

Combined Image Reconstruction for Combined PET-MR Imaging

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Positron Emission Tomography and Magnetic Resonance Imaging

Positron Emission Tomography (PET)



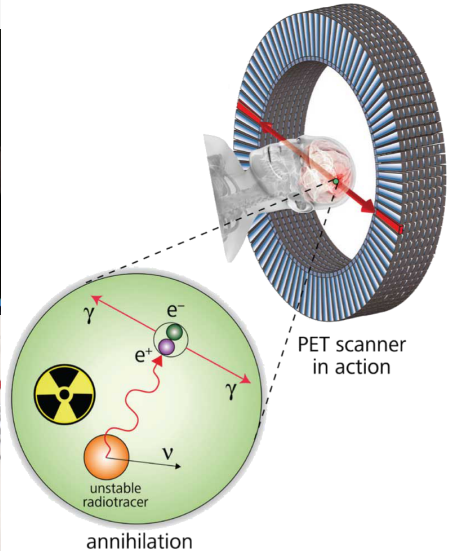
Positron Emission Tomography (PET)



Positron Emission Tomography (PET)



Positron Emission Tomography (PET)



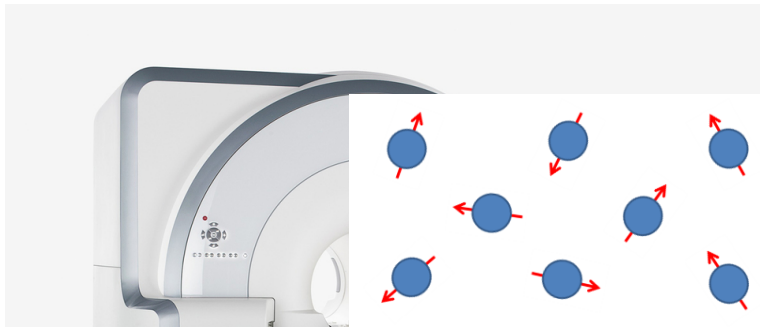
Magnetic Resonance Imaging (MRI)



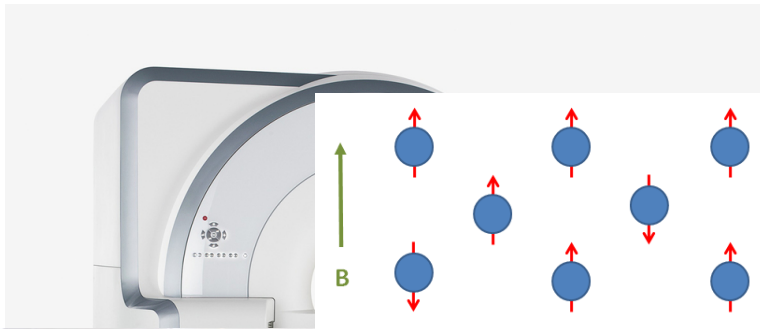
Magnetic Resonance Imaging (MRI)



Magnetic Resonance Imaging (MRI)

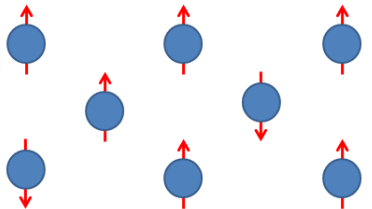
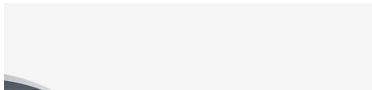
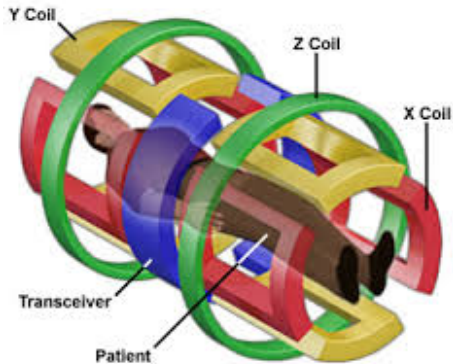


Magnetic Resonance Imaging (MRI)



Magnetic Resonance Imaging (MRI)

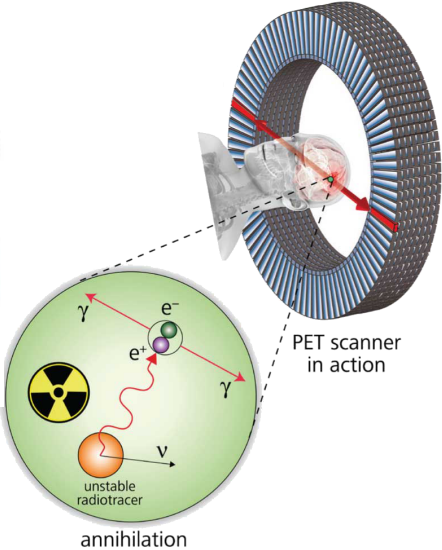
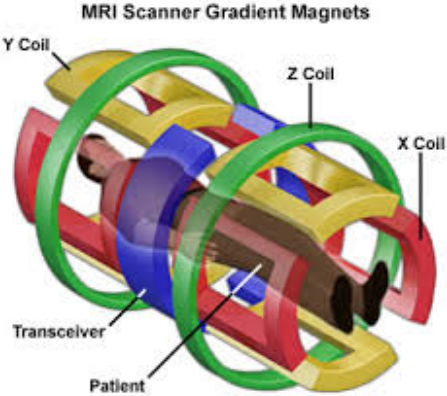
MRI Scanner Gradient Magnets



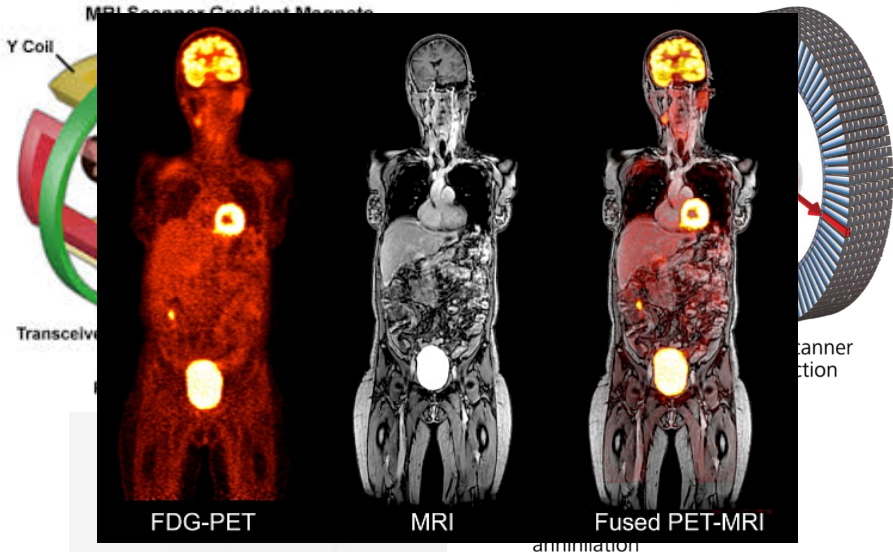
Combined PET-MR Imaging



Combined PET-MR Imaging



Combined PET-MR Imaging



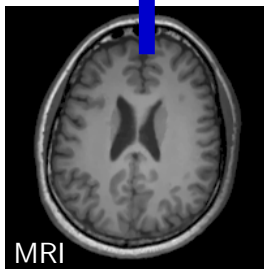
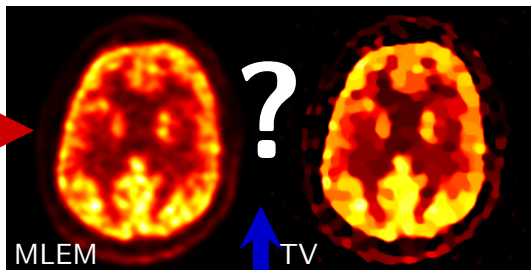
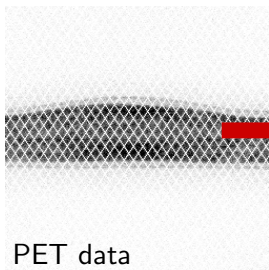
Part I:
Utilizing Resolution of MRI

