a - x_b\right) + \frac{1}{2}(Hx_a - y)^T R^{-1}(Hx_a - y)}$$

BLUE: Find x_a which maximises M

So x_a minimises J

Implementation:

Minimise the functional

$$\boxed{J(x_a) = \frac{1}{2}\left(x_a - x_b\right)^T B^{-1}\left(x_a - x_b\right) + \frac{1}{2}(Hx_a - y)^T R^{-1}(Hx_a - y)}$$

This is implemented as 3D-VAR (since 1999 in the Met Office)

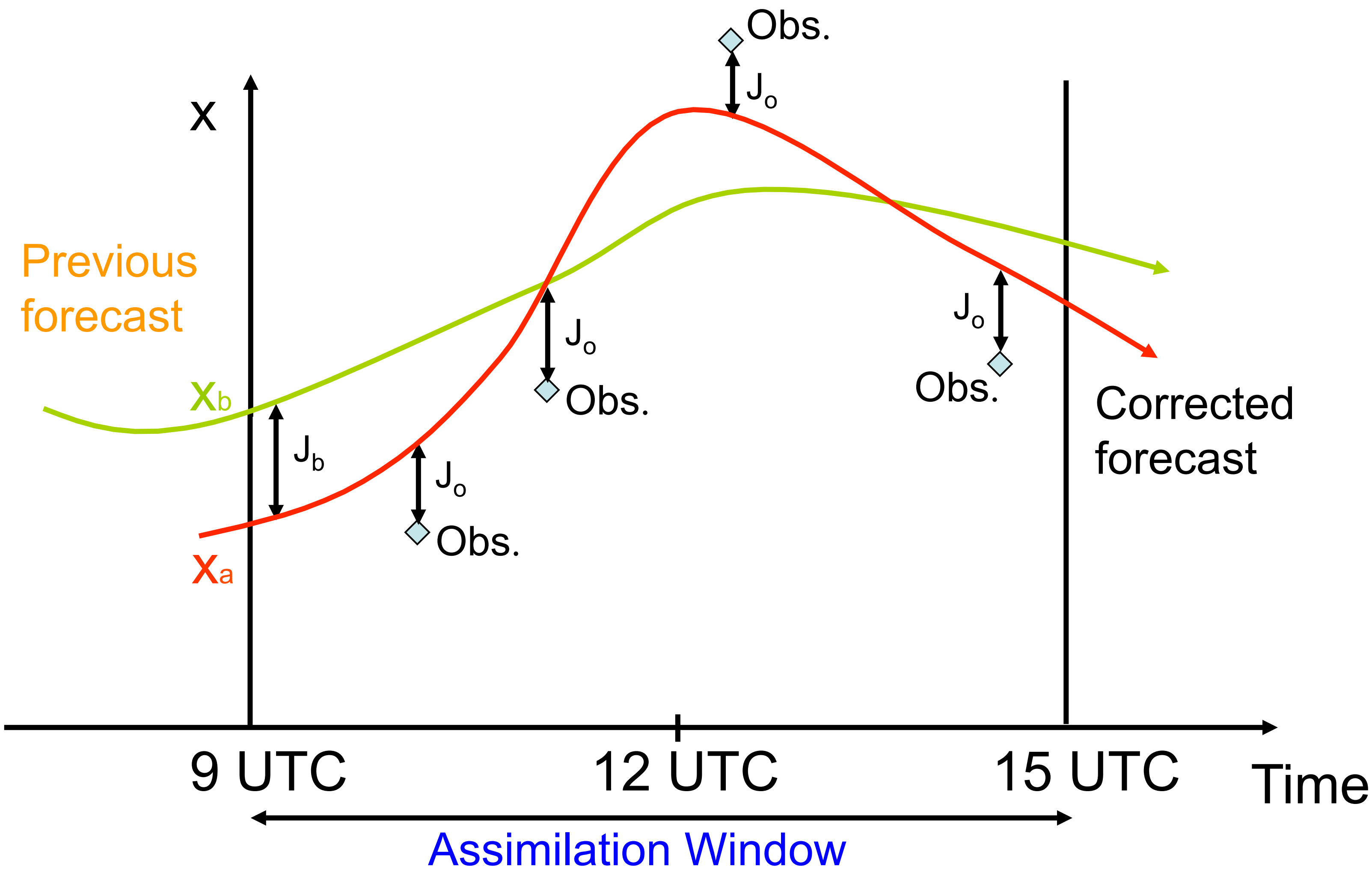
x_B **:** Background, derived from 6 hour NWP forecast

x_a : Analysis

x_f : NWP forecast using x_a as initial data

4D VAR … Preferred variational method

Use window of several observations over 6 hours



Minimise

$$J(x_0) = \frac{1}{2}\left(x_0 - x_b\right)^T B^{-1}\left(x_0 - x_b\right) + \frac{1}{2}\sum_{i=0}^{N}(Hx_i - y_i)^T R^{-1}(Hx_i - y_i)$$

Subject to the strong model constraint

$$x_{i+1} = M_i(x_i)$$

Often assume perfect model, but can also deal with certain types of model error (both random and systematic) by using a weak constraint instead

Estimation of the background and covariance errors

Good estimates of the covariance matrices R and B are important to the effectiveness of 4D-VAR

1. To get the physics correct
2. To avoid spurious correlations between parameters
3. To give well conditioned systems

NOTE: B is a very large matrix, difficult to store and very difficult to update. Impractical to calculate using the Fokker-Plank equation

Build meteorology into the calculation of B through Control Variable Transformations (CVTs)

