Nonparametric Estimation of Diffusions in the Low Frequency Setting

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May 25, 2017

Diffusions

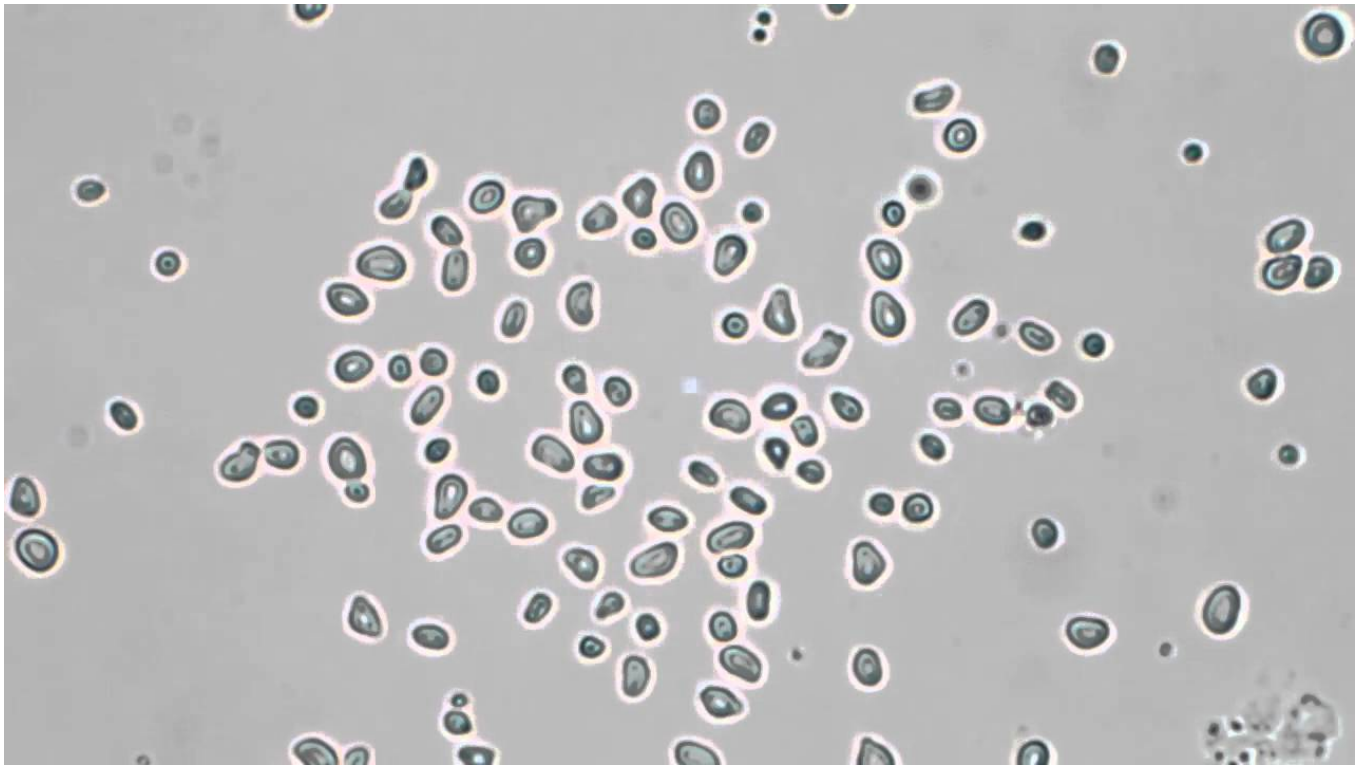

Figure 1: Pollen grain moving in water

The motion of the pollen is mathematically described by the SDE

$$dX_t = b(X_t)dt + dW_t.$$

Estimating the Drift

$$\boxed{dX_t = b(X_t)dt + dW_t}$$

Suppose one can observe the movement of the pollen at discrete times

$$X_0, X_\Delta, ..., X_{n\Delta}, \quad \Delta > 0 \text{ fixed},$$

but not the underlying drift function $b \in \{\text{some class of functions}\}$.

Question

How well can one estimate the parameter b?

Efficiency

- In finite dimensional parameter spaces, have Cramér-Rao lower bound: Let $I(\theta_0)$ be the Fisher information matrix

$$I(\theta_0) = \mathsf{Cov}_{\theta_0}\left[\frac{d}{d\theta}\log L(X,\theta)|_{\theta=\theta_0}\right].$$

  Under $\mathbb{P}_{\theta_0}$, the minimum asymptotic variance for any unbiased estimator $\hat{\theta}^{(N)}$ of is $I^{-1}(\theta_0)$, i.e.

$$n\mathsf{Var}(\hat{\theta}^{(N)}) \leq I^{-1}(\theta_0)$$

Bernstein-von-Mises theorem: Under mild assumptions, the Bayesian procedure is asymptotically efficient.

Efficiency

- In infinite dimensions, a similar notion of efficiency is more involved.
- The minimax rate of estimation for b is

$$\inf_{\hat{b}} \sup_{b \in \Theta} \mathbb{E}[\|\hat{b} - b\|_{L^2}^2]^{\frac{1}{2}} \lesssim N^{-\frac{s}{2s+3}}.$$

- Gobet, Hoffmann, Reiss (2004) propose a spectral estimation method that attains the minimax rate.
- Nickl and Söhl (2016) show that the Bayesian approach also works at the minimax rate.