Part II:
Joint PET-MRI Reconstruction

Part I:
Utilizing Resolution of MRI

Part II:
Joint PET-MRI Reconstruction

PET Reconstruction



MAP reconstruction and Total Variation

MAP reconstruction

$$u^* \in \underset{u}{\operatorname{argmin}} \left\{ \mathcal{L}(Au + r, b) + \alpha \mathcal{R}(u) \right\}$$

- ▶ total variation [Rudin, Osher, Fatemi 1992](#)

$$\mathcal{R}(u) = \operatorname{TV}(u) = \int_{\Omega} |\nabla u|$$

$$\mathcal{R}(u) = \operatorname{TV}_{\beta}(u) = \int_{\Omega} \left(\beta^2 + |\nabla u|^2 \right)^{1/2}$$

edge-preserved reconstruction

MAP reconstruction and Anatomical Information

MAP reconstruction with Anatomical Information

$$u^* \in \underset{u}{\operatorname{argmin}} \left\{ \mathcal{L}(Pu + r, b) + \alpha \mathcal{R}(u|v) \right\}$$

We want

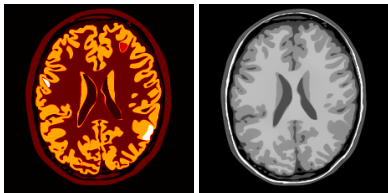
- 1) **Convexity**: $\mathcal{R}(u|v)$ should be convex in u
- 2) **No Segmentation**: should not need a segmentation of v
- 3*) **Total Variation**: $\mathcal{R}(u|v = \text{const}) = \text{TV}(u)$

PET with MRI/CT: Leahy and Yan 1991, Baete et al 2004, Pedemonte et al 2011, Bowsher et al 2004, Kazantsev et al 2014, Nuyts 2007, Somayayula et al 2005 2011, Tang and Rahmim 2009 2015 (Mutual information/ Entropy), Jiao et al 2015

EIT with CT: Kaipio et al 1999

Ehrhardt et al 2016 (under review)

Parallel Level Set Prior



Ehrhardt et al 2016 (under review)

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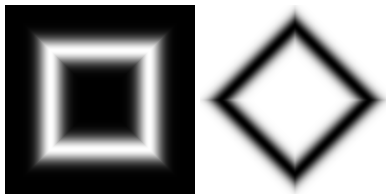
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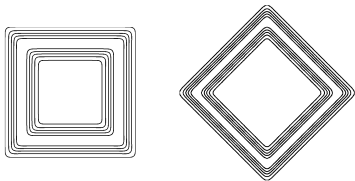
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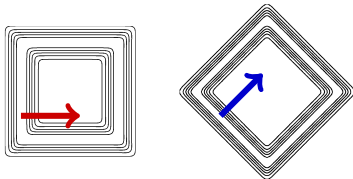
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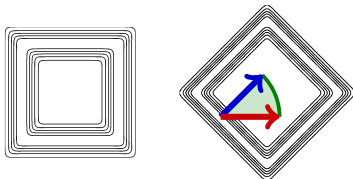
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Parallel Level Set Prior

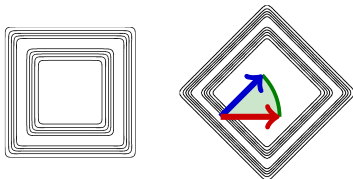


$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Ehrhardt et al 2016 (under review)

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Parallel Level Set Prior



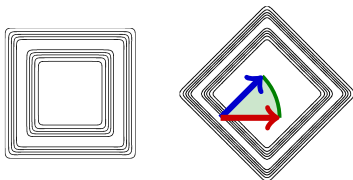
$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Measure Similar Structures

$$\mathcal{S}(u) := \left(|\nabla u|^2 - \langle \nabla u, \nabla v / |\nabla v| \rangle^2 \right)^{1/2}$$

Ehrhardt et al 2016 (under review)

Parallel Level Set Prior



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

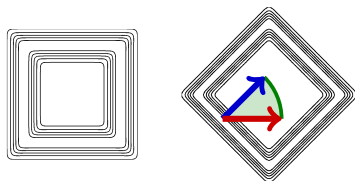
Measure Similar Structures

$$\mathcal{S}(u) := \left(|\nabla u|^2 - \langle \nabla u, \xi \rangle^2 \right)^{1/2}$$

► $\xi := \nabla v / |\nabla v|_\eta, \quad |\nabla v|_\eta := \sqrt{|\nabla v|^2 + \eta^2}, \quad \eta > 0$

Ehrhardt et al 2016 (under review)

Parallel Level Set Prior



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Measure Similar Structures

$$\mathcal{S}(u) := \left(|\nabla u|^2 - \langle \nabla u, \xi \rangle^2 \right)^{1/2}$$

- ▶ $\xi := \nabla v / |\nabla v|_\eta$, $|\nabla v|_\eta := \sqrt{|\nabla v|^2 + \eta^2}$, $\eta > 0$
- ▶ $0 \leq \mathcal{S}(u) \leq |\nabla u|$
- ▶ $\mathcal{S}(u) = 0 \Leftrightarrow u \sim v$ ($\nabla u \parallel \nabla v$)

Ehrhardt et al 2016 (under review)

Asymmetric Parallel Level Sets

$$\mathcal{S}(u) := \left(|\nabla u|^2 - \langle \nabla u, \xi \rangle^2 \right)^{1/2}$$

Asymmetric Parallel Level Sets Prior

$$\mathcal{P}(u|v) := \int_{\Omega} \left(\beta^2 + |\nabla u|^2 - \langle \nabla u, \xi \rangle^2 \right)^{1/2}, \beta > 0$$

This is **convex**, **does not need a segmentation** and **reduces to total variation**.

Ehrhardt et al 2016 (under review)

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Numerical Results

Other Methods for Anatomical Information

$$\text{TV}_{\mathcal{J}}(\mathbf{u}|\mathbf{v}) := \int_{\Omega} \left(\beta^2 + |\nabla \mathbf{u}|^2 + \gamma |\nabla \mathbf{v}|^2 \right)^{1/2}, \quad \gamma > 0$$

Sapiro and Ringach IEEE TIP 1996;

Haber and Holtzman-Gazit Surveys in Geophysics 2013;

Ehrhardt et al Inv Probl 2015, Lu et al Phys Med Bio 2015

$$\mathcal{B}(\mathbf{u}|\mathbf{v}) := \frac{1}{2} \sum_i \sum_{j \in N(i)} \omega_{i,j}(\mathbf{v}) (\mathbf{u}_i - \mathbf{u}_j)^2, \quad k \in \mathbb{N}$$

Bowsher et al IEEE NSS-MIC 2004

$$\mathcal{D}(\mathbf{u}|\mathbf{v}) := \int_{\Omega} \left(\beta^2 + |\nabla \mathbf{u}|^2 \right)^{1/2} - \langle \nabla \mathbf{u}, \boldsymbol{\xi} \rangle$$

Kazantsev et al Sensing and Imaging 2014

$$\mathcal{K}(\mathbf{u}|\mathbf{v}) := \frac{1}{2} \int_{\Omega} |\nabla \mathbf{u}|^2 - \langle \nabla \mathbf{u}, \boldsymbol{\xi} \rangle^2$$