IDEA: Choose more 'natural' physical variables χ which have uncorrelated errors so that the transformed covariance matrix is block diagonal or even the identity

Set $$\boxed{\delta x = U\chi = U_p U_v U_h \chi, \qquad B = UU^T}$$

Reduces the complexity of the system AND gives better conditioning for the linear systems

U_p^{-1} Separates physical parameters into uncorrelated ones eg. temperature, wind, balanced and unbalanced

U_v^{-1} Reduces vertical correlations by projecting onto empirical orthogonal vertical modes

U_h^{-1} Reduces horizontal correlations by projecting onto spherical harmonics

Effective, but errors arise due to lack of resolution of physical features leading to spurious correlations

[Cullen]

Eg. Problems with stable boundary and inversion layers and assimilating radiosonde data

Poor resolution leads to inaccurate predictions of fog and ice

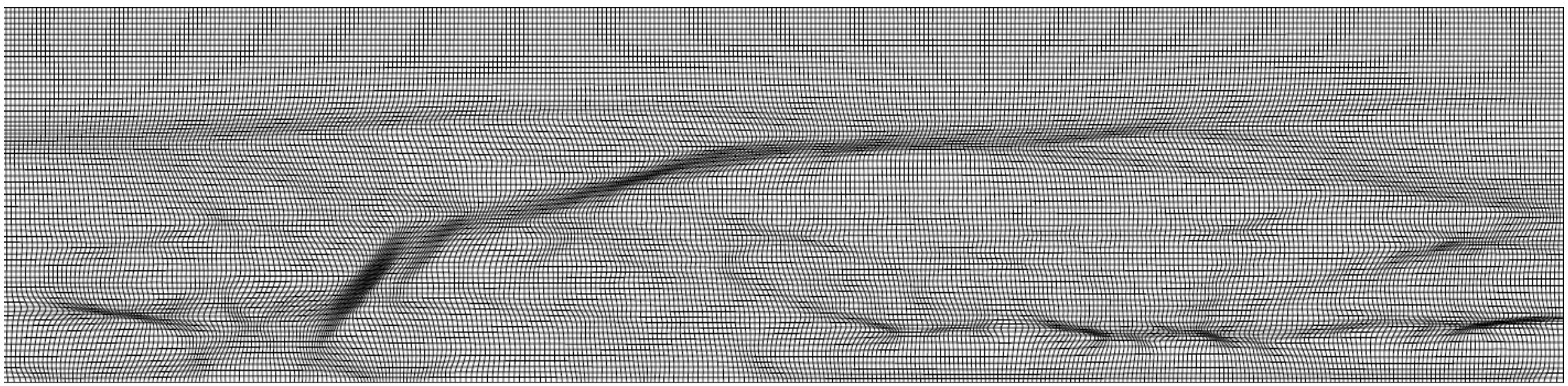

© AFP/GETTY IMAGES

Solution one: increase global resolution

VERY EXPENSIVE!!!

Solution two: locally redistribute the computational mesh to resolve the features

Cheap and effective! [Piccolo, Cullen, B,Browne, Walsh]



Adjust the vertical coordinates to concentrate points close to the inversion layer and reduce correlations

Introduce an extra transformation [Cullen and Piccolo]

$$\delta x = U\chi = U_p U_a U_v U_h \chi, \qquad B = UU^T$$

U_a^{-1}

Adaptive mesh transformation applied to latitude-longitude coordinates

Do this by using tools from adaptive mesh generation methods for PDES

Set: z original height variable

ξ new 'computational' height variable

Relate these via the equation

$$\boxed{\frac{dz}{d\xi} = M(z)}$$

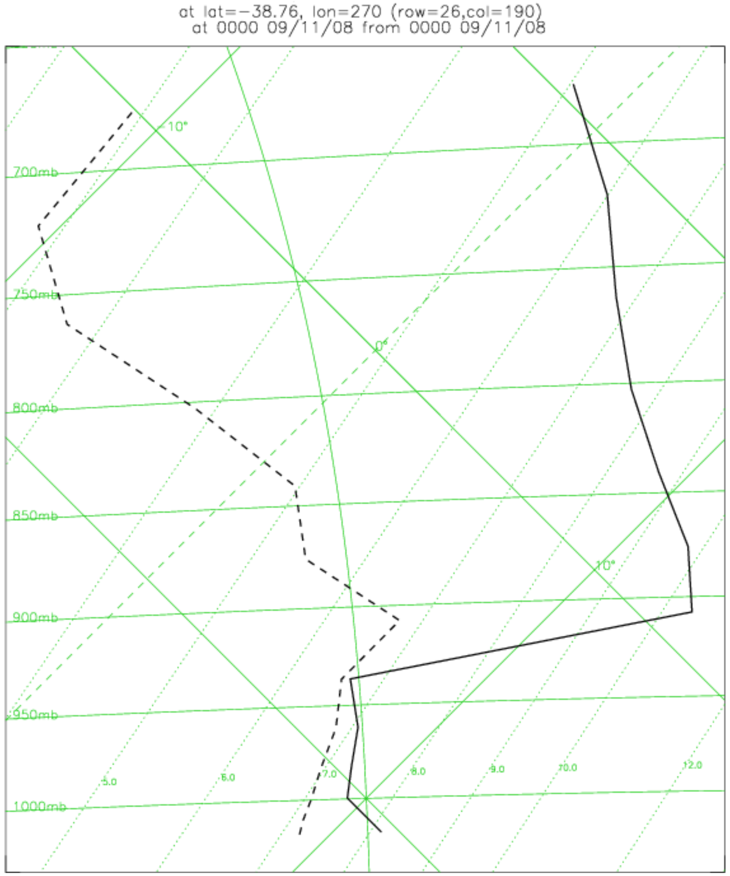
M called the 'monitor function'

[B, Huang, Russell, Walsh]

Take M large if there is active meteorology

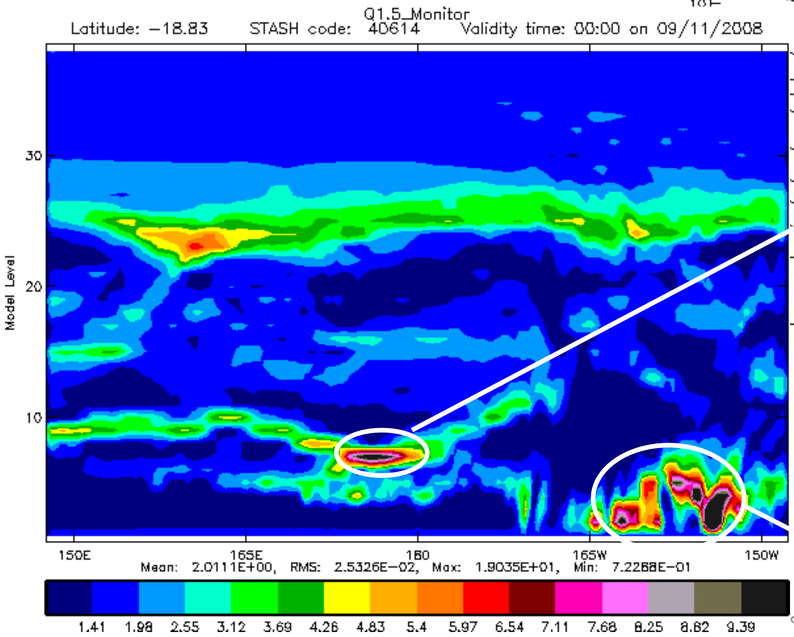
Eg. High potential vorticity

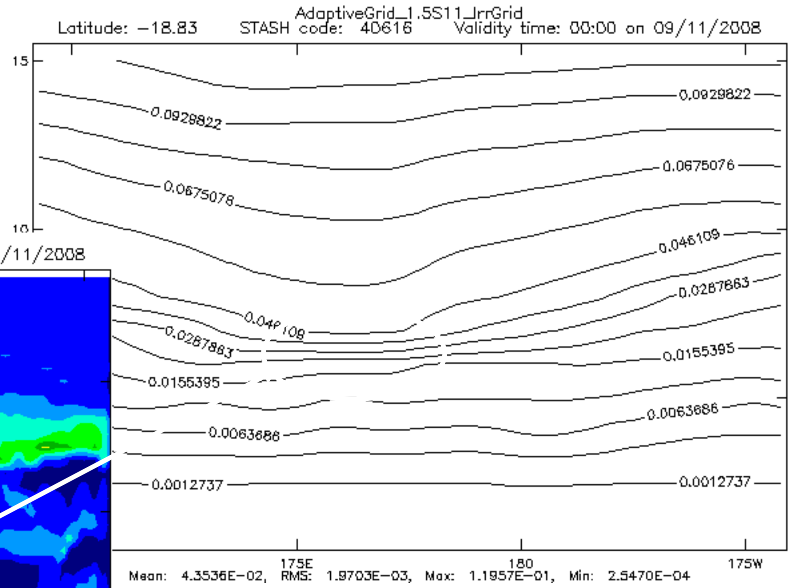
$$M = \sqrt{1 + c^2 \left(\frac{\partial \theta}{\partial z} \right)^2}$$

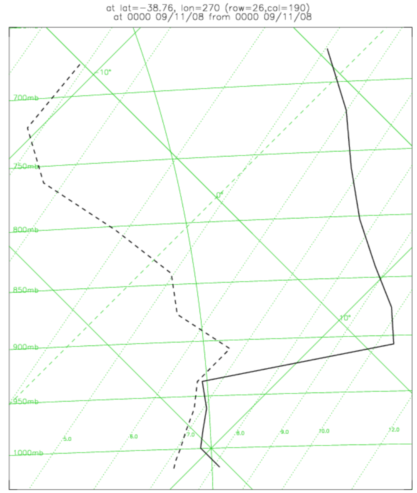


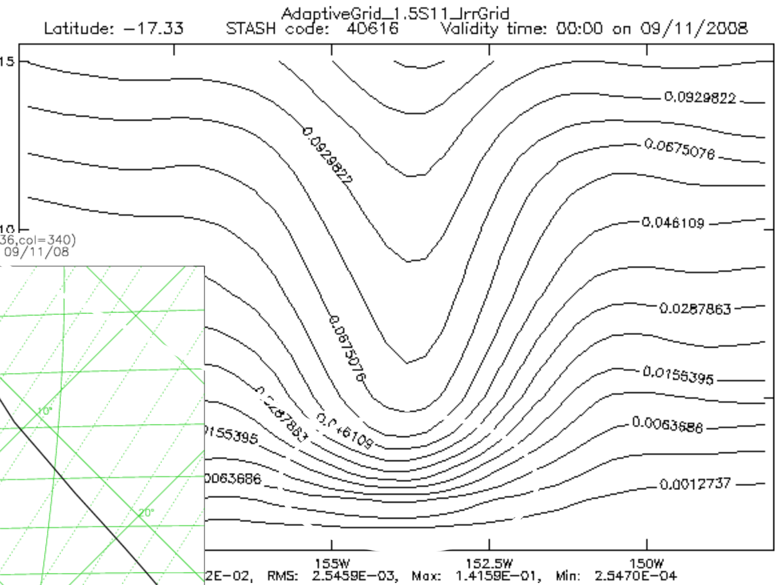
Initially use background state estimate, then update