Kaipio et al Inv Prob 1999

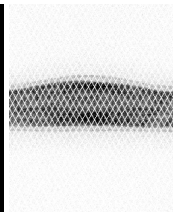
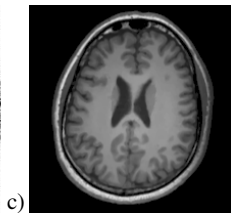
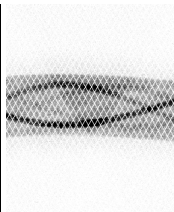
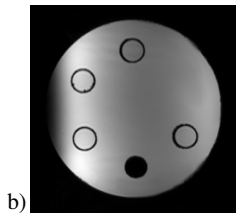
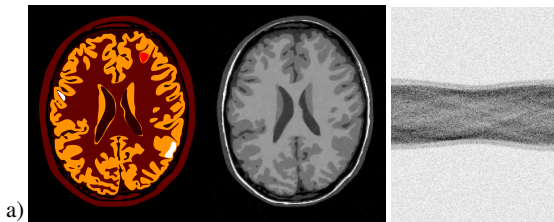
Summary of Methods

	$TV_{\mathcal{J}}$	\mathcal{B}	\mathcal{D}	\mathcal{K}	\mathcal{P}
reduces to total variation	✓	✗	✓	✗	✓
edge location dependent	✓	✓	✓	✓	✓
edge orientation dependent	✗	✗	✓	✓	✓
allows negative edge correlation	-	-	✗	✓	✓

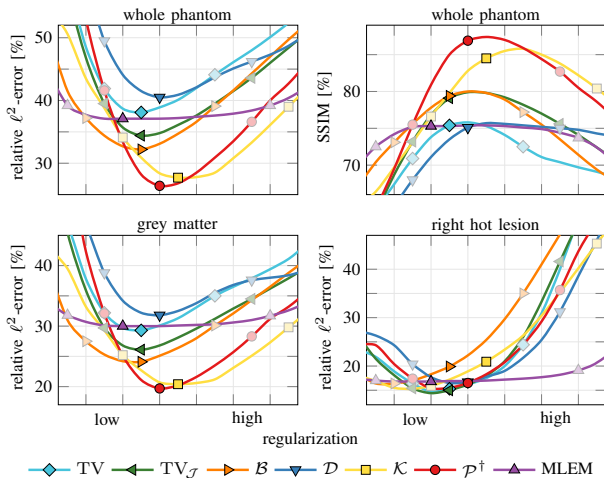
Ehrhardt et al 2016 (under review)

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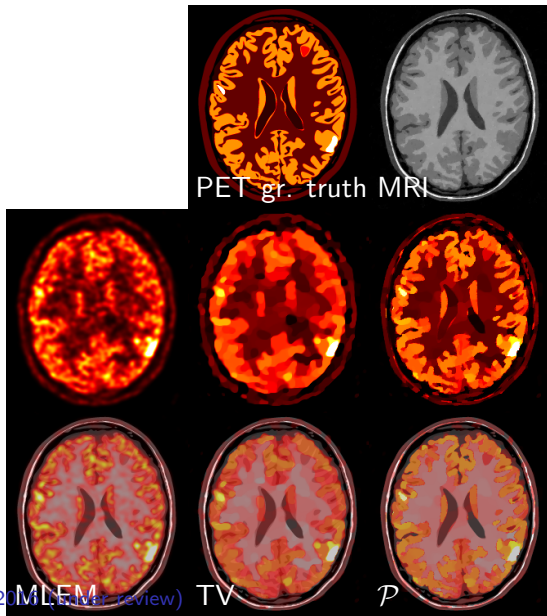
Data



Software Phantom: Quantitative Results

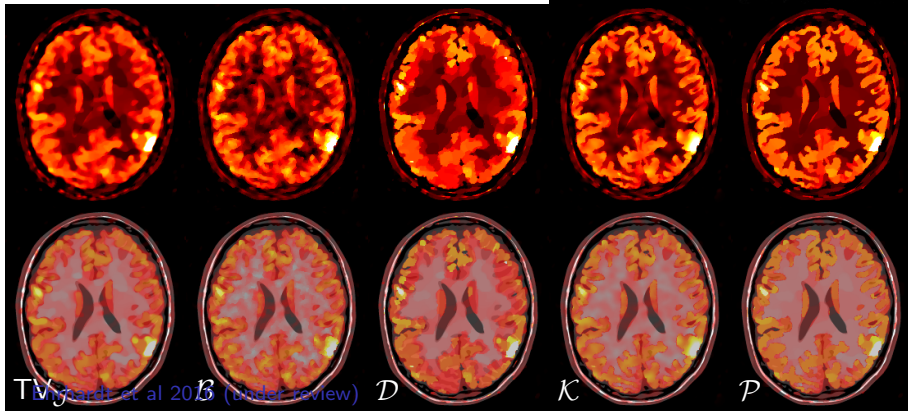
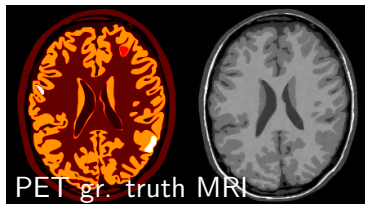


Software Phantom: Normal Recon v Anatomical Prior



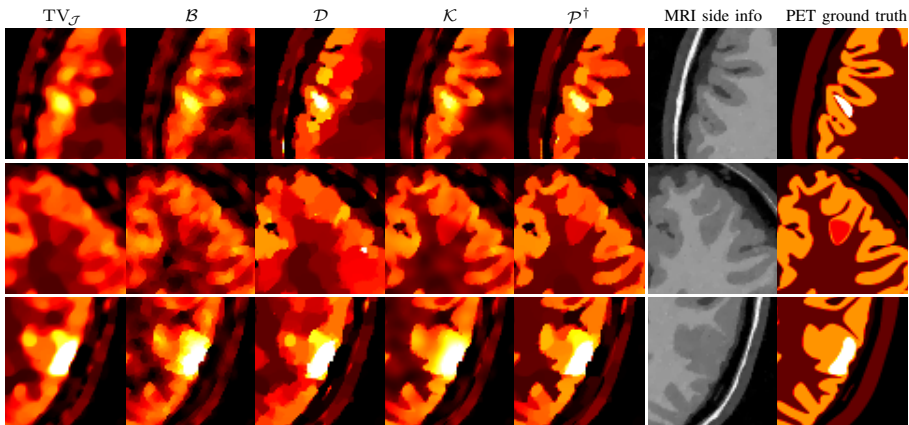
Ehrhardt et al 2011 (review)

Software Phantom: Compare Anatomical Priors



Ehrhardt et al 2016 (under review)

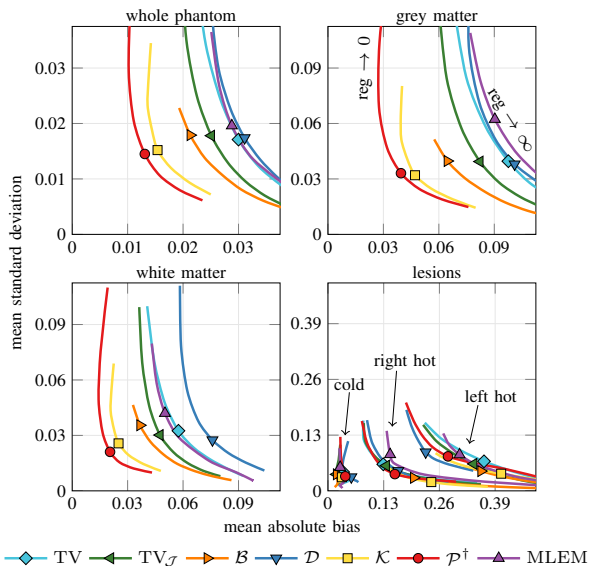
Software Phantom: Close-Ups



Ehrhardt et al 2016 (under review)