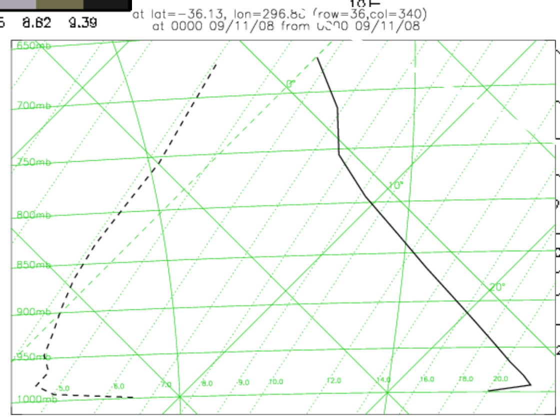
Monitor function and the Adaptive Grid







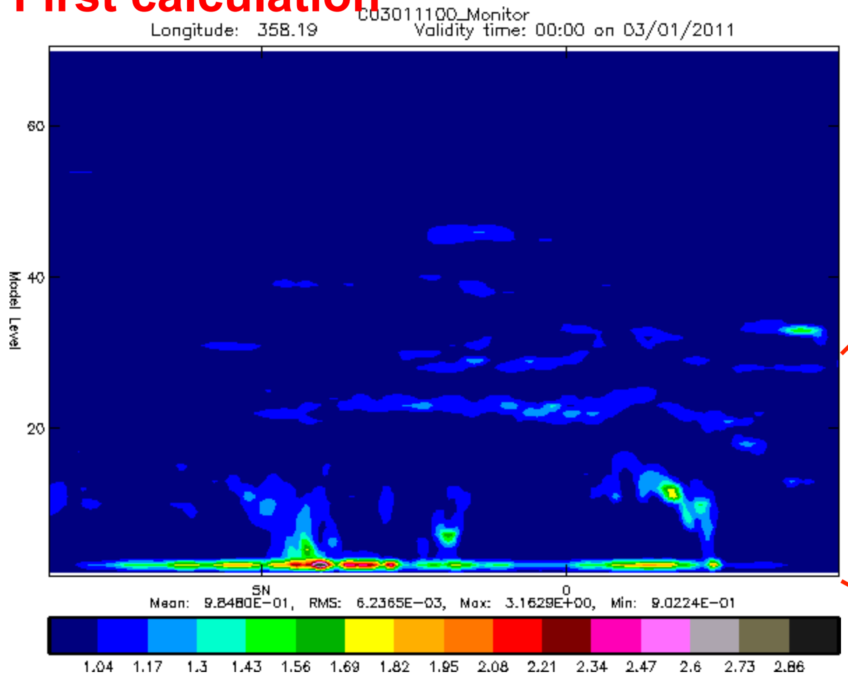




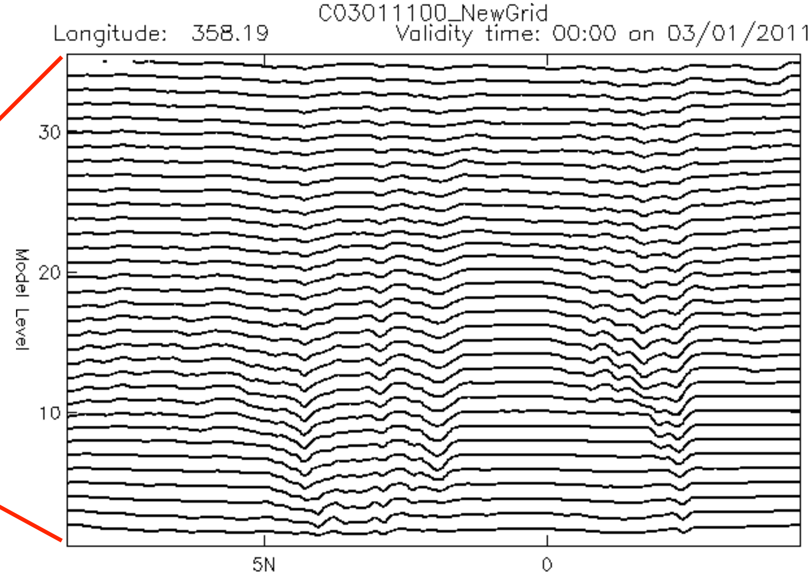
Piccolo&Cullen

QJR Met Soc 2011

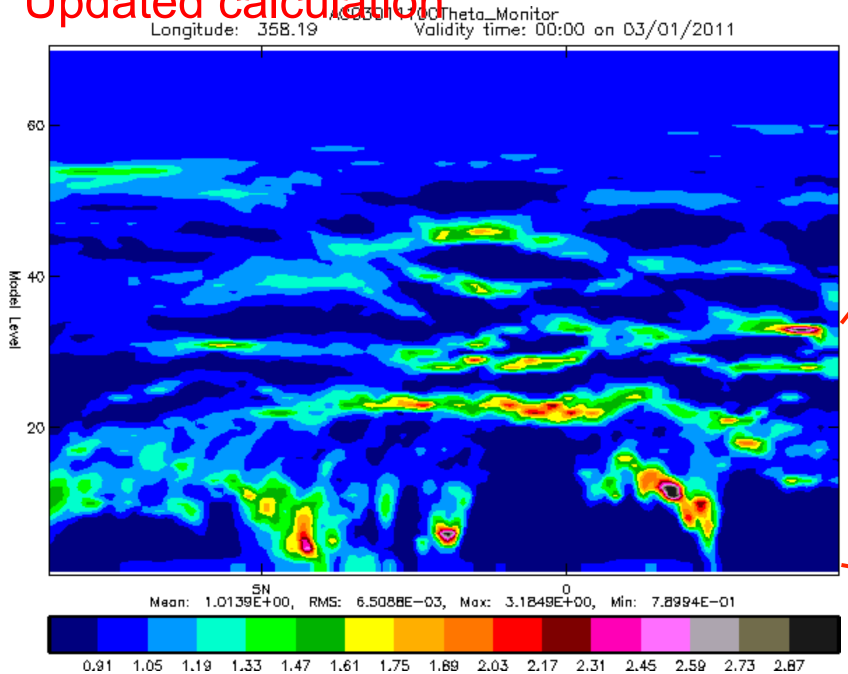
First calculation

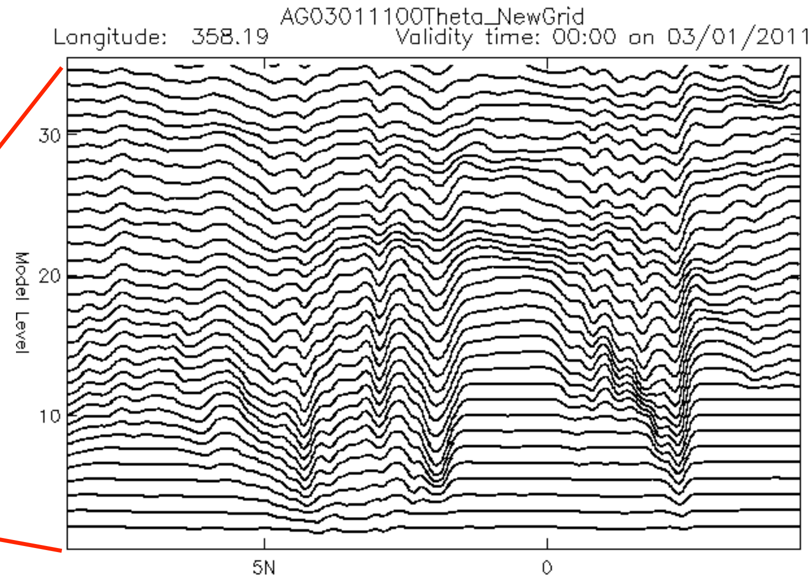


UK4 domain: 3 Jan 2011 00z



Updated calculation





Applied by Chiara Piccolo to the Met Office UK4 model

Test case: 8th Feb 2010.

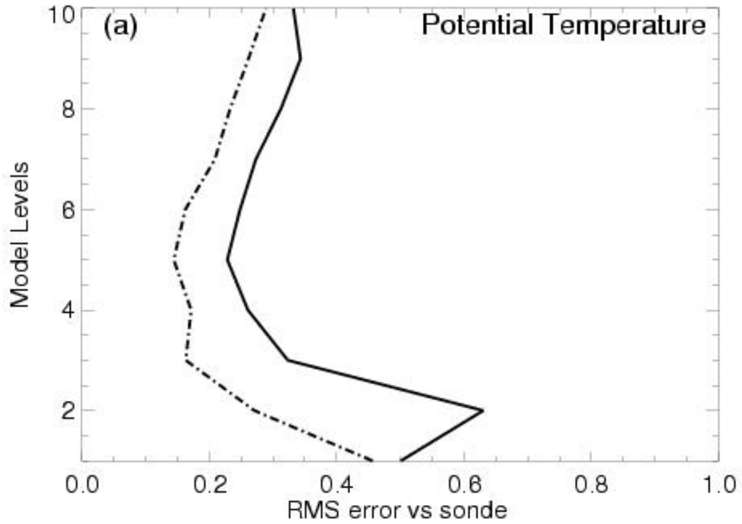
Significant reduction in RMS error especially for temperatures Piccolo&Cullen, QJR Met Soc 2011

RMS	T (K)	RH (%)	u (m/ s)	v (m/s)
Control	0.76	0.045	1.32	1.16
Test	0.64	0.045	1.29	1.16
N_{obs}	1011	901	819	819

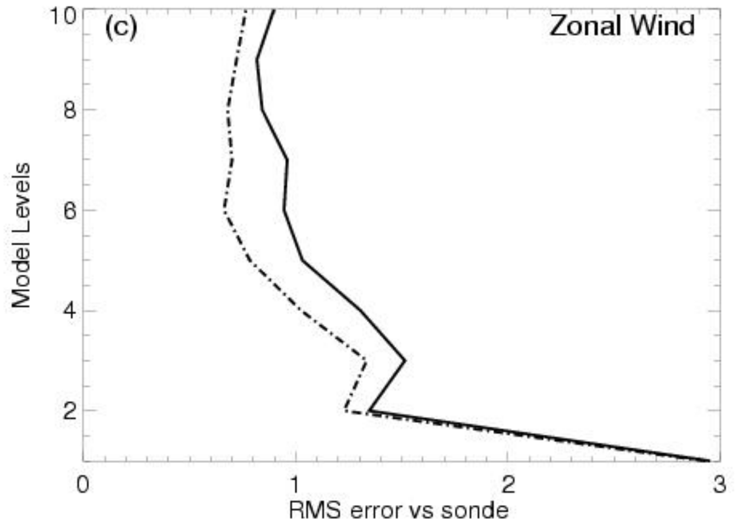
Particularly effective for the 2m temperatures