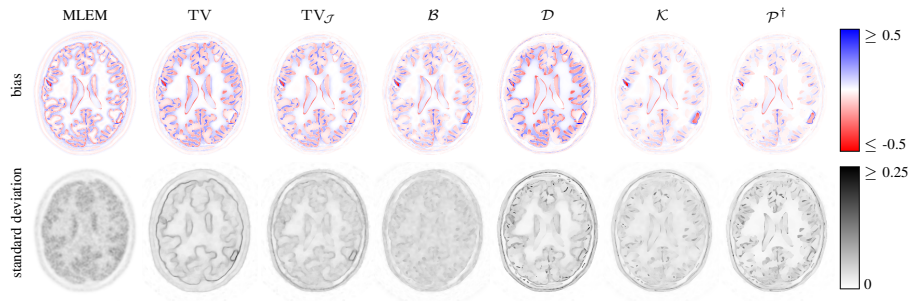
m.j.ehrhardt@damtp.cam.ac.uk

Software Phantom: Bias vs SD



Ehrhardt et al 2016 (under review)

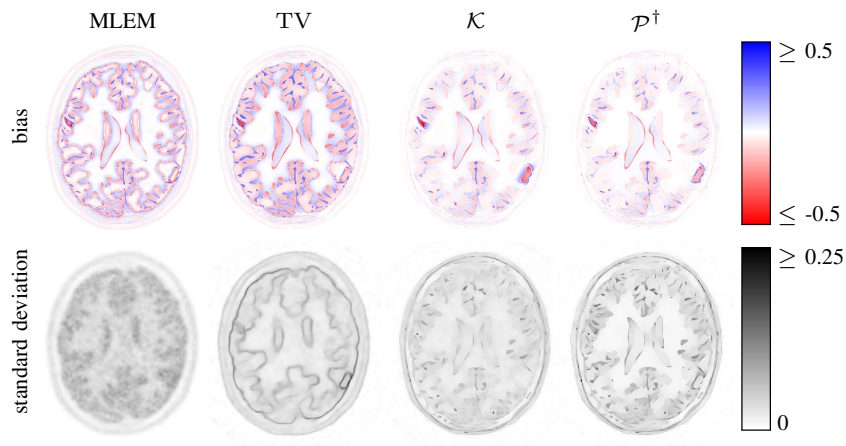
Software Phantom: Bias vs SD



Ehrhardt et al 2016 (under review)

m.j.ehrhardt@damtp.cam.ac.uk

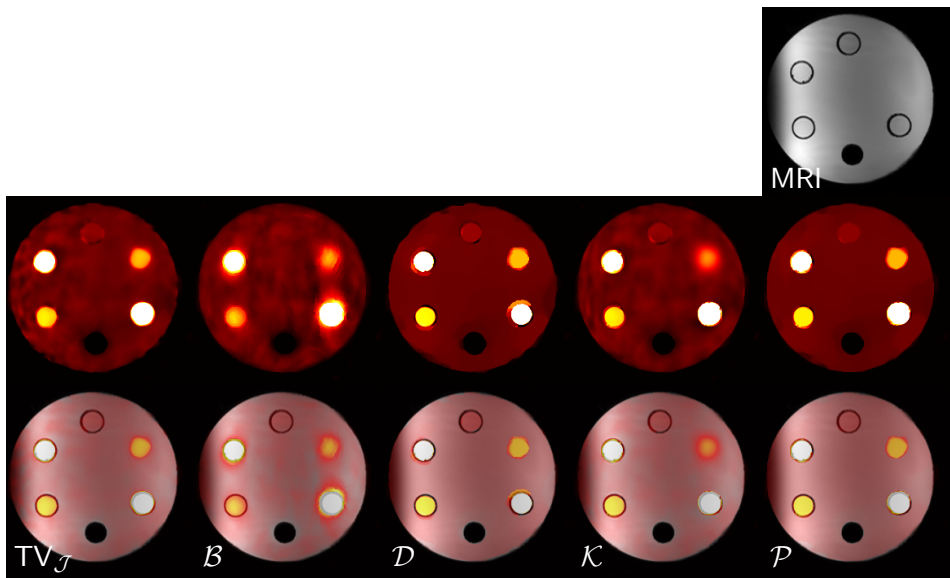
Software Phantom: Bias vs SD



Ehrhardt et al 2016 (under review)

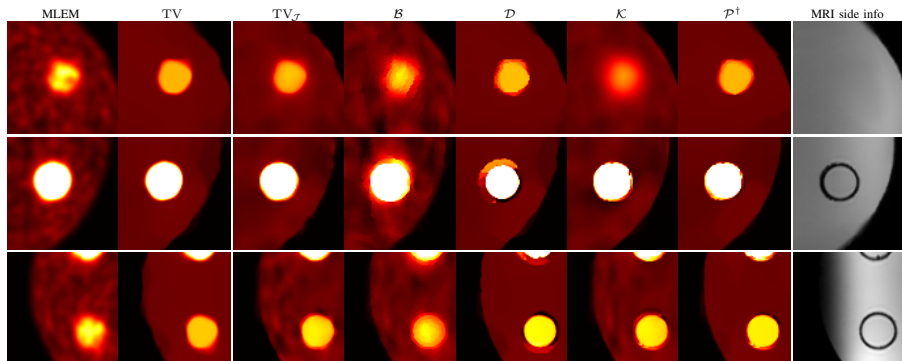
m.j.ehrhardt@damtp.cam.ac.uk

Hardware Phantom: Compare Anatomical Priors



Ehrhardt et al 2016 (under review)

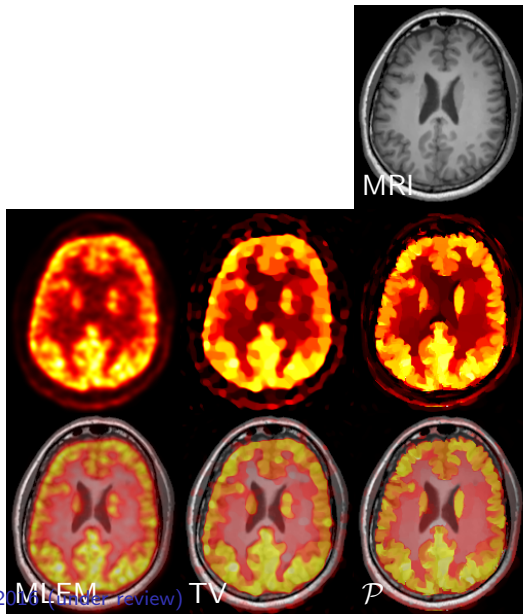
Hardware Phantom: Close-Ups



Ehrhardt et al 2016 (under review)