RMS error: Analysis - Observations

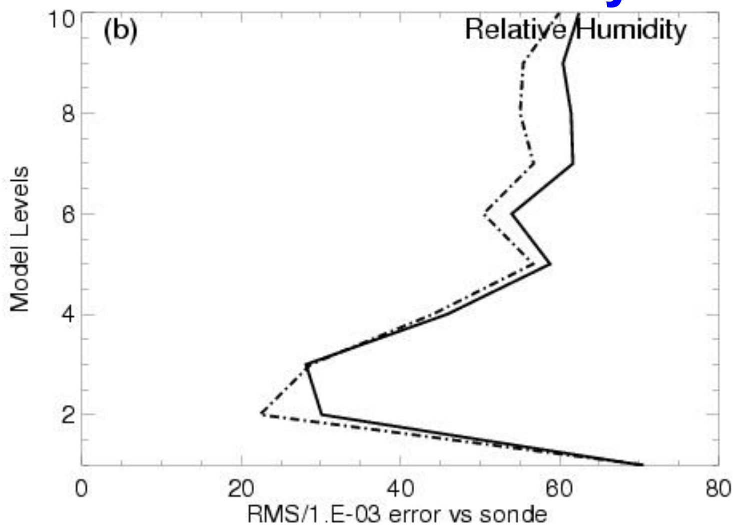
theta



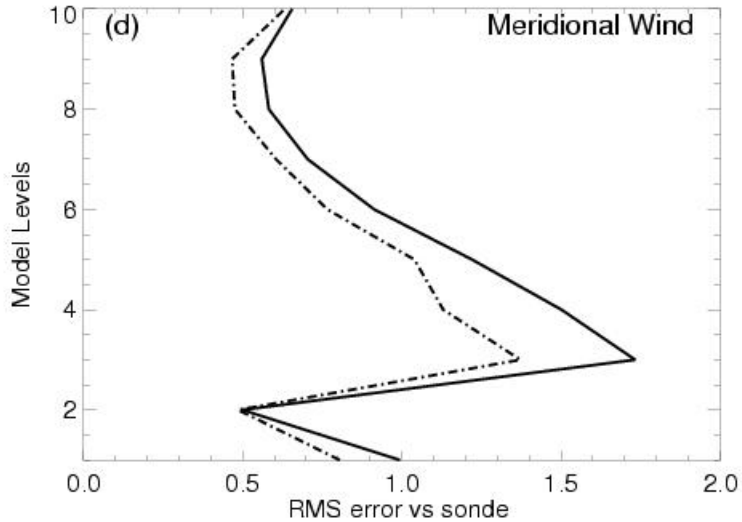
zonal wind



relative humidity

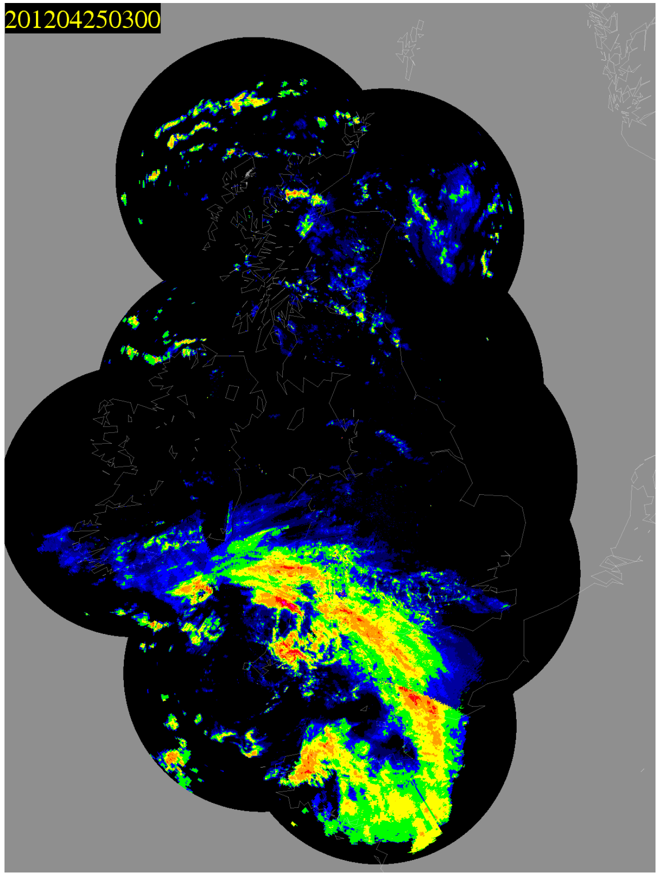


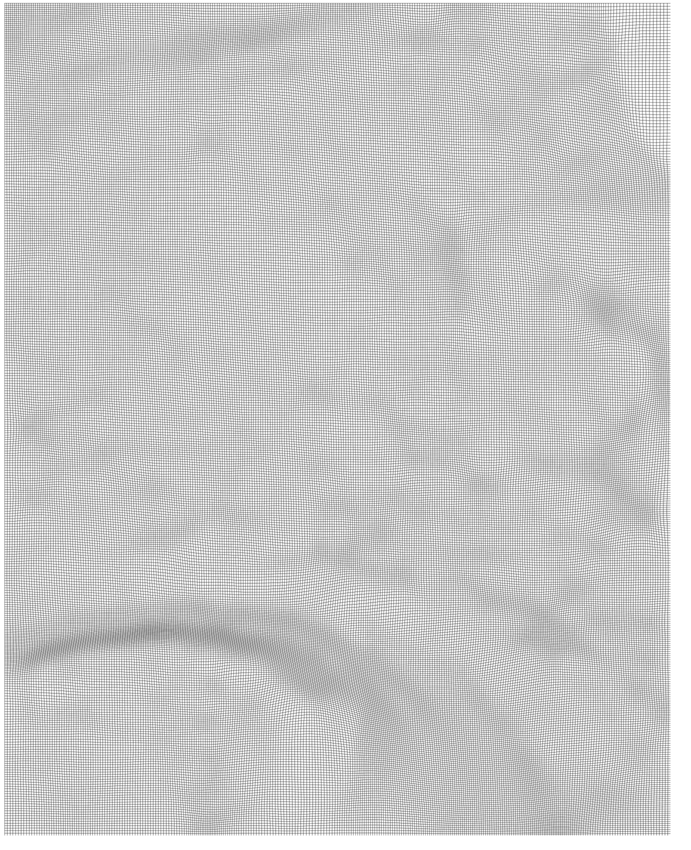
meridional wind



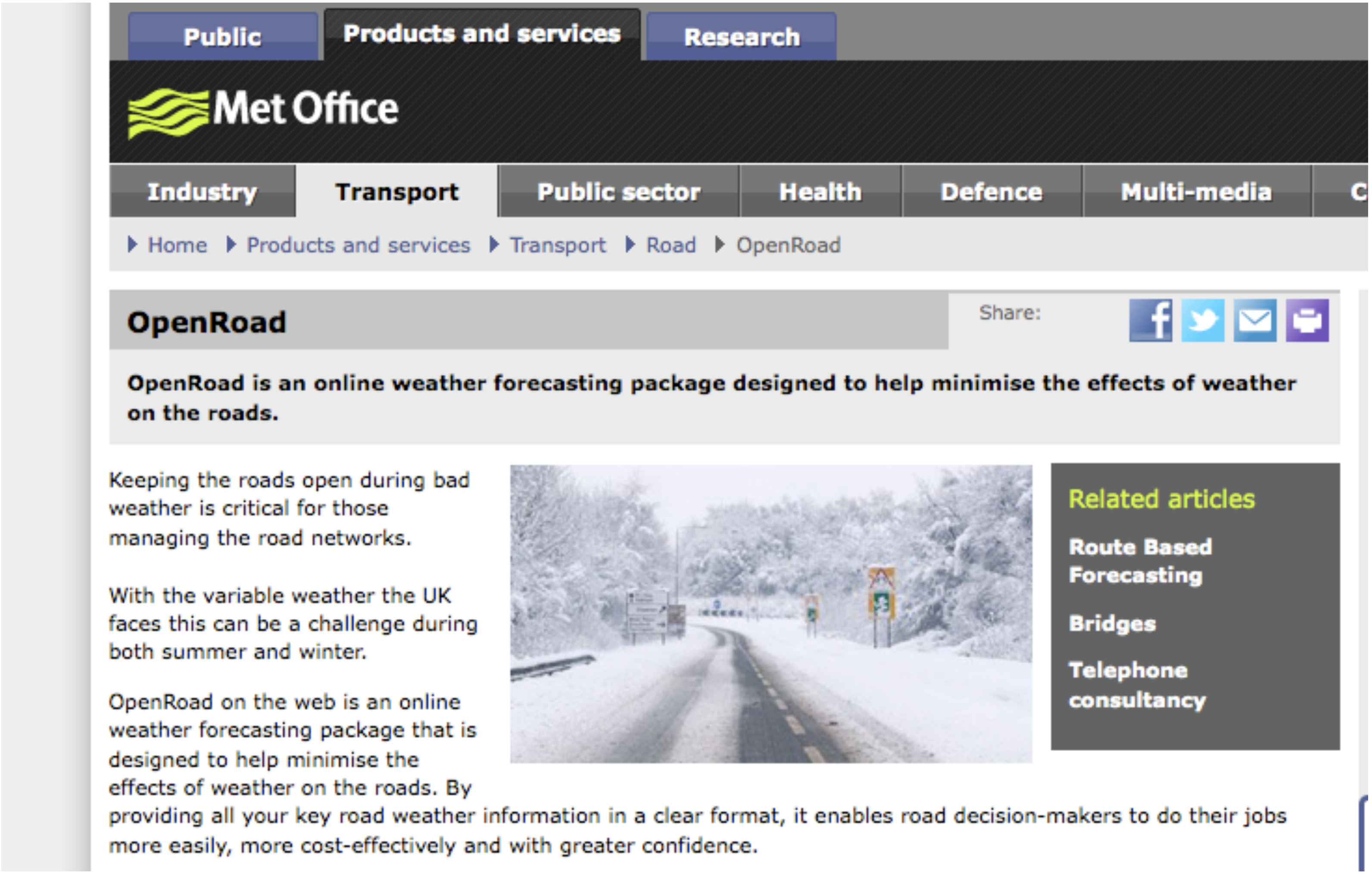
Adaptive mesh implemented operationally in November 2010.

Now extending it to a fully three dimensional implementation [B,Browne,Piccolo]





Used together with Met Office Open Road software to advise councils on road gritting over Christmas


Public | Products and services | Research

Met Office

Industry | Transport | Public sector | Health | Defence | Multi-media

Home > Products and services > Transport > Road > OpenRoad

OpenRoad

Share:

OpenRoad is an online weather forecasting package designed to help minimise the effects of weather on the roads.

Keeping the roads open during bad weather is critical for those managing the road networks.

With the variable weather the UK faces this can be a challenge during both summer and winter.

OpenRoad on the web is an online weather forecasting package that is designed to help minimise the effects of weather on the roads. By providing all your key road weather information in a clear format, it enables road decision-makers to do their jobs more easily, more cost-effectively and with greater confidence.

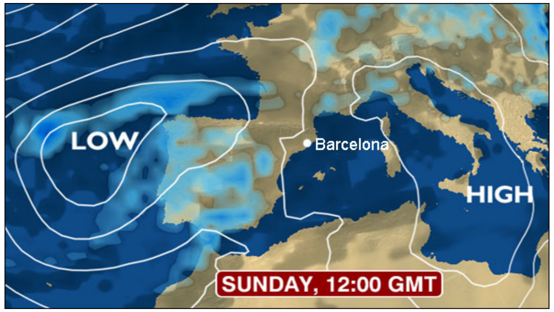
Related articles

Route Based Forecasting

Bridges

Telephone consultancy

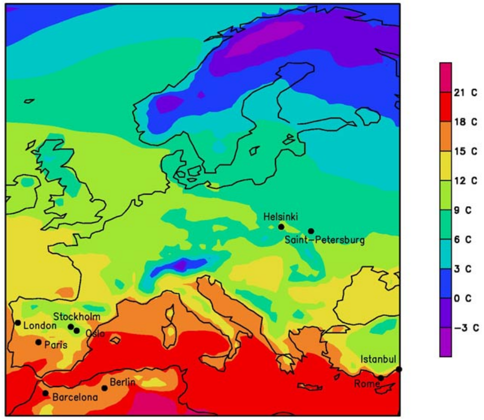
Conclusions



Data assimilation is an optimal way of merging models with data

Useful for model tuning, validation, evaluation, uncertainty quantification and reduction

Very effective in meteorology



Many other applications to Planet Earth

eg. Climate change, oil reservoir modelling, geophysics, energy management and even crowd behaviour

Solution: Find x_0 to minimise nonlinear function J

Need forward calculation to find x_i and backward solve to satisfy the constraints

VERY expensive for high dimensional problems!!! Only have limited time to do the calculation (20 mins)

Incremental 4D-Var: Cheaper!

1. Assume x_0 is close to x_B

2. Linearise J about x_B and minimise this function using an iterative method eg. BFGS

3. Repeat if needed (not usually)

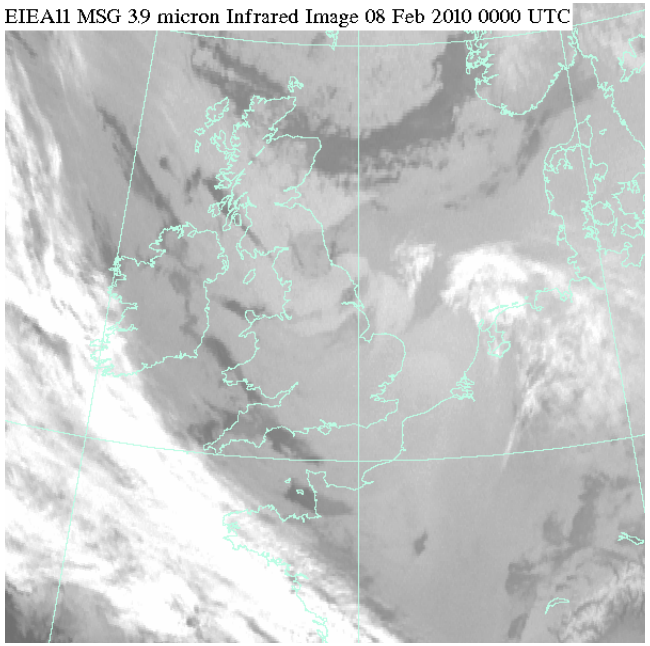
BUT: Relies on assumption of near linearity to work well

Very effective method!!

Developed at ECMWF

Met Office operational in 2004

[Lorenc, ….]



Used by many other centres

Observation Volumes in 6 hours

Category	Count	% used	Category	Count	% used
TEMPs	637	99%	Satwinds: JMA	26103	4%
PILOTs	307	99%	Satwinds: NESDIS	142478	3%
Wind Profiler	1355	39%	Satwinds: EUMETSAT	220957	1%
Land Synops	16551	99%	Scatwinds: Seawinds	436566	1%
Ships	3034	84%	Scatwinds: ERS	27075	2%
Buoys	8727	63%	Scatwinds: ASCAT	241626	4%
Amdars	64147	23%	SSMI/S	532140	1%
Aireps	7144	12%	SSMI	698048	1%
GPS-RO	776	99%	ATOVS	1127224	3%
			AIRS	75824	6%
			IASI	80280	3%

Can estimate x_a using Bayesian analyis:

Maximum likelihood estimate of data y given x_t

Posterior

$$\frac{P(x_t|y)}{P(x_t)}P(y) = P(y|x_t)$$

Prior

Likelihood

Best RMS unbiased estimate of the true state: BLUE

Minimum error variance

4D-VAR idea: Evolutionary model M (nonlinear)

Unknown initial state x_0

Times $t = t_0, t_1, t_2, \cdots$ Over a time window

Leads to state estimates $x_1, x_2, \ldots$

Data y over window

Find x_0 so that the

estimates fit the data