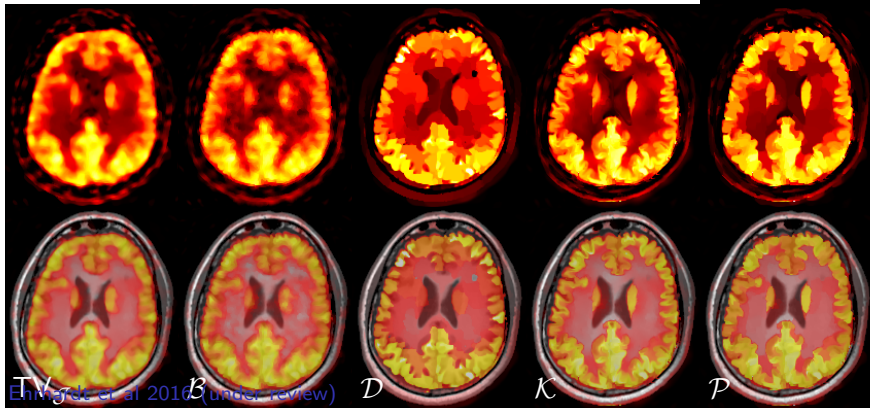
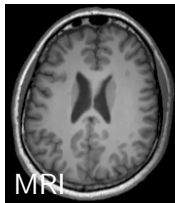
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Patient Data: Normal Recon v Anatomical Prior



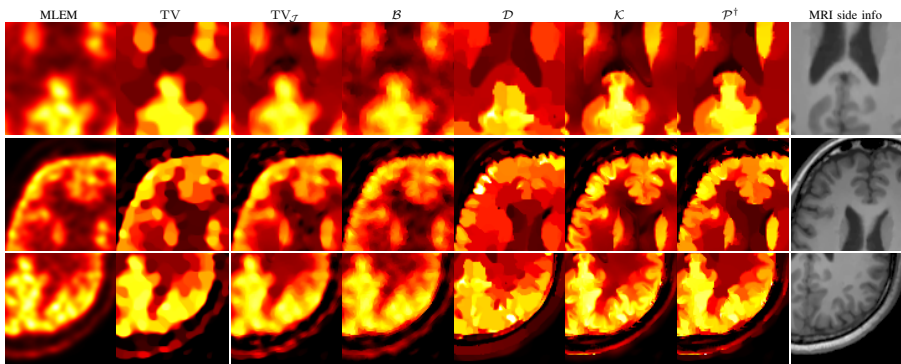
Ehrhardt et al 2016 (under review)

Patient Data: Compare Anatomical Priors



Ehrhardt et al 2016 (under review)

Patient Data: Close-Ups

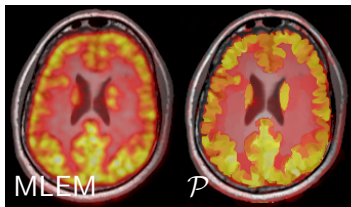


Ehrhardt et al 2016 (under review)

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Conclusions of Part I

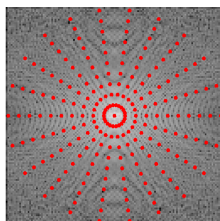
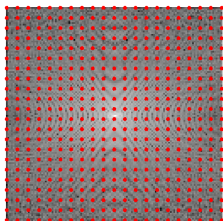
- ▶ **new prior** that can incorporate anatomical structure
 - ▶ **convex**, **no segmentation** and reduces to **total variation**
 - ▶ based on directions, not only magnitude
 - ▶ handles arbitrary intensities, no need for positive correlation
 - ▶ better in quality measures (ℓ^2 -error, SSIM, bias-vs-SD trade-off)
 - ▶ reduces bias of total variation (similar to Bregman iterations)



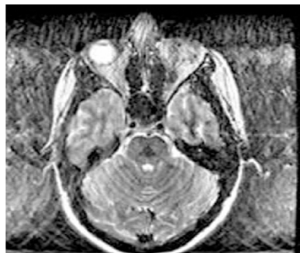
Part I:
Utilizing Resolution of MRI

Part II:
Joint PET-MRI Reconstruction

Data Acquisition in MRI

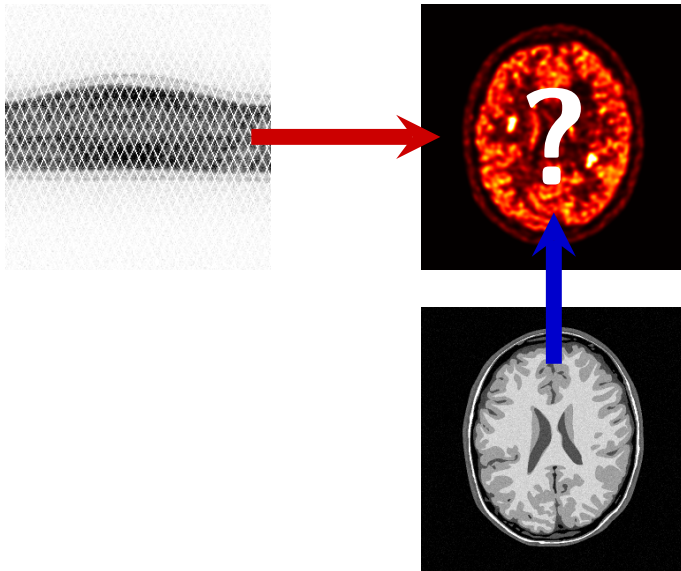


- ▶ sequential sampling of Fourier coefficients
- ▶ less data
 - ⇒ shorter acquisition time
 - ⇒ motion, patient comfort, money
- ▶ higher spatial resolution



Joint Reconstruction

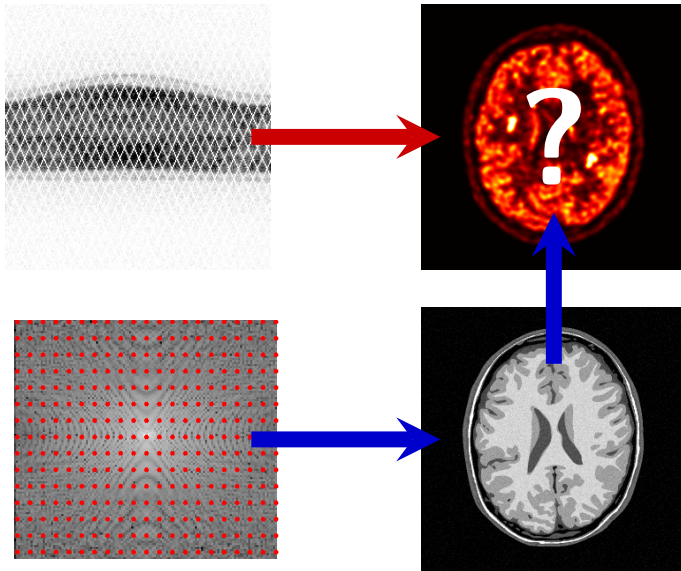
Joint Reconstruction



Ehrhardt et al Inverse Problems 2015

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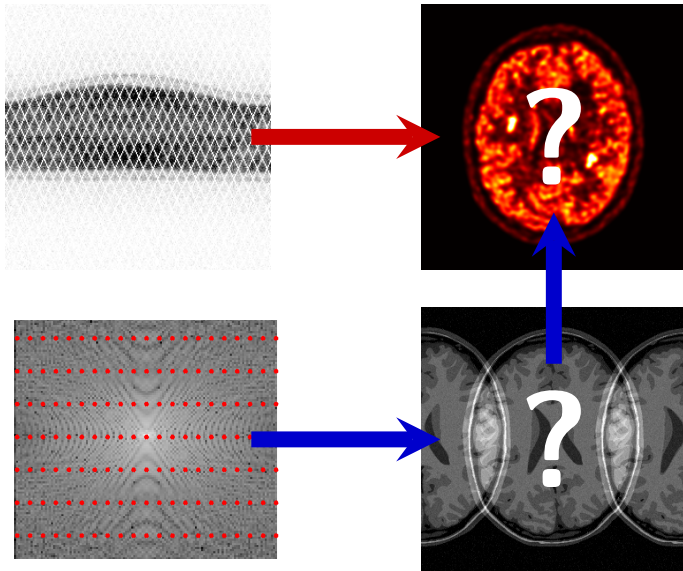
Joint Reconstruction



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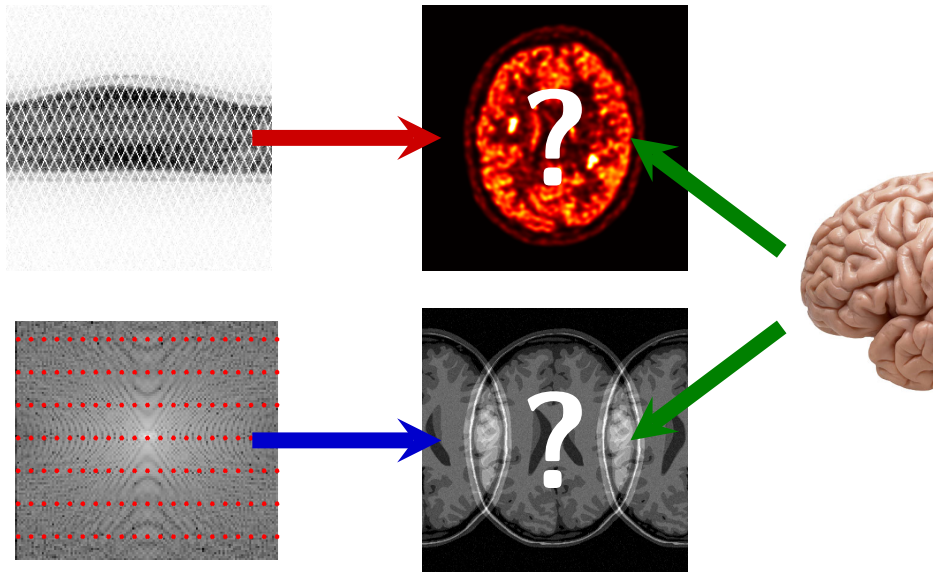
Joint Reconstruction



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Joint Reconstruction



Ehrhardt et al Inverse Problems 2015

m.j.ehrhardt@damtp.cam.ac.uk

Problem Set Up

- ▶ Reconstruct jointly PET and MRI
- ▶ Two modalities with different characteristics

MRI:

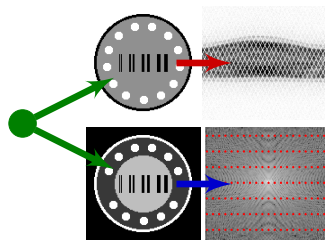
- ▶ **Undersampled Fourier** data with **Gaussian** noise
- ▶ Forward operator is not injective but pseudo inverse is well-conditioned

PET:

- ▶ **Blurry Radon** data with **Poisson** noise
- ▶ Forward operator compact, inverse is ill-conditioned

- ▶ two problems coupled by underlying anatomy

Framework for Joint Reconstruction

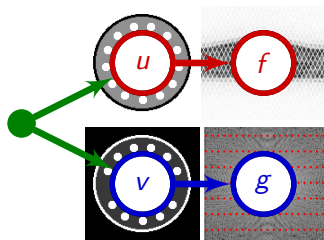


Ehrhardt et al Inverse Problems 2015

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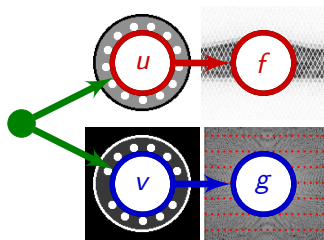
Framework for Joint Reconstruction

$$p(u, v | f, g)$$



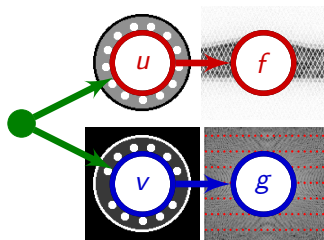
Framework for Joint Reconstruction

$$p(u, v | f, g) \propto p(f, g | u, v) p(u, v)$$



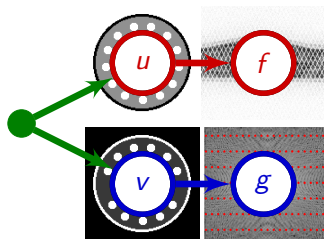
Framework for Joint Reconstruction

$$\begin{aligned} p(u, v|f, g) &\propto p(f, g|u, v)p(u, v) \\ &= p(f|u, v)p(g|u, v)p(u, v) \end{aligned}$$



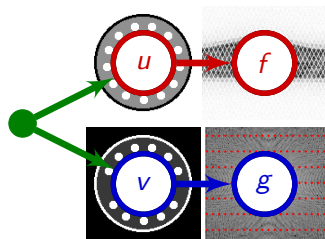
Framework for Joint Reconstruction

$$\begin{aligned} p(u, v|f, g) &\propto p(f, g|u, v)p(u, v) \\ &= p(f|u, v)p(g|u, v)p(u, v) \\ &= p(f|u)p(g|v)p(u, v) \end{aligned}$$



Framework for Joint Reconstruction

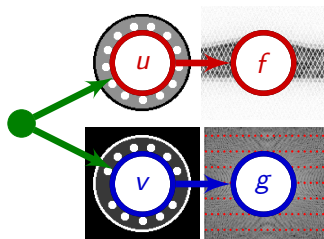
$$\begin{aligned} p(u, v|f, g) &\propto p(f, g|u, v)p(u, v) \\ &= p(f|u, v)p(g|u, v)p(u, v) \\ &= p(f|u)p(g|v)p(u, v) \end{aligned}$$



$$\underset{u, v}{\text{minimize}} \left\{ -\log p(f|u) - \log p(g|v) - \log p(u, v) \right\}$$

Framework for Joint Reconstruction

$$\begin{aligned} p(u, v|f, g) &\propto p(f, g|u, v)p(u, v) \\ &= p(f|u, v)p(g|u, v)p(u, v) \\ &= p(f|u)p(g|v)p(u, v) \end{aligned}$$

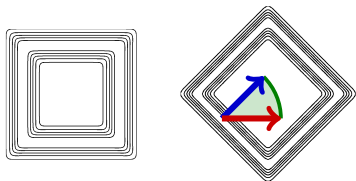


$$\begin{aligned} \underset{u, v}{\text{minimize}} \left\{ -\log p(f|u) - \log p(g|v) - \log p(u, v) \right\} \\ \propto \text{KL}(Au + b; f) + \frac{1}{2\sigma^2} \|Bv - g\|^2 - \log p(u, v) \end{aligned}$$

$$\text{KL}(x; y) := \sum_j x_j - y_j + y_j \log(y_j/x_j)$$

Parallel Level Sets

Joint Parallel Level Set Prior



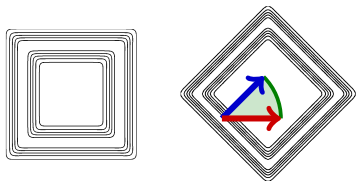
$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Measure Similar Structures

$$\mathcal{S}(u) = \left(|\nabla u|^2 - \langle \nabla u, \nabla v / |\nabla v| \rangle^2 \right)^{1/2}$$

Ehrhardt and Arridge IEEE TIP 2014, Ehrhardt 2015

Joint Parallel Level Set Prior

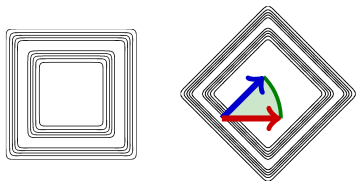


$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Measure Similar Structures

$$\mathcal{S}(u, v) = |\nabla u| |\nabla v| - \langle \nabla u, \nabla v \rangle$$

Joint Parallel Level Set Prior



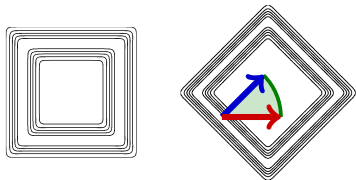
$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Measure Similar Structures

$$\mathcal{S}(u, v) = |\nabla u| |\nabla v| - \langle \nabla u, \nabla v \rangle$$

- ▶ $\mathcal{S}(u, v) \geq 0$
- ▶ $\mathcal{S}(u, v) = 0 \Leftrightarrow u \sim v \text{ (} \nabla u \parallel \nabla v \text{)}$

Joint Parallel Level Set Prior



$$\langle \nabla u, \nabla v \rangle = \cos(\theta) |\nabla u| |\nabla v|$$

Measure Similar Structures

$$\mathcal{S}(u, v) = \int_{\Omega} |\nabla u| |\nabla v| - \langle \nabla u, \nabla v \rangle$$

- ▶ $\mathcal{S}(u, v) \geq 0$
- ▶ $\mathcal{S}(u, v) = 0 \Leftrightarrow u \sim v$ ($\nabla u \parallel \nabla v$ almost everywhere)

Joint Parallel Level Set Prior

Recall,
$$\mathcal{S}(u, v) = \int |\nabla u| |\nabla v| - \langle \nabla u, \nabla v \rangle.$$

Structure is Intensity Invariant

Let $f \in C^1(\mathbb{R}, \mathbb{R})$ (with f injective). Then,

$$u \sim v \Rightarrow (\Leftrightarrow) u \sim v \circ f$$

Proof: At almost every location x , there is

$$\nabla(v \circ f)(x) = f'(v(x)) \nabla v(x) = f'(v(x)) \lambda(x) \nabla u(x) = \tilde{\lambda}(x) \nabla u(x).$$

Joint Parallel Level Set Prior

Recall, $\mathcal{S}(u, v) = \int |\nabla u| |\nabla v| - \langle \nabla u, \nabla v \rangle$.

Asymptotics

- ▶ For $|\nabla v| \approx 0$, there is

$$\mathcal{S}(u, v) \approx 0.$$

Joint Parallel Level Set Prior

Recall,
$$\mathcal{S}(u, v) = \int |\nabla u| |\nabla v| - \langle \nabla u, \nabla v \rangle.$$

Asymptotics

- ▶ For $|\nabla v| \approx 0$, there is

$$\mathcal{S}(u, v) \approx 0.$$

Parallel Level Sets Prior

$$\mathcal{S}_\beta(u, v) = \int |\nabla u|_\beta |\nabla v|_\beta - \langle \nabla u, \nabla v \rangle_{\beta^2}$$

with “smoothed” norm $|x|_\beta = \sqrt{\beta^2 + |x|^2}$.

Joint Parallel Level Set Prior

Recall, $\mathcal{S}_\beta(\mathbf{u}, \mathbf{v}) = \int |\nabla \mathbf{u}|_\beta |\nabla \mathbf{v}|_\beta - \langle \nabla \mathbf{u}, \nabla \mathbf{v} \rangle_{\beta^2}.$

Asymptotics

- ▶ For $|\nabla \mathbf{v}| \approx 0$, there is

$$\mathcal{S}(\mathbf{u}, \mathbf{v}) \approx 0.$$

Joint Parallel Level Set Prior

Recall, $\mathcal{S}_\beta(\mathbf{u}, \mathbf{v}) = \int |\nabla \mathbf{u}|_\beta |\nabla \mathbf{v}|_\beta - \langle \nabla \mathbf{u}, \nabla \mathbf{v} \rangle_{\beta^2}.$

Asymptotics

- ▶ For $|\nabla \mathbf{v}| \ll \beta$, there is $|\nabla \mathbf{v}|_\beta \approx \beta$, hence

$$\mathcal{S}_\beta(\mathbf{u}, \mathbf{v}) \approx \int \beta |\nabla \mathbf{u}|_\beta + \text{const} = \beta \text{TV}_\beta(\mathbf{u}) + \text{const}.$$

- ▶ For $|\nabla \mathbf{u}|, |\nabla \mathbf{v}| \gg \beta$, there is $|x|_\beta \approx |x|$, hence

$$\mathcal{S}_\beta(\mathbf{u}, \mathbf{v}) \approx \mathcal{S}(\mathbf{u}, \mathbf{v}).$$

Parallel Level Set Prior

Generalization

$\varphi, \psi : [0, \infty) \rightarrow [0, \infty)$, $\varphi(0) = 0$, both monotonically increasing

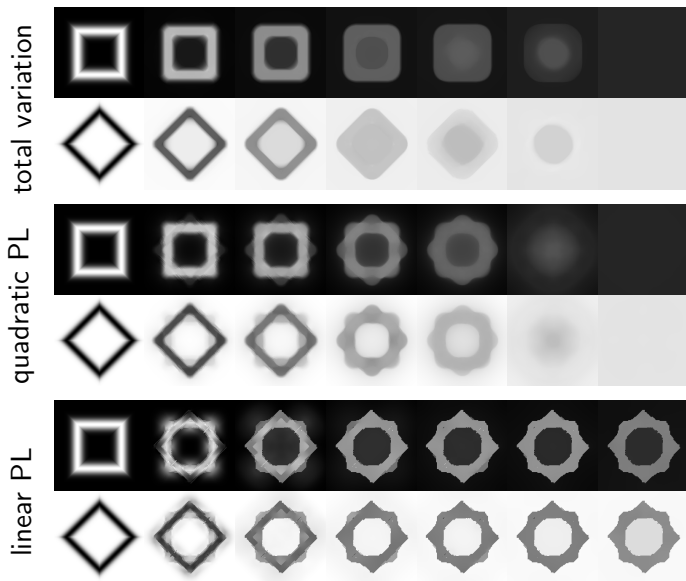
$$\mathcal{S}_{\varphi, \psi}(\mathbf{u}, \mathbf{v}) = \int \varphi \left[\psi(|\nabla \mathbf{u}|_{\beta} |\nabla \mathbf{v}|_{\beta}) - \psi(|\langle \nabla \mathbf{u}, \nabla \mathbf{v} \rangle|_{\beta^2}) \right]$$

Special cases

- ▶ **linear** parallel level sets: $\varphi(x), \psi(x) = x$
- ▶ **quadratic** parallel level sets (Nambu functional):
 $\varphi(x) = \sqrt{x}, \psi(x) = x^2$
- ▶ **cross-gradients**: $\beta = 0, \varphi(x) = x, \psi(x) = x^2$

Ehrhardt and Arridge IEEE TIP 2014, Ehrhardt et al Inverse Problems 2015,
Gallardo and Meju Geophysical Research Letters 2003; Sochen et al IEEE TIP 1998

Evolution of Test Data



Ehrhardt 2015

m.j.ehrhardt@damtp.cam.ac.uk

Generated Diffusion

Diffusivity of Parallel Level Sets

The derivative of \mathcal{S}_β with respect to u can be written as

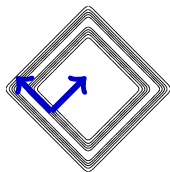
$$D\mathcal{S}_\beta[u] = -\operatorname{div} \left(K \nabla u \right).$$

Generated Diffusion

Diffusivity of Parallel Level Sets

Let R_v be Gauge coordinates for v . Then the derivative of \mathcal{S}_β with respect to u can be written as

$$D\mathcal{S}_\beta[u] = -\operatorname{div} \left(K \nabla u \right)$$



Generated Diffusion

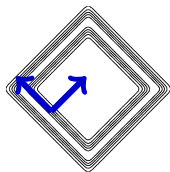
Diffusivity of Parallel Level Sets

Let R_v be Gauge coordinates for v . Then the derivative of \mathcal{S}_β with respect to u can be written as

$$D\mathcal{S}_\beta[u] = -\operatorname{div} \left(R_v \Lambda R_v^T \nabla u \right)$$

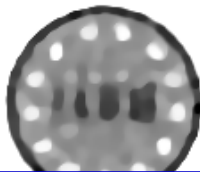
with $\Lambda = \operatorname{Diag}(\lambda^\perp, \lambda^\parallel, \dots, \lambda^\parallel)$.

- ▶ form of derivative independent of φ, ψ
- ▶ only λ^\perp and λ^\parallel depend on φ and ψ



Numerical Results

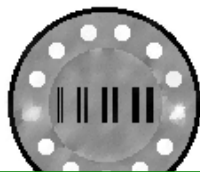
MRI sampling: full



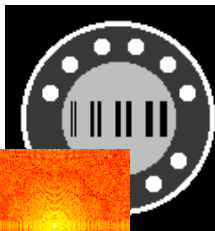
Total Variation



Quadratic PL



Linear PL



Total Variation

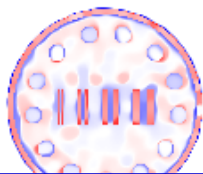
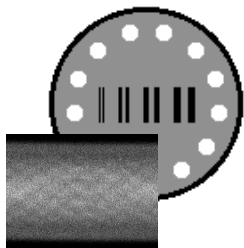


Quadratic PL



Linear PL

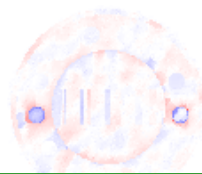
MRI sampling: full



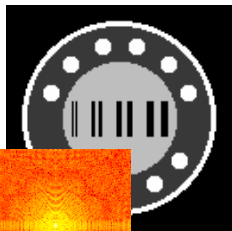
Total Variation



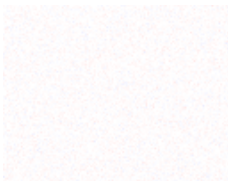
Quadratic PL



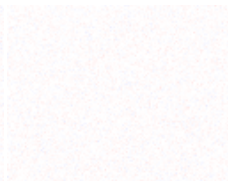
Linear PL



Total Variation

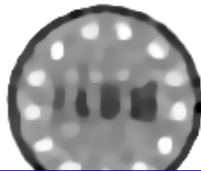


Quadratic PL



Linear PL

MRI sampling: 20 radial spokes



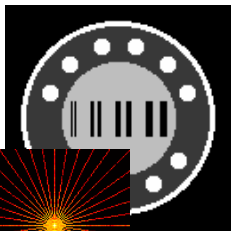
Total Variation



Quadratic PL



Linear PL



Total Variation

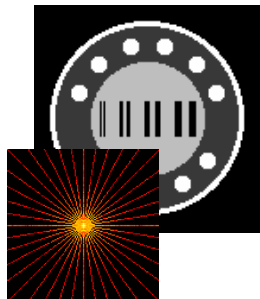
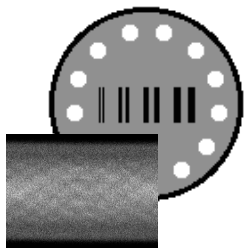


Quadratic PL



Linear PL

MRI sampling: 20 radial spokes



MRI sampling: 15 radial spokes



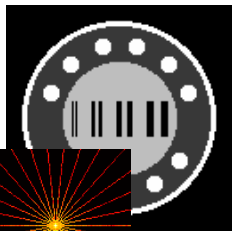
Total Variation



Quadratic PL



Linear PL



Total Variation

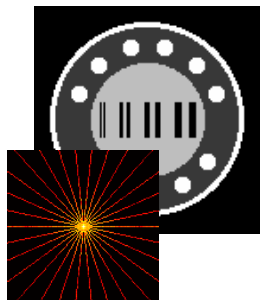
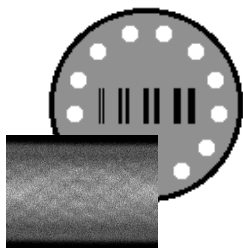


Quadratic PL

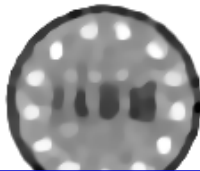
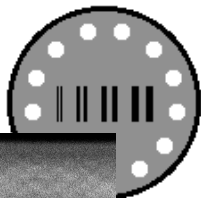


Linear PL

MRI sampling: 15 radial spokes



MRI sampling: uniform spiral



Total Variation



Quadratic PL



Linear PL



Total Variation

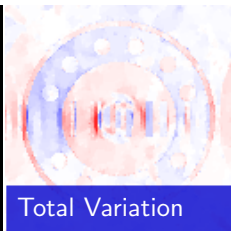
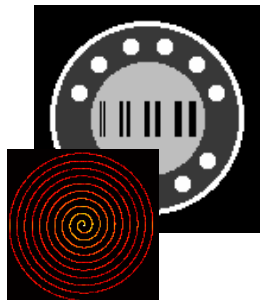
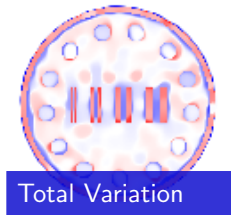
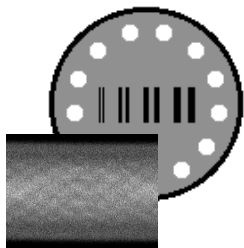


Quadratic PL

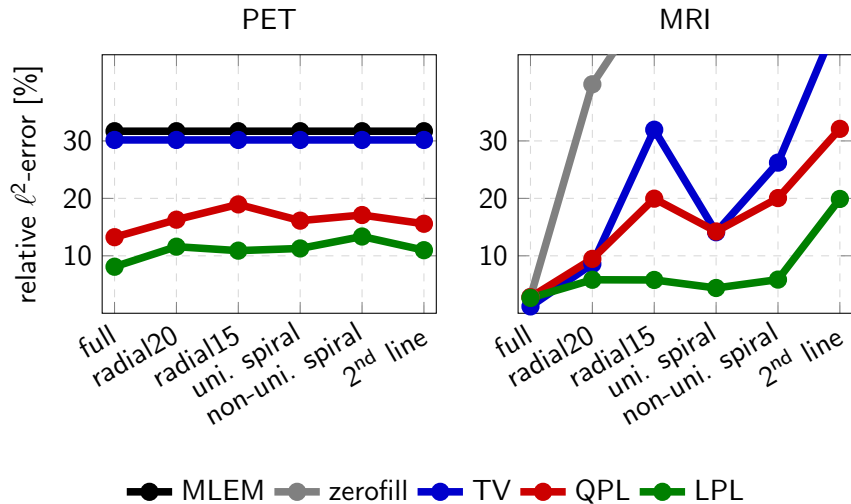


Linear PL

MRI sampling: uniform spiral

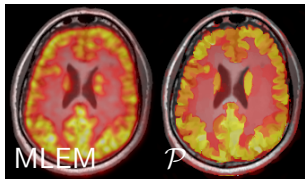


Quantitative Results



Conclusions

- ▶ Part I: **new prior** incorporates anatomical structure



- ▶ Part II: **Joint Reconstruction**

- ▶ **Parallel Level Set** prior encodes joint structure
- ▶ Minimizing PLS yields structurally coupled **anisotropic diffusion**
- ▶ Combining two inverse problems can be **beneficial to both**



separate rec



joint rec